The Research of Mathematical Modelling of the Synchronization Process of Local Power Supply Systems with Synchronous Generator

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Abstract **- Compact electrical power systems differ by their operation modes either as independent or integrated with entire consolidated electrical power system. Autonomous operation mode with no connection with an integrated power system presents its own characteristics and consequently operation and maintenance problems related specifically with control of compact power systems (CPS) operation modes. The tasks of stability of the synchronous generator synchronization process are formulated based on analysis of this paper. Less analyzed and one of the most complex is synchronization process of generators in an autonomous power system. At that part offering power system model that contains two synchronous generators with primary engines. The load is presented with statically active – inductive load. For an estimation of synchronization stability the mathematical model is developed. The model includes the synchronous generator model with the use of the full differential Park-Gorev equations and the statically active – inductive load's equations.**

Keywords**: Synchronization, generator, local power supply, mathematical model.**

I. Introduction

In several regions power supply of industrial objects and residential areas is provided simultaneously by autonomous power plants and centralized networks. Besides range of specific issues appeared in regard to integrated operation of local system and centralized network.

Efficiency of local power supply system operation is determined by applicable equipment and its appropriate operation modes. As far most significant elements of local power supply systems are autonomous power plants, their technical parameters mostly determine efficiency of the entire local power supply system. The theoretical and practical aspects of electrical technical, conversion and transformation units utilized in local systems of power supply are widely reflected in numerous studies. Development of new types of equipment is executed on the basis of fundamental researches in electrical mechanics, power engineering and is directly interconnected with improvement and upgrading of electrical machinery characteristics, regulating devices, units control and protection systems, improving its reliability and expanding functional capabilities.

Currently the attention to such technical hardware is dictated by intensive and wide implementation of concepts application of flexible controlled power lines of alternative current that in technical literature have acquired the name Flexible AC Transmission systems (FACTS).

One of the significant and preferable factors of compact power system (PS) is local availability, certain compatibility and independence regarding to the number of interruptions (disconnections) under global transient climatic changes in Latvia.

That is documented with annual statistical data for 2009 illustrated further on by charts (Fig.1.-3.).

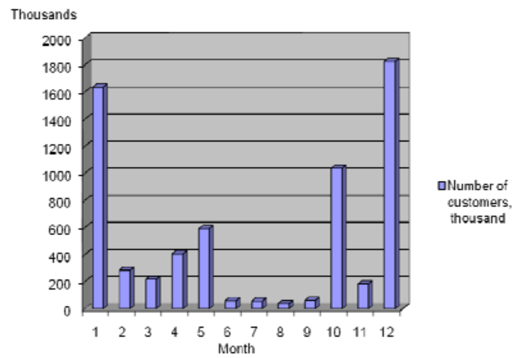

Fig.1. The total number of customers in Latvia in year2009.

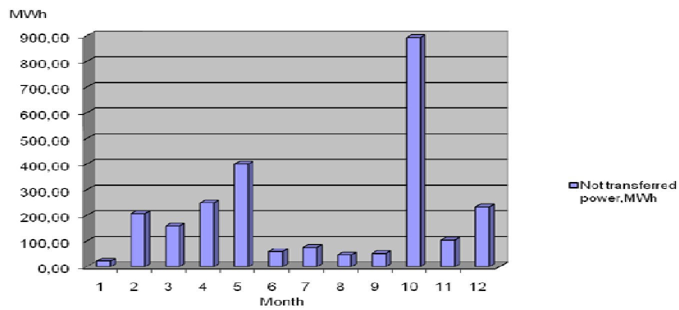

Fig.2. Data of not transferred power in Latvia in year 2009.

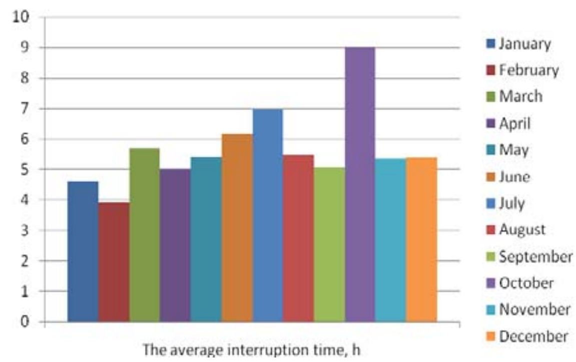

Fig.3. The total average interruption time in Latvia in year 2009.

