# **RIGA TECHNICAL UNIVERSITY**

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# MULTICRITERIA OPTIMIZATION MODEL OF POWER SYSTEM OPERATION EFFICIENCY EVALUATION (ON LATVIAN POWER SYSTEM EXAMPLE)

**Summary of Doctoral Thesis** 

**Riga 2011** 

# **RIGA TECHNICAL UNIVERSITY**

Faculty of Power and Electrical Engineering Institute of Power Engineering

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Doctoral student of program "Power Engineering"

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**Summary of Doctoral Thesis** 

Scientific supervisor Dr. sc. ing., professor A. MAHNITKO

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# DOCTORAL THESIS SUBMITTED TO RIGA TECHNICAL UNIVERSITY TO OBTAIN DOCTORAL DEGREE IN ENGINEERING

Doctoral thesis is proposed for achieving doctoral degree and will be publicly presented on the 21<sup>th</sup> of November 2011 at Faculty of Electrical Engineering of Riga Technical University, 1 Kronvalda boulevard, assembly room.

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#### CONFIRMATION

Hereby is confirmed, that I completed the following thesis, presented in Riga Technical University for doctoral degree in engineering independently. This doctoral thesis is not submitted to any other university for achieving scientific degree.

Aleksandrs Gavrilovs .....

Date: .....

The thesis is written in Latvian, includes introduction, 5 chapters, conclusions, list of references, 3 appendixes, 50 figures, 26 tables, 150 pages in total. List of references consists of 99 items.

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#### **IMPORTANCE OF THE ISSUE**

Electric power engineering is a leading sphere of infrastructure that without any alternative defines the frames of economic development opportunities; therefore it is necessary to provide prevailing increasing of generating power as well as to realize safe and qualitative supply of electric energy.

With the appearance of first electric stations and joint electric power systems (EPS) the tasks of optimization of their operation modes are formed. They include the tasks of minimum expenses achievement of electric power, decreasing of primary power resources expenses for its production, decreasing of negative influence on the environment, solving of the tasks of electric power transporting, etc.

On a par with these maintenance charges the tasks of designing mean not less: elaboration of power engineering systems optimal development strategy and selection of electric networks and its configurations, including new types of electric stations and substations, power and location, voltage, cross-section of wires, etc. This an extremely complex technical task, in the solving of which it is necessary to take into account the presence of power resources, the conditions of their transporting, the convenience of electric power and heat energy delivery to the consumers, implementation of the demands to the environment protection, choice of place for the electric station being built and other requirements.

A valuable contribution into the development of power systems regimes optimization theory has been made by such scientists like: M.J. Steiberg, T.H. Smith, V.M. Gornstein, L.A. Krumm, A.V. Bogoslovski, B.P. Mirosnicenko, A.V. Ponomarjev, V.A. Venikov, V.G. Zuravljev, D.A. Arzamascev, P.I. Bartolomej, L.S. Beljaev, N.I. Voropaj, A.Z. Gamm, A.L. Mizin, L.A. Melentjev, V.G. Derzskij, M.E. El-Hawary, G.S. Christensen, A.J. Wood, B.F. Wollenberg, Y.H. Song, J.A. Momoh and others.

Natural wish for improving of economic effectiveness of EPS and their elements modes under the market economic conditions, production of electric energy, extension and diversification of distribution and consumption processes demand the heightened requirements to the effectiveness and flexibility of the planning algorithms, operative optimization and its regimes correction.

To improve the effectiveness of maintenance and designing tasks solution at EPS with the electric stations, substations, electric transmission lines of different types belonging to different types of property forms the accounting of multicriteria models is compulsory.

The structure and organizational changes in the field of electric power having taken part during the period of last years require the review of optimization of EPS regimes control. The multi-criteria approach gives an opportunity of more complex solutions of the tasks of power systems operation regimes optimization, more accurate complying with the technical and commercial restrictions and, therefore, obtaining of the maximum close to present reality solutions.

Transfer to the formation of market electricity prices results in difference and, moreover, contradiction of optimality criteria of particular EPS entities functioning. If earlier, within the period of electrical power engineering centralized control, the key criterion of optimality was minimization of self-expenses of fuel energy utilized for electric energy production and generated energy, then under the market conditions the conventional task of optimization is divided into subtasks. The most determinative of them is maximum criteria for the income from the energy sale or minimum of tasks of electric and heating energy resources production, delivery, transmission and distribution. The criterion of prices minimization is also considered as an important.

The competitive wholesale trade of electric energy should organize transparent and effective system of management and formation of prices for the supplied (bought) volumes of electric energy. Hence, with the liberalization of power engineering market, the improvement of research methodology of operative management tasks of EPS regimes is becoming topical.

All the aspects mentioned above and the importance of their consideration define the subject of this paper, its aims and reasons of choice.

## **GOAL AND TASKS OF THE THESIS**

The goal of the paper is to investigate the opportunities of development and operation regimes of EPS, including the consideration of the questions of analysis, multicriteria optimization, implementation of modeling and decision making for the increasing of stations functional effectiveness. To achieve the goal the following tasks were considered:

- 1. To investigate basic methods of EPS regimes multicriteria optimization.
- 2. To investigate the problems of optimal EPS load distribution among electric stations under the conditions of multi-criteria goal task.
- 3. To examine principles and features of EPS operation under the conditions of competitive electric power energy market.
- 4. To propose methodology and algorithms for solution of EPS development tasks relying on the examples of solving of particular projects of Latvian power system development from the multi-criteria analysis point of view.

# METHODOLOGY OF RESEARCH

The following research methods and tools have been applied in the study:

- Newton's method for solving systems of non-linear algebraic equations describing stationary regimes of EPS;
- Lagrangian method for the optimization of non-linear tasks;
- Criteria of decision making under the conditions of uncertainty;
- Pareto optimality principle;
- Interactive environment MATLAB, foreseen for intensive calculations, data analysis and its visual representation;
- system GAMS high level programming language for development of mathematic models and solving of the optimization problems;
- MathCAD system for the solving of engineering problems and visualization and analysis of the results;
- > program package MUSTANG for the modeling of power systems regimes;
- means of Microsoft Excel program.

# SCIENTIFIC NOVELTIES OF THE RESEARCH AND MAIN RESULTS

- 1. Multi-criteria modeling methods of EPS regimes optimization tasks have been investigated that gives an opportunity to improve effectiveness of its operative management.
- 2. The mathematical model of EPS load optimal distribution is elaborated taking into account the minimum criteria of consumption of fuel, power losses in networks and emissions into atmosphere. The character of fuel consumption at power stations is investigated depending on the factor of local conditions and varying of fuel features, decreasing the emissions into atmosphere in the selected power plant area.
- 3. A research methodology and mathematic model is suggested for an optimal power distribution among EPS elements under the conditions of market relationships among power engineering entities like it is in the united EPS testing schemes of Latvia, Estonia, Lithuania, Russia and Belorussia.
- 4. The main development project of Latvian and Baltic power systems is analyzed.
- 5. The algorithm of optimal strategy selection of EPS development is elaborated at different alternatives with the use of elements of game theory.

6. A graphic approach to the solution of the task design if proposed considering different criteria and alternatives that simplify the selection to a person who makes a decision. The methodology is approved by the example of Latvian EPS development strategy selection.

# PRACTICAL VALUE OF THE WORK

The algorithms and methodology proposed in the work can be applied by:

- dispatchers for the operative dispatcher management of EPS regimes;
- producers of electric energy for daily planning of electric stations regimes;
- power engineering specialists who work in the development of electric supply schemes, their design and with the questions of EPS generating power development variants analysis;
- power energy dealers for the development of optimal trade strategy.

