Remarks to the Solution of MHD Problem on an Inflow of Conducting Fluid into a Plane Channel through the Channel's Lateral Side

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***Abstract* - An exact analytical solution of MHD problem on an inflow of conducting fluid into a plane channel through the split of finite width in channel's lateral side in a strong magnetic field is proposed. The problem is solved in Stokes and inductionless approximation by using the Fourier transform. Previously this problem was solved with an incorrect assumption. Some new numerical results for the velocity field are also presented in this paper.**

***Keywords* - MHD problem, strong magnetic field, plane channel, Fourier transform, Stokes approximation.**

I. Introduction

In the article [1] the analytical solution is presented for MHD problem on an inflow of a conducting fluid into a plane through the pane split of finite width in channel's lateral side at presence of uniform external magnetic field $\vec{B}^e$. The problem is solved both for a longitudinal and transverse magnetic fields. The solution of the problem was obtained in Stokes and inductionless approximation and it has the form of convergent improper integrals. In order to obtain the solution, the Fourier transform was used in [1] together with the assumption that the velocity and pressure gradient are equal to zero in channel in sufficient distance from the entrance region. But this assumption is not correct. In a plane channel the Poiseuille flow appears far from the entrance region in the case of longitudinal magnetic field, but the Hartmann flow appears in the case of transverse magnetic field. In the present paper the correct analytical solution of the problem is presented, besides, it is shown that the final results obtained in [1] are correct. Similarly to [1], in order to simplify the solving of the problem, the problem is divided into odd and even cases with respect to y. In paper [2] the distribution of velocity in the channel was obtained numerically for the odd problem in the case of longitudinal magnetic field. In the present article the numerical analysis of the velocity field in the channel is presented for the even and general problems both in the case of longitudinal and transverse magnetic fields.

II. Problem Statement. The Case of Sloping Magnetic Field

The plane channel with conducting fluid is located in region D: $-h < \tilde{y} < h$, $-\infty < \tilde{x} < \infty$, $-\infty < \tilde{z} < \infty$. On the channel's lateral side $\tilde{y} = -h$ there is a split in the region $-\tilde{L} < \tilde{x} < \tilde{L}$, $\tilde{y}$ =-h, $-\infty < \tilde{z} < \infty$. Conducting fluid flows into the channel through this split with constant velocity $V_0\vec{e}_y$. A strong uniform external magnetic field $\vec{B}^e$ is applied under the angle α to the split, i.e.

$$\vec{B}^e = B_0 \cos\alpha \cdot \vec{e}_x + B_0 \sin\alpha \cdot \vec{e}_y . \qquad (1)$$

The geometry of the flow is shown in Fig.1.

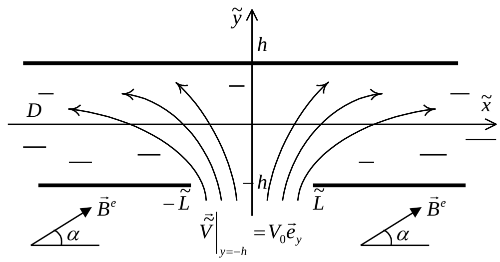

Fig.1. The geometry of the flow. The case of sloping magnetic field

The case of nonconducting walls $\tilde{y} = \pm h$ and perfectly conducting lateral sidewalls $\tilde{z} = \pm\infty$ is considered. In this case the electrical field can be assumed to be zero. This is not an essential assumption for two-dimensional flows. It is shown in [3] that in the case of stationary external magnetic field $\vec{B}^e$ located in the plane of flow, the intensity of electrical field is of constant magnitude in all the domain of the flow and the vector of this intensity is perpendicular to the plane of flow. Thus, in our problem E_x and E_y do not affect the motion of fluid and we can assume that $E_x = E_y = 0$ and $E_z = const$.

One more assumption is used below. We suppose that induced streams do not flow through the split $\tilde{y} = -h$, $-\tilde{L} < \tilde{x} < \tilde{L}$ in the region $-\infty < \tilde{y} < -h$.

We introduce dimensionless variables using h (half-width of the channel) as the scale of length, V_0 (velocity of fluid in the split in the entrance region) as the scale of velocity and B_0, V_0B_0, $\rho\nu V_0/h$ as the scales of magnetic field, electrical field and pressure, respectively, where σ, ρ, ν are, respectively, conductivity, density and viscosity of the fluid.

MHD equations in Stokes and inductionless approximation have the form (see [4]):

$$-\nabla P + \Delta\vec{V} + Ha^2(\vec{E} + \vec{V}\times\vec{e}_B)\times\vec{e}_B = 0 , \qquad (2)$$

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} = 0, \qquad (3)$$

where $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$,

$\vec{V} = V_x(x,y)\vec{e}_x + V_y(x,y)\vec{e}_y$ is the velocity of the fluid,

$P(x,y)$ is the pressure,

$\vec{e}_B = \cos\alpha \cdot \vec{e}_x + \cos\beta \cdot \vec{e}_y$ is the unit vector of external magnetic field,

$\vec{E} = E_z \cdot \vec{e}_z$ is the intensity of electrical field,

$Ha = B_0 h \sqrt{\frac{\sigma}{\rho\nu}}$ is the Hartmann number.

For determination of the constant $E_z = const$ we use the fact that this flow is the Hartmann flow in a plane channel in external magnetic field $\vec{B}^e = B_0 \sin\alpha \cdot \vec{e}_y$ as $x \to \infty$. Therefore, in the case of nonconductive channel's walls, we have

$$\vec{E} = -\frac{\sin\alpha}{2} \cdot sign(x)\vec{e}_z. \tag{4}$$

Projecting Eq.(2) onto the x and y axes, we obtain the problem in the form:

$$-\frac{\partial P}{\partial x} + \Delta V_x - Ha^2 \sin\alpha \cdot (E_z + V_x \sin\alpha - V_y \cos\alpha) = 0, \tag{5}$$

$$-\frac{\partial P}{\partial y} + \Delta V_y + Ha^2 \cos\alpha \cdot (E_z + V_x \sin\alpha - V_y \cos\alpha) = 0, \tag{6}$$

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} = 0. \tag{7}$$

Eqs. (5), (6) can be also written in the form:

$$-\frac{\partial P_m}{\partial x} + \Delta V_x - Ha^2 \sin\alpha \cdot (V_x \sin\alpha - V_y \cos\alpha) = 0, \tag{8}$$

$$-\frac{\partial P_m}{\partial y} + \Delta V_y + Ha^2 \cos\alpha \cdot (V_x \sin\alpha - V_y \cos\alpha) = 0, \tag{9}$$

where

$$P_m = P - \left(\frac{Ha^2 \sin^2\alpha}{2} x - \frac{Ha^2 \sin\alpha \cdot \cos\alpha}{2} y \right) \cdot \mathrm{sign}(x). \tag{10}$$

The boundary conditions are:

$$y = -1: \quad V_x = 0, \qquad V_y = \begin{cases} 0, & x \notin (-L, L) \\ 1, & x \in (-L, L) \end{cases}; \tag{11}$$

$$y = 1: \quad V_x = V_y = 0, \tag{12}$$

$$x \to \pm\infty: \; V_x \to V_\infty(y) \cdot \mathrm{sign}(x),$$
$$\frac{\partial P}{\partial x} \to \frac{\partial P_\infty}{\partial x} sign(x) = A \cdot sign(x) = const, \tag{13}$$

where $\vec{V}_\infty(y) = V_\infty(y) \cdot \vec{e}_x$ is the velocity of the fluid in the channel sufficiently far away from the entrance region. Depending on the magnetic field, functions $\vec{V}_\infty(y)$ and $\frac{\partial P_\infty}{\partial x}$ at $x \to \infty$ satisfy one of the below mentioned equations:

