The Research of the Processes of the Squirrel-cage Induction Motor's Direct Start-up in the Setting of the Rotor's Variable Parameters

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Abstract **– The exploitation of the squirrel-cage induction motors (SCIM) is impedingly accompanied by the emersion of the transient processes that critically impact on the condition and operation of electrical machines. The high powered induction motors with the squirrel-cage rotor have the powerful starting current and rotation torque, therefore, their starting regime exercises a significant influence on the electrical power system regime. The rotor's parameters are very essential when analysing the transient processes in the induction motors. The rotor's resistance defines the starting current and the rotation torque of the induction motor. During the start-up the resistance and the inductance of the rotor change under the current's displacement effect. The current paper is dedicated to the research of the direct start-up of the SCIM with due account for of the current's displacement in the rotor's slots. The paper displays the equivalent circuit of rotor bar and the calculations of its resistance and inductance that change under the skin effect for free form slots. Moreover, the paper presents the calculation and the comparison of the dynamic characteristics of the SCIM with the constant and variable parameters of the rotor.**

Keywords **– Induction motors, transient process, current's displacement effect, resistance, inductance, slip, starting torque.**

I. Introduction

As it is known, the induction motor's start-up characteristics (IM) could be significantly improved by using the special structures of the short-circuited (squirrel-cage) rotor that allow to increase the starting torque and to reduce the motor starting current. The constructions of the short-circuited rotor are various modifications of the deep bar rotors and bicages rotors, in which the current's displacement effect in the squirrel-cage rotors is used [1]. The distinctive feature of the above-mentioned motors is that resistance and inductance of the rotor (R_2 and X_2) depend on slip values and, therefore, at the motor start-up, when the slip changes within the range starting from $s = 1$ up till the value of the steady regime $s = 0{,}01 \div 0{,}06$, these parameters change in the rather wide range. In the majority of cases the current's displacement effect has a positive role by increasing the resistance of the rotor, decreasing the inductance of the rotor and increasing the starting torque of the motor. However, uneven distribution of the electric current density over the rotor bar could lead to undesirable effects up to and including the motor breakdown. Hence, the tracking of the impact of the current's displacement effect is considered to be a very important task during the research of the transient processes of the squirrel cage induction motors [2].

II. Description *SCIM* Mathematical Model

For the induction motor transient process analysis the Park-Gorev equations are used, the solution of which allows to estimate the current, electromagnetic torque and other values that are being changed during the transient process [3]. The Park-Gorev equations take the following form in the phase system of coordinates:

$$\begin{cases} u_{sa} = r_s \cdot i_{sa} + \dfrac{d\Psi_{sa}}{dt} \\ u_{sb} = r_s \cdot i_{sb} + \dfrac{d\Psi_{sb}}{dt} \\ u_{sc} = r_s \cdot i_{sc} + \dfrac{d\Psi_{sc}}{dt} \end{cases} \qquad (1)$$

$$\begin{cases} u_{rA} = r_r \cdot i_{rA} + \dfrac{d\Psi_{rA}}{dt} \\ u_{rB} = r_r \cdot i_{rB} + \dfrac{d\Psi_{rB}}{dt} \\ u_{rC} = r_r \cdot i_{rC} + \dfrac{d\Psi_{rC}}{dt} \end{cases} \qquad (2)$$

$$J\frac{d\omega}{dt} = M_{em} - M_{load} \qquad (3)$$

where u_{sa}, u_{sb}, u_{sc} - are the phase voltage;
i_{sa}, i_{sb}, i_{sc} - are phase stator current;
u_{rA}, u_{rB}, u_{rC} - are rotor rings voltage;
i_{rA}, i_{rB}, i_{rC} – are phase rotor current;
r_s, r_r - the stator resistance and the rotor resistance;
Ψ_{si}, Ψ_{ri} – total flux linkage with the phase windings of the stator and rotor accordingly;
M_{load} – load torque on the IM shaft;
M_{em} – IM electromagnetic torque;
$J\dfrac{d\omega}{dt}$ - dynamic torque.

