

# Analysis of Causes of Inefficiency of Stochastic Models of Dynamic System Identification

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**Abstract** – The carried out analysis of stochastic identification models has allowed to find a rigorous mathematical proof of their inefficiency. Identification of object parameters and characteristics of noise using a uniform model is impossible. It leads to erroneous results which have abstract character and cannot be decoded by using any mathematical operations. These facts have been held back many years and practical identification was in impasse. Partly it is because finding analytical solution of system of difference equations is not possible using traditional computing methods. The obtained results show that procedures of identification should be complete and should include algorithms for decoding the coefficients of difference equations.

**Keywords** – identification, identification model, stochastic model, mapping operator, numerical stability, singularity

## I. PROBLEM STATEMENT

Stochastic models described in [2], [3] are presented there as a universal approach for practical identification of analog dynamic objects. Therefore, special analysis of properties of these models for possibility of their practical application in tasks of identification of aircraft dynamic characteristics and their onboard equipment was required. Difficulties of this task are caused by the fact that transients recorded during flight test stage, are characterized by small degree of variability and, consequently, in algorithms of identification singular situations can arise which increase their sensitivity to noise. Authors in [2], [3] recommend stochastic models for wide practical application, asserting that algorithms constructed on the basis of such models are not only capable to give an estimation of object's parameters, but also an estimation of characteristics of noise. Similar recommendations cause interest for their practical application in tasks of processing the flight information.

In these models, it is offered to describe both output signal of the object  $u(t)$ ,  $y(t)$  and additive noise  $e(t)$  using rational transfer functions:

$$A(q)y(t) = \frac{B(q)}{F(q)}u(t) + \frac{C(q)}{D(q)}e(t) \quad (1)$$

In [3 (4.16)], it is stated that the choice of polynomials at  $e(t)$  can be arbitrary. As an example from (1), we shall choose the ARMAX model [3 (4.14)]:

$$y(t) = G(q)u(t) + H(q)e(t); G(q) = \frac{B(q)}{A(q)}; H(q) = \frac{1}{A(q)} \quad (2)$$

At once, we shall note that description of noise  $e(t)$  by using characteristic polynomial of the object  $A(q)$  is not a particularly reasonable variant, as its parameters should still be identified.

Topological characteristics of operators of mapping used in problems of identification have been investigated in [9]. The analysis has shown that they possess properties of isomorphism. This property was used in [4] – [8] for formation of address structures describing fragments of computing algorithms.

We shall note that descriptions of models in [3] are the same as in [2], differing only in notation. Here (1) and (2) are approximations of the initial differential equations by difference equations, which are realized on the basis of some variant of Z-transform operation. In any case, there are methodical errors of discrete approximations of input signal. They have regular character and, consequently, cannot be effectively corrected using statistical methods. They cannot be ignored, as they can result in significant energy imbalance in difference equations. In conditions of near singularity in equation system, they can cause a significant bias in the estimates of equation coefficients. Their influence cannot be ignored as it is offered in [3; p. 31, p. 82] “... assume that the input signal during the quantization interval  $T$  is constant ...” It can be true in a special case when the object is controlled by computer signals, but this control problem cannot be generalized for cases of identification.

Therefore, in [11], [12], interpolation algorithms for compensating errors of discrete approximation have been developed. Coefficients of difference equations have abstract character and the operator of mapping differential equation into difference equation has compressing properties. As a result, well-separated analog poles ( $-g_{\nu}$ ) located in the left negative half of the complex plane (stable objects are considered) are mapped by Z-transform ( $-g_{\nu} \Rightarrow \mu_{\nu} = \exp(-g_{\nu} \cdot T)$ ) into discrete poles  $\mu_{\nu}$  located in a narrow area of the right semicircle. Therefore, at the presence of noise, they become poorly separated and can be perceived as one multiple pole. As shown in [13], in this case the algorithm of decomposition of discrete transfer function has qualitatively different character. It is applied for realization of the operation of inverse Z-transform with the purpose of mapping the obtained estimates of the discrete operator into the analog form which corresponds to the actual analog object. Without such decoding operation, it is not possible to get results with meaningful physical interpretation. It is necessary to note that both solving systems of difference equations and decoding their coefficients are connected to

