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An application of Direct Filter Approach: New Economic Indicators for Latvia

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Abstract **– The paper overviews the latest development trends in direct filter approach (DFA) and applies DFA to construct three prototypical real-time zero-lag indicators for Latvian economy. The essence of DFA is to separately control for amplitude and phase shift errors. Such a filtering approach has never been implemented for Latvian data, and the resulting indicators show nice real-time properties. A particular application is construction of a light zero-lag monthly business cycle indicator for Latvia's GDP; such indicator might be welcome by macroeconomists.**

Keywords **– real time signal extraction, monthly indicator for GDP**

I. Introduction

The DFA methodology is apparently developed solely by Wildi (e.g., 1998, 2004, 2005, 2008a, 2008b, 2009, 2010, 2011). Section II introduces DFA, Section III applies the methodology to construct prototypical real-time indicators for Latvian economy, Section IV concludes, and Appendix elaborates on the (M)DFA methodology in details.

II. Direct filter approach: an overview

The direct filter approach is concerned with estimating a signal – e.g., a trend, business cycle or seasonally adjusted series – in real time.

Let us assume that

$$y_t = \sum_{j=-\infty}^{\infty} \gamma_j x_{t-j} \qquad (1)$$

is the ideal output signal of a symmetric, possibly bi-infinite filter. Since the filter in (1) generally requires bi-infinite input data, the ideal output is infeasible in practice. A real time estimate of y_t given a finite data set $\{x_1, \ldots, x_T\}$ is

$$\hat{y}_t = \sum_{j=0}^{T-1} b_j x_{t-j} \qquad (2)$$

Denote $\Gamma(\omega) = \sum_{j=-\infty}^{\infty} \gamma_j \exp(-ij\omega)$ and $\hat{\Gamma}(\omega) = \sum_{j=0}^{T-1} b_j \exp(-ij\omega)$ be the corresponding transfer functions of filters in (1) and (2). Consider stationary processes for easy exposition; generalization to integrated processes is straighforward by using pseudo spectral estimates and filter constraints, see Wildi 2008a. For a stationary process x_t, the mean square filter error can be expressed as

$$\int_{-\pi}^{\pi} |\Gamma(\omega) - \hat{\Gamma}(\omega)|^2 \, dH(\omega) = E[(y_t - \hat{y}_t)^2], \qquad (3)$$

where $H(\omega)$ is the unknown spectral distribution of x_t. A finite sample approximation of (3) is

$$\frac{2\pi}{T} \sum_{k=-[T/2]}^{[T/2]} w_k \, |\Gamma(\omega_k) - \hat{\Gamma}(\omega_k)|^2 \, S(\omega_k), \qquad (4)$$

where $\omega_k = k2\pi/T$, $[T/2]$ is the greatest integer smaller or equal to $T/2$, and the weight w_k is defined as

$$w_k = \begin{cases} 1 & \text{for } |k| \neq T/2 \\ 1/2 & \text{otherwise.} \end{cases} \qquad (5)$$

$S(\omega_k)$ in (4) can be interpreted as an estimate of the unknown spectral density of x_t, which can be any spectral estimate, e.g., it can be ARIMA-based spectral estimate. However, as discussed in Wildi (e.g., 1998, 2008a), consistency of $S(\omega_k)$ is not required because the goal is not to estimate $dH(\omega)$ but the filter mean square error (3), instead. Therefore, Wildi (2008a), among others, propose using `sufficient statistic' - periodogram - as $S(\omega_k)$ in (4):

$$S(\omega_k) := I_{Tx}(\omega_k) = \frac{1}{2\pi T} \left| \sum_{t=1}^{T} x_t \exp(-it\omega_k) \right|^2. \qquad (6)$$

Formal efficiency results are presented in Wildi (2008a, 2009). Given Wildi's subjective preference for periodogram, this paper plugs periodogram in S. However, if one feels reluctant using this inconsistent statistic, one can use his/her preferred choice in one's own application, instead.

A. Classical DFA

Wildi (1998, 2005, 2008a) proposes a decomposition of the mean square filter error into distinct components attributable to the phase and amplitude functions of the real time filter, using cosine law. For general transfer functions Γ and $\hat{\Gamma}$,

$$\begin{aligned} |\Gamma(\omega) - \hat{\Gamma}(\omega)|^2 &= A(\omega)^2 + \hat{A}(\omega)^2 \\ &- 2A(\omega)\hat{A}(\omega)\cos\left(\hat{\Phi}(\omega) - \Phi(\omega)\right) \\ &= (A(\omega) - \hat{A}(\omega))^2 \\ &+ 2A(\omega)\hat{A}(\omega)\left[1 - \cos\left(\hat{\Phi}(\omega) - \Phi(\omega)\right)\right] \end{aligned} \qquad (7)$$

Assuming Γ is symmetric and positive, $\Phi(\omega) = 0$. Inserting (7) into (4) and using $1 - \cos(\hat{\Phi}(\omega)) = 2\sin^2(\hat{\Phi}(\omega)/2)$, (4) results in

$$\frac{2\pi}{T} \sum_{k=-[T/2]}^{[T/2]} w_k (A(\omega_k) - \hat{A}(\omega_k))^2 S(\omega_k)$$

$$+\frac{2\pi}{T}\sum_{k=-[T/2]}^{[T/2]} w_k 4A(\omega_k)\hat{A}(\omega_k)\sin^2(\hat{\Phi}(\omega_k)/2)S(\omega_k) \tag{8}$$

The first part of (8) is a part of total mean square filter error attributable to the amplitude function of the real time filter. The second part of (8) is attributable to the part of the total mean square filter error due to the phase/time shift. The term $A(\omega_k)\hat{A}(\omega_k)$ in (8) is a scaling factor which accounts for the fact that the phase function does not convey level information.