# **APPROBATION OF THE WORK**

The results of the work are presented and reported at 11 international scientific conferences:

- 1. The 3rd International Conference on Electrical and Control Technologies ECT-2008, Lithuania, Kaunas, 8 9 May, 2008.
- 2. The 6th International Scientific Symposium Electric Power Engineering, Russia, St. Petersburg, 15 19 September, 2008.
- 3. The 49th International Scientific Conference. Power and Electrical Engineering and Environmental Sciences, Latvia, Riga, 14 15 October, 2008.
- 4. III Международная научно-практическая конференция "Энергосистема: управление, конкуренция, образование", Россия, Екатеринбург, 13 16 октября, 2008.
- 5. The 4th International Conference on Electrical and Control Technologies ECT-2009, Lithuania, Kaunas, 7 8 May, 2009.
- 6. Первый научно-практический семинар "Экономическая безопасность государства и научно-технические аспекты её обеспечения" ("Економічна безпека держави і науково-технологічні аспекти її забезпечення"), Украина, Киев, 21 22 октября 2009 года.
- 7. The 5th International Conference on Electrical and Control Technologies ECT-2010, Lithuania, Kaunas, 6 7 May, 2010.
- 8. International Scientific Event "Energetika 2010. Power engineering 2010". The 9th International Scientific Conference "Energy Ecology Economy 2010", Slovakia, Tatranske Matliare, 18 20 May, 2010.
- 9. XI International Scientific-Technical Conference "Problems of Present-day Electrotechnics-2010", Ukraine, Kiev, 1 3 June, 2010.
- 10. The 51st International Scientific Conference of Riga Technical University on Power and Electrical Engineering, Latvia, Riga, 14 October, 2010.
- 11. II научно-практический семинар с международным участием "Экономическая безопасность государства и научно-технические аспекты её обеспечения", Украина, Киев, 21 22 октября 2010.

# PUBLICATIONS

The results of the Doctoral thesis are presented in 16 publications:

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#### **STRUCTURE OF THE WORK**

The thesis is written in Latvian, includes introduction, 5 chapters, conclusions, list of references, 3 appendixes, 50 figures, 26 tables, 150 pages in total. List of references consists of 99 items.

#### 1. IMPORTANCE OF THE PROBLEM OF MULTICRITERIA OPTIMIZATION

This chapter suggests the analysis of principles and ideas of power systems regimes multicriteria optimization. Ob the basis of the analysis the aim functions are formed and restrictions of the optimization tasks are proposed.

In electric power as in any other fields of specialization often is obliged to meet tasks of optimization. Typical EPS optimization tasks are:

- *operating tasks* EPS stationary regimes management;
- *designing tasks* EPS development planning for different time periods.

One of the main EPS optimal management tasks under the normal conditions is the most profitable distribution of the consumers' electric load among the systems' generators. In addition to it is necessary to provide the high effectiveness of the system power, operation and financial recourses usage, safe and uninterruptable power supply, enforcement of not power engineering requirements put forward by economy.

The task is too complex, that is determined by high volumes of power engineering and its intensive development, difference of technical, economic and regime characteristics (electric stations of different types, electric transmission lines) of the separate elements of power systems. The opportunity of the effective practical solution of this problem existed only with the development of modern data bases. However, even under these conditions the successful solution of it is possible only with its decomposition, i.e. dividing the general task unto easier mutually connected series of subtasks.

The task of optimization is the simplest to be solved while investigating one criterion. In most of the cases within the disposal of the researcher there is not enough satisfactory multicriteria methods of optimization, therefore he simplifies the analogous tasks transforming those into single-criterion tasks. This is realized in the following way. From all those goals the one is extracted that could be considered as dominant. All the rest goals are replaced with some minimum necessary requirements (restrictions), the implementations of which are considered as compulsory.

Apart from optimization task's criteria number, information variables could be allocated in three groups:

- *determined variables* stated parameters, which values are exactly known (or are considered as exactly known), while defining mathematical model;
- stochastic variables occasional parameters with established distribution laws and their character;
- *indeterminate variables* with known potential values' range. Indeterminate parameters could be set only with intervals and ranges.

As there are different types of information and several conflicting goals, it is naturally that different *alternatives of the solution* exist [14]. Term "alternative" has a series of synonyms; the following terms are applied depending on the particular features of the task: variant, plan, trajectory of the system movement etc. One and the same variant (alternative) is evaluated in different way under different conditions. It is seems that one and the same alternative has

different outcome according to the system existence outward conditions. Certainly, that among the alternative it is preferable to select the best in a particular sense or in other words to find the most optimal solution of the task. These alternatives for different situations are called in different ways – *rational (efficient), effective, as optimal as possible optimal, according to Pareto.* Its general feature is that it is better in some mean than the others but it is impossible to extract from those some much better in the same mean with conventional mathematical methods.

#### Fundamentals of multi-criteria optimization

Normally it is impossible to provide fully the implementation of different goals. It is always necessary to sacrifice one full implementation for the implementation of another task, i.e. accept a compromise. The reason of it is in the conflicting of the goals. Typical example is two goals – economy and safety. As a rule, the improvement of safety takes place at economy weakening cost vice versa.

Thus, the searching for the compromise solution is the key task of multicriteria optimization [11, 16]. In addition, different goals under different conditions have different values. Therefore, the compromise solution should be based on the priority of one goal above the others. The following research methods and tools have been applied in the study:

• The Pareto optimal alternative set. Pareto optimal solution set means that any of this solutions' set can not be replaced with another better in some criterion in order not to worse the solution in some other criterion. Therefore each solution from the Pareto set is better that the others in some criteria, but worse - in other criteria. As the criteria are not comparable then among these solutions there is no that better that the others in any case. The solutions not included into Pareto set are worse at least in one criterion. That is the reason why Pareto set is called effective and distance searching with attracting of some additional procedures or conditions implemented with Pareto sets only [13].

The essence of Pareto principle can be explained with the help of simple task when there are two incomparable criteria x and y, and the optimization means the maximization of both criteria (the key essence of the problem is the same at the minimization process). In Fig. 1.1 all the allowed set of solutions is formed with the section limited with x and y axis in positive quadrant and curve *abcd*, including also the points of this curve. It is obvious that transfer to point M to any of the curve points limited with points b and d means the improvement of the solution even in one criteria without worsening in that the other. All the intermediate points on the curve b-d are simultaneously better in both criteria than the existing points within the section. But transfer from point b to d and opposite is not possible without worsening of one of the criteria. All the solutions corresponding with the points on the curve b-d, belong to Pareto optimal solutions set. In addition each solution corresponding to any other acceptable section point always can be provided with no less than one solution from Pareto set, that is better at least in one criterion and not worse in other one. For the cases of few criteria the key principle is maintained (for the space of multi-dimension criteria).



Fig. 1.1. Selection of optimal plans

• Method of scalarization. If there is some multi-dimensions aim (or criteria) vector space where for different aims (or criteria) different and non-concurrent with each other vectors are set (the modules of the vectors are defined within one and the same measurement system) then the modules can be compared in values. In separate cases it can be accepted that these vectors are close in directions and differ in modules only (vectors with larger modules correspond to the most important aims). In this way there is a possibility to measure effectiveness in an algebraic way and define the numeric advantage of one aim. In this case scalarization is possible - the replacement of vectors with scalar functions. If all the aims *i* can be described with criteria functions  $\varphi_i$ , then the scalarization means all  $\varphi_i$  replacing with one common function  $\varphi$ :

$$\varphi = \sum_{i=1}^{n} b_i \varphi_i^0 \longrightarrow extr ; \qquad \qquad \sum_{i=1}^{n} b_i = 1; \qquad \qquad \varphi_i^0 = \frac{\varphi_i}{\varphi_{i \max(\min)}},$$

where  $b_i$  – weight coefficients that show every criterion usefulness;

 $\varphi_i^0$  – criteria written in the undimension way.

As a result of scalarization the optimization becomes single criterion at some synthetic aim. Well known example of scalarization is replacement of consumer electric supply safety (precisely, non-failure process) criterion with the help of criterion of economic losses from the electric supply troubles.

