

# The choice of optimum cross section for overhead line by economic intervals' method

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**Abstract** – In this paper an approach to choosing the optimum cross section for overhead line in conditions of incomplete and uncertain information is considered. The two methods of such choice are presented: method of economic current density and economic intervals' method. The correction of the economic intervals method is offered under market conditions of costs. As example 20 kV and 110 kV overhead lines with aluminum, copper and ferroaluminum wires are selected. Universal nomograms with different standard cross section are calculated and constructed. The graphics using Mathcad software are offered.

**Keywords** – Cross section choice, annual costs, economic intervals, universal nomograms.

## I. INTRODUCTION

Economic efficiency of a construction and power system's operation as a whole substantially depends on rational construction of electric networks. In elements of electric networks about 15 % of the electric power, arriving in a network, is lost. The basic part of these losses is losses on heating in wires of overhead lines, cable veins and transformer windings, connected with presence of lines' and transformers' active resistance.

The most radical decrease way in share of the common losses could be active resistance's reduction of wires and cable veins. Possibility of decrease by active resistance is connected only with increase in cross section. On the other hand, the cross section increase is connected with growth of costs to build the transmission lines [9]. Therefore the choice of optimum cross section of wires and cables at design stage or reconstruction of networks determines the further costs in networks' exploitation. The given choice by economic considerations can be made by a method of economic current density or by a method of economic intervals. Both methods are based on criterion of minimum annual costs, however differ with accuracy of results.

## II. METHOD OF ECONOMIC CURRENT DENSITY

For the first time economic current density definition was offered by V. Thompson in 1881, during the period when electric load were rather small. There are many scientific works in this direction. The theoretical basis of the considered method is reworked and presented in [8].

With this method it is possible to define economic current density at certain values of factors and parameters. Annual costs for line with different cross sections depending on current can be found as:

$$C = \left[ (i + p_{\Sigma}) \cdot (a + b \cdot F) \cdot l + 3 \cdot I_{\max}^2 \cdot (\tau \cdot \beta' + \beta'') \cdot \frac{l}{\gamma \cdot F} \cdot 10^{-3} \right], \quad (1)$$

where  $p_{\Sigma}$  is total deductions from capital investments in line construction on amortization, repair and maintenance, r.u.;  $i$  is the market interest rate or bank credit percents, r.u.;  $I_{\max}$  is maximum line's load current in all rated period, A;  $\tau$  is the utilization time of maximum losses per year, where  $\tau = f(T_{\max})$ , h;  $\beta'$  is specific cost of electric power losses, Ls/kWh;  $\beta''$  is specific cost of capacity in maximum time of system load, Ls/kW;  $\gamma$  is specific conductivity, m/Ohm\*mm<sup>2</sup>.

Minimum of function  $C$  can be defined, making a derivative of annual costs' expression (1) by wires cross section  $F$  and equating with zero:

Introducing concept about economic current density

$$j_{ek} = \frac{I_{\max}}{F}, \quad (2)$$

then it is possible to find theoretical expression for economic current density:

$$j_{ek} = \sqrt{\frac{(i + p_{\Sigma}) \cdot b \cdot \gamma \cdot 10^3}{3 \cdot (\tau \cdot \beta' + \beta'')}}. \quad (3)$$

The choice of economic current density for wires and cables is carried out usually from the table in accordance with various types of a wire (cable) and utilization time of maximum load. This table had been offered for the first time in the early fifties of the past century by G. Grudinsky and E. Priklyonsky. At that time it was a great step forward because in costs were considered not only capital investments, but also electric power losses.

Introduction of economic current density was actual and useful. However, the choice of wires and cables by the method of economic current density does not allow receiving a global minimum of annual costs. The reason is that the method has assumptions and inaccuracies:

- the using a derivative for definition of wire optimum cross section means that the cross section continuously changes, however actually it changes discretely;

- the current density  $j_{ek}$  is resulted for rather big ranges of utilization time of maximum load;
- there is no possibility to vary factors  $i$ ,  $p_{\Sigma}$ ,  $\beta'$ ,  $\beta''$  values;
- in calculations are taken the fixed values of current density calculated in the early of the past century and real value  $j_{ek}$  can differ from resulted values in Russian normative documents [10];
- in the conditions of free prices there is no accessible information of the electrotechnical production's prices and cost of building and construction works to define factor's  $b$  value.
- in the last decades cost of electric power losses and as cost of networks has been increased several times.