1) In the case of a longitudinal external magnetic field $\vec{B}^e = B_0 \cdot \vec{e}_x$ $(\alpha = 0)$ the Poiseuille flow takes place at $x \to \pm\infty$ and the velocity $\vec{V}_\infty(y)$ satisfies equation

$$\frac{\partial P_\infty}{\partial x} = \frac{d^2 V_\infty(y)}{dy^2} = const \equiv A \tag{15}$$

2) In the case of transverse magnetic field $\vec{B}^e = B_0 \cdot \vec{e}_y$ ($\alpha = \pi/2$) the Hartmann flow takes place at $x \to \pm\infty$ and the velocity $\vec{V}_\infty(y)$ satisfies the equation

$$\frac{\partial P_\infty}{\partial x} = \frac{d^2 V_\infty(y)}{dy^2} - Ha^2 V_\infty(y) = const \equiv A \tag{16}$$

3) In the case of sloping magnetic field $\vec{B}^e = B_0 \cos\alpha \cdot \vec{e}_x + B_0 \sin\alpha \cdot \vec{e}_y$ the Hartman flow with Ha·sinα instead of Ha takes place at $x \to \infty$:

$$\frac{\partial P_\infty}{\partial x} = \frac{d^2 V_\infty(y)}{dy^2} - Ha^2 \sin^2\alpha \cdot V_\infty(y) = const \equiv A \tag{17}$$

The boundary conditions for equations (15)-(17) are:

$$y = \pm 1: \quad V_\infty(y) = 0.$$

For the solution of the problem we use complex Fourier transform with respect to x:

$$\hat{g}(\lambda, y) = F[g(x,y)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} g(x,y) e^{-i\lambda x} dx. \tag{18}$$

We introduce new functions for the velocity and pressure gradient

$$\vec{V}^{new} = \vec{V} - f(x) \cdot \vec{V}_\infty(y) \quad \text{and} \quad \frac{\partial P^{new}}{\partial x} = \frac{\partial P_m}{\partial x} - f(x)A, \tag{19}$$

where $f(x)$ is the real argument function that satisfies two conditions:

a) $f(x) \to 1$ at $x \to \infty$ and $f(x) \to -1$ at $x \to -\infty$,

b) the Fourier transform exist for $f(x)$, i.e. $f(x)$ is piecewise continuous on any finite interval and absolutely integrable for all t.

In this case

$$V_x^{new} \to 0 \quad \text{and} \quad \frac{\partial P^{new}}{\partial x} \to 0 \quad \text{as} \quad x \to \pm\infty. \tag{20}$$

As a result, the velocity of the fluid in the channel can be written in the form:

$$\vec{V} = \left(V_x^{new}(x,y) + f(x) \cdot V_\infty(y) \right) \cdot \vec{e}_x + V_y(x,y) \cdot \vec{e}_y.$$

Thus, Eq.(8) becomes:

$$-\frac{\partial P_m^{\ new}}{\partial x} - f(x)A + \Delta V_x^{\ new} + f''(x)\cdot V_\infty(y) -$$
$$-Ha^2 \sin\alpha \cdot V^* + f(x)\left(\frac{\partial^2 V_\infty(y)}{\partial y^2} - Ha^2\sin^2\alpha\cdot V_\infty(y)\right) = 0$$

where $V^* = (V_x \sin\alpha - V_y \cos\alpha)$.

Since the velocity $V_\infty(y)$ in this case satisfies Eq.(17) for the Hartmann flow, the last equation can be written in the form:

$$-\frac{\partial P_m^{\ new}}{\partial x} + \Delta V_x^{\ new} + f''(x)\cdot V_\infty(y) - Ha^2\sin\alpha\cdot V^* = 0$$

Therefore, problem (7)-(9) has the form:

$$-\frac{\partial P_m^{\ new}}{\partial x} + \Delta V_x^{\ new} - Ha^2\sin\alpha\cdot V^* + f''(x)V_\infty(y) = 0 \qquad (21)$$

$$-\frac{\partial P_m^{\ new}}{\partial y} + \Delta V_y + Ha^2\cos\alpha\cdot V^* + \frac{Ha^2\sin 2\alpha}{2\pi} f(x)V_\infty(y) = 0 \qquad (22)$$

$$\frac{\partial V_x^{\ new}}{\partial x} + \frac{\partial V_y}{\partial y} + f'(x)V_\infty(y) = 0. \qquad (23)$$

Boundary conditions are:

$$y=-1: \quad V_x^{\ new} = 0, \quad V_y = \begin{cases} 0, & x \notin (-L, L) \\ 1, & x \in (-L, L) \end{cases}; \qquad (24)$$

$$y = 1: \quad V_x^{\ new} = V_y = 0. \qquad (25)$$

$$x \to \pm\infty: \quad V_x^{\ new} \to 0, \quad \frac{\partial P^{new}}{\partial x} \to 0. \qquad (26)$$

In contrast to the solution in the article [1], it is already correct to use Fourier transform for the problem with boundary conditions (26). We apply complex Fourier transform with respect to x to problem (21)-(23) and to boundary conditions (24)-(26) and as a result we obtain the system of ordinary differential equations for Fourier transforms $\hat{V}_x(\lambda, y) = F[V_x^{\ new}(x,y)]$, $\hat{V}_y(\lambda, y) = F[V_y(x,y)]$, $\hat{P}(\lambda, y) = F[P_m^{\ new}(x,y)]$ in the form:

$$-i\lambda\hat{P} + \mathbf{L}\hat{V}_x - Ha^2\sin\alpha\cdot\hat{V}^* + \hat{f}_3(\lambda)V_\infty(y) = 0 \qquad (27)$$

$$-\frac{d\hat{P}}{dy} + \mathbf{L}\hat{V}_y + Ha^2\cos\alpha\cdot\hat{V}^* + \frac{Ha^2\sin 2\alpha}{2}\hat{f}_1(\lambda)V_\infty(y) = 0 \qquad (28)$$

$$i\lambda\hat{V}_x + \frac{d\hat{V}_y}{dy} + \hat{f}_2(\lambda)V_\infty(y) = 0, \qquad (29)$$

where $\hat{V}^* = (\hat{V}_x\sin\alpha - \hat{V}_y\cos\alpha)$, $\quad \mathbf{L} = -\lambda^2 + \frac{d^2}{dy^2}$, (30)

$$\hat{f}_1(\lambda) = F[f(x)] \qquad (31)$$

$$\hat{f}_2(\lambda) = F[f'(x)] = i\lambda\,\hat{f}_1(\lambda) \qquad (32)$$

$$\hat{f}_3(\lambda) = F[f''(x)] = i\lambda\cdot\hat{f}_2(x) = -\lambda^2\cdot\hat{f}_1(x). \qquad (33)$$