Local, compact systems of power supply are interpreted as independent isolated power supply systems of separate entities or residential areas incorporating autonomous power plants and distribution systems of limited length. The consumed power in such systems does not exceed 0,5 - 2 MW in rare cases 5-10 MW.

II. Problems of control operation modes of compact electrical power systems

Compact electrical power systems differ by their operation modes either as independent or integrated with entire consolidated electrical power system. [1]

Obviously there are three groups of such conditions that are determining the following:

1. Parallel operation with integrated power system by rigid constraints admissible for output of full capacity of compact power systems in integrated power systems including in maintenance operation modes;
2. Parallel operation with integrated power system by low constraints admissible for output and input of capacity up or 30% of full capacity of compact power system;
3. Autonomous operation mode with no connection with integrated power system.

Each conditional group creates its own specifics and, consequently, operation and maintenance problems relating particularly to the control of compact power systems (CPS) operation modes. [1] Analyzing these issues the tasks for solution are formulated.

Operation of compact power plants (systems) in parallel with integrated power system takes place by rigid constraints. Drawbacks of reliable operation in case of rigid constraints are minimal.

III. Simulation of synchronization process with mathematical model's application of two generators

The most important is generators synchronization process with limiting power system. At that part offering power system model containing two synchronous generators with primary engines.

Systems load modulation grapple with statically active – inductive load (Fig. 4.).

Synchronization process modulating condition components from two steps:

First step - SG1 generator stationary condition calculation to active – inductive load;

Second step - SG2 generator connection withal synchronization conditions noncompliance.

For an estimation of synchronization stability the mathematical model is developed. The model includes the synchronous generator model with the use of full differential Park-Gorev's equations and the statically active – inductive load's equations [3].

The mathematical model of synchronous generator is represented on the basis of differentiated equations Park-Gorev on active-induction loads in d, q, 0 reference:

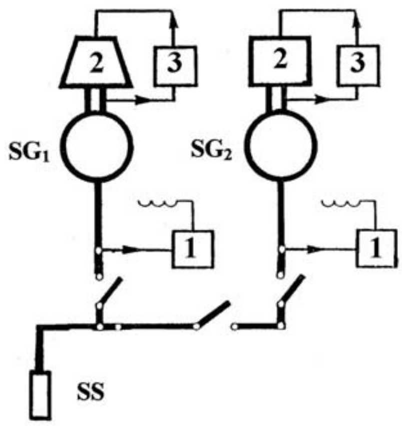

Fig. 4. Scheme of mathematical model. (1.- excitation; 2.- the engine; 3.- the engine control; SS - load).

Mathematical model of synchronous generator in d, q, 0 reference: [2, 3]

$$\left.\begin{aligned}
u_d &= \frac{d\Psi_d}{d\tau} - \Psi_q\omega + i_d R;\\
u_q &= \frac{d\Psi_q}{d\tau} + \Psi_d\,\omega + i_q R;\\
u_f &= \frac{d\Psi_f}{d\tau} + i_f\,R_f;\\
0 &= \frac{d\Psi_D}{d\tau} + i_D\,R_D;\\
0 &= \frac{d\Psi_Q}{d\tau} + i_Q\,R_Q;\\
T_M\frac{d\omega}{d\tau} &= M_T - (\Psi_q i_d - \Psi_d i_q),
\end{aligned}\right\} \quad (1)$$

where

Ψ_q , Ψ_q - are the components of full flux linkages in d, q, 0 reference;

u_d , u_q, - are the components of instantaneous values of phase voltages in d, q, 0 reference;

i_d, i_q, - are the components of instantaneous values of phase currents in d, q, 0 reference;

R_d , R_q, - are the active resistances of phase windings (in case of stator symmetry $R_d = R_q = R$);

u_f, i_f, Ψ_f, R_f - are the components of voltage at excitation winding, current in it, full flux linkage with it and active resistance of this winding;

T_M - the resulted inertia moment of a diesel engine in per unit's system;

M_T - determined from prime mover's mathematical model of internal combustion engine.