The specified above lacks of the method can lead to incorrect decisions at definition of wire optimum cross section under market conditions of prices.

### III. METHOD OF ECONOMIC INTERVALS

Considering errors of a method of economic current density, more exact method of economic intervals had been worked up. The method had been offered by V. M. Blok in 1945 [1-4]. The given technique of a cross section choice of wires and cables allows considering gradualness of cross section, any values of utilization time of maximum load, cost of energy losses and other parameters.

Total annual costs for line with different cross sections depending on maximum current are defined as follows:

$$C_i = (i + p_{\Sigma}) \cdot K_{Li} + 3 \cdot I_{maks}^2 \cdot R_i \cdot (\tau \cdot \beta' + \beta'') \cdot 10^{-3}. \quad (4)$$

It is possible to express dependence of rated costs of a transmission line from transferred power. Then it has looks like:

$$C_i = (i + p_{\Sigma}) \cdot K_{Li} + \frac{P^2}{U^2 \cdot \cos^2 \varphi} \cdot R_i \cdot (\tau \cdot \beta' + \beta'') \cdot 10^{-3}, \quad (5)$$

where  $P$  is the maximum transmitted power through line, kW;  
 $U$  is line rated voltage, kV;  
 $\cos \varphi$  is power factor (it is accepted 0,92).

Comparing annual costs for two adjacent standard cross sections of transmission line wires, it is possible to define a boundary current at which transition from smaller cross section to bigger is economically expedient [1-4].

Boundary it is possible to define from an equality's condition of total annual costs:

$$C_{i-1} = C_i. \quad (6)$$

Whence:

$$I_{ek} = \sqrt{\sigma} \cdot \sqrt{\frac{(K_{L2} - K_{L1}) \cdot 10^3}{3(R_1 - R_2)}}. \quad (7)$$

As well the top economic border of transferred power can be found from the condition (6).

Whence:

$$P_{ek} = U \cdot \cos \varphi \sqrt{\frac{(K_{L2} - K_{L1}) \cdot 10^3}{(R_1 - R_2)}} \cdot \sqrt{\sigma}. \quad (8)$$

By the given methods it is possible to construct universal economic nomograms for lines with different voltage and network fulfillments in comparison with a method of economic current density [4, 7, 8, 10].

It is necessary to notice that the choice of cross sections by economic considerations is actual and for modern market conditions of economy.

Let's define conditions for existence of economic intervals of line [3, 5, 6].

The first condition for existence of economic intervals is defined by presence of economic characteristics' crossing for lines with adjacent cross sections.

$$K_{L,i} > K_{L,(i-1)}. \quad (9)$$

In the immense form:

$$1 < \frac{K_{L,i}}{K_{L,(i+1)}}. \quad (10)$$

The second condition for existence of economic intervals is defined from an inequality:

$$I_{ek,i-(i+1)} < I_{ek,(i+1)-(i+2)}, \quad (11)$$

where  $I_{ek,i-(i+1)}$ ;  $I_{ek,(i+1)-(i+2)}$  is the boundary currents, which are defined by crossing of appropriate characteristics  $C_{L,i}$  and  $C_{L,i+1}$  or  $C_{L,i+1}$  and  $C_{L,i+2}$ .

### IV. CORRECTION OF ECONOMIC INTERVALS' METHOD IN NOWADAYS ECONOMIC CONDITIONS

Methodological principles of the technical-economical justification of design decisions in electric power engineering had been developed by soviet scientists in the second half of XX-th century, i.e., during a planned economy epoch when there was a centralized state financing of power objects.

In the conditions of market economy the investments into building of power objects are defined by own capitals of power companies, interests and financial possibilities of investors. In a present economic situation the information of construction cost for 1 km overhead transmission line, of costs for the equipment and for building and construction works is difficultly accessible. Moreover, each company has its own "price" list that proves existence impossibility of the uniform integrated indexes of line cost. That complicates process of variants' estimation as it occurs in the conditions of uncertainty of the initial information.

In modern economic conditions traditional methods of a cross section choice of overhead lines need the critical analysis and certain updating.

Due to this fact, the correction of the economic intervals method is offered.

Nowadays for various engineering firms costs for line building and construction works can differ considerably. Their share makes more than 90 % from all cost of a transmission line construction. Residuary part is the costs of metal of wires or current carrying cable veins and their isolation.