The boundary conditions in the transformed space have the form:

$$y=-1: \quad \hat{V}_x = 0, \quad \hat{V}_y = \sqrt{\frac{2}{\pi}}\frac{\sin(\lambda L)}{\lambda}; \qquad (34)$$

$$y = 1: \quad \hat{V}_x = 0, \quad \hat{V}_y = 0. \qquad (35)$$

Eliminating $\hat{V}_x$ and $\hat{P}$ from Eqs.(27) and (28), we obtain the 4th order differential equation for $\hat{V}_y$:

$$\hat{V}_y^{(4)} - a_1\hat{V}_y'' - a_2\hat{V}_y' + a_3\hat{V}_y - Z(\lambda, y) = 0 \qquad (36)$$

where

$$Z(\lambda,y) = V_\infty'(y)\left(\lambda^2\hat{f}_2(\lambda) + i\lambda\hat{f}_3(\lambda)\right) - V_\infty'(y)Ha^2\sin^2\alpha\hat{f}_2(\lambda) +$$
$$+V_\infty'''(y)\frac{Ha^2}{2}\sin 2\lambda\left(\lambda^2\hat{f}_1(\lambda) + i\lambda\,\hat{f}_2(\lambda)\right) - V_\infty'''\cdot\hat{f}_2(\lambda) \qquad (37)$$

and

$$a_1 = 2\lambda^2 + Ha^2\sin^2\alpha, \quad a_2 = i\lambda Ha^2\sin 2\alpha,$$
$$a_3 = \lambda^4 + \lambda^2 Ha^2\cos^2\alpha$$

Note, that Eq.(36) for $\hat{V}_y$ differs from the equation obtained in [1] only by term $Z(\lambda, y)$. Taking into account formulae (31)-(33) and the fact that $V_\infty(y)$ satisfies Eq.(17), we obtain

$$Z(\lambda, y) = 0.$$

Thus, the differential equation for $\hat{V}_y$ has the form:

$$\hat{V}_y^{(4)} - a_1\hat{V}_y'' - a_2\hat{V}_y' + a_3\hat{V}_y = 0. \qquad (38)$$

This differential equation completely coincides with differential equation for obtained in [1] therefore the solution of this equation is the same as in [1]:

$$\hat{V}_y(\lambda,y) = C_1e^{iBy}\sinh((+)y) + C_2e^{-iBy}\sinh((-)y) +$$
$$+C_3e^{iBy}\cosh((+)y) + C_4e^{-iBy}\cosh((-)y) \qquad (39)$$

where C1,...,C4 are arbitrary constants,

$$A = \mathrm{Re}\sqrt{D_1} = \lambda^2 + \mu^2\sin^2\lambda, \quad B = \mathrm{Im}\sqrt{D_1} = 2\lambda\mu\cos\lambda$$
$$(+) = \mu\sin\alpha + A, \quad (-) = \mu\sin\alpha - A.$$
$$D_{1,2} = \mu^2\sin^2\alpha + \lambda(\lambda \pm 2i\mu\cos\alpha), \quad Ha = 2\mu,$$
$$k_{1,2} = \mu\sin\alpha \pm \sqrt{D_1}, \quad k_{3,4} = -\mu\sin\alpha \pm \sqrt{D_2}.$$

In order to determine constants C1-C4, we have to use boundary conditions (34) and (35) and Eq. (29) which gives

$$\frac{d\hat{V}_y}{dy} = 0 \quad \text{at} \quad y = \pm 1 . \tag{40}$$

After that we can determine $\hat{V}_x$ from Eq.(29) and $-i\lambda\hat{P}$ from Eq.(27), $\partial\hat{P}/\partial y$ from (28) and using the inverse Fourier transform, we obtain solution for $\partial P_m^{\ new}/\partial x$, $\partial P_m^{\ new}/\partial y$, $V_x^{\ new}$, V_y.

Let us consider in details two special cases, i.e. the case of longitudinal magnetic field ($\alpha = 0$) and the case of transverse magnetic field ($\alpha = \pi/2$).

In order to simplify the problem and reduce the number of constants in Eq.(39) we divide the problems into two odd and even problems with respect to y, as it was done in [1]. For this purpose we consider the plane channel with two splits on the its lateral sides $y = \pm h$ in region $-\tilde{L} < \tilde{x} < \tilde{L}$ and solve two problems:

1) The odd problem (fig.2): the fluid with velocities $\mp\frac{1}{2}V_0\vec{e}_y$, flows into the channel through the both splits on $y = \pm h$.
Then the dimensionless boundary conditions are:

$$y = \pm 1: V_x = 0, \qquad V_y = \begin{cases} 0, & x \notin (-L, L) \\ \mp 1/2, & x \in (-L, L) \end{cases} \tag{41}$$

$$x \to \pm\infty: V_x \to V_\infty(y)\cdot \operatorname{sign}(x), \ \frac{\partial P}{\partial x} \to \frac{\partial P_\infty}{\partial x} = A = const . \tag{42}$$

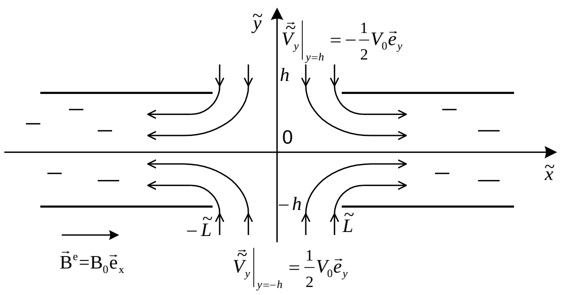

Fig.2. The geometry of the flow for the odd problem with respect to y

2) The even problem: the fluid with velocity $\frac{1}{2}V_0\vec{e}_y$, flows into the channel through the split on $y = -h$ and flows out with the same velocity through the split at $y = h$.
Then the dimensionless boundary conditions are:

$$y = \pm 1: V_x = 0, \qquad V_y = \begin{cases} 0, & x \notin (-L, L) \\ 1/2, & x \in (-L, L) \end{cases} \tag{43}$$

$$\text{and } x \to \pm\infty: \quad V_x \to 0, \ \frac{\partial P}{\partial x} \to 0 . \tag{44}$$

The solution of the general problem is equal to the sum of solutions for odd and even problems with respect to y.

In this article we consider the analytical solution only for the odd problem with respect to y due to the velocity and pressure gradient are equal to zero at $x \to \pm\infty$ for even problem and the solution in [1] for this case was obtained in correct way. We will present only new numerical results for the even problem.

III. The Case of Longitudinal Magnetic Field

In this case $\vec{B}^e = B_0\vec{e}_x$ $(\alpha = 0)$. The system of dimensionless Eqs.(7)-(9) for the case of longitudinal magnetic field can be written in the form:

$$-\frac{\partial P}{\partial x} + \Delta V_x = 0 , \tag{45}$$

$$-\frac{\partial P}{\partial y} + \Delta V_y - Ha^2 V_y = 0 , \tag{46}$$

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} = 0 . \tag{47}$$

Note that in this case $P_m = P$ since $E_z = 0$.

1) Solution of the odd problem in with respect to y

The geometry of the flow is shown in Fig.2. As in the case of slopping magnetic field, we introduce new functions for the velocity and pressure gradient (19) to solve the problem in correct way. In the case of longitudinal magnetic field, $V_\infty(y)$ is the velocity of Poiseuille flow that satisfies Eq.(15).