Assuming $\Gamma(\omega) > 0$ for all ω, $\Gamma(\omega) = A(\omega)$. Then, a generalized version of (8) can be written as

$$\frac{2\pi}{T}\sum_{k=-[T/2]}^{[T/2]} w_k (A(\omega_k)-\hat{A}(\omega_k))^2 W(\omega_k)S(\omega_k)$$

$$+(1+\lambda)\frac{2\pi}{T}\sum_{k=-[T/2]}^{[T/2]} w_k 4A(\omega_k)\hat{A}(\omega_k)\sin(\hat{\Phi}(\omega_k)/2)^2 W(\omega_k)S(\omega_k)$$

$$=\frac{2\pi}{T}\sum_{k=-[T/2]}^{[T/2]} w_k |\Gamma(\omega_k)-\hat{\Gamma}(\omega_k)|^2 W(\omega_k)S(\omega_k)$$

$$4\lambda\frac{2\pi}{T}\sum_{k=-[T/2]}^{[T/2]} w_k A(\omega_k)\hat{A}(\omega_k)\sin(\hat{\Phi}(\omega_k)/2)^2 W(\omega_k)S(\omega_k), \tag{9}$$

where $W(\omega_k) := W(\omega_k, expweight, cutoff)$:

$$W(\omega_k, expweight, cutoff) = \begin{cases} 1 & \text{if } |\omega_k| < cutoff \\ (1+|\omega_k| - cutoff)^{expweight} & \text{otherwise.} \end{cases} \tag{10}$$

$cutoff$ marks the transition between passband and stopband, and positive values of $expweight$ emphasize high-frequency components. Classical mean square optimization is for $\lambda = expweight = 0$, whereas $\lambda > 0$ emphasizes time shift error. The term $A(\omega_k)\hat{A}(\omega_k)$ in the second part of (9) implies that λ acts on the passband frequencies exclusively, and that $expweight$ does not alter the time shift error; $expweight$ emphasizes means square amplitude error by magnifying high-frequency components in the stopband.

B. Analytical approximation to classical DFA

Expression in (8) is a quadratic function of the filter parameters and therefore the solution can be obtained analytically. Expression (9), however, involves nonlinear functions of the filter parameters when $\lambda > 0$. Therefore, Wildi (2011) proposes the following analytic approximation of (9) (for notational ease, w_k has been assumed 1) :

$$\frac{2\pi}{T}\sum_{k=-[T/2]}^{[T/2]} \left|\Gamma(\omega_k) - \left[Re\left(\hat{\Gamma}(\omega_k)\right) + i\sqrt{1+4\lambda\Gamma(\omega_k)f(\omega_k)}\,Im\left(\hat{\Gamma}(\omega_k)\right)\right]\right|^2 W(\omega_k)S(\omega_k), \tag{11}$$

where $Re(\cdot)$ and $Im(\cdot)$ denote real and imaginary parts and $i=\sqrt{-1}$ is the imaginary unit. This paper follows Wildi (2011) and sets $f(\omega_k) = 1$. Expression in (11) is quadratic in filter parameters. Similarly to (8), $W(\omega_k)$ emphasizes the fit in the stopband. The term $4\lambda\Gamma(\omega_k)$ emphasizes the imaginary part of the real time filter in the passband.

Rewriting (11),

$$\frac{2\pi}{T}\sum_{k=-[T/2]}^{[T/2]} \left|\Gamma(\omega_k) - \left[Re\left(\hat{\Gamma}(\omega_k)\right) + i\sqrt{1+4\lambda\Gamma(\omega_k)}\,Im\left(\hat{\Gamma}(\omega_k)\right)\right]\right|^2 W(\omega_k)S(\omega_k)$$

$$=\frac{2\pi}{T}\sum_{k=-[T/2]}^{[T/2]} \left[\left(\Gamma(\omega_k) - Re\left(\hat{\Gamma}(\omega_k)\right)\right)^2 + Im\left(\hat{\Gamma}(\omega_k)\right)^2\right] W(\omega_k)S(\omega_k)$$

$$+4\lambda\frac{2\pi}{T}\sum_{k=-[T/2]}^{[T/2]} \Gamma(\omega_k) Im\left(\hat{\Gamma}(\omega_k)\right)^2 S(\omega_k)$$

$$=\frac{2\pi}{T}\sum_{k=-[T/2]}^{[T/2]} |\Gamma(\omega_k)-\hat{\Gamma}(\omega_k)|^2 W(\omega_k)S(\omega_k)$$

$$+4\lambda\frac{2\pi}{T}\sum_{k=-[T/2]}^{[T/2]} A(\omega_k)\hat{A}(\omega_k)^2\sin(\hat{\Phi}(\omega_k))^2 W(\omega_k)S(\omega_k). \tag{12}$$

A comparison of (9) and (12) reveals that $\hat{\Phi}(\omega_k)/2$ is replaced by $\hat{\Phi}(\omega_k)$ and an additional $\hat{A}(\omega_k)$ appears in (12). (12) can be solved analytically for any λ and $W(\omega_k)$ because $\hat{A}(\omega_k)^2\sin(\hat{\Phi}(\omega_k))^2$ is a squared imaginary part of the real time filter. when $\lambda > 0$, expression (12) is less statistically efficient than (9). However, if $\lambda = 0$, (12) reduces to classical mean square criterion and regains efficiency. Section 4 describes results using $\lambda = 0$.