Building and construction costs can be accepted constants for different voltage, but for concrete execution of lines, i.e.:

$$K_{build} = const . \quad (12)$$

The given assumption simplifies calculation process at economic intervals method and construction of universal nomograms.

Let's present total capital investments through line as follows:

$$K_L = n_f \cdot K_{met} + K_{build} , \quad (13)$$

where  $K_{met}$  is cost of wire (cable) metal on one phase;

$n_f$  is quantity of phases in a line;

$K_{build}$  is costs for building and construction works.

Then annual costs for line each cross section look like:

$$C_{Fi} = (i + p_{\Sigma}) \cdot (n_f \cdot K_{met} + K_{build}) + 3 \cdot I_{max}^2 \cdot R_i \cdot \tau \cdot (\beta' + \beta'') \cdot 10^{-3} . \quad (14)$$

Applying the condition (6), economically expedient value of boundary current looks as follows:

$$I_{ek} = \sqrt{\sigma} \cdot \sqrt{\frac{n_f \cdot (K_{met2} - K_{met1})}{3 \cdot (R_1 - R_2) \cdot 10^{-3}}} . \quad (15)$$

Costs for wire metal looks like:

$$K_{met} = F \cdot 10^{-3} \cdot l \cdot D_{met} \cdot K_{0,met} , \quad (16)$$

where  $K_{0,met}$  is cost of 1 kg metal, Ls/kg;

$D_{met}$  is density of wire metal, kg/m<sup>3</sup>;

$l$  is length of line, km.

Considering (15) and (13), economically expedient value of boundary current and power takes a form:

$$I_{ek} = \sqrt{\sigma} \cdot \sqrt{\frac{n_f \cdot K_{0,met} \cdot D_{met} \cdot (F_2 - F_1)}{3 \cdot (R_{01} - R_{02})}} . \quad (17)$$

$$P_{ek} = U \cdot \cos \varphi \cdot \sqrt{\frac{n_f \cdot K_{0,met} \cdot D_{met} \cdot (F_2 - F_1)}{(R_{01} - R_{02})}} \cdot \sqrt{\sigma} . \quad (18)$$

If value of expression (17) under the second root appears negative, then it means that  $C_{L1}=f_1(I)$  and  $C_{L2}=f_2(I)$  are not crossed, i.e. one section always is more favorable, than another at all values of the maximum load current.

In the given work the universal nomograms for 20 kV overhead lines with copper, aluminum and ferroaluminum wire by standard cross sections with 35, 50, 70, 95 mm<sup>2</sup> are calculated and constructed. As well the universal nomograms for 110 kV overhead lines with copper, aluminum and ferroaluminum wire with standard cross sections 70, 95, 120, 150, 240, 300 mm<sup>2</sup> are done.

Considering (14) and (16), specific annual costs dependence from a current for 20 kV overhead line per unit length:

$$C_{F,spec} = (i + p_{\Sigma}) \cdot (n_f \cdot F_i \cdot 10^{-3} \cdot D_{met} \cdot K_{0,met} + K_{build}) + 3 \cdot I_{max}^2 \cdot R_{0,i} \cdot \tau \cdot (\beta' + \beta'') \cdot 10^{-3} . \quad (19)$$

Considering (5) and (13), specific annual costs dependence from a power for 20 kV overhead line per unit length:

$$C_{F,spec} = (i + p_{\Sigma}) \cdot (n_f \cdot F_i \cdot 10^{-3} \cdot D_{met} \cdot K_{0,met} + K_{build}) + \frac{P^2}{U^2 \cdot \cos^2 \varphi} \cdot R_{0,i} \cdot \tau \cdot (\beta' + \beta'') \cdot 10^{-3} . \quad (20)$$

Specific annual costs dependence from a current for 20 kV overhead line with aluminum wire are constructed in Fig. 1. The same calculation of specific annual costs dependence from current for 20 kV overhead line with aluminum wire from a power is done (Fig. 2).

