Inverse Technique for the Viscoelastic Material Properties Characterisation

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***Abstract*. New non-destructive technique, namely an inverse technique based on vibration tests, to characterize nonlinear mechanical properties of adhesive layers in sandwich composites has been developed. An adhesive layer is described as a viscoelastic isotropic material with storage and loss moduli which are both frequency dependent values in a wide frequency range. An optimization based on the planning of experiments and response surface technique to minimize the error functional is applied to decrease considerably the computational expenses. The developed identification technique has been successfully applied to characterize viscoelastic material properties of 3M damping polymer ISD-112 used as a core material in sandwich panels**

***Keywords:* Adhesive layer, finite element method, inverse technique, sandwich panel, vibration test, viscoelastic material properties.**

INTRODUCTION

Due to great importance ascribed to defining performance, reliability and safety requirements for advanced composite products and services a considerable effort has been devoted to the study of their mechanical material properties and a lot of different methods have been developed in the last three decades. For many orthotropic sheet materials a measurement method based on low frequency vibrations [1] is not only the simplest approach, it is also the only approach, which does not suffer from grave difficulties of principle, when the results are used to make predictions in the same range of frequencies.

There are two other general methods which might be used - static measurements [2] and ultrasonics [3], but neither is wholly appropriate for characterization of mechanical material properties of advanced composites. The difficulties are most obvious, when it comes to the determination of damping "constants", since these would be expected to be frequency dependent.

Only some researchers [4]-[6] tried to identify the viscoelastic material properties of sandwich composites using an inverse technique based on vibration tests. The forced steady state harmonic vibrations and free vibrations have been utilized in the identification procedure in paper [4] to characterize the constitutive parameters of Voigt model and three parameters model of uniaxial viscoelasticity used for a description of viscoelastic materials applied in sandwich beams. These beams consist of a core made of polyvinyl chloride foam and two laminate faces made of glass fiber reinforced polyester. To identify parameters of aluminum honeycomb sandwich panels, an orthotropic Timoshenko beam model has been applied, and the elastic constants and modal damping ratios have been determined in paper [5] minimizing the error between experimental and analytical results. The paper [6] proposes an inverse method based on the flexural resonance frequencies and using the sandwich beam theory for the finite element modeling. It is necessary to note that the examined approaches do not give the possibility to analyze sandwich structures with high damping and to characterize their viscoelastic material properties in a wide frequency range, when storage and loss moduli are frequency dependent values.

Due to this reason the present investigations are focused on the development of new inverse technique based on vibration tests to characterize nonlinear mechanical properties of adhesive layers in different sandwich applications. The developed identification technique has been tested on aluminum panels and applied to characterize viscoelastic material properties of 3M damping polymer ISD-112 used as a core material in sandwich panels with aluminum faces.

INVERSE TECHNIQUE

The basic idea of material identification procedure developed on vibration tests and nondirect optimization methodology is that simple mathematical models (response surfaces) are determined only by the finite element solutions in the reference points of the plan of experiments. The identification parameters for all eigenfrequencies in the examined frequency range are obtained minimizing the error functional, which describes the difference between the measured and numerically calculated parameters of structural response. A significant reduction in calculations of the identification functional is achieved in this case in comparison with the conventional optimization methods, since only one design space is necessary to identify adhesive material properties in the desired frequency range.

The present inverse technique (Figure 1) consists of experimental set-up, numerical model and material identification procedure. In the first stage a plan of experiments is produced depending on the number of identified parameters and number of experiments. Then finite element analysis is performed in the reference points of experimental design and dynamic parameters of structure are calculated. In the third stage these numerical data are taken to determine simple functions using the response surface methodology. In parallel vibration experiments are carried out with the purpose to determine natural frequencies and corresponding loss factors of viscoelastic sandwich structures. An identification of material properties is performed in the final stage minimizing the error functional between experimental and numerical parameters of structural responses.

Plan of Experiments

Let us consider a criterion for elaboration of the plan of experiments independent on a mathematical model of the designing object or process [7]. The initial information for development of the plan is number of factors n and number of experiments k. The points of experiments in the domain of factors are distributed as regularly as possible (Figure 2). For this reason the following criterion is used

$$\Phi = \sum_{i=1}^{k} \sum_{j=i+1}^{k} \left(\frac{1}{l_{ij}} \right)^2 \Rightarrow \min \tag{1}$$

where l_{ij} is a distance between the points having numbers i and j ($i \neq j$). Physically it is equal to the minimum of potential energy of repulsive forces for the points with unity mass if the magnitude of these repulsive forces is inversely proportional to the distance between the points.

