Application of the Integrated Mathematical Model of Compressor and Induction Motor for Technical Diagnostics

Aleksandrs Gasparjans[1], Aleksandrs Terebkovs[2], [1-2]*Latvian Maritime Academy*, Anastasia Zhiravetska,

Riga Technical University

***Abstract* - The technical diagnostics of the electrical compressor is proposed basing on the control of electromagnetic processors in induction electric motor. The crankshaft of the compressor if hardly connected with the shaft og the motor. Therefore the present technical condition of the compressor (uneven angular velocity and acceleration, time moments of on and off of the valves, ageing of the bearings, etc.) is determined with the spectral distribution of the current consumed by the motor against the rotation angle of the compressor crankshaft. Thus the induction motor is constantly operating in the dynamic regime. The dynamic regime of the electric drive is well to be analysed by means of vector model. A widely applied on board the vessels operation cycle of double-stage compressor is considered for the investigation.**

***Keywords*: compressor, induction motor, mathematical model, diagnostics**

I. Description of the Electric Compressor Installation

Fig 1 demonstrates a differential double-stage compressor. Pistons of the first stage (low pressure) 11 and high pressure 12 are one whole element. Air with its initial pressure P_0 is inhausted through filter 1 and through inhaust valve 2 of the first stage is proceeding to the cylinder space of the low pressure. While piston 11 goes up the compressed air is extruded through the exhaust valve 3 to the pathway of the average pressure P_1. This pathway is equipped with protective valve 5, sensor of instant pressure 6, refrigerator 7, moisture oil separator 8. Cooled and cleaned air of average pressure P1 goes to the high pressure cylinder through exhaust valve 9. The compression of gas in the high pressure cylinder takes place due to the down movement of valve 11, 12.

Relatively to the compressing period in the cylinder of low pressure the compressing period in the second stage is shifted for 180^0 rotation angle of the crankshaft. It allows aligning of the rotation torque on the shaft of the compressor. The compressed gas of high pressure through the exhaust valve of the second stage 10 and refrigerator 13 goes to moisture oil separator 16 and further to receiver 17 and then to consumers.

The pathway of the high pressure has protective valve 14 and sensor of instant pressure 15. The flywheel of the compressor has sign BMT with sensor BMT 18. For defining the angular velocity and angular acceleration of the crankshaft the uniformly placed magnetic signs are used.

20 and sensor of angular velocity 19. The compressor is driven by means of three-phase induction motor 21.

Theoretical indicative pV diagram of the double-stage compressor is demonstrated in fig.2. [1], [2].

At a single-stage adiabatic compressing the expended work would be equal to the square 6-8-1-2-6, that is larger than the square of isothermal compressing 6-7-4-1-2-6. At the double-stage compressing the work of the adiabatic compressing of the first stage is characterised with square 3-9-1-2-3. After the compressing up to intermediate pressure P_1 the gas goes to intermediate refrigerator 7 (fig.1) along adiabatic curve 1-9, where it is cooled at constant pressure P_1 till the initial temperature. Cooling results in the decreasing of the gas volume for value 9-4 equal to the difference V_3-V_2. The initial point of pressure is back to the initial isotherm 1-4-7. The adiabatic pressure at the second stage will be represented with adiabatic curve 4-5 (straight line). Work of compressing. Fig.2