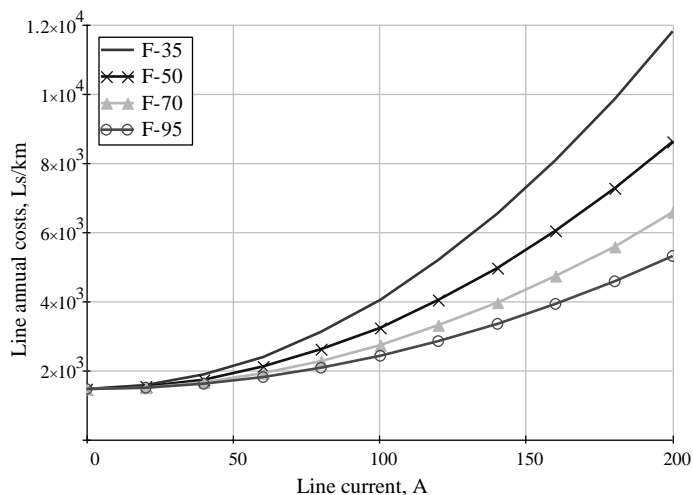


Fig. 1. Specific annual costs dependence from a current for 20 kV overhead line with aluminum wire.

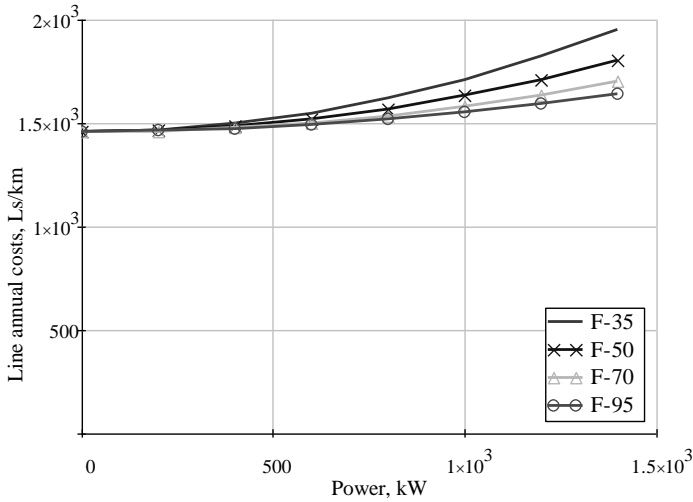


Fig. 2. Specific annual costs dependence from a power for 20 kV overhead line with aluminum wire.

The influence of components of metal cost is slightly in comparison with costs of building and construction works, therefore the given schedules are flat and zones of cross sections expediency accurately are not perceived. Because of this, using formulas (17) and (18), universal nomograms of economic intervals  $I=f(\sqrt{\sigma})$  and  $P=f(\sqrt{\sigma})$  for the nearest pair of wire cross section are constructed (Fig. 3-8). Presented nomograms allows substantially choose economic cross section of overhead lines with different voltage for aluminum, copper and ferroaluminum wires.

Choosing the cross section of wires by economic intervals curves, it is necessary to define the maximum current  $I_{max}$  of line and factor  $\sigma$ . The zone, in which gets the point with coordinates  $I_{max}, \sigma$ , meets economic cross section.

If the intersection point of coordinates  $I_{max}, \sigma$  gets directly on boundary curve of two cross sections, then it is indifferent which cross sections to choose. Both sections give identical economic benefit.

This method considers also admissible heating of wires in a normal work conditions that is reflected by a horizontal part of curve economic intervals (Fig. 5-6).

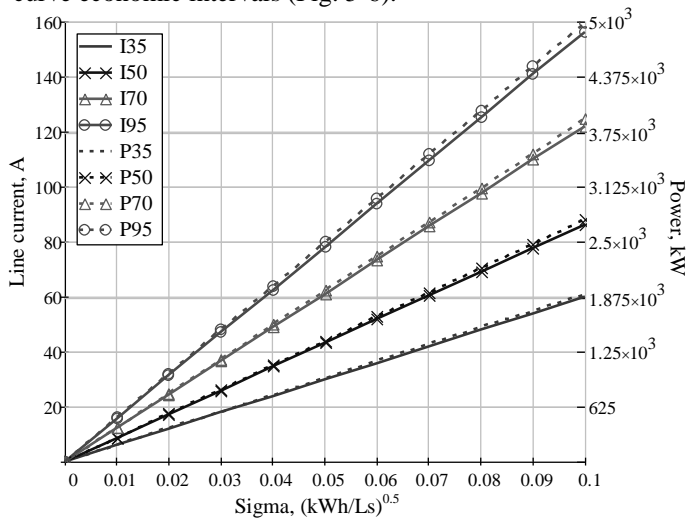


Fig. 3. Current and power universal nomograms for 20 kV overhead lines with aluminum wire.