The system of equations (21)-(23) can be written in the form:

$$-\frac{\partial P^{new}}{\partial x} + \Delta V_x^{\ new} + f''(x)\cdot V_\infty(y) = 0 , \tag{48}$$

$$-\frac{\partial P^{new}}{\partial y} + \Delta V_y - Ha^2 V_y = 0 , \tag{49}$$

$$\frac{\partial V_x^{\ new}}{\partial x} + \frac{\partial V_y}{\partial y} + f'(x)\cdot V_\infty(y) = 0 . \tag{50}$$

Boundary conditions are:

$$y = -1: V^{new}{}_x = 0, \qquad V_y = \begin{cases} 0, & x \notin (-L, L) \\ 1/2, & x \in (-L, L) \end{cases} \tag{51}$$

$$y = 1: \ V^{new}{}_x = 0, \qquad V_y = \begin{cases} 0, & x \notin (-L, L) \\ -1/2, & x \in (-L, L) \end{cases} \tag{52}$$

$$x \to \pm\infty: \quad V_x^{\ new} \to 0 , \qquad \frac{\partial P^{new}}{\partial x} \to 0 . \tag{53}$$

The system of ordinary differential equations for the Fourier transforms $\hat{V}_x(\lambda, y) = F[V_x^{\ new}(x, y)]$, $\hat{V}_y(\lambda, y) = F[V_y(x, y)]$, $\hat{P}(\lambda, y) = F[P^{new}(x, y)]$ can be obtained from (27)-(29) by substituting $\alpha = 0$ and have the form:

$$-i\lambda\hat{P}+\mathbf{L}\hat{V}_x+\hat{f}_3(\lambda)V_\infty(y)=0\,, \tag{54}$$

$$-\frac{d\hat{P}}{dy}+\mathbf{L}\hat{V}_y-Ha^2\hat{V}_y=0\,, \tag{55}$$

$$i\lambda\hat{V}_x+\frac{d\hat{V}_y}{dy}+\hat{f}_2(\lambda)V_\infty(y)=0\,, \tag{56}$$

where $\mathbf{L}=-\lambda^2+d^2/dy^2$ and the functions $\hat{f}_2(\lambda)$, $\hat{f}_3(\lambda)$ are defined by formulae (32) and (33).

Applying the Fourier transform to boundary conditions (51), (52) we obtain:

$$y=\pm 1:\ \ \hat{V}_y=\mp\frac{1}{2}\sqrt{\frac{2}{\pi}}\frac{\sin(\lambda L)}{\lambda},\ \ \hat{V}_x=0\,. \tag{57}$$

Eliminating $\hat{V}_x$ and $\hat{P}$ from Eqs.(54) and (55) we obtain the 4th order differential equation for $\hat{V}_y$ that can be obtained also from Eq.(38) by substituting $\alpha=0$.

$$\hat{V}_y^{(4)}-2\lambda^2\hat{V}_y''+(\lambda^4+\lambda^2Ha^2)\hat{V}_y=0\,. \tag{58}$$

This differential equation completely coincides with differential equation obtained in [1] for the odd problem in the case of longitudinal magnetic field therefore the solution of this equation is the same as in [1], i.e.

$$\hat{V}_y(\lambda,y)=\frac{(k_1\cosh k_1\sinh k_2y-k_2\cosh k_2\sinh k_1y)}{\Delta_1}\cdot\frac{\sin(\lambda L)}{\lambda}\,, \tag{59}$$

where $\Delta_1=k_2\cosh k_2\cdot\sinh k_1-k_1\cosh k_1\cdot\sinh k_2$,

$k_1=\sqrt{\lambda^2+i\cdot Ha\lambda},\quad k_2=\sqrt{\lambda^2-i\cdot Ha\lambda}$.

Determining $\hat{V}_x$ from Eq.(56), we obtain:

$$\hat{V}_x=\frac{i\sqrt{\lambda^4+Ha^2\lambda^2}}{\sqrt{2\pi}}\frac{(\cosh k_1\cosh k_2y-\cosh k_2\cosh k_1y)}{\Delta_1}\frac{\sin(\lambda L)}{\lambda^2}+$$
$$+\frac{i}{\lambda}\hat{f}_2(\lambda)V_\infty(y)\,, \tag{60}$$

Taking into account formulae (32) and (33), the last term of (63) can be written as

$$\frac{i}{\lambda}\hat{f}_2(\lambda)V_\infty(y)=-\hat{f}_1(\lambda)V_\infty(y)\,.$$

Thus,

$$\hat{V}_x=\frac{i\sqrt{\lambda^4+Ha^2\lambda^2}}{\sqrt{2\pi}}\frac{(\cosh k_1\cosh k_2y-\cosh k_2\cosh k_1y)}{\Delta_1}\cdot\frac{\sin(\lambda L)}{\lambda^2}-$$
$$-\hat{f}_1(\lambda)V_\infty(y)\,. \tag{61}$$

Determining $i\lambda\hat{P}$ from Eq.(54) and $\frac{\partial\hat{P}}{\partial y}$ from Eq.(55) we obtain:

$$\frac{\partial\hat{P}}{\partial y}=D\cdot\frac{(i\lambda-Ha)k_2\cosh k_2\sinh k_1y+(i\lambda+Ha)k_1\cosh k_1\sinh k_2y}{-\Delta_1} \tag{62}$$

$$i\lambda\hat{P}=D\cdot\frac{\sqrt{\lambda^4+Ha^2\lambda^2}}{\Delta_1}(\cosh k_1\cosh k_2y+\cosh k_2\cosh k_1y)+$$
$$+\left(\lambda^2\hat{f}_1(\lambda)+\hat{f}_3(\lambda)\right)-\hat{f}_1(\lambda)\cdot V_\infty''(y) \tag{63}$$

where $D=\frac{Ha}{\sqrt{2\pi}}\cdot\frac{\sin\lambda L}{\lambda}$.

In Eq.(62) $\partial\hat{P}/\partial y$ is the same as in [1], therefore the original $\partial P/\partial y$ will be the same.

Let us simplify Eq. (63). Taking into account formulae (31) and (33) it can be written in the form

$$i\lambda\hat{P}=D\cdot\frac{\sqrt{\lambda^2+Ha^2}}{\Delta_1}(\cosh k_1\cosh k_2y+\cosh k_2\cosh k_1y)-$$
$$-\hat{f}_1(\lambda)\cdot V_\infty''(y)\,. \tag{64}$$

Note, that $\hat{V}_x$ and $i\lambda\hat{P}$ differ from result obtained in [1] only by last terms. In order to obtain the solution to problem (45)-(47) we apply the inverse complex Fourier transform

$$g(x,y)=F^{-1}[\hat{g}(\lambda,x)]=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\hat{g}(\lambda,y)e^{\lambda x}dx \tag{65}$$

to functions $\hat{V}_x(\lambda,y)$, $\hat{V}_y(\lambda,y)$, $\frac{\partial\hat{P}(\lambda,y)}{\partial y}$ and $i\lambda\hat{P}$.

Applying the inverse complex Fourier transform we take into account the fact that the functions $\hat{V}_y(\lambda,y)$ and $\partial\hat{P}/\partial y$ are even functions with respect to λ, and the functions $\hat{V}_x(\lambda,y)$ and $i\lambda\hat{P}$ are odd functions with respect to λ. We also use formula (31), i.e. $F^{-1}\left[\hat{f}_1(\lambda)\right]=f(x)$, formulae (19) and the fact that $F^{-1}\left[i\lambda\hat{P}\right]=\partial P/\partial x$.