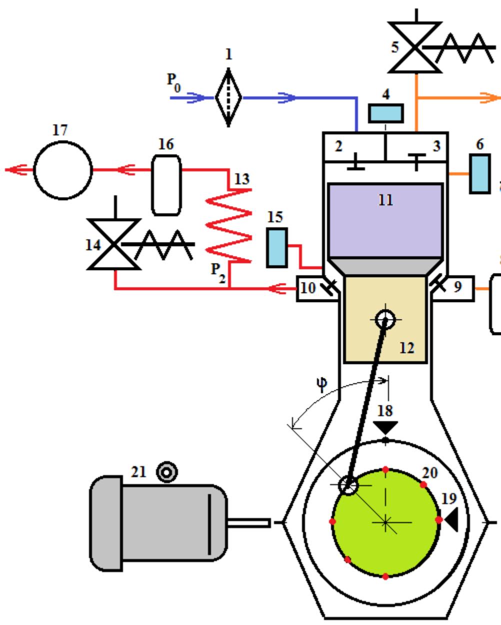

Fig.1. Electric compressing installation

indicated with square 6-5-4-3-6. Therefore at the double-stage compressing we gain work equal to the square 5-8-9-4-5, shaded in the diagram. The work for gas compressing and movement in the double-stage compressor during one turn of the crankshaft corresponds to the indicating diagram square F_i and indicating work L_i on a scale k_0 of pV diagram. (Fig.2)

$$L_i = k_o F_i, \qquad (1)$$

where k_0 - is a scale factor; F_i – square of indicating diagram (fig.3). Indicating power N_i is numerically equal to the indicating work per second.

$$N_i = L_i n, \qquad (2)$$

where n - is a rotation frequency of the crankshaft, sec^{-1}.

The operation processes in the piston compressors take place with the period equal to one turn of the crankshaft. The position of the compressor piston and changing of the cylinder volume is according to the law of a crank gear:

$$dV_{cil}/d\varphi = 0.785\,[\sin\varphi + \frac{\lambda}{2}\sin 2\varphi]\,(D^3_{cil}\phi\omega/2) \qquad (3)$$

According to the rotation angle φ the cylinder volume V_{cil} will be equal to

$$V_{cil} = 0{,}785\,(D^3\phi/2)\;[2a + 1 + \lambda/4 - \cos\varphi - \lambda/4\cos 2\varphi], \qquad (4)$$

where $\phi = S/D$; $\lambda = r/l$; S - is a piston stroke, r - is a radius of the crank, l - length of the connecting rod, φ - angle of rotation of the electro compressor shaft.

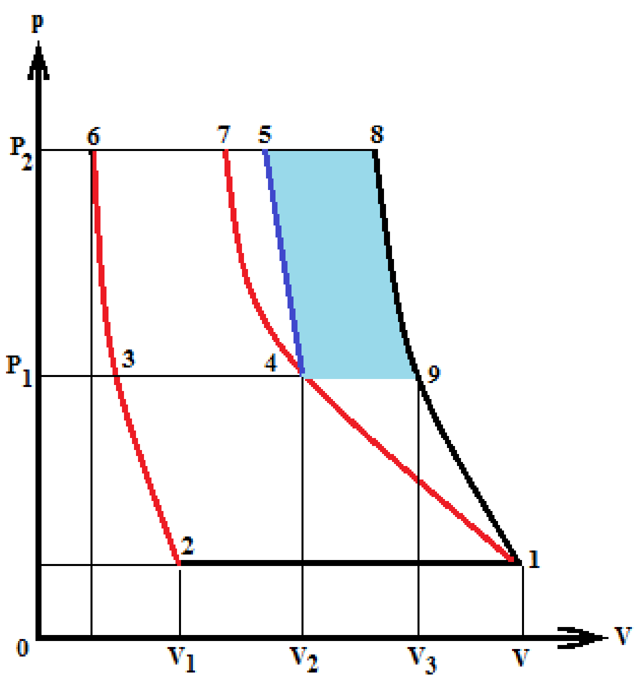

Fig.2. Theoretical indicating pV diagram .

The angle of rotation of the crankshaft φ is measured by means of sensor 19 (fig.1), the instant values of the pressure in the compressor cylinders - by means of sensors 6 and 15 (fig.1). Using the obtained values the current volume of the compressor cylinders is calculated and a real pV-diagram is created. The diagnostics of the compressor is realised comparing the obtained pV-diagram with the reference one - comparing of the squares, calculation of its squares, actual location of the points 1-9 in the reference frame «p» and «V» (fig.2). The difference between the real and ideal pV-diagrams is fixed and input for the comparison into the diagnostic matrix of failures. The matrix forms a report on the current technical condition of the compressor.