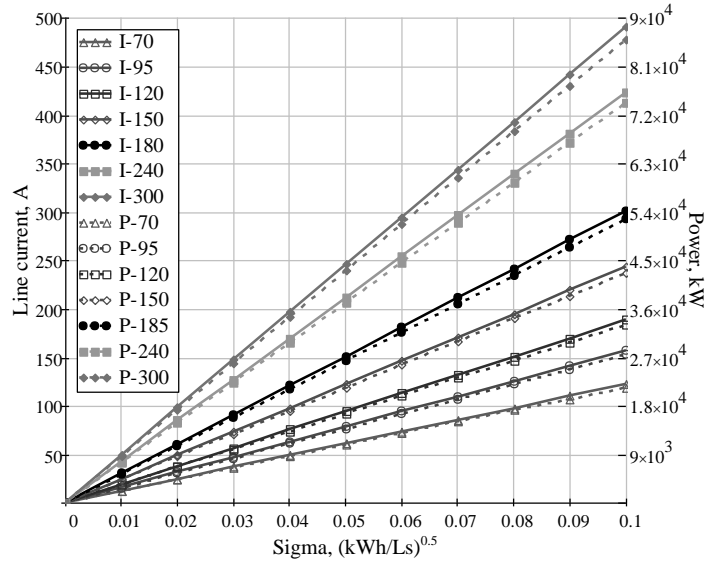


Fig. 4. Current and power universal nomograms for 110 kV overhead lines with aluminum wire.

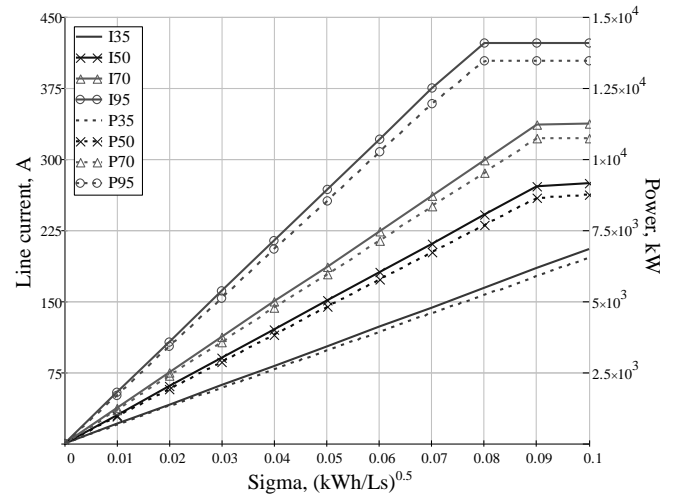


Fig. 5. Current and power universal nomograms for 20 kV overhead lines with copper wire.

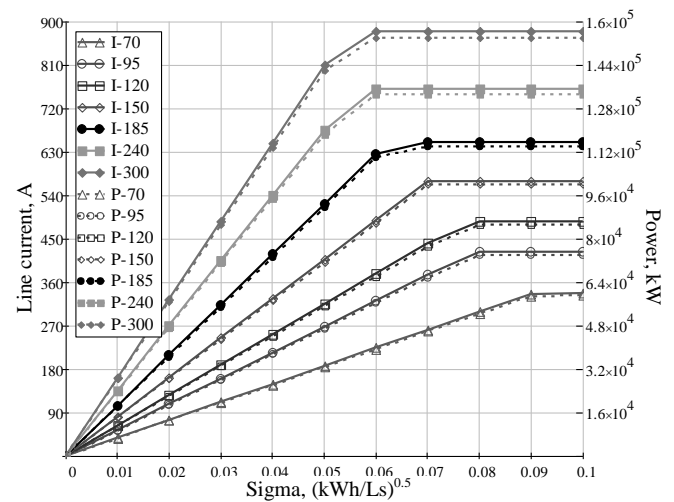


Fig. 6. Current and power universal nomograms for 110 kV overhead lines with copper wire.

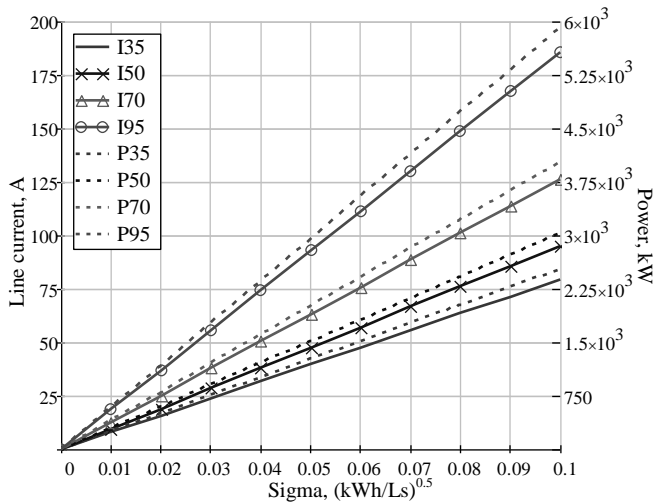


Fig. 7. Current and power universal nomograms for 20 kV overhead lines with ferroaluminum wire.