C. Multivariate DFA

The above univariate DFA has been generalized to a multivariate DFA (MDFA) in Wildi (2008a). Rewrite (4) using discrete Fourier transform (DFT) $\Xi_{Tx}(\omega_k)$:

$$\begin{aligned}&\frac{2\pi}{T}\sum_{k=-[T/2]}^{[T/2]} |\Gamma(\omega_k)-\hat{\Gamma}(\omega_k)|^2 I_{Tx}(\omega_k) = \\ &\frac{2\pi}{T}\sum_{k=-[T/2]}^{[T/2]} |\Gamma(\omega_k)\Xi_{Tx}(\omega_k)-\hat{\Gamma}(\omega_k)\Xi_{Tx}(\omega_k)|^2 .\end{aligned} \tag{13}$$

Define y_t as in (1), and assume additional m explaining variables Z_{jt}, $j=1,\ldots,m$. Then, $\hat{\Gamma}(\omega_k)\Xi_{Tx}(\omega_k)$ becomes

$$\hat{\Gamma}_X(\omega_k)\Xi_{Tx}(\omega_k) + \sum_{n=1}^{m}\hat{\Gamma}_{Z_n}(\omega_k)\Xi_{TZ_n}(\omega_k), \tag{14}$$

where

$$\hat{\Gamma}_X(\omega_k) = \left(\sum_{j=0}^{L} b_{xj}\exp(-ij\omega_k)\right)\Xi_{Tx}(\omega_k) \tag{15}$$

$$\hat{\Gamma}_{Z_n}(\omega_k) = \left(\sum_{j=0}^{L} b_{Z_n j} \exp(-ij\omega_k) \right) \Xi_{TZ_n}(\omega_k) \quad (16)$$

are the one-sided transfer functions applying to the explaining variables, and $\Xi_{Tx}(\omega_k)$, $\Xi_{TZ_n}(\omega_k)$ are the corresponding DFTs. Wildi (2008a) shows that the following extension of (13)

$$\frac{2\pi}{T} \sum_{k=-[T/2]}^{[T/2]} \left| \left(\Gamma(\omega_k) - \hat{\Gamma}_x(\omega_k) \right) \Xi_{Tx}(\omega_k) - \sum_{n=1}^{m} \hat{\Gamma}_{Z_n}(\omega_k) \Xi_{TZ_n}(\omega_k) \right|^2 \quad (17)$$

inherits efficiency properties of the univariate DFA. A multivariate version of (9) can be obtained by first rewriting (17) as

$$\frac{2\pi}{T} \sum_{k=-[T/2]}^{[T/2]} \left| \Gamma(\omega_k) \Xi_{Tx}(\omega_k) - \hat{\Gamma}_x \Xi_{Tx}(\omega_k) - \sum_{n=1}^{m} \hat{\Gamma}_{Z_n}(\omega_k) \Xi_{TZ_n}(\omega_k) \right|^2$$

$$= \frac{2\pi}{T} \sum_{k=-[T/2]}^{[T/2]} \left| \Gamma(\omega_k) - \hat{\Gamma}_x(\omega_k) - \sum_{n=1}^{m} \hat{\Gamma}_{Z_n}(\omega_k) \frac{\Xi_{TZ_n}(\omega_k)}{\Xi_{Tx}(\omega_k)} \right|^2 * |\Xi_{Tx}(\omega_k)|^2$$

$$= \frac{2\pi}{T} \sum_{k=-[T/2]}^{[T/2]} \left| \Gamma(\omega_k) - \hat{\Gamma}(\omega_k) \right|^2 |\Xi_{Tx}(\omega_k)|^2, \quad (18)$$

where

$$\hat{\Gamma}(\omega_k) := \hat{\Gamma}_x(\omega_k) + \sum_{n=1}^{m} \hat{\Gamma}_{Z_n}(\omega_k) \frac{\Xi_{TZ_n}(\omega_k)}{\Xi_{Tx}(\omega_k)}. \quad (19)$$

An analytically tractable multivariate version of customized criterion (12), thus, is

$$\frac{2\pi}{T} \sum_{k=-[T/2]}^{[T/2]} |\Gamma(\omega_k) - \hat{\Gamma}(\omega_k)|^2 W(\omega_k) |\Xi_{Tx}(\omega_k)|^2$$

$$+ 4\lambda \frac{2\pi}{T} \sum_{k=-[T/2]}^{[T/2]} A(\omega_k)\hat{A}(\omega_k)^2 \sin(\hat{\Phi}(\omega_k))^2 W(\omega_k) |\Xi_{Tx}(\omega_k)|^2 . \quad (20)$$

The ratio in (19) is a source of potential numerical instability of the solution of (20), though. Thus, Wildi (2011) comes up with the following alternative to (20) (coefficient 4 in the second part of (20) for notational simplicity is absorbed by λ, and the weighting function $W(\omega_k)$ is ignored):

$$\sum_{k=-[T/2]}^{[T/2]} \left| \Gamma(\omega_k) |\Xi_{Tx}(\omega_k)| - Re(expr) - i\sqrt{1+\lambda\Gamma(\omega_k)}\, Im(expr) \right|^2, \quad (21)$$

where

$$expr = \hat{\Gamma}_x(\omega_k) |\Xi_{Tx}(\omega_k)| + \sum_{n=1}^{m} \hat{\Gamma}_{Z_n}(\omega_k) \Xi_{TZ_n}(\omega_k) * \exp(-i \arg(\Xi_{Tx}(\omega_k))) \quad (22)$$

The paper uses the filter obtained by minimizing (21) subject to filter parameters and two potential constraints - amplitude and time shift constraints at frequency zero – discussed in Appendix.