Where

$$\Psi_d = X_d i_d + X_{ad} i_f + X_{ad} i_D;$$
$$\Psi_q = X_q i_q + X_{aq} i_Q;$$
$$\Psi_f = X_{ad} i_d + X_f i_f + X_{ad} i_D;$$
$$\Psi_D = X_{ad} i_d + X_{ad} i_f + X_D i_D;$$
$$\Psi_Q = X_{aq} i_q + X_Q i_q.$$

Equations (1) could be transformed into matrix form. Further on the equations are represented in matrix table: [4]

$$\frac{d}{d\tau}\begin{bmatrix} i_d \\ i_q \\ i_f \\ i_D \\ i_Q \end{bmatrix} = \begin{bmatrix} Q_1 & 0 & 0 & 0 & 0 \\ 0 & Q_2 & 0 & 0 & 0 \\ 0 & 0 & Q_3 & 0 & 0 \\ 0 & 0 & 0 & Q_4 & 0 \\ 0 & 0 & 0 & 0 & Q_5 \end{bmatrix} \times \begin{bmatrix} u_d \\ u_q \\ u_d \\ u_d \\ u_q \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{bmatrix} \times \begin{bmatrix} i_d \\ i_q \\ i_f \\ i_D \\ i_Q \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \end{bmatrix} =$$

$$= \begin{bmatrix} Q_1 & 0 & 0 & 0 & 0 \\ 0 & Q_2 & 0 & 0 & 0 \\ 0 & 0 & Q_3 & 0 & 0 \\ 0 & 0 & 0 & Q_4 & 0 \\ 0 & 0 & 0 & 0 & Q_5 \end{bmatrix} \times \begin{bmatrix} u_d \\ u_q \\ u_d \\ u_d \\ u_q \end{bmatrix} + \begin{bmatrix} H_1 \\ H_2 \\ H_3 \\ H_4 \\ H_5 \end{bmatrix} \qquad (2)$$

or

$$\frac{d}{dt}[I_{SG}] = [Q_{SG}][U_{SG}] + [H_{SG}] \qquad (3)$$

where

$$a_{11} = -R_a Q_1, \quad a_{12} = X_q \omega Q_1, \quad a_{13} = -R_f Q_3, \quad a_{14} = -R_D Q_4, \quad a_{15} = X_{aq} \omega Q_1;$$
$$a_{21} = -\omega X_d Q_2, \quad a_{22} = -R_a Q_2, \quad a_{23} = -X_{ad} \omega Q_2, \quad a_{24} = -X_{ad} \omega Q_2, \quad a_{25} = -R_Q Q_5;$$
$$a_{31} = -R_a Q_3, \quad a_{32} = X_q \omega Q_3, \quad a_{33} = -\frac{X_d X_D - X_{ad}^2}{\Delta d} R_f, \quad a_{34} = \frac{X_s X_{ad}}{\Delta d} X_D, \quad a_{35} = X_{aq} \omega Q_3;$$
$$a_{41} = -R_a Q_4, \quad a_{42} = X_q \omega Q_4' \quad a_{43} = \frac{X_s X_{ad}}{\Delta d} R_f, \quad a_{44} = -\frac{X_d X_B - X_{ad}^2}{\Delta d} R_D, \quad a_{45} = X_{aq} \omega Q_4;$$
$$a_{51} = -X_d \omega Q_5, \quad a_{52} = -R_a Q_5, \quad a_{53} = -X_{ad} \omega Q_5, \quad a_{54} = -X_{ad} \omega Q_5, \quad a_{55} = -\frac{X_q}{\Delta q} R_Q.$$
$$Q_1 = \frac{X_D X_f - X_{ad}^2}{\Delta d}; \quad Q_2 = \frac{X_Q}{\Delta q}; \quad Q_3 = -\frac{X_{sD} X_{ad}}{\Delta d}; \quad Q_4 = -\frac{X_{sf} X_{ad}}{\Delta d}; \quad Q_5 = -\frac{X_{aq}}{\Delta q},$$
$$\Delta d = X_d X_f X_D - X_{ad}^2 (X_d + X_D + X_B - 2X_{ad}),$$
$$\Delta q = X_q X_Q - X_{aq}^2$$