For each number of factors n and number of experiments k it is possible to elaborate a plan of experiments, but it needs much computer time. Therefore each plan of experiment is developed only once and it can be used for various designing

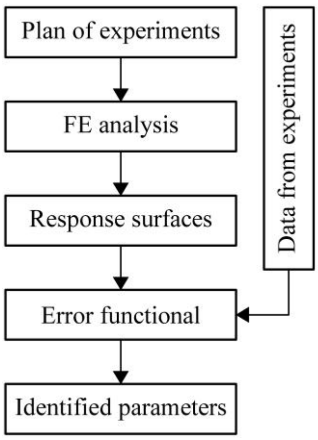


Fig. 1. Inverse procedure

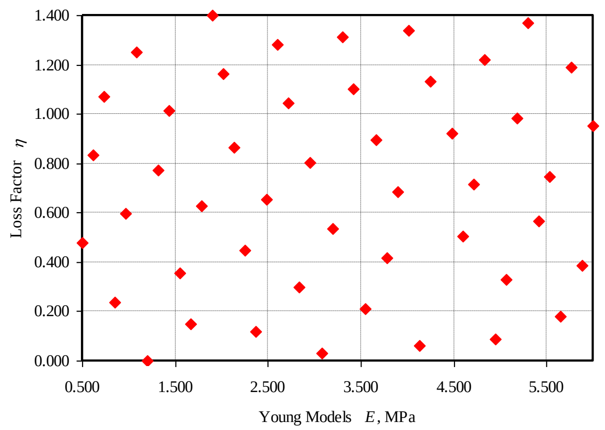


Fig. 2. Plan of experiments: n=2, k=48.

cases. The plan of experiments is characterized by the matrix of plan B_{ij}, when the domain of factors is determined as $x_j \in [x_j^{\min}, x_j^{\max}]$ and the points of experiments are calculated by the following expression:

$$x_j^{(i)} = x_j^{\min} + \frac{1}{k-1}\left(x_j^{\max} - x_j^{\min}\right)\left(B_{ij} - 1\right) \tag{2}$$
$$i = 1,2,\ldots,k, \quad j = 1,2,\ldots,n$$

Finite Element Analysis

In the present investigations the finite element method is used for the modeling and dynamic analysis of sandwich panels with adhesive core layers.

Finite element modeling is based on the first order shear deformation theory including rotation around the normal. In this case the widely known expressions of displacements have the following form:

$$u = u_0 + z\gamma_x \quad , \quad v = v_0 + z\gamma_y \quad , \quad w = w_0 \tag{3}$$

where u_0, v_0, w_0 are the displacements in a reference plane, z is the coordinate of the point of interest from a reference plane γ_x, γ_y are the rotations connected with the transverse shear deformations. For sandwiches this hypothesis is applied separately for each layer (Figure 3). This case corresponds to the broken line model [8] and satisfies the following displacement continuity conditions between the layers

$$\begin{aligned} u^{(1)} &= u^{(2)}\Big|_{z=z_1} \quad , \quad u^{(2)} = u^{(3)}\Big|_{z=z_2} \\ v^{(1)} &= v^{(2)}\Big|_{z=z_1} \quad , \quad v^{(2)} = v^{(3)}\Big|_{z=z_2} \\ w^{(1)} &= w^{(2)}\Big|_{z=z_1} \quad , \quad w^{(2)} = w^{(3)}\Big|_{z=z_2} \end{aligned} \tag{4}$$

where, the numbers of layers are given in brackets.

To describe the rheological behaviour of viscoelastic materials, the complex modulus representation [9] is used. Using this model, the constitutive relations will be expressed in the frequency domain as follows

$$\sigma_0 = E^*(\omega)\varepsilon_0 = E(\omega)[1 + \mathrm{i}\eta(\omega)]\varepsilon_0, \quad \eta(\omega) = \frac{E''(\omega)}{E(\omega)} \tag{5}$$

where σ_0 and ε_0 are an amplitude of the harmonically time dependent stress and strain respectively, E^* is the complex modulus of elasticity, E, E'' are the real and imaginary parts of the complex modulus of elasticity, η is a loss factor and ω is a frequency. It is necessary to note that the storage and loss moduli in this case are frequency dependent values.