occurrence of almost singular situations, which lower the reliability of results. Probably for this reason, the majority of authors restrict themselves to abstract numbers. Thus, illusions concerning the ease of practical application of identification algorithms are caused by the erroneous statements, such as in [2; 1.2.8], that discrete poles lay outside the unit circle  $|\mu_\nu| > 1 \nu=1,2,\dots,N$  and consequently are well separated. But there are more important reasons that make practical application of stochastic models (1) and (2) impossible. Proof of this fact could not be derived using traditional computing algorithms. It needed a new mathematical approach based on application of computing symbolical combinatory models. They were developed since 2003, and the proofs have been derived in [4], [5] in 2008. Their modification is presented in publication [1], which is the mathematical basis for the results described in this paper.

## II. OPERATOR-BASED DESCRIPTION OF STOCHASTIC MODEL

For the model (2), represented in the analog domain, equations of Laplace transform for the signal and transfer functions will be as follows:

$$y_{UW}(p) = W(p) \cdot u(p) = \frac{R_U(p)R_W(p)}{Q_U(p) \cdot Q_W(p)} \quad (3)$$

$$Q_U(p) = \prod_{i=1}^{N_U} (p + a_{U_i}) \quad Q_W(p) = \prod_{i=1}^{N_W} (p + a_{W_i}) \quad (4)$$

$$y_H(p) \Rightarrow H(p) = \frac{R_H(p)}{Q_H(p)}; \quad Q_H(p) = \prod_{i=1}^{N_H} (p + a_{H_i}) \quad (5)$$

From that follows that output signal (3) is the reaction of a formal object with the operator (3) on the input impulse in the form of  $\delta$  function. But the same signal operates at the input of operator  $H(p)$ , on the basis of which an attempt to determine the characteristics of noise is made. In this case, one can assume that operator  $H(p)$  is included in the model (2) in parallel with the operator (3). Then, the output signal can be represented as decomposition into the sum of elementary partial components:

$$y(p) = y_{UW}(p) + y_H(p) \Rightarrow \sum_{i=1}^{N_U} \frac{S_{U_i}}{(p + a_{U_i})} + \sum_{i=1}^{N_W} \frac{S_{W_i}}{(p + a_{W_i})} + \sum_{i=1}^{N_H} \frac{S_{H_i}}{(p + a_{H_i})} \quad (6)$$

Let's represent this expression more concisely:

$$y(p) = \sum_{i=1}^{NU+NW+NH} \frac{S_i}{(p + a_i)} \quad (7)$$

$$y(t_0 + kT) = \sum_{i=1}^{NU+NW+NH} S_i \exp(-a_i t_0) \cdot q_i^k$$

$$q_i = \exp(-a_i T) \quad (8)$$

$$y(t_0 + kT) = \sum_{i=1}^{NU+NW+NH} C_i \cdot q_i^k$$

$$C_i = S_i \cdot \exp(-a_i t_0) \quad (9)$$

It is obvious that the system of difference equations should be formed on the basis of the vector of coefficients of discrete operator's characteristic equation. Taking into account that the input signal is  $\delta$  function, this operator found is on the basis of Z-transform of the analog operator:

$$G(z) = Fz * \left[ \frac{R_U(p)R_W(p) \cdot R_H(p)}{Q_U(p) \cdot Q_W(p) \cdot Q_H(p)} \right] \Rightarrow \frac{R_Z(z)}{B(z)} \quad (10)$$

$$B(z) \Rightarrow \{Fz * [Q_U(p)]\} \cdot \{Fz * [Q_W(p)]\} \cdot \{Fz * [Q_H(p)]\} \quad (11)$$