$$B_1 = Q_3 u_B; \quad B_2 = 0; \quad B_3 = \frac{X_d X_D - X_{ad}^2}{\Delta d} u_B; \quad B_4 = -\frac{X_s X_{ad}}{\Delta d} u_B; \quad B_5 = 0.$$

$$[u_1] = -[L_{StL}]\frac{d}{dt}[I_{StL}] - [Z_{StL}] \times [I_{StL}],$$

(4)

$$[Z_{StL}] = \begin{bmatrix} R_{StL} & -X_{StL} \\ X_{StL} & R_{StL} \end{bmatrix}. \ [L_{StL}] = \begin{bmatrix} X_{StL} & 0 \\ 0 & X_{StL} \end{bmatrix}; \quad [u_1] = -\begin{bmatrix} u_{d1} \\ u_{g1} \end{bmatrix};$$

where

Applying all the systems elements of expressions (4) we obtain the system of current equations of synchronous generator stator.

Equation according to Kirchhoff's law for estimated point:

$$[I_{SG1}] + [I_{SG2}] = [I_{StL}] \qquad (5)$$

For mathematical model the equations of synchronous generator are completed therefore the currents are expressed by differentiated functions and equations (4) shall be differentiated.

$$\frac{d[I_{SG1}]}{dt} + \frac{d[I_{SG2}]}{dt} = \frac{d[I_{StL}]}{dt} \qquad (6)$$

Equations of current flows of stator (2), in the main expression form:

$$\frac{d[I_{SGi}]}{dt} = [Q_{SGi}] \times [U_1] + [H_{SGi}] \qquad (7)$$

Applying (4) and (6) into equations (7) we obtain:

$$[U_1] = -[L_{StL}] \times [m_{SG1} Q_{SG1} + m_{SG2} Q_{SG2}] \times [U_1] - [L_{StL}] \times [m_{SG1} H_{SG1} + m_{SG2} H_{SG2}] -$$
$$- [Z_{StL}] \times [m_{SG1} I_{SG1} + m_{SG2} I_{SG2}] \qquad (8)$$

Under transformation we obtain the following:

$$([1] + [L_{StL}] \times [m_{SG1} Q_{SG1} + m_{SG2} Q_{SG2}]\ ) \times [U_1] = -[L_{StL}] \times [m_{SG1} H_{SG1} + m_{SG2} H_{SG2}] -$$
$$- [Z_{StL}] \times [m_{SG1} I_{SG1} + m_{SG2} I_{SG2}] \qquad (9)$$

Solving equation (9), we obtain the system of variables of voltage for mathematical model of synchronous generator.

$$[U_1] = [([1] + [L_{StL}] \times [m_{SG1} Q_{SG1} + m_{SG2} Q_{SG2}])]^{-1} \times (-[L_{StL}] \times [m_{SG1} H_{SG1} + m_{SG2} H_{SG2}] -$$
$$- [Z_{StL}] \times [m_{SG1} I_{SG1} + m_{SG2} I_{SG2}]) \qquad (10)$$

For system modelling the parameters of synchronous generator are used in per unit system (p.u.):

$\sum m_{SGi} = 1$,This is: $m_{SG1} + m_{SG2} = 1$ (11)

Under the constant load on the first generator; $m_{SG1} = 1.0$

If at the moment of second generator actuation the first remains under the load, $m_{SG1} = 0.9$ if, and $m_{SG2} = 0.1$ (accept for the calculations), than process of synchronization is represented below in Fig. 5. - 8.