The solution of the similar system that is nonlinear and contains circulating coefficients, of the differential equations in the phase coordinates is difficult. One of the possible methods how to obtain the equation with the constant coefficients is the method of the transformation of the coordinates that is based on the theory of two reactions related to all voltages, currents, flux linkages decomposition along two mutually perpendicular coordinate axis. The current paper presents the IM mathematical model that has been generated in the set of coordinate axes *d, q, 0*, i.e. the set of the axes is statical as relative to the rotor and rotates relative to the stator with the rotor's rotation speed (ω_κ=

ω_r). Subsequently, the modified Park-Gorev's equations take the following form:

$$\begin{bmatrix} u_{sd} \\ u_{sq} \\ u_{s0} \end{bmatrix} = R_s \begin{bmatrix} i_{sd} \\ i_{sq} \\ i_{s0} \end{bmatrix} + \frac{d}{dt}\begin{bmatrix} \Psi_{sd} \\ \Psi_{sq} \\ \Psi_{s0} \end{bmatrix} + \begin{bmatrix} -\Psi_{sq} \\ \Psi_{sd} \\ 0 \end{bmatrix} \cdot \frac{d\theta_k}{dt} \tag{4}$$

$$\begin{bmatrix} u_{rd} \\ u_{rq} \\ u_{r0} \end{bmatrix} = R_r \begin{bmatrix} i_{rd} \\ i_{rq} \\ i_{r0} \end{bmatrix} + \frac{d}{dt}\begin{bmatrix} \Psi_{rd} \\ \Psi_{rq} \\ \Psi_{r0} \end{bmatrix} + \begin{bmatrix} -\Psi_{rq} \\ \Psi_{rd} \\ 0 \end{bmatrix} \cdot \left(\frac{d\theta_k}{dt} - \frac{d\theta}{dt} \right) \tag{5}$$

$$T_M \frac{d\omega}{dt} = \frac{3}{2} L_{ad}\left(i_{rd} \cdot i_{sq} - i_{rq} \cdot i_{sd}\right) - M_{load} \tag{6}$$

where θ_k - the angle between longitudinal axis d of the set of coordinate axes *d, q, 0,* rotating with the free speed, and magnetic axis of the phase motor's stator ;

θ_k - θ- the angle between longitudinal axis d and magnetic axis of the phase motor's rotor ;

$$\frac{d\theta_k}{dt} = \omega_k, \quad \frac{d\theta}{dt} = \omega.$$

Usually, solving the equations (4) - (6), it is asumed that rotor's parameters R_r (R_2) and X_r (X_2) are the constant values. However, the following approach is incorrect and it could contribute significant errors into the calculation data of high power induction motors. In the correct analysis of the transient process the functional dependence $R_2(s)$ and $X_2(s)$ should be applied.

The numerical methods that allow to solve the problem of the defining the rotor parameters to a high accuracy (these are mesh method, finite element method FEM, secondary sources methods and etc.) are labour intensive and time consuming, that eliminates the possibility to build up on their basis simple engineering calculation methods [4],[5],[6].

In the calculation practice the transformation of the parameters of the IM is based on k_r and k_x coefficients, that are defined as the function of the slip of IM on the assumptions that do not considerate the real picture during the transient process. Such an approach in the case of the deep bar rotors and bicage rotors does not provide the required calculation accuracy [2].

III. The squirrel-cage induction motor equivalent circuit

The current paper offers a general approach and presents a numerical method of calculation of the variable parameters of the rotor slots with arbitrary configuration [1].

The rotor circuit equivalent scheme with the variable parameters that are influenced by the current's displacement effect could be presented in the form of the multi-link circuit with the constant and independent of the current displacement impedances (fig. 1).