During the classification of estimates of identification parameters, it is necessary to take into account that the solution of the system of difference equations should be formed on the basis of unions of sets of operator parameters, contained in the stochastic model:

$$a_i \in \{D_U^{(NU)} \cup D_W^{(NW)} \cup D_H^{(NH)}\} \quad (12)$$

$$q_i \in \{Q_U^{(NU)} \cup Q_W^{(NW)} \cup Q_H^{(NH)}\} \quad (13)$$

Elements of the set (13) are formed on the basis of nonlinear operator of mapping the analog poles into the area of right unit semicircle:

$$Q_U^{(NU)} \Rightarrow \varphi \text{Exp} * [-T \cdot D_U^{(NU)}] \quad (14)$$

$$Q_W^{(NW)} \Rightarrow \varphi \text{Exp} * [-T \cdot D_W^{(NW)}] \quad (15)$$

$$Q_H^{(NH)} \Rightarrow \varphi \text{Exp} * [-T \cdot D_H^{(NH)}] \quad (16)$$

Obviously, the total characteristic discrete polynomial is equal to the product of partial polynomials:

$$B(z) = B_U(z) \cdot B_W(z) \cdot B_H(z) \Rightarrow \left( \sum_{i=0}^{NU} \beta_{U_i} z^i \right) \cdot \left( \sum_{i=0}^{NW} \beta_{W_i} z^i \right) \cdot \left( \sum_{i=0}^{NH} \beta_{H_i} z^i \right) \quad (17)$$

Zero discrepancy in the difference equation system is reached if the vector (17) is orthogonal to the equation system row vectors:

$$\left\{ \begin{array}{l} 1) \sum_{j=1}^N \beta_j \cdot y(t_0 + jT) = -y[t_0 + (N+1)T] \\ \dots \\ r) \sum_{j=1}^N \beta_j \cdot y(t_0 + rT + jT) = -y[t_0 + (r+N+1)T] \end{array} \right. \quad (18)$$

That is, the following relation must hold:

$$[Y; \bar{y}] \cdot \bar{\beta} = \bar{0}; \quad \bar{\beta} \in B(z); \quad B(z) = \sum_{i=0}^{NU+NW+NH} \beta_i \cdot z_i^k \quad (19)$$

The equation system (18) is formed from measurements of process (7), and the vector  $\bar{\beta}$  is formed according to (17).

### III. SYMBOLICAL COMBINATORY MODEL FOR FORMATION OF FRAGMENTS OF COMPUTING ALGORITHM

For solving this problem, we apply methods of formation of address structures [1] using fragments of computing operations on a matrix the elements of which are described by:

$$[A(t_0)]_{i,j} = S_j \exp[a_j(t_0 + T(i-1))] \quad (20)$$

Using designations (8) and (9), we shall get a matrix that is a function of time passed since the system of the equations (18) was formed:

$$A^{(m \times n)} = \begin{bmatrix} C_1 & C_2 & \dots & C_n \\ C_1 q_1 & C_1 q_2 & \dots & C_n q_n \\ \dots & \dots & \dots & \dots \\ C_1 q_1^{m-1} & C_2 q_2^{m-1} & \dots & C_n q_n^{m-1} \end{bmatrix} \quad (21)$$

Let's find the vector of results of application of the minor operator on the set of associative matrixes:

$$\bar{\alpha} \Rightarrow \psi \text{Minr} * \overline{\text{Sec}(m, n)} \quad (22)$$

Matrixes are formed on the components  $[\text{Sec}(m, n)]_{i=(1 \ 2 \ \dots \ m)}$  of numerical address sequence [1]:

$$\psi \text{NumSec} * (m, n) \Rightarrow \overline{\text{Sec}(m, n)} \quad (23)$$