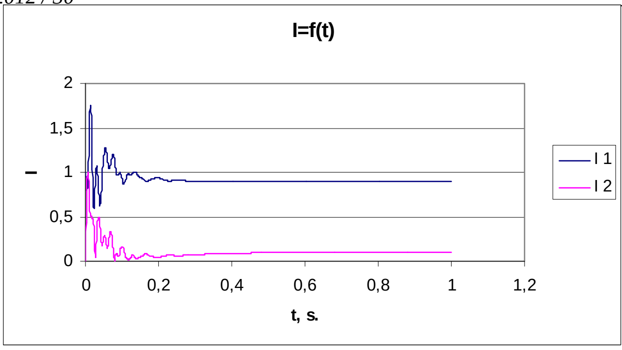

Fig.5. Synchronization results of two generators – currents by time:

I1- the first generator current;

I2- second generator current.

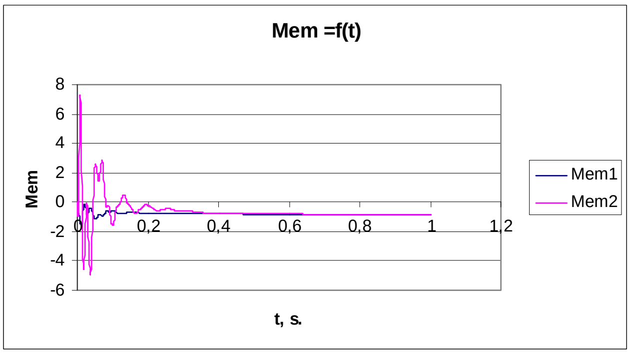

Fig.6. Synchronization results of two generators – moments by time:

Mem1- the first generator torque;

Mem2- second generator tongue

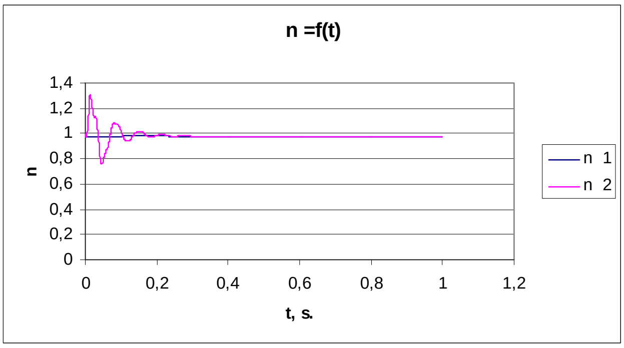

Fig.7. Synchronization results of two generators – frequency by time:

n1 – the first generator;

n2 – second generator.

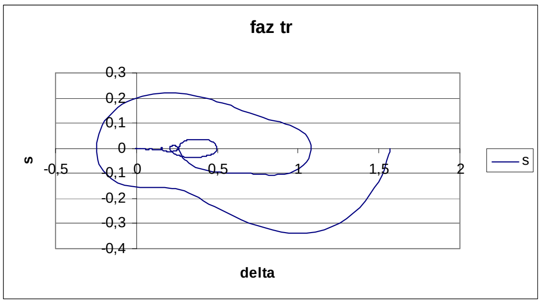

Fig.8. Synchronization results of two generators – phase trajectory.

The model has shown availability of two generators.

On the developed program's basis it is possible to define synchronization conditions of two generators with high level of reliability. To determine possible deviation limits on rotation frequency and mismatch corner of exact synchronization. Also the offered program allows to estimate a size of stator currents and electromagnetic torques of synchronous generators.

IV. Conclusions

The existing development trends of compact - electrical power plants envisage demand of sustainable development of hardware and methods on process control of synchronization of generators and sub-systems as one of the most significant tasks to raise operation efficiency of these systems.

Theoretical preconditions and viable practical analogues for creation of more efficient systems of ideal automatic synchronization of generators and sub-systems of PS are included in the developed research and supplement theory on construction of adaptive systems on programming control of objects relocation.

Regarding theoretical aspect it is significant to substantiate methods applicability and relevance of construction adaptive control systems of technical objects to the tasks of synchronization of generators and sub-systems of compact PS. Practically the whole range of methodological and technical tasks should be solved for application of this method on the generators and compact - PS synchronization.

The tendency of wide application of flexible controlled systems during the latest years observed in energy sector allows to upgrade the task positions of ideal synchronization of compact PS. Application of these systems as power elements allows in more flexible way to form and synthesize stipulations of systems on ideal synchronization of compact - PS.