III. New economic indicators for Latvia

A. Targeting cyclical fluctuations in industrial production

All data used are real time vintages. Some variables contain interpolated values. For example, Fig. 1 shows one of the explanatory variables whose first 100 monthly observations are interpolated from quarterly data. Such variables should be carefully prepared for the best outcome, but meanwhile I take those data as such, without proper treatment of seasonal effects, log transform, cointegration, etc. Industrial production trend published by GD ECFIN is plotted in Fig. 2.

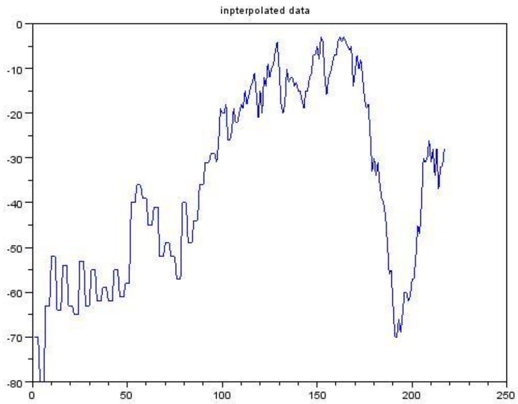

Fig. 1. Interpolated explanatory data.

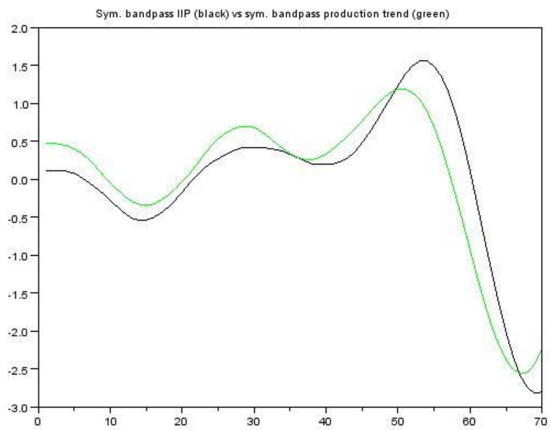

Fig. 2. Bandpass of IIP (black) versus bandpass of industrial production trend (green).

The resulting real-time indicator for Latvia's industrial production – with an emphasis of timely turning point detection – is shown in Fig. 3.

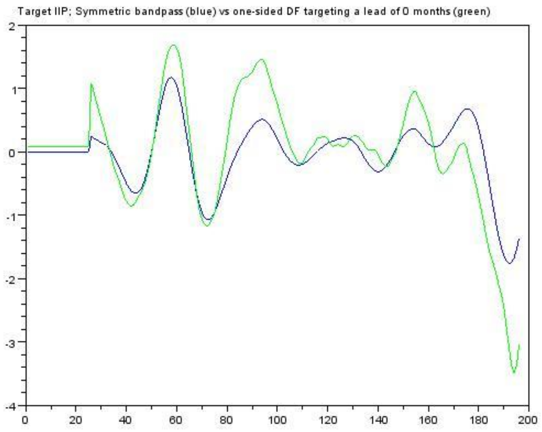

Fig. 3. symmetric bandpass of industrial production trend (black) versus its real-time estimate (green).

Fig. 3 shows that the real-time indicator (green) estimates turning points precisely (without lag) and that its amplitude is slightly misplaced. Particularly, we can sacrifice amplitude error to get better phase fit.

B. Targeting monthly cyclical fluctuations in quarterly GDP

Gross domestic product has large publishing lags and revisions. Therefore, it is beneficial to have a zero-lag monthly indicator tracking cyclical movements in quarterly GDP. Fig. 4 shows a prototypical indicator (green) that tracks turning points in GDP in real-time setting.

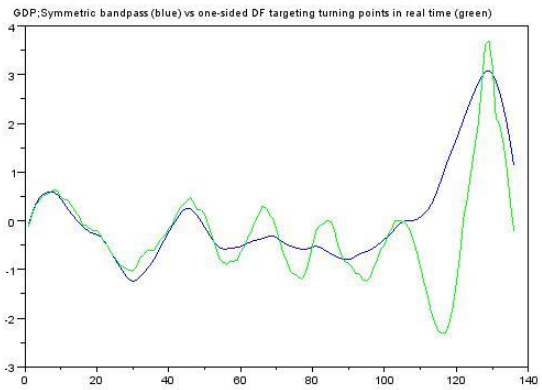

Fig. 4. Quarterly GDP (black) versus monthly indicator (green) tracking cyclical turning points in GDP in real time.

C. A light monthly indicator tracking level in year-on-year GDP

Fig. 5 shows a prototypical real-time zero-lag monthly indicator tracking level of quarterly year-on-year GDP values. This indicator might especially be useful for economists who usually look at y-o-y GDP.