The instant resistive torque on the compressor shaft is not constant and depends on the shaft rotation angle φ. Therefore the rotation torque of the induction motor is not constant. The induction motor operates in dynamic mode the parameters of which are strictly connected with the dynamic parameters of the compressor. It is convenient to connect the mathematical model of the piston compressor with that of three-phase induction motor.

II. MATHEMATICAL MODEL OF THREE-PHASE INDUCTION MOTOR WITH SQUIRREL-CAGE ROTOR IN DYNAMIC REGIMES.

The investigation and analysis of the induction motors behaviour in dynamic regimes results in the necessity to develop so called vector models of these machines. Mathematical description of electromechanical transformation of power in electrical machines is based on the development of Kirchhoff's differential equations and their solutions [5]. In matrix type the equations of voltages for three phases of stator A, B, C and three-phases of rotor a, b, c for the model (fig.3) can be written as:

$$[R]\cdot[i(t)] + \tfrac{d}{dt}[\psi(t)] - [U(t)] = 0, \qquad (5)$$

where:
$[U(t)]$ - matrix of voltages ($U_A;U_B;U_C;0;0;0$).

$$u_A = \sqrt{2}\cdot E_f\cdot\sin(\omega_s\cdot t),$$
$$u_B = \sqrt{2}\cdot E_f\cdot\sin(\omega_s\cdot t - \tfrac{2\pi}{3}), \qquad (6)$$
$$u_C = \sqrt{2}\cdot E_f\cdot\sin(\omega_s\cdot t + \tfrac{2\pi}{3}),$$

E_f - phase rms value of the supply voltage;

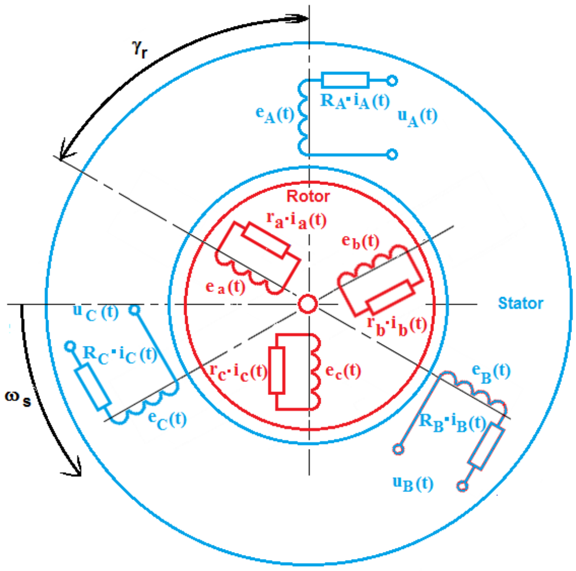

Fig.3. Equivalent physical model of induction motor

$\omega_s = p * f * \tilde{\pi}$ - angular velocity of the stator field rotation, with the number of poles pairs - p , at the network frequency f_s,

[R] - square diagonal matrix of active resistances ($R_A;R_B;R_C;R_a;R_b;R_c$);

[i (t)] - - column matrix of the instant values of the phases currents;

$$[\psi(t)] = \begin{bmatrix} \Psi_A(t) \\ \Psi_B(i) \\ \Psi_C(t) \\ \Psi_a(t) \\ \Psi_b(t) \\ \Psi_c(t) \end{bmatrix}$$ - column matrix of the instant values of the phase flux linkages. (6)

Conventionally the flux linkages of the phases can be divided into flux linkages produced with phase currents stationary according to the considered phase and those rotating with the speed of the rotor.