The coefficients of the vector corresponding to this component are formed on the basis of the model:

$$\alpha_i = [C_{i1} C_{i2} \dots C_{im}] \cdot \exp[t_0(a_{i1} + a_{i2} + \dots + a_{im})]$$

$$\cdot Fg(Q_i); \quad Q_i = (q_{i1}, q_{i2}, \dots, q_{im}) \quad (24)$$

The initial address structure for formation of  $Fg(Q_i)$  has the form of numerical series:

$$\text{Sec}(2, N) \Rightarrow \psi \text{NumSec} * (2, N) \quad (25)$$

Using the operator of allocation, it is filled with elements of set  $Q$ , and then operators of arithmetic operations are put into it as lexicographic multipliers [10]:

$$Fg(Q) \Rightarrow [\Phi A(0, \times) * \Phi A(11, -)] * \\ * [\varphi \text{Comm}(Q) * \text{Sec}(2, N)] \quad (26)$$

They allow implementing the principles of information monitoring between the space of address models and the space of operations. Transformations on the basis of unifying specification of subsets and transition to new address structure allows transforming the address model into a new form that is more convenient for programming the fragments of computing algorithm. Such structures arise at the formation of vector of results of application of the operator  $\psi \text{Minr}$  on the set of associative matrixes (22).

The matrix (21) is formed from discrete poles of operators included in the model (2). Therefore, mapping of structures (26) according to the principle of information monitoring will yield results which will be formed according to  $b_i = b_{1i} \cdot b_{2i}$ :

$$b_{1i} \Rightarrow \left[ \prod_{J1} (q_{W, j1} - q_{W, j2}) \right] \cdot \left[ \prod_{J2} (q_{U, j1} - q_{U, j2}) \right] \cdot \left[ \prod_{J3} (q_{H, j1} - q_{H, j2}) \right] \quad (27)$$

$$b_{2i} \Rightarrow \left[ \prod_{J1} (q_{W, j1} - q_{U, j2}) \right] \cdot \left[ \prod_{J2} (q_{W, j1} - q_{H, j2}) \right] \cdot \left[ \prod_{J3} (q_{U, j1} - q_{H, j2}) \right] \quad (28)$$

Let's find the expression for vector (22) for the case when the matrix of difference equation system is formed from measurements of transient process at the output of stochastic model (2). Its elements are determined by powers of partial elements (13.) Operator (22) is realized on the basis of address graph structure [1; eq. 18]:

$$\text{Adress} \{A^{(m \times m)}\} \Rightarrow \left\{ \overline{L^{(k)}} \times \circ [\psi \text{Column} * \text{Sec}(n, m)] \right\} \quad (29)$$

Symbolical model of information allocation in its branches is described by the equation:

$$V_K \Rightarrow \left\{ \tilde{Q}_{1j} * \psi \text{Accom} [f(r_{i1} + L_{j1})] \right\} \times \circ$$

$$\times \circ \left\{ \tilde{Q}_{2j} * \psi \text{Accom} [f(r_{i2} + L_{j2})] \right\} \times \circ$$

$$\dots \times \circ \left\{ \tilde{Q}_{mj} * \psi \text{Accom} [f(r_{im} + L_{jm})] \right\} \quad (30)$$

Here, it is shown that distribution of powers on elements (13) is done by allocation of subset  $\tilde{Q}_j$  of the set (13) in the branches of graph. Functions, depending on the time moments when measurements of process (9) (in this case, row and column indices) are made, are put into the rule of allocation. In [6] – [8], it was proved that address structure (30) has properties of decomposition and can be formed on the basis of number series  $Sec(n, m)$  [1; eq. 5]. Therefore, according to [1; eq. 15, 18], we have:

$$\psi \text{Minr} * A^{(n)} \Rightarrow \sum_{i=1}^r \varphi \text{Column} * \left[ \tilde{L}_j \times \circ \text{Sec}(n, m)_i \right] \quad (31)$$

Here,  $n$  – order of matrix,  $m$  – order of model (1).