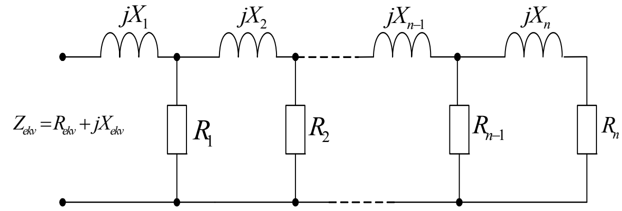


Fig 1. The rotor bar equivalent scheme.

The calculation method of the rotor bar displacement of the multi-linked scheme of impedances is based on the following. Knowing the configuration of the magnetic lines of flux leakage in the slot, we will present a massive electrical conductor (a squirrel-cage winding rotor bar) divided into a great number of elementary layers isolated from each other by the thin layer of the insulation.

Let us presume that there is a plain-parallel lines filed in the slot, and electric current density along the line of flux is constant. With regards to a relevantly low height of the elementary layers these allowances do not bring into any considerable errors of the calculations. With the account of these assumption the parameters of the equivalent scheme (fig. 1) are resistance R_i and inductance X_i of the elementary layer *i:*

$$R_i = \rho_i \cdot l / q_i \tag{7}$$

where ρ– resistivity of the rotor bar material;

q_i - cross sectional area of the *i* layer;

l - the length of the *i* elementary layer.

$$X_i = \omega_2 \mu_0 \lambda_i \cdot l = 2\pi f_1 s \mu_0 \lambda_i \cdot l \tag{8}$$

where ω_2 – current's angular frequency of the rotor bar;

λ_i - geometrical conductivity of the magnet tube the borders of which are defined by the *i* elementary layer.

IV. The calculations of the rotor's resistance and inductance

As an example for calculation purposes, we have chosen the induction motor 4AH355S4U3 with the deep slots in the rotor. On the basis of the equivalent scheme the functional dependences $R_2(s)$ and $X_2(s)$ for the chosen IM have been obtained (fig. 2 and fig. 3).

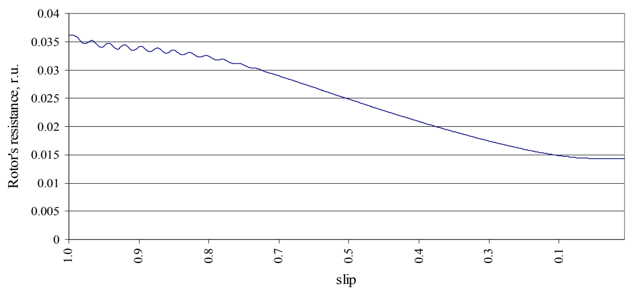


Fig. 2. The rotor's resistance at the start-up time.

From the obtained dependence (fig. 2) it is obvious that during the start-up moment *t=0 (s=1)* the rotor's resistance reaches its maximum $R''_{2start} = 0{,}03613$ *r.u.* With the acceleration of the motor, the slip reduces as well as the value of the resistance reaching its nominal value $R''_{2nom} = 0{,}01427$ *r.u.* The received starting and nominal resistance values practically coincide with the data for the chosen SCIM ($R''_{2start} = 0{,}038$ *r.u.* and $R''_{2nom} = 0{,}014$ *r.u.*).

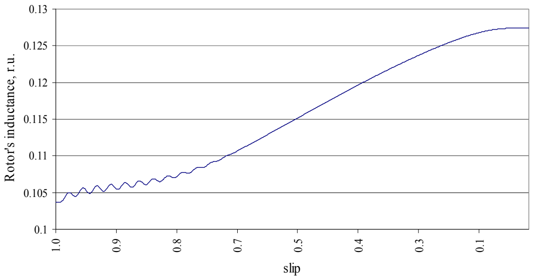


Fig. 3. The rotor's inductance at the start-up time.