$$\begin{aligned} &\psi_A(t) = \psi_{As}(t) + \psi_{Ar}(t),\\ &\psi_B(t) = \psi_{Bs}(t) + \psi_{Br}(t),\\ &\psi_C(t) = \psi_{Cs}(t) + \psi_{Cr}(t),\\ &\psi_a(t) = \psi_{ar}(t) + \psi_{aS}(t),\\ &\psi_b(t) = \psi_{br}(t) + \psi_{bS}(t),\\ &\psi_c(t) = \psi_{cr}(t) + \psi_{cS}(t), \end{aligned} \quad (7)$$

where:

$$\Psi_{As}(t) = (l_1 + \tfrac{2}{3}L_1)i_A(t) + \tfrac{2}{3}L_1\cos(\rho)i_B(t) + \tfrac{2}{3}L_1\cos(2\rho)i_C(t);$$
$$\Psi_{Bs}(t) = \tfrac{2}{3}L_1\cos(\rho)i_A(t) + (l_1 + \tfrac{2}{3}L_1)i_B(t) + \tfrac{2}{3}L_1\cos(\rho)i_C(t);$$
$$\Psi_{Cs}(t) = \tfrac{2}{3}L_1\cos(\rho)i_A(t) + \tfrac{2}{3}L_1\cos(\rho)i_B(t) + (l_1 + \tfrac{2}{3}L_1)i_C(t);$$

$$\begin{aligned}
&\Psi_{Ar}(t) = \tfrac{2}{3}M\cos[p\gamma_r(t)]i_a(t) + \tfrac{2}{3}M\cos[p\gamma_r(t) +\\
&+ \rho]i_b(t) + \tfrac{2}{3}M\cos[p\gamma_r(t) - \rho]i_c(t);\\
&\Psi_{Br}(t) = \tfrac{2}{3}M\cos[p\gamma_r(t) - \rho]i_a(t) + \tfrac{2}{3}M\cos[p\gamma_r(t)]i_b(t) +\\
&+ \tfrac{2}{3}M\cos[p\gamma_r(t) + \rho]i_c(t);\\
&\Psi_{Cr}(t) = \tfrac{2}{3}M\cos[p\gamma_r(t) + \rho]i_a(t) + \tfrac{2}{3}M\cos[p\gamma_r(t) -\\
&- \rho]i_b(t) + + \tfrac{2}{3}M\cos[p\gamma_r(t)]i_c(t);\\
&\Psi_{ar}(t) = (l_2 + \tfrac{2}{3}L_2)i_a(t) + \tfrac{2}{3}L_2\cos(\rho)i_b(t) + \tfrac{2}{3}L_2\cos(2\rho)i_c(t);\\
&\Psi_{br}(t) = \tfrac{2}{3}L_2\cos(\rho)i_a(t) + (l_2 + \tfrac{2}{3}L_2)i_b(t) + \tfrac{2}{3}L_2\cos(\rho)i_c(t);\\
&\Psi_{cr}(t) = \tfrac{2}{3}L_2\cos(2\rho)i_a(t) + \tfrac{2}{3}L_2\cos(\rho)i_b(t) + (l_2 + \tfrac{2}{3}L_2)i_c(t);\\
&\Psi_{as}(t) = \tfrac{2}{3}M\cos[p\gamma_r(t)]i_A(t) + \tfrac{2}{3}M\cos[p\gamma_r(t) -\\
&- \rho]i_B(t) + + \tfrac{2}{3}M\cos[p\gamma_r(t) + \rho]i_C(t);\\
&\Psi_{bs}(t) = \tfrac{2}{3}M\cos[p\gamma_r(t) + \rho]i_A(t) + \tfrac{2}{3}M\cos[p\gamma_r(t)]i_B(t) +\\
&+ \tfrac{2}{3}M\cos[p\gamma_r(t) - \rho]i_C(t);\\
&\Psi_{cr}(t) = \tfrac{2}{3}M\cos[p\gamma_r(t) - \rho]i_A(t) + \tfrac{2}{3}M\cos[p\gamma_r(t) + \rho]i_B(t) +\\
&+ \tfrac{2}{3}M\cos[p\gamma_r(t)]i_c(t).
\end{aligned} \quad (8)$$

Summing up of the components of equations (7,8) results in the full flux linkages of the phases.