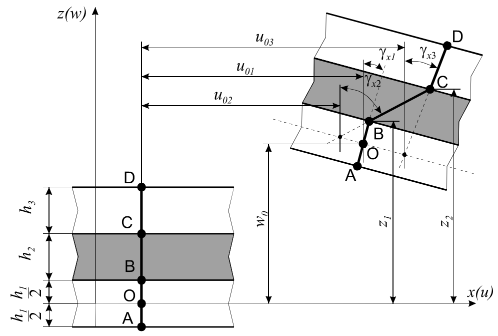


Fig. 3. Kinematic assumptions for a sandwich plate.

In the method of complex eigenvalues damped eigenfrequencies and corresponding loss factors are determined from the free vibration analysis of a structure

$$\left[\mathbf{K}^*(\omega) - \omega^{*2}\mathbf{M}\right]\overline{\mathbf{X}}^* = 0 \tag{6}$$

where $\mathbf{M}$ is the mass matrix of the structure, $\mathbf{K}^*(\omega) = \mathbf{K}(\omega) + i\mathbf{K}''(\omega)$ is the complex stiffness matrix of the structure, and $\omega^* = \omega + i\omega''$ is the complex eigenfrequency. The real part ω represents the damped eigenfrequency of the structure and the imaginary part ω'' specifies the rate of decay of the dynamic process. The matrix $\mathbf{K}(\omega)$ is determined using the storage moduli $E(\omega)$ and $G(\omega)$, while $\mathbf{K}''(\omega)$ is calculated using the imaginary parts of the complex moduli $E''(\omega) = \eta_E(\omega)E(\omega)$ and $G''(\omega) = \eta_G(\omega)G(\omega)$, where $\eta_E(\omega)$ and $\eta_G(\omega)$ represent the material loss factors.

Equation (6) can be written as a nonlinear generalised eigenvalue problem

$$\mathbf{K}^*(\omega)\overline{\mathbf{X}}^* = \lambda^*\mathbf{M}\overline{\mathbf{X}}^* \tag{7}$$

where $\lambda^* = \omega^{*2}$ is a complex eigenvalue and $\overline{\mathbf{X}}^*$ is a complex eigenvector. The solutions of Eq. (5) start at a constant frequency ($\omega = \text{const}$). Then, for each step, the linear generalised eigenvalue problem with $\mathbf{K}^*(\omega) = \text{const}$ is solved using the Lanczos method [16], which is programmed in a truncated version, where the generalised eigenvalue problem is transformed into a standard eigenvalue problem with a reduced order symmetric three diagonal matrix. Orthogonal projection operations are used with greater economy and elegance using elementary reflection matrices. An iteration process terminates, when the following condition is satisfied

$$\frac{|\omega_{i+1} - \omega_i|}{\omega_i} * 100\% \le \xi \tag{8}$$

where ξ is a desired precision and ω_{i+1} is the real part of the eigenfrequency of the structure calculated from the linear generalised eigenvalue problem for the storage and loss moduli at the frequency ω_i, which were obtained from the same equation in the previous step. The modal loss factors of the structure for each vibration mode are determined using the following equation

$$\eta_n = \frac{\lambda_n''}{\lambda_n} \tag{7}$$

This approach gives the possibility to preserve the frequency dependence of viscoelastic materials and to calculate structures with high damping.

Response Surface Method

In the present approach a form of the equation of egression is unknown previously [7]. There are two requirements for the equation of regression: accuracy and reliability. Accuracy is characterized as a minimum of standard deviation of the table data from the values given by the equation of regression. Increasing a number of terms in the equation of regression it is possible to obtain a complete agreement between the table data and values given by the equation of regression. However it is necessary to note that prediction in intervals between the table points can be not so good. For an improvement of prediction, it is necessary to decrease a distance between the points of experiments by increasing the number of experiments or by decreasing the domain of factors. Reliability of the equation of regression can be characterized by an affirmation that standard deviations for the table points and for any other points are approximately the same. Obviously the reliability is greater for a smaller number of terms in the equation of regression.