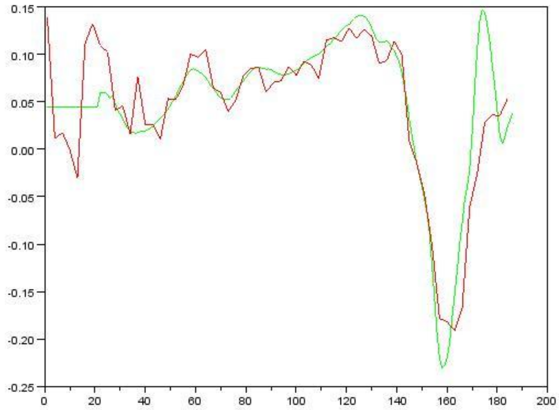

Fig. 5. Interpolated y-o-y GDP (red) versus real-time zero-lag monthly indicator (green) tracking level of y-o-y GDP.

IV. Conclusion

This paper overviews latest developments in real-time filtering, and proposes three prototypical real-time indicators for Latvian economy:

- real-time indicator tracking turning points in Latvian industrial production;
- monthly real-time indicator tracking turning points in quarterly GDP;
- monthly real-time indicator tracking the level of year-on-year GDP.

V. Appendix

A. Univariate DFA

Rewrite (11) (with $f(\omega_k)=1$)

$$\sum_{k=-[T/2]}^{[T/2]} |\,\Gamma(\omega_k)-Re(\hat{\Gamma}(\omega_k))-i\sqrt{1+\lambda\Gamma(\omega_k)}Im(\hat{\Gamma}(\omega_k))|^2\, I_{Tx}(\omega_k)$$

$$=\sum_{k=-[T/2]}^{[T/2]}\left(\begin{array}{l}\left[\Gamma(\omega_k)-Re(\hat{\Gamma}(\omega_k))\right]^2\\ +(1+\lambda\Gamma(\omega_k))Im\left(\hat{\Gamma}(\omega_k)\right)^2\end{array}\right)I_{Tx}(\omega_k) \qquad (23)$$

Differentiate (23) w.r.t. filter parameters:

$$\sum_{k=-[T/2]}^{[T/2]}((\Gamma(\omega_k)-Re(\hat{\Gamma}(\omega_k)))(-d/db_j(Re(\hat{\Gamma}(\omega_k))))$$

$$+(1+\lambda\Gamma(\omega_k))Im(\hat{\Gamma}(\omega_k))d/db_j(Im(\hat{\Gamma}(\omega_k))))I_{Tx}(\omega_k)=0 \qquad (24)$$

Since

$$\frac{d}{db_j}Re(\hat{\Gamma}(\omega_k))=\cos(-j\omega_k) \qquad (25)$$

and

$$\frac{d}{db_j} Im(\hat{\Gamma}(\omega_k)) = \sin(-j\omega_k) \quad (26)$$

(24) becomes

$$\sum_{k=-[T/2]}^{[T/2]} \begin{pmatrix} (\Gamma(\omega_k) - Re(\hat{\Gamma}(\omega_k)))(-\cos(-j\omega_k)) \\ +(1+\lambda\Gamma(\omega_k))Im(\hat{\Gamma}(\omega_k))\sin(-j\omega_k) \end{pmatrix} I_{Tx}(\omega_k) = 0 \quad (27)$$

or

$$\sum_{k=-[T/2]}^{[T/2]} \Gamma(\omega_k)\cos(-j\omega_k) I_{Tx}(\omega_k)$$

$$= \sum_{k=-[T/2]}^{[T/2]} \begin{pmatrix} Re(\hat{\Gamma}(\omega_k))\cos(-j\omega_k) \\ +(1+\lambda\Gamma(\omega_k))Im(\hat{\Gamma}(\omega_k))\sin(-j\omega_k) \end{pmatrix} I_{Tx}(\omega_k). \quad (28)$$

Since

$$Re(\hat{\Gamma}(\omega_k)) = \sum_l b_l \cos(-l\omega_k) \quad (29)$$

and

$$Im(\hat{\Gamma}(\omega_k)) = \sum_l b_l \sin(-l\omega_k), \quad (30)$$

the r.h.s. of (28) is

$$\begin{aligned} & b_0 \sum_{k=-[T/2]}^{[T/2]} (\cos(-j\omega_k)\cos(-0\omega_k) \\ & +(1+\lambda\Gamma(\omega_k))\sin(-j\omega_k)\sin(-0\omega_k)) I_{Tx}(\omega_k) + \\ & b_1 \sum_{k=-[T/2]}^{[T/2]} (\cos(-j\omega_k)\cos(-1\omega_k) \\ & +(1+\lambda\Gamma(\omega_k))\sin(-j\omega_k)\sin(-1\omega_k)) I_{Tx}(\omega_k) + \\ & \vdots \\ & b_L \sum_{k=-[T/2]}^{[T/2]} (\cos(-j\omega_k)\cos(-L\omega_k) \\ & +(1+\lambda\Gamma(\omega_k))\sin(-j\omega_k)\sin(-L\omega_k)) I_{Tx}(\omega_k). \end{aligned} \quad (31)$$

Let $b = C^{-1}V$. Then, the r.h.s. of (28) implies

$$C = \begin{pmatrix} \sum_{k=-[T/2]}^{[T/2]} (\cos(-j\omega_k)\cos(-l\omega_k) \\ +(1+\lambda\Gamma(\omega_k))\sin(-j\omega_k)\sin(-l\omega_k)) I_{Tx}(\omega_k) \end{pmatrix}_{jl}, \quad (32)$$

where $0 \le j,l \le L$. The l.h.s. of (28) implies

$$V = \left(\sum_{k=-[T/2]}^{[T/2]} \Gamma(\omega_k)\cos(-j\omega_k) I_{Tx}(\omega_k) \right)_j, \quad (33)$$

where $j = 0,\ldots,L$.

