Optimal Control of Pantograph-Catenary Systems using MatLab

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Abstract

This paper continues research of the pantograph-catenary system started in previous papers [1,2] and presented in the First International Conference on Railway Technology [3] . The main purpose of the study is to use the computer to optimize pantograph-catenary system of a high-speed train by reducing power consumption when basic parameters of pantograph and catenary (contact network) are changing in time randomly.

A linear model of pantograph-catenary system is considered where the upper and lower blocks of pantograph and catenary are modelled using lumped masses, springs and shock absorbers. Input and output system signals are measured when the train moves. These signals are processed by parametric identification algorithms to determine current values of the system matrices. State matrices are used in the Riccati equation to calculate controller coefficients. Adaptive controllers provide dynamic stability of the system when its parameters are changing in time and random external perturbations are present.

Keywords: pantograph-catenary system, Riccati equation, mechanical multibody system, parametric identification, controller, adaptive pantograph.

1 Introduction

Since October 1964 when first high-speed railway line with 210 km/h speed was commissioned in Japan have gone nearly 50 years. Many of high-speed rail problems have been resolved successfully during this time period. In coming years experts from leading industrialized countries (Japan, France and others) plan to increase train speed to 500 km/h. At the same time scientists have identified those problems that need to be addressed to further enhance progress in this area [4]. In the following block diagram (Figure 1) the solution of these problems will be investigated by modelling with computer technology. As can be seen from the structure, one of the main problems is the study of models with time-varying parameters of the main components of this high-speed rail system.

In present paper some results about dynamic characteristics of above mentioned block-scheme components with time-changing parameters, pantograph and catenary, are considered.

In contrast to previously published results on pantograph-catenary system [5, 6] we present results about the system with optimal control methods based on parameter identification and adaptive control algorithms. These methods of dynamic systems were investigated by the authors for various purposes [1, 2], [7,8].

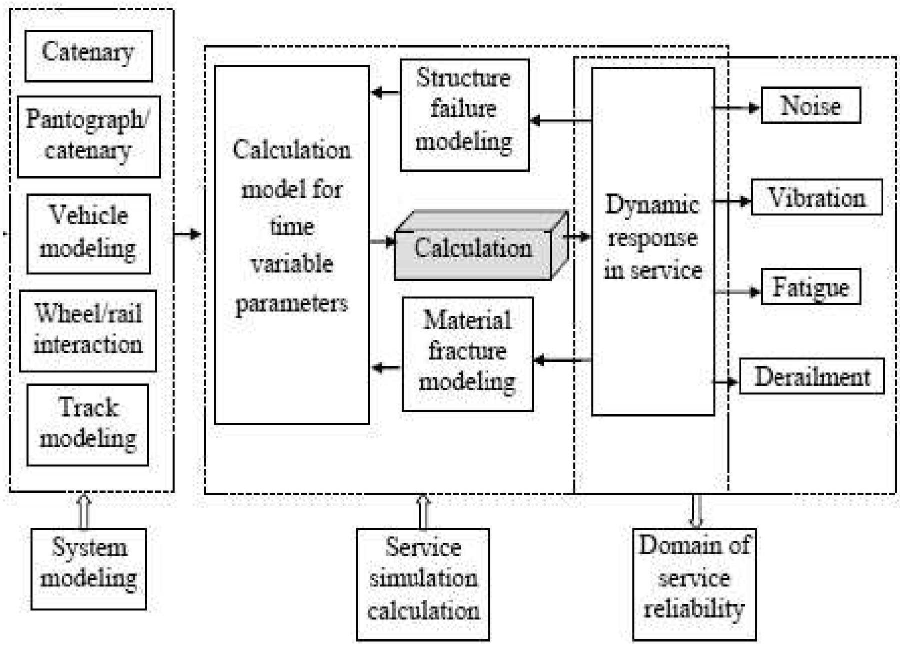


Figure 1. Computer simulation of service reliability for high-speed trains

In this paper parameters of pantograph-catenary model vary randomly in time, and the system is periodically subjected to external perturbations.