From the obtained dependence (fig. 3) it is obvious that during the start-up moment *t=0* (*s=1*) the inductance of the rotor has its minimum $X''_{2start} = 0{,}1034$ *r.u.* With the acceleration of the motor, the slip reduces whereas the value of the inductance increases up to $X''_{2nom} = 0{,}1289$ *r.u.* The received starting and nominal inductance values practically coincide with the catalogue data for the chosen IM ($X''_{2start} = 0{,}08$ *r.u.* and $X''_{2nom} = 0{,}14$ *r.u.*).

V. THE CALCULATIONS OF THE DYNAMIC CHARACTERISTICS OF THE SCIM

On the basis of the equivalent scheme the dynamic characteristics with Park-Gorev equations with the variable parameters of the rotor for the chosen SCIM 4AH355S4U3 have been obtained. The Park-Gorev equations in relative units are solved.

Load simulation is provided in relative units as follows:

$$M_{load} = M_c + k \cdot \omega^2$$

where $M_c = 0$ - static torque; *k = 0.702* – coefficient.

Moment of inertia *J = 5,8* $kg \cdot m^2$ is simulated with mechanical time constant:

$$T_M = \frac{J \cdot n_{nom}}{9{,}55 \cdot M_{nom}} \cdot 314{,}159 = \frac{5{,}8 \cdot 1500}{9{,}55 \cdot 2032} \cdot 314{,}159 = 140.74$$

Dynamic characteristics *I=f(t),* M_{em}*=f(t), n=f(t)* are presented in fig.4.

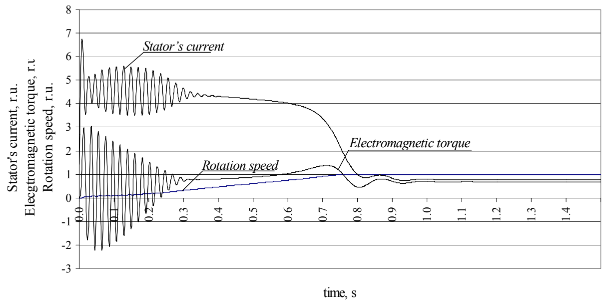


Fig. 4. Dynamic characteristics of the IM 4AH355S4У3 with the variable parameters of the rotor.

Then influence of the current displacement effect on the dynamic characteristics *I=f(t),* M_{em}*=f(t), n=f(t)* are calculated for the model of idealized generalized IM with the constant parameters of the rotor. SCIM 4AH355S4U3 parameters are used in relative units for nominal mode of operation:

stator's resistance *0.019 r.u.;*
stator's inductance *0.11 r.u.;*
rotor's resistance *0.014 r.u.;*
rotor's inductance *0.14 r.u.;*
magnetizing inductance *4.6 r.u.*

The comparison of the dynamic characteristics are presented in fig.5, fig. 6 and fig.7.

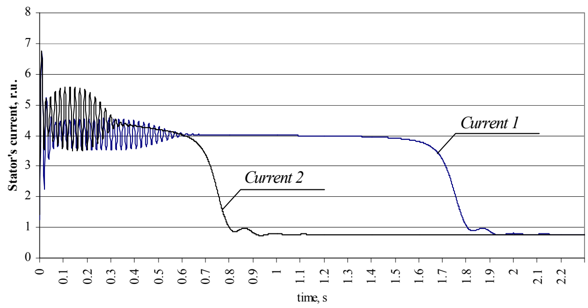


Fig. 5. Stator's current for SCIM 4AH355S4U3:
1 – mode of operation with constant rotor's parameters R_2 and X_2;
2 – mode of operation with variable rotor's parameters R_2 and X_2.