B. Multivariate DFA

Consider the following multivariate version of (11) (for notational simplicity the multiplier 4 is concatenated into λ and the weighting function $W(\omega_k)$ is ignored):

$$\begin{aligned} & \sum_{k=-[T/2]}^{[T/2]} |\Gamma(\omega_k)\Xi_{Tx}(\omega_k) - Re\begin{pmatrix} \hat{\Gamma}(\omega_k)\Xi_{Tx}(\omega_k) \\ +\sum_{n=1}^{m}\hat{\Gamma}_{Z_n}(\omega_k)\Xi_{TZ_n}(\omega_k) \end{pmatrix} \\ & -i\sqrt{1+\lambda\Gamma(\omega_k)}Im\left(\hat{\Gamma}(\omega_k)\Xi_{Tx}(\omega_k) + \sum_{n=1}^{m}\hat{\Gamma}_{Z_n}(\omega_k)\Xi_{TZ_n}(\omega_k)\right)|^2 \\ & = \sum_{k=-[T/2]}^{[T/2]} \begin{pmatrix} \Gamma(\omega_k)Re(\Xi_{Tx}(\omega_k)) - Re(\hat{\Gamma}_x(\omega_k)\Xi_{Tx}(\omega_k)) \\ -\sum_{n=1}^{m} Re(\hat{\Gamma}_{Z_n}(\omega_k)\Xi_{TZ_n}(\omega_k)) \end{pmatrix}^2 \\ & + \sum_{k=-[T/2]}^{[T/2]} (\Gamma(\omega_k)Im(\Xi_{Tx}(\omega_k)) - \\ & \sqrt{1+\lambda\Gamma(\omega_k)}[Im(\hat{\Gamma}_x(\omega_k)\Xi_{Tx}(\omega_k)) \\ & + \sum_{n=1}^{m} Im(\hat{\Gamma}_{Z_n}(\omega_k)\Xi_{TZ_n}(\omega_k))])^2 \end{aligned} \quad (34)$$

Wildi (2011) identifies two problems with (34):

- a nuisance term $\Gamma(\omega_k)Im(\Xi_{Tx}(\omega_k))$ appears in the imaginary part in (34);
- requiring a smaller imaginary part of the aggregate filter,

$$Im\left(\hat{\Gamma}_x(\omega_k)\Xi_{Tx}(\omega_k) + \sum_{n=1}^{m}\hat{\Gamma}_{Z_n}(\omega_k)\Xi_{TZ_n}(\omega_k)\right)$$

by augmenting λ would not necessarily lead to the expected improvement because the target signal $\Gamma(\omega_k)\Xi_{Tx}(\omega_k)$ is a complex number with a non-vanishing imaginary part, too.

Both problems could be avoided in (18) - (19) by isolating $\Xi_{Tx}(\omega_k)$ outside of the filter expression. Wildi (2011) notes that one does not need to `isolate' the whole DFT: its argument would be sufficient. So, the modified expression is

$$\sum_{k=-[T/2]}^{[T/2]} \left| \begin{matrix} \Gamma(\omega_k)|\Xi_{Tx}(\omega_k)| - Re(expr) \\ -i\sqrt{1+\lambda\Gamma(\omega_k)}Im(expr) \end{matrix} \right|^2 * \left|\exp(i\arg(\Xi_{Tx}(\omega_k)))\right|^2, \quad (35)$$

where

$$\begin{aligned} expr &= \hat{\Gamma}_x(\omega_k)|\Xi_{Tx}(\omega_k)| \\ &+ \sum_{n=1}^{m}\hat{\Gamma}_{Z_n}(\omega_k)\Xi_{TZ_n}(\omega_k)\exp(-i\arg(\Xi_{Tx}(\omega_k))). \end{aligned} \quad (36)$$

In (35), only $\exp(i\arg(\Xi_{Tx}(\omega_k)))$ is isolated. Since $|\exp(i\arg(\Xi_{Tx}(\omega_k)))|^2 = 1$, (35) becomes

$$\sum_{k=-[T/2]}^{[T/2]} \left| \begin{matrix} \Gamma(\omega_k)|\Xi_{Tx}(\omega_k)| - Re(expr) \\ -i\sqrt{1+\lambda\Gamma(\omega_k)}Im(expr) \end{matrix} \right|^2. \quad (37)$$

In (37), the nuisance term has vanished in the imaginary part; imposing a smaller imaginary part of the aggregate multivariate filter by augmenting λ would meet the target signal $\Gamma(\omega_k)|\Xi_{Tx}(\omega_k)|$ which now is real; and this expression is stable numerically.