The equation of regression can be written in the following form

$$y = \sum_{i=1}^{p} A_i f_i(x_j) \tag{10}$$

where A_i are the coefficients of the regression equation; $f_i(x_j)$ are the functions from the pool of simple functions θ_1, θ_2, … θ_m which are assumed to satisfy:

$$\theta_m\left(x_j\right) = \prod_{i=1}^{s} x_j^{\xi_{mi}} \tag{11}$$

where ξ_{mi} is a positive or negative integer, including zero. Synthesis of the equation from the bank of simple functions is carried out in two stages: selection of perspective functions from the bank and then step by step elimination of the selected functions.

In the first stage, all variants are tested with the least square method and the function, which leads to a minimum of the sum of deviations, is chosen for each variant. In the second stage, the elimination is carried out using the standard deviation

$$\sigma_0 = \sqrt{\frac{S}{k-p+1}}, \quad \sigma = \sqrt{\frac{1}{k-1}\sum_{i=1}^{k}\left(y_i - \frac{1}{k}\sum_{j=1}^{k} y_j\right)^2} \qquad (12)$$

or the correlation coefficient given by:

$$c = \left(1 - \frac{\sigma}{\sigma_0}\right) \times 100\% \qquad (13)$$

where k is the number of experimental points, p is the number of selected perspective functions and S is the minimum sum of deviations. It is more convenient to characterize an accuracy of the equation of regression by the correlation coefficient (Figure 4). If insignificant functions are eliminated from the equation of regression, a reduction of the correlation coefficient is negligible. If in the equation of regression only significant functions are presented, an elimination of one of them leads to important decrease of the correlation coefficient.

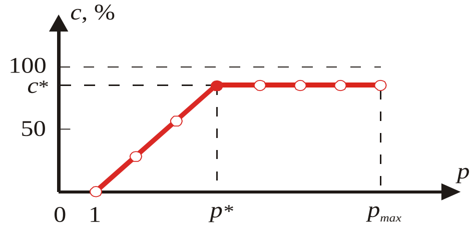


Fig. 4. Diagram of elimination for the correlation coefficient

Experimental Analysis

The experimental set-up used for vibration testing of sandwich panels (Figure 5) presents the impulse technique, where an excitation is accomplished by PCB impulse hammer with a built in force transducer. The structural response is detected by three accelerometers located on the panel as presented in Fig. 5. Both the input and output signals are converted to the frequency domain by fast Fourier transformation in a signal analyzer and the frequency response functions are created. After that, these frequency response functions are exported to the modal analysis program, where natural frequencies and corresponding loss factors are calculated.

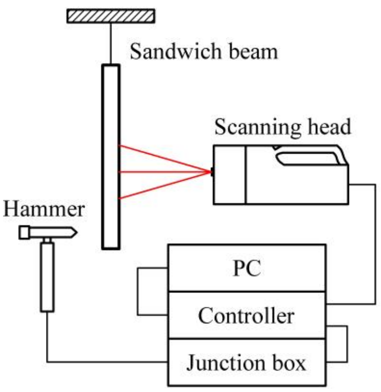


Fig. 5 Experimental set-up

Error Functional Minimization

The error functional between experimental and numerical parameters of structural responses is written in the case of identification of viscoelastic material properties for each eigenfrequency

$$\Phi_n(x) = \frac{\left(f_n^{EXP} - f_n^{FEM}\right)^2}{\left(f_n^{EXP}\right)^2} + \frac{\left(\eta_n^{EXP} - \eta_n^{FEM}\right)^2}{\left(\eta_n^{EXP}\right)^2} \Rightarrow \min \quad (14)$$

To minimise the error functional, the following constrained nonlinear optimisation problems must be solved:

$$\begin{aligned} &\min \Phi(x),\ H_i(x) \geq 0,\ G_j(x) = 0 \\ &i = 1,\ 2,\ \ldots L;\ j = 1,\ 2,\ \ldots J \end{aligned} \qquad (15)$$

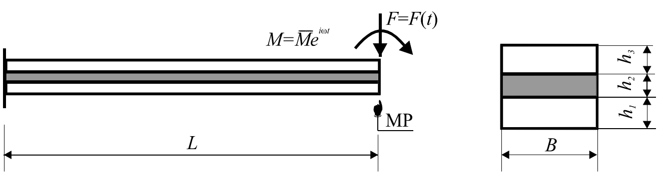
where I and J are the numbers of inequality and equality constraints. This problem is replaced with the unconstrained minimization problem in which the constraints are taken into account with the penalty functions. New version of random search method [11] is used for a solution of the formulated optimization problem. An application of the curve fitting procedure is required additionally to obtain the frequency dependent viscoelastic material properties of adhesive layers.