Nevertheless, very often for the simplification of calculations the scalarization is adapted to the conditions when the numerical values of weight factor are selected on the way of the expert that contains much more voluntarism than the logically based expert evaluation.

• Method of preference. This method is applied under the conditions of uncertainty and based on the marked set of probabilistically-optimal plan according to each criterion existence. The scheme of calculation is the following. Let us accept the values of all criteria minimum permissible  $y_i = y_2, y_3, ..., y_n$ , excluding one  $y_1$  – the most important. With the help of the selected most important criterion  $y_1$  the single-criterion optimization can be realized ( $y_i = \min y_i$ ) and set of probabilistically-optimal plans  $R_1$  is formed according to the given criterion.

All the criteria replaced with restrictions are put into a series according to the satisfaction preference  $y_i = y_2$ ,  $y_3$ ,...,  $y_n$  values, where  $y_2$  – the first important criterion (according to  $y_1$ ) etc. At set  $R_1$  the optimization is realized according to  $y_2$  criterion. It results in obtaining a subset  $R_2 \subseteq R_1$  that is probabilistically-optimal not only according to criterion  $y_1$ , but also according to the criterion  $y_2$ .

This procedure should be repeated for all the rest of the criteria, resulting in gradual decreasing of the set of probabilistically-optimal plans according to the scheme  $\Pi_{opt} = R_n \subseteq R_{n-1} \subseteq \ldots \subseteq R_2 \subseteq R_1$ .

As an example Fig. 1.2 demonstrates set of probabilistically-optimal plans *S* and correspondent subset of probabilistically-optimal plans:  $R_1$  – for criterion  $y_1$ ;  $R_2$  – for criterion  $y_1$  and  $y_2$ ;  $R_3$  – for criteria  $y_1$ ,  $y_2$ ,  $y_3$ ;  $\Pi_{opt} = R_4$  – for criteria  $y_1$ ,  $y_2$ ,  $y_3$ ,  $y_4$ .



Fig. 1.2. Sets of probabilistically-optimal plans according to the

• **Random search methods.** The Monte Carlo method is one of the simplest random search methods that are used in conditioned optimization tasks. The essence of the method is to get quantity of given random points' in the admissible area, to establish and to calculate values of function in them, but then to define the point with efficiency function's extreme value.

In general the Monte Carlo method is complex of methods that allows reducing calculation of the task to multiple probations. Usually such probations bring to calculations with incidental numbers.

The Monte Carlo method is used to research stochastic things and to calculate determined tasks. In electric power the first type of tasks is used for optimization of power system regimes, schemes and parameters, to research functioning safety of systems and elements, to forecast systems' development, to create the optimal management algorithm and other. For such tasks' solution corresponding stochastic models are developed, that use statistical probations according to definite logical schemes. To define determined tasks with statistical probation method, the stochastic model of solution should be prepared.

There are also other different random search methods. One of them casually selects movement direction on the every step (Fig. 1.3).



Fig. 1.3. Random search methods

In practice such algorithm is used [19]:

**Step 1.** The first point  $x^0 = x^1$  in the variable factor space is defined.

**Step 2.** The probation  $\rho$  and the working *a* step values ( $a > \rho$ ) are assigned for the random vector  $\xi$ .

**Step 3.** Coordinates  $\xi_1, ..., \xi_n$  of the vector  $\xi$  are calculated, that is probation vector's  $\rho$  length, using the evenness allocation low on the *n*-dimension sphere.

**Step 4.** Perform two probation experiments in points  $x^k$  and  $(x^k + \xi)$ , where k – the number of step; it means that from the beginning it is  $x^k = x$ , but after this  $x^k + \xi = x^1 + \xi$ .

References in those points are compared and function is formed:

$$\psi = sign\{f(x^{k} + \xi) - f(x^{k})\}; \qquad (\psi = sign\{f(x^{1} + \xi) - f(x^{1})\}).$$

If  $\psi > 0$ , then move to steps  $(3 \div 6)$  is performed (also searching experiment number should be defined). If  $\psi < 0$ , then conversion to the step 5 is performed.

**Step 5.** Working step is realized (with the length a) in the direction of the efficiency function's value decrease:

$$x^{k+1} = x^k - \psi \frac{a}{\rho}, \qquad \left(x^{k+1} = x^k - \psi \frac{a}{\rho}\right).$$

**Step 6.** The point  $x^{k+1}(x^2)$  is accepted as the new starting point and move to steps  $(3 \div 6)$  is performed, if criterion on the step 7 is not realized. If  $f(x^k) = f(x^k + \xi)$ , then movement direction determination is made according to random number again, for example:

- in the direction  $x^k + \xi$  if even number fall out;
- in the direction  $x^k \xi$  if odd number fall out.

**Step 7.** As the minimum area entry criteria consider frequentative situation  $f(x^k + \xi) > f(x^k)$  repeat (for example, repeat 100 times).

The choice of the solution under the conditions of uncertainty and multicriteria is usually made by a decision making person (DMP) in compliance with expert board point of view. It is formed with the help of a particular expert evaluation and the method of evaluation achievement from the experts of the final board.

#### **Optimization of Power system regimes multipurpose structure**

The tasks of EPS optimization are contiguous with solving of a lot of contradictory problems. The uncertainty of the output information in the making of almost every task decision defines an uncertainty zone of optimal solution. Involving of additional criteria gives an opportunity not to take into account the problem of optimal solution uncertainty selection. A simultaneous registering of several criteria is an optimal tool for reduction of optimal solution uncertainty zone.

The management of power systems regimes consists of the following functions: prognosis of the conditions, defining of the purposes and criteria, development, making of the decision and its realization, evaluation of the results. The condition of the power system is characterized with a particular structure of the operating generators, networks topologies, condition of commutation equipment, state of relay protection and protective automation, regulations of regimes and maintaining, consumed power and voltages within the units of the network, currents in the branches. If the output condition in the power system is marked with vector

$$S_{0} = \left(S_{0}^{s}, S_{0}^{k}, S_{0}^{e}, S_{0}^{ek}\right),$$

where  $S_0^s, S_0^k, S_0^e, S_0^{ek}$  – are the vectors of output condition that correspondently determine the stability, quality, economy and ecology of the development, then *j* realization of technical solution,  $j \in m$ , in the optimization of the regime structure results in transition of the power system to a new *j* condition  $S_j = (S_j^s, S_j^k, S_j^e, S_j^{ek})$ . In addition, the following restrictions should be implemented:  $S_j^s \in \psi^s$ ,  $S_j^k \in \psi^k$ ,  $S_j^e \in \psi^e$ ,  $S_j^{ek} \in \psi^{ek}$ , where  $\psi^s, \psi^k, \psi^e, \psi^{ek}$  – are the sets of vector condition permissible values determining stability, quality, economy and ecology regulations of the power system regime correspondently [18].

The structuring of the purpose gives the possibility to form a partial criteria vector the numerical values of which determine the preset level of purpose achievement. In addition a part of the criteria  $k_1 \subset k$  is maximized as far as possible, but the other part  $k_2 \subset k$  is minimized. It results in the task of optimization of power systems regimes multicriteria structure: from the given final variants set of the realized technical solutions A = (a, b, ..., j, ...m) select the variant that could be optimal in several criteria. The mathematical model of the selection of the technical solution optimal variant looks like the following

$$opt k(A) = \{\max_{A} k_1(A), \min_{A} k_2(A)\},\$$

$$W(Y, D) = 0,\$$

$$X^{\min} \le X \le X^{\max},\$$

$$Y^{\min} \le Y \le Y^{\max},\$$
(1.1)

$$D = (S, B, X), X = (P, Q, U, k_T), k_1 \cup k_2 = k,$$

where W(Y, D) – vector-function of the power unbalance in the network segments; Y – vector of the dependent variables; D – vector of the output data; S – the complex load vector in the segments of the network; B – matrix of the network self-conductivity and mutual conductivity; X – matrix of independent regimes parameters; P, Q – active and reactive power of generators; U – voltage in the network segments;  $k_T$  – transformation factors of the transformers.