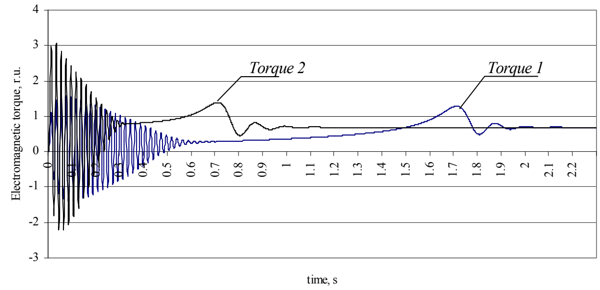
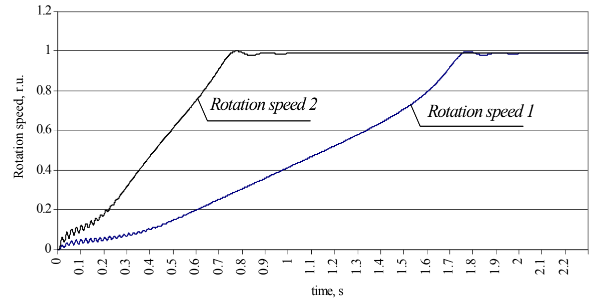


Fig. 6. Electromagnetic torque for SCIM 4AH355S4U3:
1 – mode of operation with constant rotor's parameters R_2 and X_2;
2 – mode of operation with variable rotor's parameters R_2 and X_2.

Fig. 7. Rotation speed for SCIM 4AH355S4U3:



1 – mode of operation with constant rotor's parameters R_2 and X_2;
2 – mode of operation with variable rotor's parameters R_2 and X_2.

From the obtained dependences (fig. 4, fig. 5, fig. 6) it is obvious that transient process proceeds faster in the mode with variable rotor's parameters; IM reaches its steady mode of operation during $t_{start} = 0{,}9s$. In mode of operation with constant rotor's parameters IM reaches its steady mode during $t_{start} = 1{,}8s$.

As it is seen from *I=f(t)* characteristics (fig. 5), the first current's peaks are the same independent on the mode of operation ($I_{max} = 6.4\,r.u.$ at $t = 0.009\,s$). In steady mode current's values are identical and reaches its rated value $I_{nom} = 0.77\,r.u.$ Current's relation is:

$$\frac{I_{start}}{I_{nom}} = \frac{6.738}{0.77} = 8.75$$

As it is seen from M_{em}*=f(t)* characteristics (fig. 6), in time interval *t=0÷0.25 s* oscillations of electromagnetic torque occur synchronously in both modes of operation. In steady-state condition of operation torque values are identical and reaches its nominal value $M_{nom} = 0.687\,r.u.$ However, start torque's value is a lot more in mode of operation with variable rotor's parameters.

Start torque relations are:

$\frac{M_{start}}{M_{nom}} = \frac{1{,}61}{0{,}687} = 2{,}35$ - mode of operation with constant rotor's parameters;

$\frac{M_{start}}{M_{nom}} = \frac{3{,}059}{0{,}687} = 4{,}45$ - mode of operation with variable rotor's parameters.

This research shows that the variable parameters effect on the dynamic characteristics of induction motor's direct start-up.

VI. The calculations of the dynamic characteristics of the SCIM with program complex *PSIM*

Simulation of SCIM start-up in settings of the rotor's **constant parameters** $R_2'=0.014$ *r.u.* and $X_2'=0.14$ *r.u.* is carried out by the scheme presented in fig. 8.

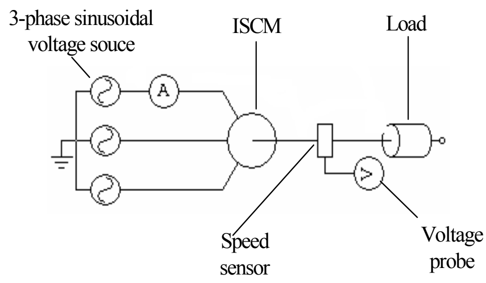


Fig. 8. SCIM start-up with rotor's constant parameters simulation scheme.