The solution of the phases voltages equations (5) together with the equation of rotor movement allows define the character of the transient process, calculate the currents, flux linkages and the electromagnetic torque of the motor [5, 6]:

$$\begin{aligned} &J[\tfrac{d}{dt}\Omega(t)] + M = M_R,\\ &\tfrac{d}{dt}\gamma_r(t) - \Omega(t) = 0, \end{aligned} \quad (9)$$

where:

J - the moment of inertia of rotor;

M - electromagnetic torque influencing the rotor;

M_R – the rotating torque of the electro compressor,

$\Omega(t)$ - the rotor rotation speed.

The values of the windings currents are calculated according to the expressions given in paper [4].

The expression for the electromagnetic torque influencing the rotor:

$$\begin{aligned} &M = \tfrac{p}{\sqrt{3}}\{[\Psi_C(t) - \Psi_B(t))] \cdot i_A(t) + [\Psi_A(t) -\\ &- \Psi_C(t)] \cdot i_B(t) + [\Psi_A(t) - \Psi_B(t)] \cdot i_C(t)\} \end{aligned} \quad (10)$$

On the other hand the rotating torque of the compressor is:

$$M = \frac{N}{\omega}; \quad (11)$$

$$N = \frac{N_i}{\eta_k \cdot \eta_m}, \quad (12)$$

where N_i – indicating power of compressor,

η_k, η_m – is an efficiency factor of the compressor and electric motor.

An algorithm and programming package of the mathematical approach under consideration was developed for analysis and numerical solution of the mathematical model of induction motor in dynamic regimes. For the solution of differential equations systems for the mathematical model under consideration the method of Runge-Kutta is supposed to be the most optimal where the range of the step is defined complying with the pre-set values of local and global quantisation errors.

The full flux linkage of the phases and the position of the main magnetic flux vector can be obtained from the instant values of stator currents and voltages.

III. Block-diagram of the measurement part of the compressor diagnostic system

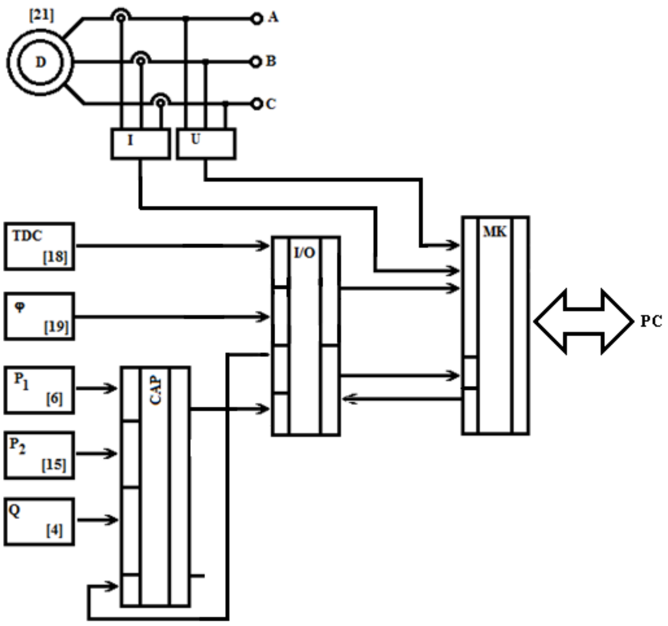

Fig.4. The block diagram of the measurement part

The measurement part of the diagnostic system has two channels. The channel of measuring of compressor parameters consists of 5 sensors - sensors of instant value of pressures of the first and second stages (P_1 and P_2), sensor of vibration of accelerometer Q, sensor of the upper dead point TCD, sensor of the angle position φ of the crankshaft. The data in square brackets define positions of these sensors in fig.1. The analogue signals from the sensors of instant pressure of the first (P_1) and the second stage (P_2) come to the inputs of analogue-digital converter CAP. This is the same place where the signals from the vibro-sensor Q come. Pulse (digit) signals from the sensors of upper dead point TCD and angle position φ of the crankshaft come to the interface I/O. The information in digital form from the CAP output also comes to the interface I/O. The output if I/O interface is connected to microcontroller MK and further to computer C. Computer processes the obtained information. The second channel exists for the measurement of the instant values of the currents and voltages of induction electric motor [21 in fig.1]. The instant values of the currents and voltages of induction motor stator are measured by means of transformer sensors I and U and with their converters transfer the signals in digital form to microcontroller MK. Computer C (fig.4) calculates full flux linkages of the phases, electromagnetic rotating torque, its comparison with resistive torque of the compressor, etc. The current position of the rotor of induction motor (compressor crankshaft), angular velocity (angular acceleration), and transition through the upper dead point is followed by sensors 18 and 19 (fig.1).