The task of optimization of power systems regimes multicriteria structure can be solved:

- in the case of certainty, if in the model (1.1) the vector of output data D is presented with the determined information vector  $D_d$ ;
- in the case of uncertainty, if the vector of output data *D* is presented with indeterminate information vector  $D_H$ , where  $D_H^{\min} \le D_H \le D_H^{\max}$ ;
- in the casual regulations, if the vector of output data D is presented with casual information vector  $D_B$ .

### 2. THE CHOICE OF POWER SYSTEM OPTIMAL DEVELOPMENT STRATEGY

In the given chapter, basing on official sources and documents, the problems and development trends of electric power in Latvia are considered. The design task solving technique is presented basing on the example of the choice of power system development strategy applying the criteria of game theory.

#### The issues of power supply and development trends in Latvia

The power system of Latvia operates in the so-called 750/330 kV electrical ring including interconnected power systems of Lithuania, Estonia, Russia and Belorussia (Fig. 2.1). The output capability of power plants in Latvia is currently insufficient to satisfy the demand. The needed energy is supplied by the power plants of neighbouring states. The analysis of available information on the development prospective of the adjoining states' power systems showed that if the existing power plants are not reconstructed and new power plants are not constructed, after year 2015 the Baltic States will face the shortage of electric power [25, 26].

This can limit the volumes of import, increase the prices and, in worst case, limit the consumption of energy in Latvia.



Fig. 2.1. High voltage power network scheme of Eastern Europe (220 – 750 kV)

In order to compensate power shortage in Latvia, the following practical solutions are available:

- natural gas power plant built on the basis of the old block of the TEC-2 in Riga (the applied technology combined cycle gas turbine CCGT);
- power plant using solid fuel (black coal, peat, biomass or fuel mixtures). The applied technologies are pulverized combustion (PC), fluidized bed combustion (FBC) or combined cycle technology with solid fuel gasification;
- ▶ in the longer term atomic fuel application can be considered in Latvia [5].

According to the scientific calculations of power company professionals from the Baltic States, the full integration of the Baltic power systems in the European power system network supposes the realization of at least three projects of power supply system interconnection (Fig. 2.1):

- the second stage of submarine cable *"Estlink 2"* between Estonia and Finland (approx. 650 MW);
- submarine cable "*NordBalt*" between Lithuania and Sweden (till 1000 MW);
- land interconnection *"PowerBridge"* between the power systems of Lithuania and Poland (electrical transfer power till 1000 MW).

In the view of power supply politics in EU, only longsighted and planned basic power development (refurbishment and construction) policy, total market technical, economical and regulative integration, as well as the improvement of resource delivery factors, can help to reduce the dependence of Latvia from the growing import of power supply. Without base capacity neither internal nor external markets can operate, and the power supply safety of the Baltic region cannot be provided.

#### The issue of power system development strategy choice

Earlier almost all power system optimization tasks were completed using single criterion – the present value of costs. Variant with the smallest present value of costs was preferred as the best opportunity. In turn, considering the project investment efficiency and choosing the sources of financing the effects of such measures should be taken into account. In practice, the choice must be done basing on several criteria.

If the nature of optimization task supposes single criterion, the output information will appear uncertain. For example, load power changes in perspective, losses due to power supply interruptions and the loss of quality, fuel cost in the long view, damage to the environment as a result of plant refurbishment, etc.

The optimization measures of power supply taken at the stage of project and exploitation include the choice of power system development scenarios, minimization of power losses, safety measures, choice of substation location, etc. The multicriteria tasks are solved showing power systems as a part of complicated system with multi-function aims and uncertainty of information [23].

The optimization task solution by multicriteria models in conditions of uncertain information is possible if applying game theory.

#### Power supply development scenario comparison in Kurzeme region, Latvia

The power supply system design assignment is considered using the specific example of power system development in Kurzeme region, Latvia [2]. The following power system development strategies are analyzed (Fig. 2.2):

- $\varphi_1$  the construction of 330 kV overhead line (OL) Ventspils Grobina and upgrade of Ventspils port facilities (coal power plant construction in Ventspils);
- $\varphi_2$  the construction of 330 kV OL Liepaja Grobina and upgrade of Liepaja port facilities (coal power plant construction in Liepaja);
- $\varphi_3$  the construction of direct current cable line (CL) Nybro (Sweden) Ventspils.



Fig. 2.2. Power supply development alternatives in Kurzeme region, Latvia

The basic goals of power system development are the provision of high-quality and safe power supply at lowest costs, as well as the minimum impact on the environment. So, choosing the optimal development strategy, the following criteria have been put forward:

 $C_{\Sigma}$  – discounted annual costs;

 $A_R$  – undelivered amount of electricity in the year due to emergency failures;

 $C_{ek}$  – discounted annual costs for the condemnation of land and natural resources recovery.

Decision making supposes the choice of the situation with the most beneficial outcome from the variety of variants under the uncertainty of indefiniteness. These tasks can be described using matrix games in which the player interacts with the environment.

In given case the game with environment is considered, i.e. a matrix game in which the player (power supply company) interacts with the environment, that is not interested in his loss considering the uncertainty of environmental state.

There is a range of criteria used to choose the optimal strategy under the circumstances of indefiniteness [19, 22]. In given case the following criteria are applied:

• **Wald criterion** is oriented on the worst development prospective of the plan and guarantees profit at minimal risks:

$$a^*_{ij} = \max \min a_{ij},$$
 (2.1)

where  $\alpha_{ij}$  – result of situation, when player chooses strategy *i* at state *j* of the environment;

• **Hurwitz criterion** allows choosing a strategy that is somewhere between pessimism and optimism:

$$a_{ij}^{*} = \max_{i} [\lambda \min_{i} a_{ij} + (1 - \lambda) \max_{i} a_{ij}], \qquad (2.2)$$

where  $\lambda$  – loyalty coefficient that is chosen in the interval [0, 1];

• **Savage criterion.** The essence of criterion is the choice of such strategy that would not allow major losses which it might lead to:

$$r_{ij}^* = \min \max r_{ij},$$
 (2.3)

where  $r_{ij} = \max_{i} a_{ij} - a_{ij}$  - elements of the risks' matrix;

• Laplace criterion. The given criterion is basing on the principle of insufficient substantiation. As the probabilities of natural state are unknown, it can be assumed that they remain authentic:

$$a_{ij}^* = \max_i \frac{1}{n} \sum_{j=1}^n a_{ij} , \qquad (2.4)$$

where n – considered factors number while comparing development strategies.

According to the results of calculation, the choice between the following solutions must be done:

- > realize the alternative  $\varphi_1$  according to Wald criterion;
- > realize the alternative  $\varphi_3$  according to Hurwitz criterion, as loyalty coefficient  $\lambda$  is closer to zero (the strategy of "strong optimism") and the alternative  $\varphi_1$ , as  $\lambda$  is closer to one (pessimistic strategy);
- > realize the alternative  $\varphi_1$  according to Savage criterion;
- > realize the alternative  $\varphi_1$  according to Laplace criterion.

The choice of decision-making criterion appears one of the most complicated and responsible stages in operation investigation. Moreover, there are no general advices. Nevertheless, if no risks are acceptable, Wald criterion is applied. And otherwise, if some sort of risk is acceptable, Savage criterion is used [29].

The stability of the choice of development variant can be assessed basing on the analysis of some criteria. The variant with the maximum number of matches is chosen. In such a manner the result of calculations by different criteria recommends  $\varphi_1$  more often, so, basing on the existing data, the power supply company has to build a 330 kV OL Ventspils – Grobina and improve Ventspils port which is the part of solid fuel power plant project in Ventspils, in order to enhance power supply in Kurzeme at minimal risks.