As a result, we have the solution to problem (45)-(47) for the odd case in the form of convergent improper integrals that coincide with the solution of the problem obtained in [1]:

$$V_x=\frac{1}{\pi}\int_0^\infty B\cdot\frac{(\cosh k_2\cosh k_1y-\cosh k_1\cosh k_2y)\sin\lambda L\sin\lambda x}{\Delta_1\cdot\lambda}d\lambda$$

$$V_y=\frac{1}{\pi}\int_0^\infty\frac{(k_1\cosh k_1\sinh k_2y-k_2\cosh k_2\sinh k_1y)\sin\lambda L\cos\lambda x}{\Delta_1\cdot\lambda}d\lambda$$

$$\frac{\partial P}{\partial x}=\frac{iHa}{\pi}\int_0^\infty B\cdot\frac{(\cosh k_1\cosh k_2y+\cosh k_2\cosh k_1y)\sin\lambda L\sin\lambda x d\lambda}{\Delta_1}$$

where $B=\sqrt{\lambda^2+Ha^2}$,

$\Delta_1=k_2\cosh k_2\cdot\sinh k_1-k_1\cosh k_1\cdot\sinh k_2$

$k_1=\sqrt{\lambda^2+i\cdot Ha\lambda},\quad k_2=\sqrt{\lambda^2-i\cdot Ha\lambda}$.

For the odd problem the profiles of the velocity component V_x was presented in [2] for the Hartmann numbers Ha=10, Ha=20 and Ha=50. It was shown that the velocity component

V_x has the M-shaped profile in the channel's entrance region and the M-shaped profiles become more pronounced as the Hartmann number increases. The qualitative explanation of this phenomenon is also given in [2].

2) Numerical results for the even problem with respect to y

As it was mentioned before, the solution in [1] for this case was obtained in correct way, due to the velocity and pressure gradient are equal to zero at $x \to \pm\infty$. We represent here only the new numerical results for even case. The solution for velocity component obtained in [1] has the form

$$V_x = \frac{1}{\pi}\int_0^\infty B\frac{(\sinh k_1 \sinh k_2 y - \sinh k_2 \sinh k_1 y)}{\Delta_2} \cdot \frac{\sin\lambda L \sin\lambda x}{\lambda} d\lambda \ ,$$

$$V_y = \frac{1}{\pi}\int_0^\infty \frac{(k_2 \sinh k_2 \cosh k_1 y - k_1 \sinh k_1 \cosh k_2 y)\sin\lambda L \cos\lambda x}{\Delta_2 \lambda} d\lambda$$

where $B = \sqrt{\lambda^2 + Ha^2}$

$$\Delta_2 = k_2 \sinh k_2 \cdot \cosh k_1 - k_1 \sinh k_1 \cdot \cosh k_2 \ ,$$

$$k_1 = \sqrt{\lambda^2 + i \cdot Ha\lambda}, \quad k_2 = \sqrt{\lambda^2 - i \cdot Ha\lambda} \ .$$

Fig.3 presents the profiles of the velocity component V_x calculated for the Hartmann numbers Ha=10 and Ha=50 for $-1 \le y \le 0$. Note that in this case the function V_x is an odd function with respect to y.

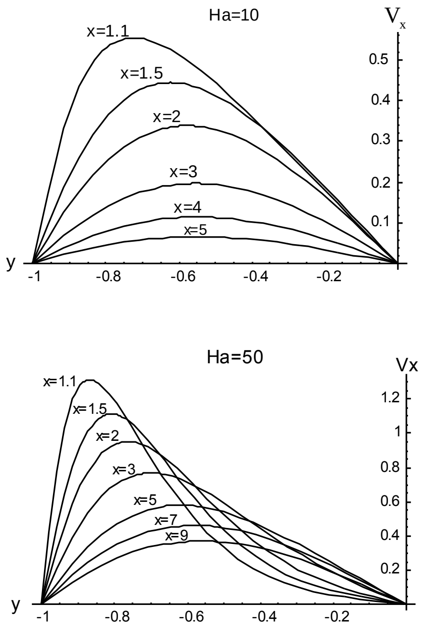

Fig.3. Profiles for the component $V_x(x,y)$ of the velocity $\vec{V}$ for the even problem and $\vec{B}^e = B_0\vec{e}_x$

It can be seen from Fig.3 that $V_x \to 0$ at $x \to \infty$ and the magnitude of the velocity component V_x slower approach zero as the Hartmann number increases. In addition, in the channel's entrance region the component $V_x(x,y)$ of velocity has the M-shaped profile for small x ($1 < x < 5$) and large Hartmann numbers ($Ha = 50$).

3) Numerical results for the general problem in the case of longitudinal magnetic field.

The solutions to general problem (45)-(47), (11)-(14) for $\alpha = 0$ are equal to the sum of solutions for even and odd problems with respect to y. The profiles of the velocity component V_x are shown in Fig.4 for the Hartmann numbers Ha=10 and Ha=50.

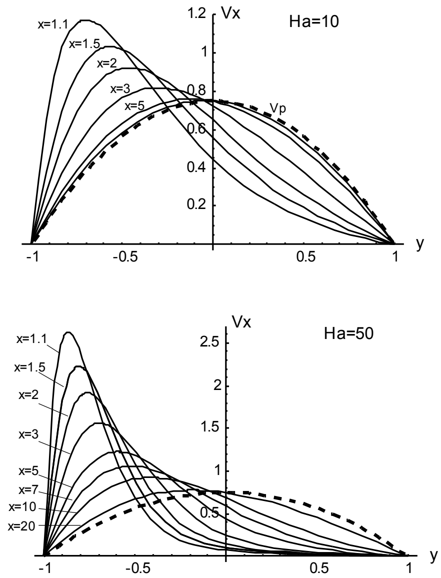

Fig.4. Profiles for the x component of the velocity $\vec{V}$ for $\vec{B}^e = B_0\vec{e}_x$
(V_p is the Poiseuille flow)

One can see that near to the entrance split the flow mostly occurs along the wall with the split (at $y = -1$). As the Hartmann number increases, the layer of the flow is getting narrower and the velocity is increasing in this layer. The Poiseuille flow takes place at a distance from the entrance split. In addition, when the Hartmann number grows (i.e. with the increase of the intensity of the magnetic field) the flow in the channel slowly approaches Poiseuille flow. For instance, for $Ha = 10$ Poiseuille flow takes place already at $x = 6$, for $Ha = 20$ at $x = 12$ and for $Ha = 50$ only at $x > 20$. So L_{init} increases as the Hartmann number grows. L_{init} is the length of the initial part, where the x component of the velocity $\vec{V}(x,y)$ of fluid in channel differs from the velocity of the Poiseuille flow V_P by less than 1%.

IV. THE CASE OF TRANSVERSE MAGNETIC FIELD.

In this case $\vec{B}^e = B_0\vec{e}_y$ $(\alpha = \pi/2)$. The system of dimensionless equations (7)-(9) for the case of transverse magnetic field can be written in the form:

$$-\frac{\partial P_m}{\partial x}+\Delta V_x-Ha^2V_x=0\ , \tag{66}$$

$$-\frac{\partial P_m}{\partial y}+\Delta V_y=0\,, \tag{67}$$

$$\frac{\partial V_x}{\partial x}+\frac{\partial V_y}{\partial y}=0\,. \tag{68}$$

In this case $P_m = P-\frac{Ha^2}{2}x\cdot\operatorname{sign}(x)$ (see formula (14)).

1) Solution of the odd problem in with respect to y

The geometry of the flow is shown in Fig.2, but the external magnetic field is perpendicular to the channel's walls.

As in the case of slopping magnetic field, we introduce new functions for the velocity and pressure gradient (19). In the case of transverse magnetic field, $V_\infty(y)$ is the velocity of Hartmann flow that satisfies Eq.(16).