Denote

$$\tilde{\Xi}_{Tx}(\omega_k) = |\Xi_{Tx}(\omega_k)|$$
$$\tilde{\Xi}_{TZ_n}(\omega_k) = \Xi_{TZ_n}(\omega_k)\exp(-i\arg(\Xi_{Tx}(\omega_k))). \quad (38)$$

Differentiate (37) w.r.t. b_j^m (the j-th MA coefficient of the filter applied to Z_{mt}) and equate to zero:

$$\sum_{k=-[T/2]}^{[T/2]}\begin{pmatrix}\Gamma(\omega_k)\tilde{\Xi}_{Tx}(\omega_k) - Re\left(\hat{\Gamma}_x(\omega_k)\tilde{\Xi}_{Tx}(\omega_k)\right)\\ -\sum_{n=1}^{m}Re\left(\hat{\Gamma}_{Z_n}(\omega_k)\tilde{\Xi}_{TZ_n}(\omega_k)\right)\end{pmatrix}*$$
$$(-1)Re\left(\exp(ij\omega_k)\tilde{\Xi}_{TZ_n}(\omega_k)\right)$$
$$-\sum_{k=-[T/2]}^{[T/2]}\left(\sqrt{1+\lambda\Gamma(\omega_k)}\begin{bmatrix}Im\left(\hat{\Gamma}_x(\omega_k)\tilde{\Xi}_{Tx}(\omega_k)\right)\\ +\sum_{n=1}^{m}Im\left(\hat{\Gamma}_{Z_n}(\omega_k)\tilde{\Xi}_{TZ_n}(\omega_k)\right)\end{bmatrix}\right)*$$
$$(-1)Im\left(\exp(ij\omega_k)\tilde{\Xi}_{TZ_n}(\omega_k)\right) = 0, \quad (39)$$

where $\tilde{\Xi}_{TZ_m}(\omega_k) = \tilde{\Xi}_{Tx}$ if $m = 0$. Rearranging (39),

$$\sum_{k=-[T/2]}^{[T/2]}\Gamma(\omega_k)\tilde{\Xi}_{Tx}(\omega_k)Re\left(\exp(ij\omega_k)\tilde{\Xi}_{TZ_m}(\omega_k)\right)$$
$$= \sum_{k=-[T/2]}^{[T/2]}\left(Re\left(\hat{\Gamma}_x(\omega_k)\tilde{\Xi}_{Tx}(\omega_k)\right) + \sum_{n=1}^{m}Re\left(\hat{\Gamma}_{Z_n}(\omega_k)\tilde{\Xi}_{TZ_n}(\omega_k)\right)\right)$$
$$*Re\left(\exp(ij\omega_k)\tilde{\Xi}_{TZ_m}(\omega_k)\right)$$
$$+\sum_{k=-[T/2]}^{[T/2]}(1+\lambda\Gamma(\omega_k))\begin{pmatrix}Im\left(\hat{\Gamma}_x(\omega_k)\tilde{\Xi}_{Tx}(\omega_k)\right)\\ +\sum_{n=1}^{m}Im\left(\hat{\Gamma}_{Z_n}(\omega_k)\tilde{\Xi}_{TZ_n}(\omega_k)\right)\end{pmatrix} \quad (40)$$
$$*Im\left(\exp(ij\omega_k)\tilde{\Xi}_{TZ_m}(\omega_k)\right).$$

(40) reduces to the classical univariate mean square DFA criterion when $m = \lambda = 0$; the term in l.h.s. of (40) becomes

$$\Gamma(\omega_k)\tilde{\Xi}_{Tx}(\omega_k)Re\left(\exp(ij\omega_k)\tilde{\Xi}_{TZ_m}(\omega_k)\right)$$
$$= \Gamma(\omega_k)\cos(-j\omega_k)I_{Tx}(\omega_k), \quad (41)$$

which corresponds to (33). The r.h.s. simplifies to

$$Re\left(\hat{\Gamma}_x(\omega_k)\tilde{\Xi}_{Tx}(\omega_k)\right)Re\left(\exp(ij\omega_k)\tilde{\Xi}_{Tx}(\omega_k)\right)$$
$$+Im\left(\hat{\Gamma}_x(\omega_k)\tilde{\Xi}_{Tx}(\omega_k)\right)Im\left(\exp(ij\omega_k)\tilde{\Xi}_{Tx}(\omega_k)\right)$$

$$= Re\left(\hat{\Gamma}_x(\omega_k)\tilde{\Xi}_{Tx}(\omega_k)\right)Re\left(\overline{\exp(ij\omega_k)\tilde{\Xi}_{Tx}(\omega_k)}\right)$$
$$-Im\left(\hat{\Gamma}_x(\omega_k)\tilde{\Xi}_{Tx}(\omega_k)\right)Im\left(\overline{\exp(ij\omega_k)\tilde{\Xi}_{Tx}(\omega_k)}\right)$$
$$= Re\left(\hat{\Gamma}_x(\omega_k)\overline{\exp(ij\omega_k)}\right)I_{Tx}(\omega_k), \quad (42)$$

which corresponds to (32). The r.h.s. of (40) attributes the following weight to the filter coefficient b_l^u :

$$Re\left(\exp(il\omega_k)\Xi_{TZ_u}(\omega_k)\right)Re\left(\exp(ij\omega_k)\Xi_{TZ_m}(\omega_k)\right)$$
$$+(1+\lambda\Gamma(\omega_k))Im\left(\exp(il\omega_k)\Xi_{TZ_u}(\omega_k)\right) \quad (43)$$
$$*Im\left(\exp(ij\omega_k)\Xi_{TZ_m}(\omega_k)\right),$$

where $\Xi_{TZ_u}(\omega_k) = \Xi_{Tx}(\omega_k)$ for $u = 0$. This generalized C matrix reduces to (32) in the univariate case.