#### 3. THE CLASSICAL MODEL OF POWER SYSTEM OPERATION OPTIMIZATION

In the given chapter the EPS operation task and possible solutions are presented, considering the fuel consumption by power plants, voltage losses and harm to the environment minimization. Besides this, the method of fuel consumption assessment is presented, allowing reducing the harm to the environment in the region of power plant.

Power system operation management task requires a solution that will provide the minimal costs of power production, transportation and distribution. This means that the task supposes minimization of power resource costs C(P). In turn, the minimal consumption can be provided by means of optimal use of limited hydro resources.

At normal EPS operation management there is a necessity for a cost-efficient distribution of load between the sources, i.e. active power  $P_i^t$  must be defined in order to provide minimal power source costs.

Consuming more water energy than water source W allows is impossible, but lesser consumption appears profitless. Ineffectively consumed water means that the system will produce the needed power by means of combusting auxiliary fuel. The fuel consumption  $B_i$  of power plant *i* depends on its active power and the power device. Both of them depend on time. Consequently, total fuel costs are represented by function of time [1, 3]:

$$C(P) = \sum_{i=1}^{N_T} c_i \int_0^T B_i(t) dt , \qquad (3.1)$$

where  $c_i$  –fuel price for station *i*;  $N_T$  – amount of thermal power plants.

The full use of water resource in period T is:

$$\sum_{j=1}^{N_{H}} \left( \int_{0}^{T} W_{qj}(t) dt - W_{j} \right) = 0, \qquad (3.2)$$

where  $N_H$  – HPP amount;  $W_j$  – water accumulation of plant j in T period;  $W_{qj}(t)$  – water

consumption depending on active power of generators and devices.

In such a manner the task of power system operation optimization is completed by means of defining minimal costs C(P) according to (3.1) upon conditions of (3.2). The given conditions link variables and mean that the optimum values in given period can't be defined only on the basis of system information.

In case there are no hydroelectric power plants in the system, i.e. there is no condition (3.2), the solution of optimization task becomes even easier, as operation in *T* periods will appear optimal in case the costs remain optimal:

$$\Delta C^{t}(P)_{\min} = \min\left(\sum_{i=1}^{N_{T}} B_{i}^{t} \cdot c_{i}\right), \qquad (3.3)$$

where  $B_i$  – conventional fuel consumption at power in time moment  $P_i^t$ .

Thereat, the optimum operation is searched in the area formed by operation limitations. The most common operation limitations are:

$$P_{i\min} \leq P_{qi} \leq P_{i\max}; \quad Q_{i\min} \leq Q_{qi} \leq Q_{i\max}; \quad U_{i\min} \leq U_i \leq U_{i\max}; \quad P_{L\min} \leq P_L \leq P_{L\max}$$

#### Evaluation of fuel consumption by means of air pollution

In case of poor weather conditions the harm brought to the environment may appear huge. If the powerful thermal power plant is situated in a region with high population density, sanitary service can ask to reduce the power of plant in order to minimize the level of pollution. This is the reason why fuel consumption by power plants should be assessed, in order to reduce the harm brought to the environment in the region of power plant [18]. Fig. 3.1 shows a certain power system consisting of two parts –  $\mathbf{A}$  and  $\mathbf{B}$ . The subsystem  $\mathbf{B}$  is situated in an area of high population density where there is a need to reduce the volumes of hazardous emission. This can be achieved by distributing power between power system devices, considering pollution of air. The fuel overconsumption should be evaluated.



Fig. 3.1. The model of energy system with two subsystems A and B

The best power distribution is achieved using operation multi-purpose optimization condition [15]:

$$\sum_{i}^{n} B_{i} \cdot c_{i} + \sum_{i}^{n} Y_{i} \to \min, \qquad (3.4)$$

where  $B_i$  – conventional fuel consumption in subsystem *i*, *t.c.f./st*;  $c_i$  – conventional fuel price c.m.u./t;  $Y_i$  – the harm of emission, c.m.u./t hazardous substances.

Assuming that fuel prices for power plants are equal, the condition (3.4) is matched by

$$\sum_{i}^{n} B_{i} \cdot \tilde{c}_{i} = \min , \qquad (3.5)$$

where  $\tilde{c}_i = (c_i + \Delta c_i)$  – the fuel price on power plant *i*, adjusted considering damage  $\Delta c_i$ .

At the permanency of operating devices' structure the condition (3.5) can be rewritten in the form of the incremental fuel cost rate of power plants equality:

$$\mathcal{E}_i \cdot \tilde{c}_i = idem; \qquad i \in n , \qquad (3.6)$$

where  $\varepsilon_i$  – incremental fuel cost rate (the rate of fuel consumption change changing power), c.f.t./MWh.

The condition (3.6), for the case when  $c_A > c_B$ , is represented by Fig. 3.2. Horizontal line O-O meets this condition and shows the optimum subsystem loads  $P_{oA}$  and  $P_{oB}$ .

In order to reduce the consumption of fuel, the increments of growth must be adjusted, i.e. unload the subsystem **B** for  $\Delta P$  rate and to increase the load of subsystem **A** for the same rate (it is assumed that the power limitations of power plants suppose such opportunity). The given operation mode will be named reference sample (Fig. 3.2, line  $O_1 - O_1$ ).

The unload of subsystem **B** provides fuel economy, which is illustrated by the painted area of Fig. 3.2, consisting of rectangle and triangle  $\Delta b$ . Similarly, the loading of subsystem A increases fuel consumption, which is represented by the painted area of Fig. 3.2, consisting of the same rectangle and triangle  $\Delta a$ . All this shows that the auxiliary fuel consumption  $\Delta B$  will be needed in order to reduce hazardous emissions. The amount of consumed fuel  $\Delta B$  is represented by the sum of two curved triangles:  $\Delta B = \Delta a + \Delta b$ , or shape *cdef* area.

It is not complicated to prove that:

$$\Delta B = 0.5 \cdot \varepsilon_{no}^2 \cdot \left(1 - c^*\right)^2 \cdot ctg \ E \ , \tag{3.7}$$

where  $\varepsilon_{no}$  – increment fuel rate at non-optimum loading of system **B**;  $c^* = \frac{\tilde{c}}{\tilde{c}_{\max}}$ ;

$$ctg E = \frac{ctg \alpha \cdot ctg \beta}{ctg \alpha + ctg \beta}$$
 – represents the bend of equivalent representative curve;  $ctg \alpha$  and

 $ctg\beta$  – the incline of power station **A** and **B** differential representative curves.

The received equity (3.7) represents the following conclusions:

- Fuel consumption appears at any  $c^*$  different from 1 (this matches the case when the limitations do not affect the power distribution between the electric power plants);
- Fuel consumption is proportionate to the incremental rate square of subsystem **B** and grows as the average system load grows, because when the load increases, incremental consumption rate and steepness of equivalent representative curve increase.



Fig. 3.2. The definition of fuel consumption considering ecological outcomes.
Figures identify representative curves of increment rates: *1* – subsystem A; 2 – subsystem B (real); 3 – subsystem B, adjusted considering price and hazard

#### The task of EPS optimization and solution instances

The classical EPS operation optimization and the method of fuel consumption assessment are shown in the scheme of test power system consisting of three equivalent thermal circulations (Fig. 3.3) [7, 8, 24]. The nominal voltage is 330 kV. Reactive power flows are not considered in optimization process.