Let's consider the case when the vector (22) is formed on the set of matrices, the address position components of which consist of two regular fragments [1; eq. 40].

Application of operator (31) to any of these matrices  $B$  is implemented on the basis of graph (31) in which the sections are formed from two conjugate numerical series. They are represented in the form of direct lexicographic product:

$$\psi \text{Minr} * B \Rightarrow \sum_{i=1}^r \varphi \text{Column} *$$

$$* \left\{ \tilde{L}_j \times \circ [\text{Sec}(n_1, m)_i \otimes \circ \text{Sec}(n_2, m)_i] \right\} \quad (32)$$

From [4], [5], and [1; eq. 40] follows that for one of the matrices, corresponding to one of the components for (23), we have:

$$\psi \text{Minr} * B_j \Rightarrow \bigcup_{k=1}^N \left[ \overline{S_{1j}} \times \circ Fg(\tilde{Q}_{1j}^{(n1)}) \right] \otimes \circ$$

$$\otimes \circ \left[ \overline{S_{2j}} \times \circ Fg(\tilde{Q}_{2j}^{(n2)}) \right] \quad (33)$$

Here vectors  $\overline{S_1}$  and  $\overline{S_2}$  are formed from products of elements from subsets of discrete poles  $\tilde{Q}_{1j}^{(n1)}$  and  $\tilde{Q}_{2j}^{(n2)}$ , belonging to the set (13.) These subsets are formed on all position components of numerical series. Taking into account the linear properties of the operator, we get the value of coefficient of the vector (24):

$$\psi \text{Minr} * B \Rightarrow \sum_j \bigcup_{k=1}^N \left[ \overline{S_{1j}} \times \circ Fg(\tilde{Q}_{1j}^{(n1)}) \right] \otimes \circ$$

$$\otimes \circ \left[ \overline{S_{2j}} \times \circ Fg(\tilde{Q}_{2j}^{(n2)}) \right] \quad (33)$$

From (33), vectors are formed for the set of associative matrixes from which the Cartesian product for two functionals which are formed on the basis of basic positional component of rows and columns is formed:

$$\overline{\Omega} \times \circ \overline{\Omega}^{*T} \Rightarrow \left[ \overline{\psi \text{Minr} * \{R\}} \right] \times \circ \left[ \overline{\psi \text{Minr} * \{R\}} \right]^T \quad (34)$$

Here,  $R$  designates sets of the associative matrices derived from the matrix of system of difference equations (18.) Structures  $Fg(\tilde{Q}_{1j}^{(n1)})$  and  $Fg(\tilde{Q}_{2j}^{(n2)})$  in (34) are formed from expressions (27) and (28).

From (34) follows that elements of the matrix of system of difference equations (18) include products of every possible differences of discrete poles of the operator of noise and the operator of input signal. It means that their introduction in model (2) distorts estimations of solution vector of the system related to the parameters of object. This error can be estimated from the value of the vector (33) which uses numerical series into which the components of at least one element from sets (14) or (16) is contained:

$$Q_U^{(NU)} \Rightarrow \varphi \text{Exp} * \left[ -T \cdot D_U^{(NU)} \right]$$

$$Q_H^{(NH)} \Rightarrow \varphi \text{Exp} * \left[ -T \cdot D_H^{(NH)} \right]$$

Therefore, the error can be characterized by the functional made from these components:

$$\rho \Rightarrow \sum_{i=2}^r \varphi \text{Column} * \left[ \tilde{L}_j \times \circ \text{Sec}(n, m)_i \right] \quad (35)$$

#### IV. ANALYSIS OF REASONS OF INAPPLICABILITY OF STOCHASTIC MODELS OF IDENTIFICATION

Identification of analog dynamic object is reduced to finding estimations of its transfer function:

$$W(p) = \frac{R(p)}{Q(p)} = \frac{p^n + q_{n-1}p^{n-1} + \dots + q_1p + q_0}{b_m p^m + b_{m-1}p^{m-1} + \dots + b_1p + b_0} \quad (36)$$