SCIM 4AH355S4U3 parameters used for simulation are:
line voltage 660 V;
supplying source's frequency 50 Hz;
stator's resistance 22.42 mΩ;
stator's inductance 0.4134 mH;
rotor's resistance (nominal value) 16.52 mΩ;
rotor's inductance (nominal value) 0.5261 mH;
magnetizing inductance 17,29 mH;
number of poles - 4;
moment of inertia $J = 5{,}8\ kg \cdot m^2$

Load torque: $M_{load} = M_c + k \cdot \omega^2$

where $M_c = 0$ - static torque;

$$M_{load} = M_{nom} \cdot 0{,}702 = 2032 \cdot 0.702 = 1426.46\,Nm;$$

$$\omega = \frac{2 \cdot \pi \cdot n}{60} = \frac{2 \cdot \pi \cdot 1482}{60} = 1551/s$$

angular mechanical speed;

$$k = \frac{M_{load}}{\omega^2} = \frac{1426.46}{155^2} = 0.059 - \text{coefficient.}$$

The obtained dynamic characteristics *I=f(t),* M_{em}*=f(t), n=f(t)* are presented in fig. 9.

From the dynamic characteristics the following relations are obtained.

Start torque relation is:

$$\frac{M_{start}}{M_{nom}} = \frac{3648}{1429} = 2{,}55.$$

Start current relation is:

$$\frac{I_{start}}{I_{nom}} = \frac{3071}{314} = 9.78 \cdot$$

Start-up duration is $t_{start} = 1.6s$.

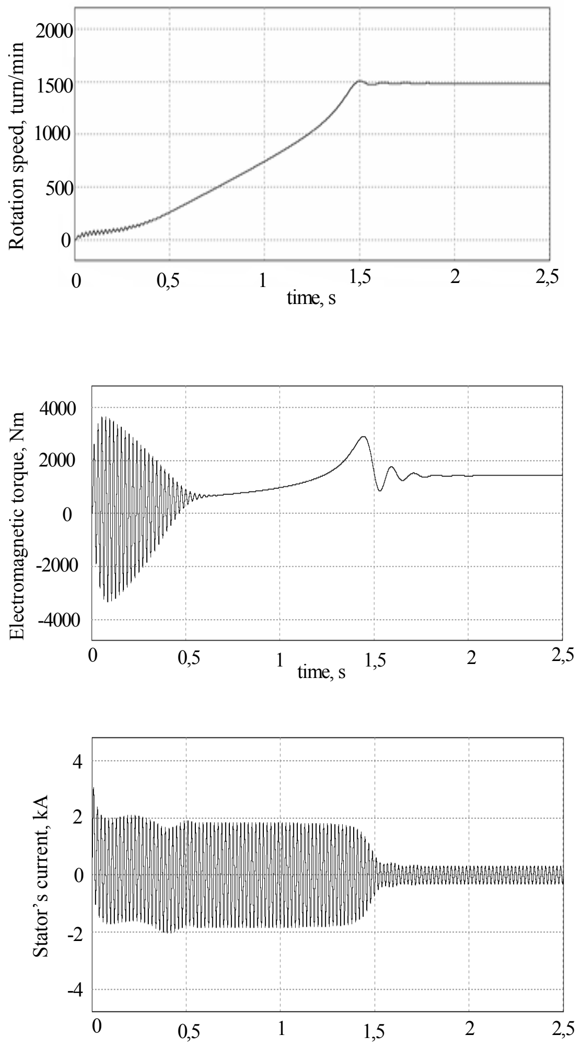


Fig. 9. Dynamic characteristics of the SCIM 4AH355S4У3 with the constant parameters of the rotor *(PSIM)*.