Fig. 3.3. Test power system scheme

The optimization of EPS operation was completed by means of three efficiency functions:

$$\sum_{i} c_i \cdot B_i \to \min, \quad i \in N;$$
(3.8)

$$\sum_{i} c_{i} \cdot B_{i} + \Delta P \cdot (\tau \cdot \beta' + \beta'') \to \min, \quad i \in N;$$
(3.9)

$$\sum_{i} c_{i} \cdot B_{i} + \sum_{i} c_{k} \cdot M_{\Sigma i} + \Delta P \cdot (\tau \cdot \beta' + \beta'') \to \min, \quad i \in \mathbb{N}, \qquad (3.10)$$

under the limitations on generation:

$$P_{\min} \le P_i \le P_{\max}, \tag{3.11}$$

considering the active power balance of EPS:

$$\sum_{i} P_i + \Delta P - \sum_{i} P_{sl_i} = 0, \qquad i \in N, \qquad (3.12)$$

where  $\Delta P$  – total active power losses in the network, MW;  $\tau$  – amount of hours of maximal losses, h/year;  $\beta'$  – electricity price including power losses,  $\ell/MWh$ ;  $\beta''$  – the power unit price at system maximum,  $\ell/MW$ ;  $M_{\Sigma i}$  – hazardous emissions, t of hazardous emissions/year;  $c_k$  – specific harm from ashes, sulphur oxide and nitrogen oxide emissions,  $\ell/t$  hazardous emissions;  $P_{g_i}$  – active power generated in the node i, MW;  $N_{sl_i}$  – active load power in the node i, MW; N – number of nodes in the network.

It can be concluded from the calculations that the total costs increased by 24.5%, considering air pollution, fuel consumption and power losses in network (expression (3.10)) in comparison with the results of function (3.8). The given result proves the concern that the more precise the mathematical model of the task is, the more real the outcome.

# Fuel overconsumption in the EPS, decreasing emissions into atmosphere on the selected power plant

Fuel consumption is assessed considering environmental hazards, assuming that electric power station Nr.1 (Fig. 3.3) is situated in a region with high population density, where negative impacts should be reduced.

As pollution depends on various factors, the use of any sound values appears uneasy. It is much more important to figure out the trend of parameter change and the margins of additional fuel costs. In order to assess the character of change, coefficient  $\alpha_m$  was used in calculations. The given coefficient  $\alpha_m$  is used to reflect the growth of pollution that increases  $\alpha_m$  times when the weather conditions worsen (still air, low clouds, increased background pollution, smog, etc.) Parameter  $\alpha_m$  is affected by population density, the presence of industry, agriculture and weather conditions near power plant [7, 8, 24].

The results of calculations are illustrated by graphs in Fig. 3.4. The minimums of curves are realized in several points and divide consumption curve into ascending and descending lines. These points coincide with EPS reference sample operation mode ( $\Delta B = 0$ ). Then, as the coefficient  $\alpha_m$  of environmental conditions grows that is equal to the specific costs of hazard increase, the power plant Nr.1 notable unloading take place in reference to the optimum state. This leads to the overload of power plants Nr.2 and Nr.3 and the escalation of fuel consumption, which exceeds the economy effect on power plant Nr.1. Consequently, the increase of consumption  $\Delta B$  within the reduction of hazards in the area of power plant Nr.1 takes place. The fuel consumption grows with  $\alpha_m$ , unless the limitations (3.11) of power plants Nr.2 and Nr.3 are achieved. It is represented by graph horizontal character of curves in Fig. 3.4.



Fig. 3.4. Additional fuel consumption in EPS decreasing the negative ecological effect in the area of power plant Nr.1, depending on the local condition coefficient  $\alpha_m$  varying: a – fuel ash parameter  $A^d$ ; b – fuel sulphur parameter  $S^d$ 

Fig. 3.5 *a* shows the reduction of pollution from hazardous emissions of plant Nr.1 depending on the coefficient of local conditions  $\alpha_m$  at various fuel parameters.

Fig. 3.5 *b* illustrates the comparison of emission change with an additional consumption: percentage reduction of emissions by 1% of fuel consumption increase; this is shown by relation  $\Delta Y/\Delta B$  depending on local factors (variable parameter).



Fig. 3.5. Decrease of harm from hazardous emissions in the area of power plant Nr.1 (*a*) and its comparison with additional fuel consumption in the system (*b*), depending on local conditions coefficient  $\alpha_m$  at various fuel parameters

#### 4. EPS OPTIMIZATION TASK IN MARKET CONDITIONS

In the given chapter the method of EPS operation optimization in market conditions is presented. It was also approbated on the basis of equivalent interconnected power system of the Baltic States, Russia and Belorussia.

Market relations evolved in economics a long time ago. Nevertheless, in electrical power industry they appeared much more later than in other branches even in developed nations.

In case of centralized management the main emphasis is put on the safety of power supply, which is provided by means of subjecting all EPS elements to one control centre. Thereat, safety, the main functional factor of EPS, is provided at the expense of economical efficiency [21]. Vertical integrated management system has the following disadvantages:

- the efficient energy emission minimal cost criterion is used in operation mode optimization;
- the monopolistic state in the branch does not promote the technical (technological) development and does not motivate subjects to implement cost effective but sometimes quite expensive technologies.

The mentioned factors defined the necessity of moving to the new competition-driven management system [12]. Together with modern economical conditions in energy industry and the escalation of power wholesale, the task, criteria, optimization methods and approaches changed. Conversion to the formation of electric power tariff formation led to the fact that functional criteria of nodes in the united power market are different and sometimes even contradictory. Earlier, when electrical power was managed centrally, the optimal criterion was the minimization of costs related to fuel consumption required to produce power. Now, under market conditions, the optimization task is subdivided into a range of tasks. These are defined by criteria related to electrical energy maximum sales or minimum costs related to the production, transformation, transportation and division of electrical power. The criterion of minimal price is also important.

# The analysis of operation efficiency under the conditions of power market using Pareto principle and Nash equilibrium

The peculiarities of EPS operation under the circumstances of market and competition are considered in the model of EPS with a free electrical power price.

The total costs for EPS nodes look as follows [9]:

$$C_{\Sigma_{i}}(x_{i}, y_{ij}, y_{ji}, c_{ij}, c_{ji}) = C_{i}(x_{i}) - \sum_{j=1}^{n} a_{ij}c_{ij}y_{ij} + \sum_{j=1}^{n} a_{ji}c_{ji}y_{ji} + \frac{1}{2}\sum_{j=1}^{n} a_{ij}\pi_{ij}c_{ij}y_{ij} + \frac{1}{2}\sum_{j=1}^{n} a_{ji}\pi_{ji}c_{ji}y_{ji}, \quad (4.1)$$

where  $C_i$  – costs related to EPS function generation in nodes;  $x_i$  – generation of power in node i;  $y_{ij}$  – power flow from node i to j;  $c_{ij}$  – the price of power that corresponds to the power flow  $y_{ij}$ ;  $a_{ij}$  – elements of junction matrix;  $\pi_{ji}$  – coefficients of power losses in line between nodes j and i.

The first element of the expression (4.1) conforms to  $x_i$  electrical energy node production costs. The second element represents profit from the sold electrical energy, but the third element is related to the costs of energy purchase. It is assumed that energy loss costs are divided uniformly between the sellers and the buyers. In connection to this, there are fourth and fifth element in the expression (4.1).

Accordingly, the considered multi-purpose optimization task is formulated as follows:

$$x_{i} = P_{i} + \sum_{j=1}^{n} a_{ij} y_{ij} - \sum_{j=1}^{n} a_{ji} (1 - \pi_{ji}) y_{ji}, \quad i = \overline{1, n},$$
(4.3)

$$y_{ij} y_{ji} = 0, \qquad i, j = \overline{1, n},$$
 (4.4)

$$C'_{i}(x_{i})y_{ij} \le c_{ij}y_{ij} \le C'_{j}(x_{j})y_{ij}, \quad i, j = \overline{1, n},$$
(4.5)

$$0 \le x_i \le \overline{x}_i, \qquad i = 1, n, \tag{4.6}$$

$$0 \le y_{ij} \le \overline{y}_{ij}, \quad i = 1, n, \tag{4.7}$$

where  $P_i$  – load in the node *i*;  $\overline{x}_i$  – maximal power of plant *i*;  $\overline{y}_{ij}$  – maximal transferred power between nodes *i* and *j*.