Taking into account the discrete character of analog signal processing, instead, one has to find estimations of factors of the following discrete operator:

$$D(z) = \varphi Z(T) * W(p) \Rightarrow$$

$$\Rightarrow \frac{\alpha_m z^m + \alpha_{m-1} z^{m-1} + \dots + \alpha_1 z + \alpha_0}{z^n + \beta_{n-1} z^{n-1} + \dots + \beta_1 z + \beta_0} = \frac{y(z)}{x(z)} \quad (37)$$

which is an approximation of the operator (36) on the basis of which the system of difference equations is formed

$$[X] \cdot \bar{\alpha} + [Y] \cdot \bar{\beta} = \bar{y} \quad (38)$$

that is used for finding coefficients of (37).

Here, in the operator  $\varphi Z(T)$  that is based on operation of Z-transform, an interpolating operator is included, for smoothing the mistakes of discrete approximation. Algorithms of mathematical transformations between (36) and (37) are investigated in [11], – [13]:

$$\begin{aligned} \varphi Z(T) * W(p) &\Rightarrow D(z) \\ \varphi Z^{-1}(T) * D(z) &\Rightarrow W(p) \end{aligned} \quad (39)$$

However, coefficients (38) have no physical interpretation and represent abstract numbers and, consequently, they are unsuitable for practical use and must be decoded. In [2], [3], this problem is ignored. For test modes (38) [4], [7], [12] it is possible to write down system of equations for definition of factors of the characteristic polynomial (2):

$$\begin{aligned} [Y] \cdot \bar{\beta} = \bar{y}; \quad [Y]_{rL} = \sum_{k=1}^n C_k q_k^r \cdot q_k^L \\ q_k = \exp(-a_i T); \quad \bar{\beta} \Rightarrow H \cdot \bar{y}; \quad H = [Y]^{-1} \end{aligned} \quad (40)$$

Finding  $H$  is complicated because of singular character of  $Y$ . In SC models [4], [7], operations of division by small numbers are eliminated and autonomy of computing stages is provided. Therefore, they are well adapted for solving almost singular problems. These advantages are incorporated in graph-based symbolical combinatory model (SC model) [1], [6], [10]:

$$\begin{aligned} \varphi Gr(\bar{r}, \bar{L}): \tilde{q}_1^{(n)} &\Rightarrow \varphi Dpv * [\varphi Kc(m_1) * \tilde{q}_1] \times \dots \\ \dots \times \varphi Dpv * [\varphi Kc(m_k) * \tilde{q}_k]; &\quad \sum_{i=1}^k m_i = n \\ \varphi Dpv(\bar{r}, \bar{L}) * \tilde{q}_i & \varphi Kc(m_i) * \tilde{q}_i; \quad \tilde{q}_i \end{aligned} \quad (41)$$

The model is implemented on the basis of mutual mapping of mathematical constructs between symbolical area "S" and arithmetic space "A" [1], [5], [6]. It has been proved that in the area of originals, equation (6) is written as:

$$H \Rightarrow \text{Diag}(\bar{fg}) \cdot C \cdot \text{Diag}(\bar{fg}); \quad [\bar{fg}]_i = \sum^N \prod_{k,r} (q_k - q_r) \quad (42)$$

Here, elements of diagonal matrices will consist of products of differences of discrete poles of the operator (37), which are less than one, and matrix  $C$  is formed from weight coefficients

of transient. The theory and applications of equation (42) are described in [1], [6], [7].