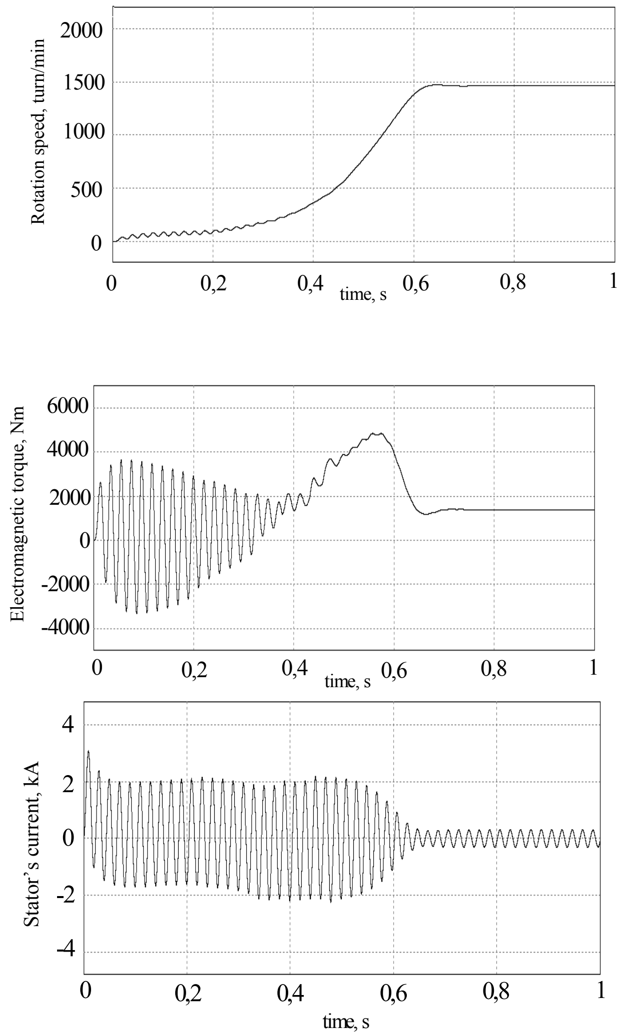


Fig. 10. Dynamic characteristics of the SCIM 4AH355S4У3 with the variable parameters of the rotor *(PSIM)*.

SCIM start-up simulation with program *PSIM* in settings of rotor's variable parameters R_2 и X_2 is based on scheme (fig. 8).

In each phase rotor's circuit five sections are introduced, containing resistance (*5.66 m*Ω) and inductance (0.*045 mH*). When SCIM reaches certain rotation speed the inductance is included in rotor's circuit, but resistance is replaced on short-circuit. At start-up moment the nominal resistance *16.52 m*Ω and starting inductance *0.3 mH* are set in. Thereby, the resistance's decrease and the inductance's increase during when SCIM start-up are simulated.

Dynamic characteristics of the SCIM 4AH355S4У3 with the variable parameters of the rotor are presented in fig.10.

From the dynamic characteristics the follow relations are obtained.

Start torque relation is:

$$\frac{M_{start}}{M_{nom}} = \frac{3632}{1394} = 2{,}61$$

Start current relation is:

$$\frac{I_{start}}{I_{nom}} = \frac{3065}{308} = 9{,}95$$

Start-up duration is $t_{start} = 0{,}7 \div 0{,}8s$.

VI. Conclusions

1. During the start-up the resistance of rotor decreases under the current's displacement effect in the rotor bars. Moreover the starting value of resistance depends on the frequency of supply source and the slip.

2. During the start-up the inductance of the rotor increase under the current's displacement effect in rotor's bars. What is more, the starting value of inductance depends on the frequency of supply source and the slip too.
3. The obtained algorithm allows to calculate accurately the resistance and inductance during the SCIM start-up. Also it makes possible to take into account changes in the calculations of the transient processes. Comparison of results obtained by different methods confirms the validity of the calculations.
4. Accounting the variables rotor's parameters alters the course of the transient process. The duration of the transient process becomes shorter by increasing the average torque developed by SCIM during the start-up.