The aim of the whole system is to provide the defined power consumption, but the goal of every node is to minimize the costs of electrical energy production, which will further allow gaining maximal profit from selling energy in every node. Both of these aims can lead to different levels of energy production in every node. Two approaches are used to consider the optimal state of the whole system and single nodes. The first approach conforms to Pareto optimal solution, but the second – to the Nash equilibrium point [28].

On the assumption of (4.1), Pareto task can be formulated:

$$\sum_{i=1}^{n} C_{i}(x_{i}) + \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} \pi_{ij} c_{ij} y_{ij} \to \min, \qquad (4.8)$$

considering limitations (4.3) - (4.7).

The minimization of the expression (4.8) can be explained as follows: Pareto optimal solution minimizes total production costs (the first element in the expression (4.8)) and costs related to power losses (the second element in the (4.8)).

In turn, Nash task is focused on function minimization

$$\sum_{i=1}^{n} C_{i}(x_{i}) - \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} c_{ij} y_{ij} + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} \pi_{ij} c_{ij} y_{ij} \to \min, \qquad (4.9)$$

considering limitations (4.3) - (4.7).

The peculiarity of the expression (4.9) is that the first element (production costs) and the third element (total loss costs) are minimized, but the second element (profit from sold electrical energy), due to negative sign, is maximized.

The considered mathematical programming tasks (4.8) and (4.9) are not concave, so various local optimums can exist. In order to define the global optimum, the method of global optimization is applied [4, 30].

The abovementioned EPS operation mode modelling and assumption method approbation was realized basing on the real interconnected EPS calculation model (Fig. 4.1), that conforms to the real model of Latvian (node 3), Russian (node 1), Estonian and Finnish (node 2), Lithuanian (node 4), Belorussian (node 6) and Kaliningrad region (Russia; node 5) EPS [20]. Within the framework of the work power systems of every country were considered as separate nodes, except for Estonia and Finland (due to the interconnection "Estlink"), where systems were united as one equivalent node. Power generation and consumption by countries, the power line resistance parameters are based on actual data gained from the database of JSC "Latvenergo".



Fig. 4.1. The calculation model of the interconnected Baltic, Russian and Belorussian power systems

As expenditure function  $C_i(x_i)$  of the real EPS is confidential information and in official way is unavailable, than in the considered interconnected power system model (Fig. 4.1) arbitrary expenditure functions' values were accepted in the form of the second power polynomials.

Both approaches for solving Pareto task (4.8) and Nash task (4.9) with arbitrary expenditure functions were realized in "GAMS" (the General Algebraic Modelling System) [10, 17, 31].

1. The optimal provision of claimed fuel consumption by EPS, in accordance with Pareto task (4.8), considering limitations (4.3) – (4.7), is shown in Fig. 4.2 *a*). Power flow prices and marginal prices in nodes are shown in Fig. 4.2 *c*).





Fig. 4.2. Solution of Pareto task: a) – optimal provision of claimed fuel consumption; b) – equilibrium price obtaining; c) – marginal prices in nodes and power flow prices

2. The results of Nash task (4.9) modelling in "GAMS", considering limitations (4.3) – (4.7), are shown in Fig. 4.3.



a)



Fig. 4.3. Nash task solution: a) – the optimal distribution of active power in system; b) – equilibrium price obtaining; c) – marginal prices in nodes and power flow prices

Basing on the received results, it can be stated that the optimal task by Pareto (goal function (4.8)) leads to an unprofitable market state for power suppliers in power system (Fig. 4.2 c)). This is indicated by flow prices that are lower than the market price:  $c_{ij} = 0.762 < C^* = 0.993$  (Fig. 4.2 b)). In connection to a fact that node *I* sells power at the price comparable to the price of its own node, the given seller does not gain any profit. Such result is explained by the fact that the goal function (4.8) task does not have the element describing profit maximization from selling power. The obtained market state is unfavourable for nodes in short supply – 3 and 5 – which are forced to compensate deficient power at a higher price compared to their claimed prices. This is connected with system deficiency and the absence of other power sources in system for the mentioned nodes. Nevertheless, the described situation is favourable for node 4, which has the opportunity to buy cheaper power compared to the own produced. Pareto optimal market state leads to a situation when it is impossible to improve the state of any market member without reducing the well-being of at least one other member.

In turn, the records of power market member profit maximization in model (4.9) led to the state of market corresponding to the results of market:  $c_{ij} = C^* = 0.887$  (Fig. 4.3 *b*)). In the obtained power system operation mode node *1* supply power at the price equal to the claimed price of nodes 2 and 4 (Fig. 4.3 *c*)). This price is maximum possible, which is acceptable for

nodes 2 and 4 to buy deficient power amount. In the same time, if there are no other sources, nodes 3 and 5 are forced to buy deficient power volumes at the highest price. Nash equilibrium leads to the state of market when the change of own power strategy is unfavourable to any market member.

Proceeding from the results of calculations, profit record in mathematical model (4.9) led to the reduction of total costs of nodes 3, 4 and 5, but increased the expenditures of members 1, 2 and 6, in comparison with mode (4.8). In addition, total system costs increased.

## 5. THE APPLICATION OF THE PARETO PRINCIPLE WHEN CHOOSING THE OPTIMAL DEVELOPMENT VARIANT OF EPS

In the given chapter the method allowing graphical generation of optimal Pareto solution range is considered, solving the EPS development projection task in accordance with the specific set of criteria and strategies.

As it has already been mentioned, when criteria are numerically and qualitatively incomparable, Pareto principle is used. Although alternatives that satisfy Pareto principle should theoretically be treated as equally optimal, there are opportunities of practical comparison and further disallowance. This approach appears useful, because the amount of alternatives sometimes is quite big and the only choice must be made. This kind of alternative analysis is named as interference into the set of compromises [27].

The example presents the comparison of two possible variants of power system development in the Baltic States (Fig. 5.1) [6]:

 $\varphi_1$  – construction of the DC high-voltage submarine CL Klaipeda (Lithuania) – Nybro (Sweden) "NordBalt" (400 km) with transferred power 750 MW;

 $\varphi_2$  – construction of DC CL Sindi (Estonia) – Dundaga (Latvia) (180 km) and 330 kV OL Grobina – Ventspils – Dundaga – Tume – Riga (310 km).



Fig. 5.1. Power system development alternatives in the Baltic States considered in the example

In order to provide a qualitative and safe power supply at lower costs, as well as to minimize the impact on the environment, chosen alternatives must satisfy Pareto principle with respect of the following four criteria:

- $C_{K}$  annual capital expenses;
- $C_E$  annual operating costs;
- $C_{ek}$  costs related to the environment pollution;
- $C_R$  national economic losses due to line's emergency outage.

Scalar criterion expression is

$$F = p_1 C_K + p_2 C_E + p_3 C_{ek} + p_4 C_R, (5.1)$$

where

$$p_1 + p_2 + p_3 + p_4 = 1 \tag{5.2}$$

is in force for weight coefficients  $p_i$ .

The problem lays in a fact that weight coefficients  $p_j$  are unknown. Moreover, according to the Pareto principle, one opportunity can be favourable at one weight coefficients, but the other may match other weight coefficient values.

Using the graphical interpretation of the Pareto optimal solution, DMP can see the opportunity of compromise between the given criteria. He chooses three most significant criteria or those criteria with the most variable values or those closest to the critical ones and applies limitations to the rest of criteria. Pareto multitude construction task is solved and then DMP chooses the best variant. After that DMP chooses another criteria of high enough values, others transfers to the category of limitations and using those limitations constructs the Pareto set again applying three other criteria. After several cycles of such procedure, DMP gains enough information about the opportunities of compromise between the criteria, and is able to choose the appropriate variant. The described method represents generalization in three-dimensional space.