IDENTIFICATION EXAMPLES AND RESULTS VERIFICATION

The present inverse technique is tested and applied to characterise the viscoelastic material properties of a 3M damping polymer (ISD-112) used as a core material in sandwich panels.

Inverse Technique Testing

The sandwich beam shown in Figure. 6 has been chosen to test the performance of the inverse technique developed in this study. It has the following dimensions: width B=0.05 m, length L=0.3 m and thickness of layers h_1=0.0012 m, h_2=0.0001016 m, h_3=0.0008 m. The external layers are made out of aluminium 2024 T6 with characteristics: E =64 GPa, υ =0.32, and ρ =2695·Ns^2/m^4. Free-free boundary conditions. The structural dynamic characteristics, eigenfrequencies and corresponding loss factors (Table 1) have been obtained from physical vibration experiment by an impulse technique (Figure 5).



MP-measurement point

Fig. 6 Sandwich beam.

TABLE 1

DYNAMIC CHARACTERISTICS VERIFICATION FOR SANDWICH PANEL

#	f_n^{EXP} Hz	f_n^{FEM} Hz	Δ, %	η_n^{EXP}	η_n^{FEM}	Δ, %
1	94.58	93.71	0.92	0.17	0.177	4.12
2	225.8	225.3	0.22	0.243	0.246	1.23
3	414.6	414.6	0.00	0.251	0.258	2.79
4	652.1	653.1	0.15	0.251	0.254	1.20
5	937.6	938.1	0.05	0.238	0.240	0.84
6	1270	1270	0.00	0.223	0.222	0.45
7	1651	1649	0.12	0.205	0.201	1.95

To describe the viscoelastic isotropic material properties only one material parameter is necessary. This is modulus of elasticity $E^*(\omega) = E(\omega) + iE''(\omega)$. However in this case it is a complex value consisting of storage $E(\omega)$ and loss $E''(\omega)$ parts, which are both frequency dependent. As known material parameters, Poisson ratio υ=0.49 and density ρ=1000·(Ns2/m4) are taken into consideration. The boarders of identified parameters are taken in the present analysis as follows E = 0.5−6 (MPa) and η = 0 −1.5 .

The plan of experiments is produced for 2 design parameters and 98 experiments (Fig. 2). Then the finite element analysis is performed in 48 experimental points and seven first dynamic characteristics are determined. Employing these numerical values, the approximating functions (response surfaces) for all eigenfrequencies and corresponding loss factors have been obtained with the correlation coefficients higher than 94.3%. As an example these approximations with correlation coefficients are given below for the first eigenfrequency and corresponding loss factor:

$c = 95.8\%$

$$f_1 = 106.2 - 2.309 z_2 + 4.805 z_1 + 0.3119 z_2^2 + \frac{4.736 z_2}{z_1} + \frac{0.1597}{z_1^2} - \frac{0.2371 z_2^2}{z_1}$$

where $z_1 = 0.16667\text{E}$ and $z_2 = -1.0 + 0.27559\text{E}''$

$c = 94.3\%$

$$\eta = 0.2048 - 0.1477 z_1 + 0.5184 z_2 + 0.3119 z_2^2 - \frac{0.1995 z_2}{z_1} - 0.2808 z_1 z_2 - \frac{0.1983 z_2^2}{z_1} - 0.1351 z_1 z_2^2$$

where $z_1 = 0.16667\text{E}$ and $z_2 = -1.0 + 0.27559\text{E}''$

Minimizing the error functional (14), material properties of 3M viscoelastic damping polymer ISD-112 are found for each eigenfrequency. These values are presented in Fig. 7 by points. Applying the curve fitting procedure, the following shear modulus and material loss factor as functions on frequency are obtained in the frequency range f=94.58-1651(Hz)

G=2.682–0.9225/z (MPa)

where z=0.3635+0.0003855f

η=1.226–0.08378/z–0.3881 e-16/z^{16}

where z=0.1+0.0005451f

These dependencies are presented graphically in Fig. 7 with solid lines and they are used later in the finite element analysis to verify the identified material properties. Table 1 shows a good correlation between experimental and numerical dynamic characteristics.