The system of equations (21)-(23) can be written in the form:

$$-\frac{\partial P^{new}}{\partial x}+\Delta V_x^{new}+f''(x)V_\infty(y)-Ha^2V_x=0\,, \tag{69}$$

$$-\frac{\partial P^{new}}{\partial y}+\Delta V_y=0\,, \tag{70}$$

$$\frac{\partial V_x^{new}}{\partial x}+\frac{\partial V_y}{\partial y}+f'(x)V_\infty(y)=0\,. \tag{71}$$

Boundary conditions are:

$$y=-1:\ V^{new}{}_x=0,\qquad V_y=\begin{cases}0, & x\notin(-L,L)\\ 1/2, & x\in(-L,L)\end{cases} \tag{72}$$

$$y=1:\ V^{new}{}_x=0,\qquad V_y=\begin{cases}0, & x\notin(-L,L)\\ -1/2, & x\in(-L,L)\end{cases} \tag{73}$$

$$x\to\pm\infty:\quad V_x^{new}\to 0\,,\qquad \frac{\partial P^{new}}{\partial x}\to 0\,. \tag{74}$$

The system of ordinary differential equations for the Fourier transforms $\hat V_x(\lambda,y)=F[V_x^{new}(x,y)]$, $\hat V_y(\lambda,y)=F[V_y(x,y)]$, $\hat P(\lambda,y)=F[P^{new}(x,y)]$ can be obtained from (27)-(29) by substituting $\alpha=\pi/2$. As a result, we have:

$$-i\lambda\hat P+\mathbf{L}\hat V_x-Ha^2\hat V_x+\hat f_3(\lambda)V_\infty(y)=0\,, \tag{75}$$

$$-\frac{d\hat P}{dy}+\mathbf{L}\hat V_y=0\,, \tag{76}$$

$$i\lambda\hat V_x+\frac{d\hat V_y}{dy}+\hat f_2(\lambda)V_\infty(y)=0\,, \tag{77}$$

where $\mathbf{L}=-\lambda^2+\frac{d^2}{dy^2}$ and the functions $\hat f_2(\lambda)$, $\hat f_3(\lambda)$ are defined by formulae (32) and (33).

Applying the Fourier transform to the boundary conditions we obtain:

$$y=\pm1:\quad \hat V_y=\mp\frac{1}{2}\sqrt{\frac{2}{\pi}}\frac{\sin(\lambda L)}{\lambda}\,,\quad \hat V_x=0\,. \tag{78}$$

Eliminating $\hat V_x$ and $\hat P$ from Eqs.(75) and (76) we obtain the 4th order differential equation for $\hat V_y$, that can be obtained also from Eq.(38) by substituting $\alpha=\frac{\pi}{2}$:

$$\hat V_y^{(4)}-(2\lambda^2+Ha^2)\hat V_y''+\lambda^4\hat V_y=0\,. \tag{79}$$

Due to this differential equation completely coincide with differential equation obtained in [1] for the odd problem in the case of transverse magnetic field, the solution of this equation is the same as in [1], i.e.

$$\hat V_y(\lambda,y)=\frac{1}{\sqrt{2\pi}}\cdot\frac{(k_1\cosh k_1\sinh k_2y-k_2\cosh k_2\sinh k_1y)}{\Delta_1}\cdot\frac{\sin(\lambda L)}{\lambda} \tag{80}$$

where $\Delta_1=k_2\cosh k_2\cdot\sinh k_1-k_1\cosh k_1\cdot\sinh k_2$.

$k_1=\mu+\sqrt{\mu^2+\lambda^2}$, $k_2=\mu-\sqrt{\mu^2+\lambda^2}$,

$k_3=-k_1$, $k_4=-k_2$.

Determining $\hat V_x$ from Eq.(77) we obtain:

$$\hat V_x(\lambda,y)=\frac{i}{\sqrt{2\pi}}\cdot\frac{(\cosh k_2\cdot\cosh k_1y-\cosh k_1\cdot\cosh k_2y)\sin(\lambda L)}{\Delta_1}+$$
$$+\frac{i}{\lambda}\hat f_2(\lambda)V_\infty(y)\,. \tag{81}$$

Taking into account formulae (32) and (31), the last term of (81) can be written in the form

$$\frac{i}{\lambda}\hat f_2(\lambda)V_\infty(y)=-\hat f_1(\lambda)\,.$$

Thus,

$$\hat V_x=\frac{i\cdot(\cosh k_2\cdot\cosh k_1y-\cosh k_1\cdot\cosh k_2y)\cdot\sin(\lambda L)}{\sqrt{2\pi}\cdot\Delta_1}-\hat f_1(\lambda)V_\infty(y) \tag{82}$$

Determining $i\lambda\hat P$ from Eq. (75) and $\partial\hat P/\partial y$ from Eq.(76) we obtain:

$$\frac{\partial\hat P}{\partial y}=-\frac{Ha}{\sqrt{2\pi}}\frac{\lambda}{\Delta_1}(\cosh k_1\cdot\sinh k_2y-\cosh k_2\cdot\sinh k_1y)\cdot\sin(\lambda L) \tag{83}$$

and

$$i\lambda\hat P=-\frac{iHa}{\sqrt{2\pi}}\frac{(k_1\cosh k_1\cosh k_2y-k_2\cosh k_2\cosh k_1y)}{\Delta_1}\cdot\sin(\lambda L)+S \tag{84}$$

where

$$S = \hat{f}_3(\lambda)\cdot V_\infty(y) - i\lambda \hat{f}_2(\lambda)V_\infty + i\frac{\hat{f}_2(\lambda)}{\lambda}\left(-Ha^2 V_\infty + V_\infty''\right) \qquad (85)$$

Taking into account formulae (32), (33) and the fact that function V_∞ satisfies Eq.(16), we obtain

$$S = -\hat{f}_1(\lambda)\cdot A.$$

The solution for $\hat{V}_x$ and $i\lambda\hat{P}$ differ from result obtained in [1] only by last terms. In order to obtain the solution to problem (66)-(68) we apply the inverse complex Fourier transform (65) to the functions $\hat{V}_x(\lambda, y)$, $\hat{V}_y(\lambda, y)$, $\dfrac{\partial \hat{P}(\lambda, y)}{\partial y}$ and $i\lambda\hat{P}$.

Applying the inverse complex Fourier transform, we take into account the fact that the functions $\hat{V}_y(\lambda, y)$ and $\partial\hat{P}/\partial y$ are even functions with respect to λ and the functions $\hat{V}_x(\lambda, y)$ and $i\lambda\hat{P}$ are odd functions with respect to λ.

We also use formula (31), i.e. $F^{-1}\left[\hat{f}_1(\lambda)\right] = f(x)$, formulae (19) and the fact that $F^{-1}\left[i\lambda\hat{P}\right] = \dfrac{\partial P}{\partial x}$.