The solutions of the given example using two comparable opportunities can be explained graphically. Fig. 5.2 shows zoning results when the value of the weight coefficient  $p_4$  is  $p_4 = 0.25$ . Different  $p_4$  values are represented by planes parallel to plane *P*. Figure *aofebc* volume limited by plane  $F_{\varphi_1} = F_{\varphi_2}$  and coordinate axis, conforms to  $\varphi_1$  alternative efficiency condition  $F_{\varphi_1} < F_{\varphi_2}$ , but figure *cbghfe* volume is corresponded by  $\varphi_2$  alternative condition  $F_{\varphi_2} < F_{\varphi_1}$ .

Comparing volumes of figures *aofebc* and *cbghfe*, the best alternative can be stated. As  $V_{cbghfe} > V_{aofebc}$ , in case when  $p_4 = 0.25$  (Fig. 5.2)  $\varphi_2$  alternative is chosen as the best one – construction of CL Sindi – Dundaga and OL Grobina – Ventspils – Dundaga – Tume – Riga, because shape's *cbghfe* volume exceeds the volume of the shape *aofebc*.

Chart in Fig. 5.3 shows volumes of shapes  $V_{aofebc}$  (where  $F_{\varphi_1} < F_{\varphi_2}$  is in force) and  $V_{cbghfe}$ ( $F_{\varphi_2} < F_{\varphi_1}$ ) at various values of weight coefficient  $p_4$ : from  $p_4 = 0$  to  $p_4 = 0.75$ . As it can be seen from Fig. 5.3, both alternatives reach the same meaning ( $F_{\varphi_1} = F_{\varphi_2}$ ) in case when weight coefficient of national economic losses  $p_4 = 0.63$ . In the given point volumes of both shapes are equal  $V_{aofebc} = V_{cbghfe}$ . As the significance of national economic loss criterion grows ( $p_4 > 0.63$ ), the best alternative is  $\varphi_1$  – the construction of CL Klaipeda – Nybro, because shape *aofebc* volume is grater than the volume of the shape *cbghfe* ( $V_{aofebc} > V_{cbghfe}$ ).



Fig. 5.2. Zoning of solutions in the area of criteria ( $p_4 = 0.25$ )



Fig. 5.3. Volumes of shapes *aofebc* ( $F_{\varphi_1} < F_{\varphi_2}$ ) and *cbghfe* ( $F_{\varphi_2} < F_{\varphi_1}$ ) at different weight coefficient  $p_4$  variations

The presented procedure obtained through the synthesis of multicriteria task solution methods – Pareto and preference methods – has the following advantages: apart from constructing the Pareto set and visualization method, it allows operation with more than three criteria, while four- and more dimensional Pareto set creation construction is impossible. Apart from the method of preferences, the presented procedure reduces the number of stages and allows assessing the possibility of compromise between various criteria on every stage; thereat, the final outcome will appear Pareto-optimal by three criteria, which are considered as the most significant. The peguliarities of such method can be used for the solution of any

multi-purpose optimization task where the number of criteria exceeds three in cases when it is impossible to complete a quantitative comparison of criteria relative significance.

# 6. CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK

- 1. The development of power systems and tasks of maintenance have multicriteria character. The power system should be economic; the consumers should be provided with safe and qualitative power supply. Also it should have minimum negative effect on the environment. In addition many other local regulations and conditions should be taken into account.
- 2. One of the basic tasks in the field of power engineering is the optimization of EPS operation regime. Together with this maintaining task the task of designing is also of high importance. Multicriteria approach to its solution gives the possibility to demonstrate its mathematical model with several criteria functions and restrictions (depending on the type of the tasks for solving) including restrictions of system, regime, finances, resources and other restrictions influencing the given task solution.
- 3. The multicriteria option gives opportunity to approach the task of EPS regimes' optimization in a complex way, obtaining the better solutions correspondent to the reality.
- 4. The optimization criterion within the centralized control period of electric power engineering is minimization of cost price of fuel, consumed for the electric energy production, and amount of the produced electric energy.
- 5. Decentralization of power engineering management caused statement of the changes optimization tasks and the necessity to consider new optimization criteria. The liberalization of power engineering field made the problem of profit from electric energy sale maximization, transmission and distribution. The final purpose of the market relations development is decreasing of electric energy price for consumers.
- 6. In the paper there are analysed the criteria significant for the decision making person within the task of EPS regimes optimal planning and development. Based on the criteria analysis a formalized purpose functions description is proposed for its further possible application in the solution of multi-purpose optimization tasks.
- 7. The problems of Latvian power supply strategy and EPS development projects are analysed on the basis of official sources and documentation. It is stressed that the solution of the existing problems in the area of electric power supply will accelerate the process of Latvian power system integration into the other power systems of the European Union, providing the access to the market of electrical energy. The realization of the planned measures for the increasing of generated power (coal station, APP, wind power parks) as well as the participation of Latvia in the realization of mutually united system with those of neighbours will give the possibility to increase its power safety.
- 8. The methodology of project tasks solutions under the conditions of uncertainty with the use of game theory is suggested. It is demonstrated in the work that the selection of decision making criterion is the most complex and crucial point within the investigation of operation and often it depends on the interests of the decision making person.
- 9. The author proposes the mathematical model of the load optimal distribution among the EPS electric stations taking into account the minimization criteria of fuel consumption at the stations, power losses in electric networks and emission into environment. The fuel consumption at the selected electric station is evaluated for the decreasing of emissions according to the factor of local conditions and variations of fuel features.
- 10. The methodology of EPS optimal power distribution among the entities of the electric energy market under the conditions of market relationships with the use of Pareto optimal principle and Nash balance is demonstrated in the work. The calculation with the use of the example of the tested scheme (equivalent of Latvian, Lithuanian, Estonian (with

Finland), Russian and Belorussian integrated EPS) is made for the analysis of the obtained results and practical approbation of the developed methodology.

- 11. The task of methodology development if stated and solved giving the possibility to obtain Pareto set of optimal solutions in graphic form for the selection of optimal power system development variant for particular criteria and strategy structure. The graphic approach to the task solution is demonstrated on the example of Baltic EPS development strategy selection.
- 12. Using the visual graphic interpretation of the Pareto optimal solution the decision making person can easily observe the opportunity of compromise among the criteria under consideration. The proposed methodology gives the possibility to evaluate this compromise possibility and obtain a solution for the tasks when the number of criteria is more than three making in addition the limitation with considering three-dimension set. These features allow application of the proposed methodology in the multicriteria optimization tasks when the number of criteria is more than three.
- 13. The work demonstrates the examples of operating and designing tasks calculations using simplified EPS mathematical models. In future the investigation is planned to be added with technically and economically more accurate mathematical models.
- 14. The methodologies of EPS regimes optimization considered in the paper do not take into account the distribution of reactive power in the network. For homogenous networks this distribution is realized independently. It is necessary to investigate additionally possible influence of reactive power onto the results of the optimization in the case of inhomogeneous networks (for example, in the EPS equivalent model of Baltic, Russia and Belorussia).
- 15. Determining optimal load distribution among the sectors of EPS under the conditions of electric power market with the example of Baltic, Russian and Belorussian power grid calculation, the consumption functions are accepted in the type of quadric polynomial that does not correspond to the real situation as it is not possible to obtain it in official form. If the real task functions were obtained, the real prices for the electric energy in particular sectors (states) would be possible to model.
- 16. The optimization of EPS regimes under the market conditions with the equivalent model of Baltic, Russian and Belorussian unified EPS has been realized with unknown consumer prices in the sectors. If the prices were known, then the formation of accurate demand and supply regulations and determination of real balanced prices would be possible. The result of modelling could give opportunity to obtain real power distribution in the unified EPS that could guarantee the participant of the market with maximum profit.

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