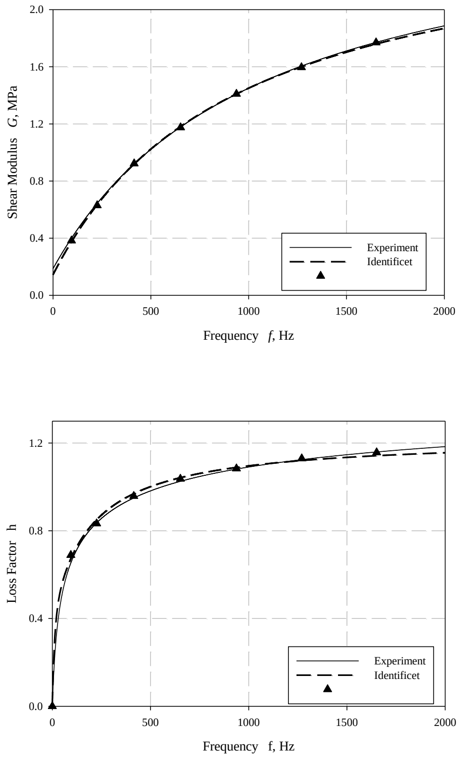


Fig. 7 Identified viscoelastic material properties.

CONCLUSIONS

New inverse technique operating on vibration tests has been developed to characterize nonlinear mechanical properties of adhesive layers in sandwich composites. The optimization approach based on the planning of experiments and response surface technique to minimize the error functional has been applied in this case to decrease considerably the computational efforts. The present methodology gave the possibility to preserve the frequency dependences for storage and loss moduli of viscoelastic adhesive materials in a wide frequency range and to analyze structures with high damping. The developed inverse technique has been applied to characterize viscoelastic material properties of 3M damping polymer ISD-112 used as a core material in sandwich panels. Good correlation between experimental and numerical results has been observed during verification of identified material properties.

It is necessary to note that an examined approach, like any other inverse approach based on vibration tests, has non-destructive character and the same material properties for test specimen and construction due to the same technology are used for their production. The present inverse technique due to its universality can be applied to characterize isotropic, orthotropic, elastic or viscoelastic material properties of advanced composites and structures, and viscoelastic material properties obtained in the present investigations can be used easily in other analyses, not presented in the identification procedure, to study the dynamic effects of structures with high damping properties.

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Eduards Skuķis, Pāvels Akišins, Evgenijs Barkanovs. Apgrieztā procedūra vizkoelastīgo materiāla īpašību noteikšanai.
Jaunā nesagraujošā metode – apgrieztā procedūra, kas balstīta uz vibrāciju testiem – bija izstrādāta „sendviča" tipa kompozīta līmes slāņu nelineāro mehānisko īpašību noteikšanai. Līmes slānis tika aprakstīts kā viskozelastīgs izotrops materiāls ar frekvenču atkarīgiem uzkrājuma un zuduma moduļiem. Optimizācijas procedūrā, kura balstās uz eksperimentu plānošanas un atsauces virsmu metodiku, tiek minimizēts kļūdu funkcionālis, kurš apraksta starpību starp parauga pašsvārstību frekvences vērtībām, iegūtām fiziskā eksperimenta gaitā un ar aproksimācijas funkcijām. Netiešā optimizācija ļauj ievērojami ietaupīt laika resursus, kas nepieciešami materiāla īpašību identifikācijai. Izstrādātā metodika tika veiksmīgi pielietota līmes ISD-112, kura ir plaši izmantojama „sendviča" tipa paneļos, īpašību noteikšanai.

Эдуард Скукис, Павел Акишин, Евгений Барканов. Обратная процедура для определения вязкоупругих свойств материала.
Новый неразрушающий метод – обратная процедура, основанная на вибрационных тестах – был разработан для определения нелинейных механических свойств клеевых слоёв композитов типа «сендвич». Клеевой слой рассматривается как вязкоупругий изотропный материал с частотно-зависимыми модулями накопления и потерь. В оптимизационной процедуре, основанной на планировании экспериментов и методе поверхностей отклика, минимизируется функционал, описывающий разность между значениями частот собственных колебаний образца, полученными в ходе физического эксперимента и с помощью аппроксимизационных функций. Косвенная оптимизация позволяет существенно сократить временные ресурсы, необходимые для идентификации свойств материала. Разработанная методика была успешно использована для определения свойств клея ISD-112, который широко применяется в «сэндвич»-панелях.