As a result, we have the solution to problem (45)-(48) for the odd problem for the transverse magnetic field in the form of convergent improper integrals that coincide with the solution of the problem obtained in [1]:

$$V_x = \frac{1}{\pi}\int_0^\infty \frac{(\cosh k_1\cdot\cosh k_2 y - \cosh k_2\cdot\cosh k_1 y)\cdot\sin\lambda L\sin\lambda x}{\Delta_1}d\lambda$$

$$V_y = \frac{1}{\pi}\int_0^\infty \frac{(k_1\cosh k_1\cdot\sinh k_2 y - k_2\cosh k_2\cdot\sinh k_1 y)\sin\lambda L\cos\lambda x}{\Delta_1\cdot\lambda}d\lambda$$

$$\frac{\partial P}{\partial x} = \frac{Ha}{\pi}\int_0^\infty \frac{(k_2\cosh k_2\cosh k_1 y - k_1\cosh k_1\cosh k_2 y)\cdot\sin\lambda L\sin\lambda x}{\Delta_1}d\lambda$$

$$\frac{\partial P}{\partial y} = \frac{Ha}{\pi}\int_0^\infty \frac{\lambda(\cosh k_2\cdot\sinh k_1 y - \cosh k_1\cdot\sinh k_2 y)\cdot\sin\lambda L\cos\lambda x}{\Delta_1}d\lambda$$

where $\Delta_1 = k_2\cosh k_2\cdot\sinh k_1 - k_1\cosh k_1\sinh k_2$

$$k_1 = \mu + \sqrt{\mu^2+\lambda^2}\quad,\quad k_2 = \mu - \sqrt{\mu^2+\lambda^2}.$$

2) Numerical results for the odd problem

Fig.5 plots the profiles of the velocity component V_x for the Hartmann number Ha=10 and Ha=50 for L=1 (Fig.5A) and for L=4 ((Fig.5B). Note that in this figure the velocity component V_x is shown only for $0 \le y \le 1$, since the component V_x is an even function with respect to y.

It can be seen from Fig.5 that V_x has the M-shaped profiles only near to the entrance hole ($1 \le r \le 1.1$ at Ha=10 and $1 \le r < 1.1$ at Ha=50) for L=1.

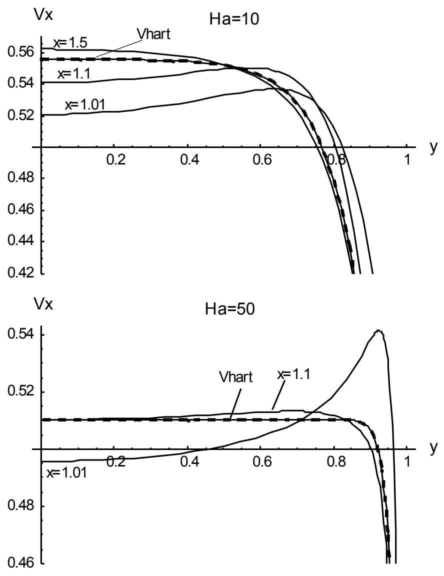

Fig.5A. Profiles for the x component $V_x(x, y)$ of the velocity $\vec{V}$ for odd case and $\vec{B}^e = B_0\vec{e}_y$ if L=1 (--- Vhart is the Hartmann flow)

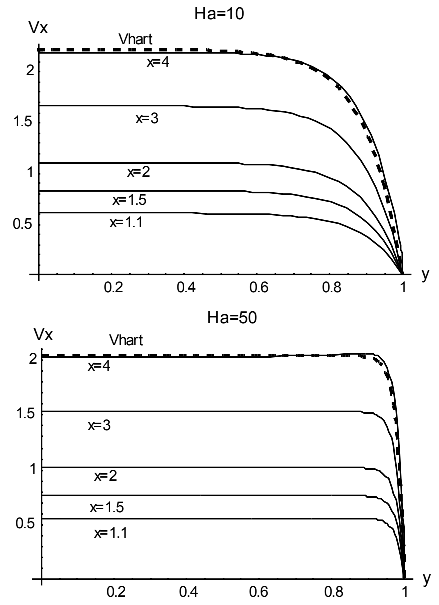

Fig.5B. Profiles for the x component $V_x(x, y)$ of the velocity $\vec{V}$ for the odd case and $\vec{B}^e = B_0\vec{e}_y$ if L=4 (V_{hart} is the Hartmann flow)

However, even at small distance from the entrance, the flow approaches the Hartmann flow in a plane channel in transverse magnetic field. With increasing L the length of initial part increases. For L=4 the Hartmann flow takes place only at x=4. Note that in the present problem the initial part of the channel is defined to be the part where the x-component of the velocity $\vec{V}(x, y)$ differs from the Hartmann flow V_{hart} by less than 1%. In addition, at L=4, velocity component V_r don't have M-shaped profiles.

3) Numerical results for the even problem with respect to y

As it was mentioned before, the solution in [1] for this case was obtained in correct way, due to the velocity and pressure gradient are equal to zero at $x \to \pm\infty$. We represent here new numerical results for even problem. The solution for velocity component obtained in [1] for the transverse magnetic field has the form

$$V_x = \frac{1}{\pi}\int_0^\infty \frac{(\sinh k_2 \cdot \sinh k_1 y - \sinh k_1 \cdot \sinh k_2 y)\cdot \sin\lambda L \sin\lambda x}{\Delta_2} d\lambda$$

$$V_y = \frac{1}{\pi}\int_0^\infty \frac{(k_2 \sinh k_2 \cosh k_1 y - k_1 \sinh k_1 \cosh k_2 y)\sin\lambda L \cos\lambda x}{\Delta_2 \cdot \lambda} d\lambda$$

$$\Delta_2 = k_2 \sinh k_2 \cdot \cosh k_1 - k_1 \cdot \sinh k_1 \cdot \cosh k_2 .$$

Fig.6 plots the x profiles of the velocity component V_x for the Hartmann numbers Ha=10 and Ha=50 for $-1 \le y \le 0$. Note that in this case the function V_x is an odd function with respect to y.

One can see from Fig.6 that V_x differs from zero only near the entrance region ($r \le 2$ for $Ha = 10$ and $r < 1.5$ for $Ha = 50$). In addition, in Fig.6 for some values of x the component V_x is negative at $-1 < y < 0$ and Ha=10. However, since the fluid inflows into the channel through the hole on $y = -1$, the x-component of the velocity must be positive for $-1 < y < 0$ at Ha=0. It means that there exists an opposite flow in the region in transverse magnetic field. It happens due to a vortex generated in the channel (see Fig.7).

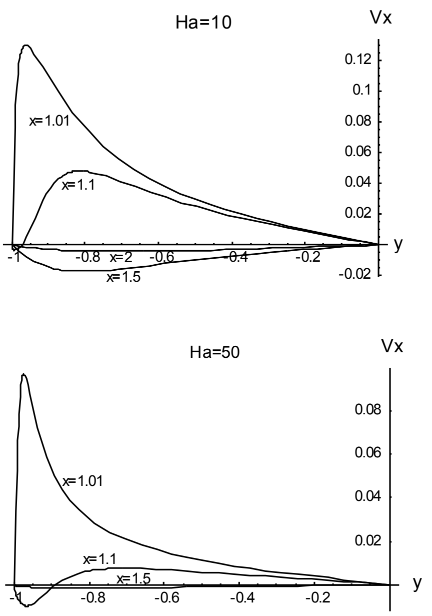

Fig. 6: Velocity profiles for the x component V_x in the even case for $\vec{B}^e = B_0\vec{e}_y$, L=1

One can see from Fig.6 that V_x differs from zero only near the entrance region ($r \le 2$ for $Ha = 10$ and $r < 1.5$ for $Ha = 50$). In addition, in Fig.6 for some values of x the component V_x is negative at $-1 < y < 0$ and Ha=10. However, since the fluid inflows into the channel through the hole on $y = -1$, the x-component of the velocity must be positive for $-1 < y < 0$ at Ha=0. It means that there exists an opposite flow in the region in transverse magnetic field. It happens due to a vortex generated in the channel (see Fig.7). Note that the velocity of fluid in this vortex is very small. The vector field of velocity for Ha=10 is shown in Fig.7.