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Jūlija Maksimkina. Procesu izpēte tieši palaižot īsslēgto asinhrono dzinēju ar mainīgiem rotora parametriem
Asinhrono dzinēju (AD) dinamiskās raksturlīknes nosaka strāvas un momenta izsitienus pārejas procesu laikā. Tāpat tās nosaka rotācijas frekvences uzspiestā režīmā. Parametri dinamiskajos režīmos mainās strāvas izspiešanas dēļ rotora stieņos. Sevišķi tas attiecas uz dzinējiem ar aizvērtām rievām, kuru dziļums ir lielāks par viļņa iespiešanās dziļumu. Praktiskos aprēķinos AD parametru maiņu novēro, izmantojot koeficientus k_r un k_x, kuri neievēro realitāti pārejas procesa laikā. Dziļrievu un dubultrievu rotoru gadījumos šāda pieeja nenodrošina nepieciešamo aprēķinu precizitāti. Šajā darbā tiek piedāvāta vispārējā pieeja un skaitliskā metode parametru aprēķināšanai rotoriem ar brīvas formas rievām. Rotora ķēdes aizvietošanas shēma satur daudzposmu ķēdi, kuru veido pretestības, kas nav atkarīgas no strāvas izspiešanas. Aizvietošanas shēmai aprēķināta kompleksā ekvivalentā pretestība, kuras reālajai daļai atbilst rotora aktīvā, bet imaginārajai – rotora induktīvā, no slīdes atkarīgā, pretestība. Šīs pretestības tiek izmantotas Goreva-Parka vienādojumu risināšanā. Uz aizvietošanas shēmas pamata sastādīta kompleksās ekvivalentās pretestības un dinamisko raksturlīkņu aprēķinu programma. Goreva-Parka vienādojumos rotora parametri R_r (R_2) un $X_r(X_2)$ ir mainīgie lielumi. Dots AD dinamisko raksturlīkņu aprēķinu rezultātu salīdzinājums ar mainīgiem un nemainīgiem rotora parametriem. Ar kataloga datiem salīdzināti rotora parametru R_2'', X_2'' aprēķinu rezultāti; tiek veikts dinamisko raksturlīkņu aprēķins ar mainīgiem un nemainīgiem rotora parametriem ar *PSIM* programmas kompleksu, dots dinamisko raksturlīkņu aprēķinu rezultātu salīdzinājums.

Юлия Максимкина. Исследование процессов прямого пуска асинхронного короткозамкнутого двигателя при переменных параметрах ротора
Динамические характеристики асинхронных двигателей (АД) представляют значительный интерес при определении бросков токов и ударных моментов при переходных процессах, а также времени выхода на установившуюся частоту вращения. Расчет динамических характеристик АД затруднен необходимолстью учета изменения параметров машины, вызванных эффектом вытеснения тока в стержнях ротора. Особенно это касается АД, имеющих на роторе закрытые пазы различной конфигурации, высота которых превышает глубину проникновения волны. В расчетной практике изменение параметров АД учитывается коэффициентами k_r и k_x, которые определяются как функции скольжения машины при допущениях, которые не учитывают реальную картину при переходном процессе. Такой подход в случае глубокопазных и двухклеточных роторов не обеспечивает требуемую точность расчетов.
В данной работе предложен общий подход и приведен численный метод расчета переменных параметров пазов ротора произвольной конфигурации. Схема замещения роторной цепи с изменяющимися под влиянием эффекта вытеснения тока параметрями представлена в виде многозвеньевой цепи с постоянными, не зависящими от вытеснения тока, сопротивлениями. Для полученной схемы замещения рассчитано комплексное эквивалентное сопротивление, которое содержит реальную (активное сопротивлениеротора) и мнимую (индуктивное сопротивление ротора) части, зависящие от величины скольжения. Данные активное и индуктивное сопротивлениярotoраиспользуются для решения уравнений Парка-Горева. На базе схемы замещения составлена программа расчета комплексного эквивалентногосопротивления и динамических характеристик АД с использованием уравнений Парка-Горева, в которых параметры ротора R_r (R_2) и $X_r(X_2)$ переменные величины. Проведено сравнение результатов расчета динамических характерик АД с учетом и без учета вытеснения тока в пазах ротора. Проведено сравнение результатов расчета параметров ротора R_2'', X_2'' с данными каталога. Проведен расчет динамических характерик АД с учетом и без учета вытеснения тока в пазах ротора с помощью программного комплекса *PSIM*.