Recorded input and output signals of the system during it functioning in real time provide information about system states. These signals are processed using parametric identification algorithms, which include the following main tasks:
- measurement of input and output signals of the system at a given time interval;
- calculation of state vector of pantograph model;
- calculation of parameters current values of the system mathematical model;
- calculation of active pantograph controller parameters for every time interval where the state vector of pantograph and the mathematical model parameters are calculated;
- correction of pantograph controller coefficients when the model parameters and the state vector are changing at a given time interval.

Solutions of parametric identification problem for the system by means of pantograph state vector allow to calculate coefficients of adaptive controller based on the Riccati equation [2]. Created on these principles adaptive controller guarantees optimal dynamic characteristics of the pantograph-catenary system both in case of parameters random changes in time and in case of external disturbances (drums) on the system.

Based on Matlab/Simulink simulation results of the developed control algorithms for pantograph-catenary system with time-varying parameters confirm effectiveness of the system compared to other methods of solving the problem using for example a second-order sliding mode control scheme [5].

2 Mathematical model of the system with varying parameters

2.1 Catenary mathematical model with time-varying parameters

Catenary mathematical model is usually represented by the 2nd order differential equation with time-varying parameters [1, 8] :

$$M_i(t)\ddot{z}_i(t) + C_i(t)\dot{z}_i(t) + K_i(t)z_i(t) = Q_i(t) \ , \tag{1.1}$$

where

$z_i(t)$ – amplitude of *i*-th modal component,

$M_i(t)$ – mass of *i*-th modal component,

$C_i(t)$ – damping coefficient of *i*-th modal component,

$K_i(t)$ – stiffness coefficient of *i*-th modal component,

$Q_i(t)$ – forcing function of *i*-th modal component.

Taking the frequency of oscillations for the *i*-th component of the model by $\omega_i(t)$ ($\omega_i(t) = \sqrt{\frac{K_i(t)}{M_i(t)}}$), and the damping factor with respect to other model parameters by $\varepsilon_i(t)$, Equation (1.1) takes the form [1]:

$$M_i(t)\ddot{z}_i(t) + 2M_i(t)\varepsilon_i(t)\dot{z}_i(t) + M_i(t)\omega_i^2(t)z_i(t) = Q_i(t) \ . \tag{1.2}$$

For a given shape and vibration frequency of catenary signals time response for every component is defined by Equation (1.2), and the output signal of the entire catenary model is equal to the sum of all $M_i(t)$ components.

2.2 Pantograph mathematical model with variable parameters

Mathematical model of passive pantograph is defined by 2nd order differential equation with time-varying parameters [1] :

$$
\begin{aligned}
&m(t)\ddot{y}_p(t) = -F_c(t) - w(t)(y_p(t) - y_f(t)) - u(t)(\dot{y}_p(t) - \dot{y}_f(t)),\\
&M(t)\ddot{y}_f(t) = F_s(t) + w(t)(y_p(t) - y_f(t)) + u(t)(\dot{y}_p(t) - \dot{y}_f(t)) -\\
&\quad v(\dot{y}_f(t) - \dot{y}_r(t)) - t_m(y_f(t) - y_r(t)),
\end{aligned}
\tag{1.3}
$$

where

$y_p(t)$ – displacement of the head,

$y_f(t)$ – displacement of the frame,

$y_r(t)$ – displacement of the vehicle roof,

$F_c(t)$ – contact force acting on the pantograph head,

$F_s(t)$ – permanent lifting of static force.