From here follows that computing operations from analog area are mapped into operation with very small numbers in the area of difference equations and influence of noise thus grows nonlinearly. The similar structure has been proved also for the method of least squares in [5]. It was proved that improvements of accuracy generally occur because of stronger compression of the area of calculations. For example, for 7<sup>th</sup> order  $H$ , if  $|q_k - q_r| < 0.1$ , the elements  $[\bar{fg}]_i < 10^{-21}$  in (42). In the field of such small numbers it is not realistically possible to generate Hessian matrices from second derivatives of discrepancy equations (40) [2; p. 39], [3; p. 247], for application of gradient method in stochastic models [14]:

$$y(z) = D(z)u(z) + B(z)e(z) \quad (43)$$

The idea of finding simultaneously the estimations of parameters of object  $D(z)$  and noise  $B(z)$ , which is introduced on subjective assumptions, will lead to situation where diagonal elements of (42) will contain with mutual differences of poles of operators  $D(z)$  and  $B(z)$ . First, it will even more increase the compression of area of calculations. Besides, it will be essentially impossible to identify the poles. The poles of operator of noise  $B(z)$  contained in the elements  $[\bar{fg}]_i$ , further compress the area of calculations and increase the influence of experimental noise. Therefore, in the field of such small numbers that are overpowered by noise, there can be no hope for existence of global extremum and for convergence of gradient method. Second, from (42) follows that separating parameters of object and noise is essentially impossible. Therefore, solving various modifications of stochastic models [3] results in abstract numbers, describing physically impossible objects.

Properties of numerical stability of SC models have been used in development of alternative models of identification of characteristics of aerospace objects. Necessity of solving such problem has arisen because of dynamism of transients in such objects is strongly limited [7], [8]. For this purpose, special dampers of angular fluctuations of vehicle are used. Therefore, matrices in (38), as a rule, are ill-conditioned. For objects working in modes of normal functioning, increasing the performance of identification algorithms requires development of formalized mathematical methods for the parallelization [1], [7], [8].

## V. CONCLUSIONS

The reasons of inefficiency of stochastic models are connected to the occurrence of methodical errors in the elements of inverse matrix of difference equation system. They arise because of the introduction of operators of noise in the model.

From the derived proof follows that the elements of inverse matrix include products of all possible distances between discrete poles of operators entered into the model. Therefore, solutions of difference equation systems have abstract

character and cannot be decoded using any mathematical operations. This fact completely prevents application of stochastic models for estimating the parameters of real technical objects. The desire to identify both the parameters of the object and the characteristics of noise using a unified model is erroneous.

From the obtained results follows that unreasonable attempt to raise the order of equation system in order to achieve greater accuracy of identification is erroneous. In this case, the degree of singularity of computing algorithm increases nonlinearly depending on the order, which rapidly increases the sensitivity of algorithm to noise.

Procedure of identification should not be stopped after finding the solution of the system of difference equations. The obtained numbers have an abstract character and cannot characterize the state of the technical object. Procedure should be continued, applying algorithms for decoding these numbers. However, it is necessary to take into account that discrete poles found by solving equation systems are located in a narrow area inside the right unit semicircle. Their separation worsens and that also negatively affects the accuracy of operators of restoration of the initial analog description of real object.

Application of symbolical combinatory models that describe fragments of computing algorithms in proofs has shown that they can serve as effective means for computer programming.

Introducing in the model additional operators, such as the operator of noise, leads to formation of a false extremum in the functional of discrepancy of difference equations. It is the reason of biased estimations of model's discrete poles. As they are located in a narrow area of a unit semicircle, relative errors are large. Therefore, restoration of object's analog transfer function is impossible because of the loss of usability of gradient methods.

The analysis of properties of stochastic models of identification, which in [2], [3] are recommended for wide practical application, has been made with the purpose of studying the possibilities of their use in tasks of processing of flight information at the aircraft flight test stage. With the help of symbolical combinatory models (SC models), analytical expressions of solution of system of difference equations of identification, which are used in [2], [3], have been derived. The results have shown that solution is found in the field of small numbers because it contains combinations of products of differences of discrete poles of object. It proves the fact that computing operations in stochastic models are done in the field of small numbers where influence of noise is increased. In that case even methodical errors of discrete approximation can lead to significant distortions of results. In the field of such small numbers, it is impossible to construct functional of Hessian matrices for effective movement on the discrepancy gradient. The impossibility of application of gradient method is visible from an example when the joint system of difference equations is solved. In this case, discrepancy is equal to zero and the necessary information for construction of functional

on the basis of Hessian matrices calculated from the first and second derivative is absent.