In this paper local power system mathematical model with 2 generators was presented.

The model has shown synchronization and availability of two generators.

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Elēna Ketnere, Kristīna Bērziņa, Aleksandrs Mesņajevs. Lokālo barošanas sistēmu ar sinhrono ģeneratoru sinhronizācijas procesa matemātiskā modeļa izpēte
Kompaktā energosistēma tiek interpretēta kā neatkarīgas izolētās energoapgādes sistēma, kura ietver sevī autonomas elektrostacijas un ierobežota garuma sadales sistēmas atsevišķu personu patērētājiem vai dzīvojamo rajonu patērētājiem. Šo sistēmu jauda nepārsniedz 0,5 - 2 MW, retos gadījumos 5-10 MW. Kompaktās energosistēmas var iedalīt trijās grupās:

- ar paralēlu darbību, ar vienotu elektroenerģijas sistēmu, ar neelastīgo saikni un pieļaujamo pilnu kompakto energosistēmu jaudas piegādi, tai skaitā remonta režīma laikā;
- ar paralēlu darbību, ar vienotu energosistēmu, ar elastīgo saikni un pieļaujamo 30% no kompaktās energosistēmas pilnas jaudas pieņemšanu un piegādi;
- autonomā darba režīmā strādājošu bez savienojuma ar vienotu energosistēmu.

Katrai grupai ir savas īpatnības un līdz ar to noteiktas ekspluatācijas un uzturēšanas problēmas, kas saistītas ar kompakto energosistēmu (KES) ekspluatācijas režīmu kontroli. Balstoties uz Parka-Goreva diferenciālvienādojumiem, tika izveidots divu sinhrono ģeneratoru matemātiskais modelis d, q, 0 asīs ar aktīvi-induktīvo slodzi. Izstrādātā programma palīdz ar augstu ticamības pakāpi noteikt divu ģeneratoru sinhronizācijas nosacījumus. Atrisināts uzdevums, lai noteiktu iespējamās griešanās ātruma un fāžu nobīdes leņķa novirzes precīzas sinhronizācijas gadījumā. Turklāt izstrādātā programma ļauj novērtēt sinhronā ģeneratora statora strāvas un elektromagnētiskā momenta lielumus. Šis pētījums pierāda divu paralēli ieslēgto ģeneratoru matemātiskā modeļa efektivitāti.

Елена Кетнере, Кристина Берзиня, Александр Месняев. Исследования математической модели процесса синхронизации локальных систем питания с синхронным генератором
Под компактными системами электроснабжения - КЭС понимаются изолированные системы электроснабжения отдельных предприятий или населенных пунктов, содержащие автономные электростанции и распределительные сети ограниченной протяженности.
Потребляемая мощность в таких системах не превышает 0,5-2 МВт, реже 5-10 МВт.
Компактные - энергосистемы различаются по условиям их раздельной, или совместной работы с мощными энергосистемами.
С достаточной ясностью выделяются три группы таких условий, определяющих:

- параллельную работу с мощной энергосистемой по сильным связям, допускающим выдачу полной мощности источников компактных - ЭС в мощных энергосистемах в том числе в ремонтных режимах;
- параллельную работу с мощными энергосистемами по слабым связям, допускающим выдачу и прием до 30% полной мощности компактных - ЭС;
- автономный режим работы, когда связь с мощными энергосистемами полностью отсутствует.

Каждая группа условий создает свои особенности и, соответственно, проблемы эксплуатации, связанные, прежде всего с управлением режимами компактных - ЭС.
На основе дифференциальных уравнений Парка - Горева разработана математическая модель двух синхронных генераторов соизмеримой мощности в осях d, q, 0 при наличии активно- индуктивной нагрузки.
На основе разработанной программы можно с высокой степенью достоверности определить условия синхронизации двух генераторов. Чтобы определить возможные пределы отклонений от частоты вращения и несоответствие углу точной синхронизации. Кроме того, предлагаемая программа позволяет оценить размер тока статора и электромагнитного момента синхронных генераторов. Это исследование доказывает работоспособность математической модели двух генераторов.