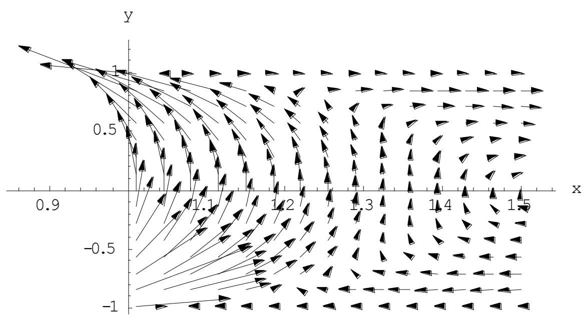

Fig. 7. Velocity field in the even case for $\vec{B}^e = B_0\vec{e}_y$, L=1 at Ha=10

4) Numerical results for the general case

The solution to general problem (66)-(68) at α=π/2 with boundary conditions (11), (14) is equal to the sum of the solutions of odd and even problems with respect to y. Fig. 8 plots the results of calculation of the x-component $V_x(x, y)$ of the velocity for the general problem for the Hartmann numbers Ha=10 and Ha=50.

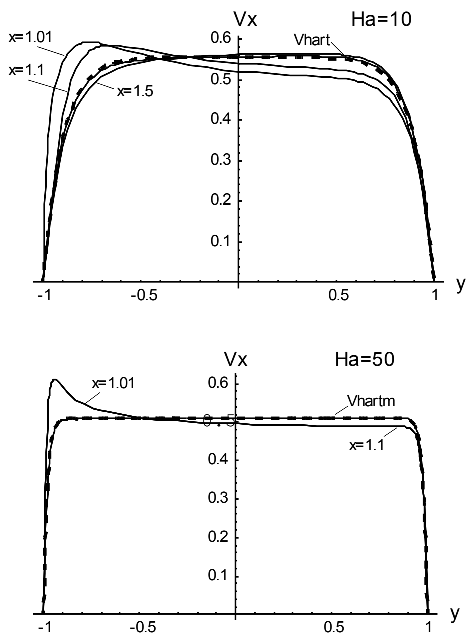

Fig. 8: Velocity profiles for the x component V_x for the general problem and $\mathbf{B}^e = B_0\mathbf{e}_y$, L=1

One can see that, similarly to the previous case, the profiles of the velocity component V_x differ from the Hartmann flow profiles only near the entrance region. For Ha=10 the flow approaches the Hartmann flow at $x \geq 2$ and in the case Ha=50 the Hartmann flow takes the place at $x \geq 1.5$

V. CONCLUSIONS

1.In this paper a correct analytical solution on an inflow of conducting fluid into a plane channel through the split of finite width in channel's lateral side in a strong magnetic field is presented. The problem is solved in Stokes and inductionless approximation by using Fourier transform. In order to solve the problem correctly, a new function of velocity and new function of pressure gradient are introduced. It is shown, that the final results obtained in [1], despite the incorrect assumption, nevertheless are correct.

2. The problem is solved by its dividing into the odd and even cases with respect to y axis. The solution of general problem is equal to the sum of even and odd problems with respect to y.

3.New numerical results are presented for the velocity field both for the longitudinal and transverse magnetic fields.

4.Velocity profiles of the component V_x are obtained numerically at Ha=10 and Ha=50 for the even problem with respect to y. For even with respect to y problem in transverse magnetic field there exists an opposite flow in the region of transverse magnetic field. It happens due to a vortex generated in the channel. Note that the velocity of fluid in this vortex is very small**.**

5.Velocity profiles of the component V_x are obtained numerically at Ha=10 and Ha=50 also for the general problem both for the longitudinal and transverse magnetic fields.

In the strong longitudinal magnetic field flows mostly happen along the wall with a split. On increasing the Hartmann number, the layer of a flow is getting narrower.

For the transverse magnetic field, the profiles of the velocity component V_x differ from the Hartmann flow profiles only near the entrance region. For Ha=10 the flow approaches the Hartmann flow at $x \geq 2$ and for Ha=50 the Hartmann flow takes place at $x \geq 1.5$

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Jeļena Liģere. Piezīmes par analītisko risinājumu MHD problēmām par vadītspējīga šķidruma ietecēšanu plakanā kanālā caur kanāla malējo sienu
Šajā darbā tiek aprakstīts korekts analītiskais risinājums uzdevumam par vadītspējīga šķidruma ietecēšanu plakanā kanālā caur galīga platuma spraugu kanāla malējā sienā. Problēma tiek risināta Stoksa un bezindukcijas tuvinājumā, izmantojot Furjē transformāciju. Sīkāk tiek aplūkoti paralēla magnētiskā lauka un perpendikulāra magnētiskā lauka gadījumi. Šis uzdevums jau ir atrisināts rakstā (Antimirov M., Ligere E. Analytical solutions for the problems of the flowing into of the conducting fluid through the lateral side of the plane channel in a strong magnetic field//Magnetohydrodynamics.–2000.-Vol.36, No.1.-pp.47-60), bet risinājumam bija izmantots nekorekts pieņēmums. Tas ir, minētajā rakstā bija pieņemts, ka šķidruma ātrums un spiediena gradients ir vienādi ar nulli pietiekošā attālumā no ieejas reģiona. Īstenībā, pietiekoši tālu no ieejas spraugas, plūsma pārvēršas par Puazeļa plūsmu paralēla magnētiskā lauka gadījumā un par Hartmana plūsmu perpendikulāra magnētiskā lauka gadījumā. Dotajā rakstā tiek prezentēts analītiski precīzs risinājums, un tiek pierādīts, ka atrisinājums, kas ir iegūts minētajā rakstā, ir pareizs neskatoties uz nekorekto pieņēmumu. Lai vienkāršotu risinājumu, uzdevums sadalīts divos apakšuzdevumos: viens ir pāra uzdevums attiecībā pret y un otrais - nepāra uzdevums. Šajā darbā ir iegūti arī jaunie skaitliski rezultāti. Tas ir, skaitliski tiek izpētīts šķidruma ātrumu lauks kanālā pāra uzdevumam attiecībā pret y un vispārīgam uzdevumam, kuram atrisinājums ir vienāds ar atrisinājumu summu pāra un nepāra gadījumiem.

Елена Лигере. Дополнения к решению МГД задачи о втекании проводящей жидкости в плоский канал через боковую стенку канала
В данной работе приводится корректное решение МГД задачи о втекании проводящей жидкости в плоский канал через щель конечной ширины в боковой стенке канала. Проблема решается в Стоксовом и безындукционном приближении, используя преобразование Фурье. В деталях рассматриваются случаи продольного и поперечного магнитных полей. Эта задача ранее решалась в работе (Antimirov M., Ligere E. Analytical solutions for the problems of the flowing into of the conducting fluid through the lateral side of the plane channel in a strong magnetic field//Magnetohydrodynamics.–2000.-Vol.36, No.1.-pp.47-60) с использованием некорректных допущений. А именно, предполагалось что скорость и градиент давления равны нулю в канале на достаточном удалении от входного отверстия. На самом же деле, на некотором расстоянии от щели течение в канале переходит в течение Пуазеля в случае продольного магнитного поля, и в течение Гартмана при поперечном магнитном поле. В данной статье задача решена аналитически строго и показано, что конечные результаты, полученные в ранее упомянутой статье, являются правильными, несмотря на неточное решение. Для упрощения решения, задача разбивается на две подзадачи - четную задачу относительно оси y и нечетную. В работе приводятся также новые численные результаты. А именно, численно изучается поле скоростей для четной относительно y задачи и для общей задачи, решение которой равно сумме решений четной и нечетной задач.