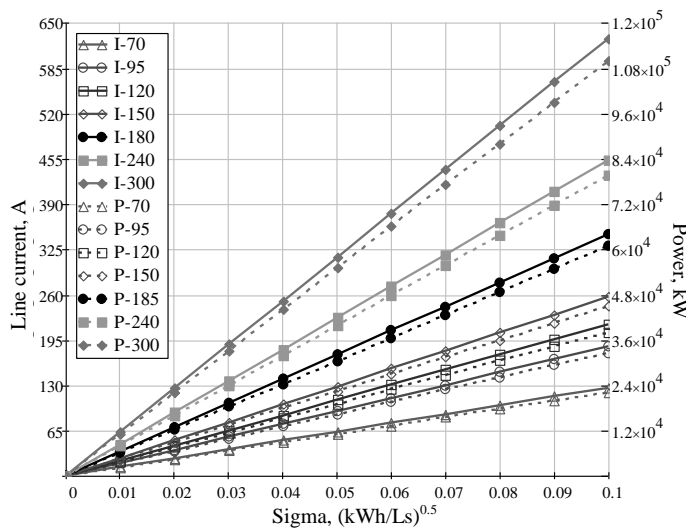


Fig. 8. Current and power universal nomograms for 110 kV overhead lines with ferroaluminum wire.

In Fig.3-8. the symbols  $I-N$  and  $P-N$  are used, where are line current and power of wire with  $N$  cross section accordingly.

At the detailed analysis of a economic intervals method the algorithm of a choice of transmission overhead line optimum cross section for various voltage is made in Fig. 9.

### V.CONCLUSIONS

1. In modern economic conditions traditional methods by economic considerations of a cross section choice of overhead lines need the critical analysis and certain updating. Previous calculated nomograms are not actual nowadays and selected at that time lines' cross sections are overrated.
2. The presented correction of economic intervals method makes it possible to select optimal overhead line's cross section in the early stages of the projection.
3. Calculated nomograms of current and power are universal for different voltages and conductor material.

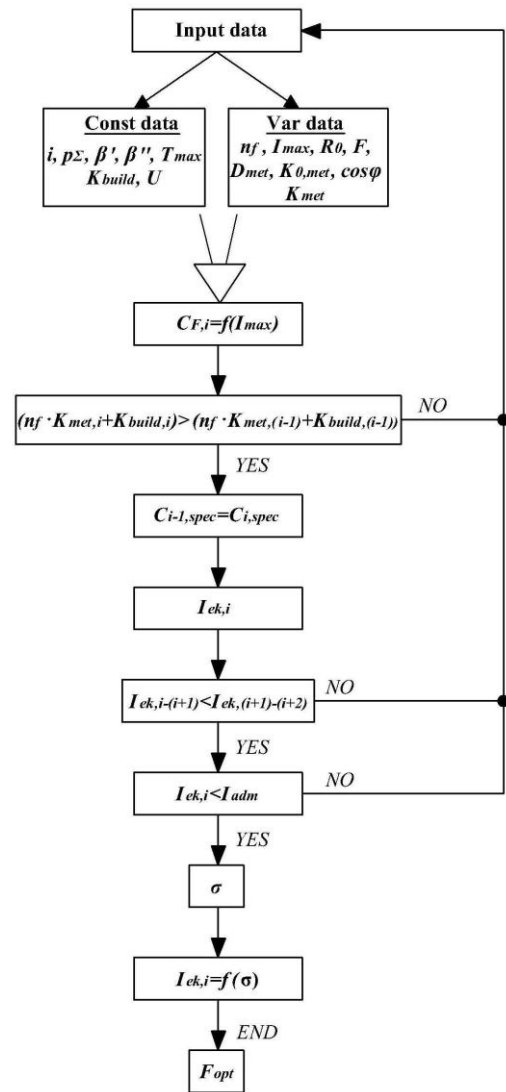


Fig. 9. Algorithm of optimal cross section choice by economic intervals method.

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