IV. Conclusions

A simultaneous investigation of the diagnostic parameters of a compressor and induction motor and their comparison with reference quantities on the basis of a unified mathematical model gives an opportunity of full evaluation of the current technical condition of an electric compressor installation and prediction of its residual life.

References

[1] Б.С. Фотин, И.Б.Пирумов, И.К.Прилуцкий, П.И. Пластинин. Поршневые компрессоры: Учеб. пособие для студентов вузов. –Л.: Машиностроение. Ленингр. Отд-ние, 1987.-372 с.: ил.

[2] М.М.Соколов. Автоматизированный электропривод общепромышленных механизмов. М.: Энергия, 1969. Изд. 2-е, перераб. и дополненное.

[3] A.Zhiravetska, A.Gasparjans, A.Terebkovs. Integrated mathematical model of ship electric compressor installations. 13th International Conference "Maritime transport and Infastructure-2011". Riga, 2011.

[4] A.Gasparjan, J.Greivulis, A.Terebkov. Diagnostics of Ship Power Installations in Transient Regimes. 11th International Power Electronics and Motion Control Conference. Riga.Latvia. 2004.

[5] Иванов-Смоленский А.В. Электрические машины. М.: Энергия.1980.

[6] Важнов А.И. Переходные процессы в машинах переменного тока. Л.: Энергия. 1980.

[7] Greivulis J., Gasparian A., Terebkov A. Complex diagnostics system of Ship Electrical Machines // IEEE -Elecship 98. Proceedings of International Conference on Electric Ship, 1998. Istanbul, TURKEY // Istanbul: Middle East Technical University Turkey, 1998.

Aleksandrs Gasparjans - professor of Latvian Maritime Academy. He graduated the Riga Polytechnical institute in 1970. Qualification - Engineer -electromechanic. Engineer Science Doctor of Latvia from 1992. Latvian Maritime Academy professor from 2004.Research Work Directions:

1. Ship power and electrical mashines reliability, control and tehnical diagnostics;
2. Numerical magnetic field calculations in electrical machines.

Total number of scientific publications 109.

Aleksandr Terebkov, Dr.Sc.Ing. Assoc. professor of Latvian Maritime Academy. He was graduated form Riga Technical University in 1971, obtained Master degree in 1996. The scientific degree of Doctor of sciences he obtained in 2002. The scientific interests are technical diagnostics of power installations, questions of electromagnetic compatibility, power supply sources. Address: 5B Flotes str., Riga, LV-1016, Latvia. E-mail: aleksandrs.terebkovs@latja.lv Phone: +37167161124

Anastasia Zhiravetska, Dr.Sc.Ing. Defended PhD paper in 1999 in Riga Technical University (RTU), Latvia. From2005 she is an associated professor at Industrial Electronics and Electrical Engineering Institute of RTU. She conducts lecturers and practical classes for IEEI bachelor, master and post-graduate students. Fields of scientific interests are power electronics, electric drives, motion control and English for professional and academic purposes, application of modern technologies in translation use of terminology in all these areas. She teaches students and participates in the development of different Latvian local and international projects.