C. Constraints

The filter is subject to two potential constraints. The first order restriction

$$b_1^n + b_2^n + \ldots + b_L^n = w^n \quad (44)$$

imposes amplitude constraint in frequency zero according to $\hat{\Gamma}_{Z_n}(0) = w^n$ ($\hat{\Gamma}_{Z_m} = \hat{\Gamma}_x$ if $m = 0$).

The second order restriction imposes vanishing time shift in frequency zero - the derivative of the transfer function in frequency zero must vanish:

$$\left.\frac{\partial}{\partial\omega}\right|_{\omega=0}\sum_{j=1}^{L}b_j^n\exp(i(j-1)\omega) = 0, \quad (45)$$

which results in the following coefficient constraint:

$$b_2^n + 2b_3^n + 3b_4^n + \ldots + (L-1)b_L^n = 0. \quad (46)$$

Imposing both constraints simultaneously leads to

$$b_{L-1}^n = -(L-1)b_1^n - (L-2)b_2^n - \ldots - 2b_{L-2}^n + (L-1)w^n$$
$$b_L^n = (L-2)b_1^n + (L-3)b_2^n + (L-4)b_3^n + \ldots + b_{L-2}^n$$
$$-(L-2)w^n$$

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Ginters Bušs. Tiešās filtrēšanas pielietojums: jauni indikatori Latvijas ekonomikai
Šajā rakstā es apskatu tiešās filtrēšanas pieeju, kuras būtība ir sadalīt vidējo kvadrātisko kļūdu divās daļās – amplitūdas kļūdā un fāzes kļūdā. Tādējādi, tiešās filtrēšanas pieeja ļauj izveidot indikatorus, kas pieļauj lielāku amplitūdas kļūdu, taču samazina fāzes kļūdu – filtra rezultātam ir mazāka laika nobīde nekā citām alternatīvām pieejām. Tiešās filtrēšanas pieejas autors ir Marks Vildi. Vildi ir izstrādājis filtrēšanas metodi ar statistiski precīzu amplitūdas un fāzes kļūdas nošķirtību, taču optimizēšanas metode ir nelineāra parametros, līdz ar to, to ir apgrūtinoši pielietot praksē. Tādējādi, Vildi ir izstrādājis optimālās filtrēšanas analītisku aproksimāciju, kuru viegli ir pielietot praksē tās ātruma dēļ – filtrēt ir tikpat ātri kā aprēķināt regresiju ar mazāko kvadrātu metodi. Šī analītiskā aproksimācija, konkrēti – aproksimācija, kas vērsta nevis uz fāzes kļūdas minimizēšanu, bet gan uz filtra imaginārās daļas minimizēšanu (kas padara amplitūdas un fāzes kļūdas nošķirtību nepilnīgu/neoptimālu), tiek izmantota šajā rakstā. Kā pielietojumu es ilustrēju trīs dažādus reālā laika Latvijas ekonomikas indikatorus:
i) reālā laika indikators, kas novērtē pagrieziena punktus Latvijas rūpniecības produkcijā;
ii) reālā laika indikators, kas novērtē Latvijas ikmēneša IKP pagrieziena punktus;
iii)ikmēneša reālā laika indikators, kas novērtē Latvijas gads-pret-gadu IKP tendences.
Šie indikatori izmanto tikai nerevidētus (skaitā:4-5) ievaddatus un tādēļ tie ir viegli uzturami un atkārtojami.

Гинтер Буш. Приложение прямой фильтрации: новые показатели для латвийской экономики
В настоящей статье я рассматриваю подход прямой фильтрации, суть которого заключается в разделении среднеквадратичной ошибки на две части - ошибка амплитуды и ошибка фазы. Таким образом, подход прямой фильтрации позволяет создавать индикаторы, которые допускают более высокую ошибку амплитуды, но снижает ошибку фазы - результат фильтра отстает меньше по сравнению с другими альтернативными подходами. Автором подхода прямой фильтрации является Марк Вилди. Вилди разработал метод фильтрации с четким исключением ошибки амплитуды и фазы, но этот метод является нелинейной оптимизацией параметров, и, следовательно, её трудно применить на практике. Поэтому, Вилди разработал аналитическую аппроксимацию оптимального фильтра, которую легко применить на практике в по причине её скорости - фильтр такой же быстрый, как и метод наименьших квадратов. Эта аналитическая аппроксимация, которая минимизирует не ошибку фазы, а ошибку имагирной части фильтра (поэтому разделение ошибки амплитуды и ошибки фазы уже неявляется неполным/неоптимальным), использована в этой статье. Для иллюстрации использования подхода имеются три различных показателя латвийской экономики в реальном времени:
I) индикатор для оценки поворотных моментов латвийского промышленного производства в режиме реального времени;
II) ежемесячный индикатор для оценки поворотных точек Латвийского ВВП в режиме реального времени;
III) ежемесячный индикатор для оценки латвийского годового тренда ВВП в режиме реального времени.
Эти показатели используют только неаудированные (в том числе, 4-5) данные, и поэтому их легко поддерживать и воспроизводить.