Introduction of any variables in difference equations, which according to the author will help to find estimations of characteristics of noise, is an incorrect approach. It is visible from the derived solution of system of difference equations, obtained using the SC model. The abstract character of coefficients of solution thus increases even more and they cannot be decoded by any mathematical method. It makes stochastic models unsuitable for application in tasks of processing of flight information with the purpose of control and diagnosis of aircraft onboard equipment.

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#### **Genādijs Burovs. Dinamisko sistēmu identifikācijas stohastisko modeļu neefektivitātes iemeslu analīze**

Ieviešot stohastiskajā identifikācijas modelī papildus trokšņa operatoru nav iespējams iegūt drošus analogā objekta parametru novērtējumus. To identifikācija tiek veikta atrisinot diferencu vienādojumu sistēmu, kas tiek veidota uz diskrētā pārejas operatora pamata. Trokšņu operatora diskrētie poli sajaucas ar objekta operatora poliēm. Tie atrodas šaurā kompleksās telpas vienības apļa apgabalā. Šajā gadījumā apgrūtinās polu izšķiršana, kas padara neiespējamu objekta analogā operatora noteikšanu. Šī operācija ir nepieciešama, lai dekodētu diferencu vienādojumu koeficientus, kam ir abstrakts raksturs un kas nav praktiski izmantojami. Ieviešot modelī papildus operatoru, nav iespējams izmantot gradientu identifikācijas metodes, tā kā parametru iestatīšana notiek attiecībā pret neīstu vienādojuma nesaistes funkcionāla ekstrēmu. Iegūt stingrus analītiskus pierādījumus kļuva iespējams izmantojot jaunu matemātisko metodi – simboliskos kombinatoriskos modeļus skaitļošanas algoritmu fragmentu aprakstam. Šādi modeļi parādīja augstu efektivitāti praktisku uzdevumu risināšanā. Tā piemēram tika atrisināts polinomiālās aproksimācijas uzdevums, kurā ar 100% precizitāti tika atrasta inversā matrica gandrīz deģenerētai 20.kārtas Hilberta matricai, kaut arī tiek uzskatīts, ka šādu uzdevumu iespējams atrisināt tikai matricām ar kārtu, kas nav lielāka par desmit.

#### **Геннадий Буров. Анализ причин неэффективности стохастических моделей идентификации динамических систем**

Введение в стохастическую модель идентификации дополнительного оператора шумов не позволяет получить достоверные оценки параметров аналогового объекта. Их идентификация производится путем решения системы разностных уравнений, формируемой на основе дискретного передаточного оператора. Дискретные полюсы оператора шумов перемешиваются с полюсами оператора объекта. Они лежат в узкой области единичного полукруга комплексной плоскости. В этом случае ухудшается различимость полюсов, что делает невозможным восстановление аналогового оператора объекта. Эта операция является обязательной для дешифрирования коэффициентов разностного уравнения, которые несут абстрактный характер и непригодны для практического использования. Введение в модель дополнительного оператора приводит к неработоспособности градиентных методов идентификации, так как настройка параметров модели производится относительно ложного экстремума функционала невязки уравнения. Получение строгих аналитических доказательств стало возможным благодаря применению нового математического аппарата – символьных комбинаторных моделей для описания фрагментов вычислительных алгоритмов. Такие модели показали высокую эффективность при решении прикладных задач. Например, была решена задача полиномиальной аппроксимации, в которой была получена со 100% точностью обратная матрица для почти вырожденной матрицы Гильберта 20-го порядка, хотя считается, что такую задачу можно решить лишь до десятого порядка.