Address: b.Kronvalda 1, LV-1048 Riga, Latvia

Phone: +371 67089917

e-mail: zhiravetska@eef.rtu.lv

Aleksandrs Gasparjans, Aleksandrs Terebkovs, Anastasija Žiravecka. Kompresora un asinhronā dzinēja apvienotā matemātiskā modeļa pielietošana tehniskajai diagnostikai

Rakstā ir aprakstīta divpakāpju elektriskā kompresora tehniskās diagnostikas sistēma. Elektriskā dzinēja un kompresora apvienotais matemātiskais modelis tika pamatots kopā ar mērīšanas daļas blokshēmu. Tiek piedāvāta elektriskā kompresora tehniskās diagnostikas sistēma, pamatota ar elektromagnētisko procesu vadību asinhronajā dzinējā. Kompresora kloķvārpsta ir stingri savienota ar elektriskā dzinēja vārpstu. Tādējādi kompresora darbības cikla esošie stāvokļi (nevienmērīgie leņķiskie ātrums un paātrinājums, vārstu atvēršanas un aizvēršanas momenti, gultņu novecošanās utt.) nosaka elektriskajā dzinējā atkarībā no kompresora kloķvārpstas pagrieziena leņķa patērētās strāvas spektrālo sastāvu. Tādējādi asinhronais elektrodzinējs visu laiku darbojas dinamiskā režīmā. Dzinēja dinamisko režīmu ir izdevīgi analizēt ar vektoriālā modeļa palīdzību. Elektrisko mašīnu enerģijas elektromehāniskās pārveidošanas procesu matemātiskais apraksts tiek pamatots ar Kirhhofa diferenciālo vienādojumu sastādīšanu un risināšanu. Lai izanalizētu un skaitliski atrisinātu asinhronā dzinēja matemātisko modeli dinamiskajos režīmos, tika izstrādāts aplūkotā matemātiskā aparāta algoritms un programmu komplekss. Par visoptimālāko metodi šā modeļa diferenciālo vienādojumu sistēmas risinājumam akceptēta Runge-Kuttes metode ar mainīgo soli, kur soļa vērtība tiek noteikta no diskretizēšanas lokālās un globālās kļūdas uzdoto vērtību apmierināšanas nosacījuma. Tika aplūkots kuģos plaši pielietotā divpakāpju kompresora darba cikls. Kompresora un asinhronā dzinēja diagnostikas parametru vienlaicīga izpēte un salīdzināšana ar to etalonu parametriem uz apvienotā matemātiskā modeļa bāzes ļauj vispusīgi novērtēt kompresora iekārtas esošo stāvokli un prognozēt paliekošo resursu.

Александр Гаспарян, Александр Теребков, Анастасия Жиравецкая. Применение объединенной математической модели компрессора и асинхронного двигателя для технической диагностики

Описана система технической диагностики двухступенчатой электрокомпрессорной установки. Дано математическое обоснование объединённой математической модели электродвигателя и компрессора, блок-схема измерительной части. Предлагается система технической диагностики электрокомпрессора, основанная на контроле электромагнитных процессов в асинхронном электродвигателе. Коленчатый вал компрессора жестко связан с валом электродвигателя. Поэтому текущее техническое состояние компрессора (неравномерные угловая скорость и угловое ускорение, моменты открытия и закрытия клапанов, износ подшипников и т.д.) определяет спектральный состав потребляемого электродвигателем тока в зависимости от угла поворота коленчатого вала компрессора. Т.о. асинхронный электродвигатель всё время работает в динамическом режиме. Динамический режим электродвигателя удобно анилизировать с помощью векторной модели. Математическое описание процессов электромеханического преобразования энергии в электрических машинах основано на составлении и решении дифференциальных уравнений Кирхгофа. Для анализа и численного решения математической модели асинхронного двигателя в динамических режимах, разработан алгоритм и программный комплекс представленного математического аппарата. Наиболее оптимальным методом решения систем дифференциальных уравнений, представленной математической модели, является метод Рунге-Кутта с переменным шагом, в котором величина шага определяется из условия удовлетворения заданных значений локальной и глобальной ошибок дискретизации. Рассмотрен рабочий цикл двухступенчатого компрессора, широко применяемый на морских судах. Одновременное исследование диагностических параметров компрессора и асинхронного электродвигателя и их сравнение с эталонными значениями на базе объединённой математической модели позволяет произвести полную оценку текущего технического состояния электрокомпрессорной установки и прогнозировать её остаточный ресурс.