Assuming equation (1.3) with constant coefficients *m*, *M*, *w*, and *u*, Laplace Transform can be applied with zero initial conditions for Y_p, Y_f, and F_c. As a result, we receive

$$
\begin{aligned}
&(ms^2 + us + w)Y_p + F_c = (us + w)Y_f,\\
&(Ms^2 + (u+v)s + (w + t_m))Y_f = (us + w)Y_p
\end{aligned}
\tag{1.4}
$$

According to Equation (1.4), transfer function of the pantograph can be found. Evaluating Y_p / X_0 and Y_f / X_0 we obtain

$$
\frac{Y_p(s)}{X_0(s)} = \frac{Ms^2 + (u+v)s + w + t_m}{\Delta} K_w \tag{1.5}
$$

$$
\frac{Y_f(s)}{X_0(s)} = \frac{us + w}{\Delta} K_w, \tag{1.6}
$$

where

$$
\begin{aligned}
\Delta = {}& Mms^4 + (Mu + m(u+v))s^3 + (M(w + K_w) + m(w + t_m) + u(u+v) - u^2)s^2 +\\
&((u+v)(w + K_w) + u(w + t_m) - 2uw)s + ((w + t_m)(w + K_w) - w^2).
\end{aligned}
$$

The model (1.5) and (1.6) of two-mass system is a system of two degrees of freedom [1]. Although the actual pantograph is much more complicated, this model is sufficient to represent dynamic characteristics of the pantograph system with two degrees of freedom.

To analyse pantograph’s dynamic characteristics, specific (nominal) parameters of passive pantograph should be used. A block diagram of passive pantograph is shown in [1]; the parameters for this version are as follows:

$K_w = 178.58\,kg/m,\ w = 178.59\,kg/m,\ u = 3.57\,kg \cdot \sec/m,$
$t = 34287.3\,kg/m,\ v = 21.43\,kg \cdot \sec/m,\ m = 1.11\,kg \cdot \sec^2/m,$
$M = 1.66\,kg \cdot \sec^2/m.$

Further analysis of the system provides a pantograph with one degree of freedom defined by the following transfer function (1.5):

$$H_{px}(s) = \frac{Y_p(s)}{X_0(s)} = \frac{161.3s^2 + 2428s + 3.35 \cdot 10^6}{s^4 + 18.28s^3 + 20794s^2 + 66231s - 17341} \tag{1.7}$$

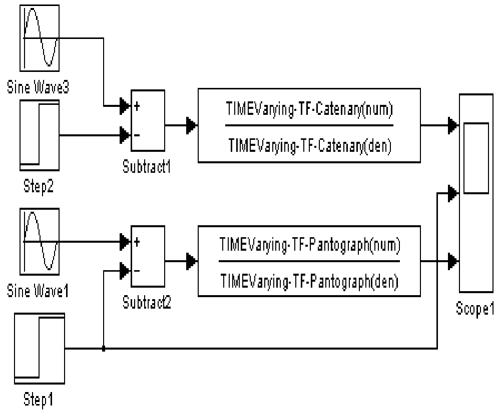
2.3 System “pantograph-catenary” model with variable parameters

System software MATLAB/Simulink allows simulating dynamic systems with time-varying parameters. We illustrate this by example of catenary dynamic characteristics with time-varying parameters modeling by Equation (1.2).

This simulation includes the following steps:

a) representation the system model with time-varying parameters to the model with constant parameters;
b) formation the transfer function for the system with given (nominal) values of the coefficients (M, C, K, Q) in Equation (1.1);
c) to form of the corresponding S-function in MATLAB / Simulink based on the original transfer function, which provides changing of the coefficients in time in the desired range of frequencies and amplitudes;
d) to add necessary subfunctions of random variations in amplitudes of the simulated output signal to the S-function ;
e) to study of the dynamic system under external disturbances at different time intervals of operation.
f) to create the combined pantograph-catenary system with time-varying parameters.

The research block-scheme of the catenary dynamic model with time-varying parameters is shown in Figure 1a. The transfer function of the catenary (upper photo in the Figure 1a) and transfer function of the pantograph (bottom photo in the Figure 1a) are represented. From the graphics in Figure 1b one can conclude that both devices are unstable.



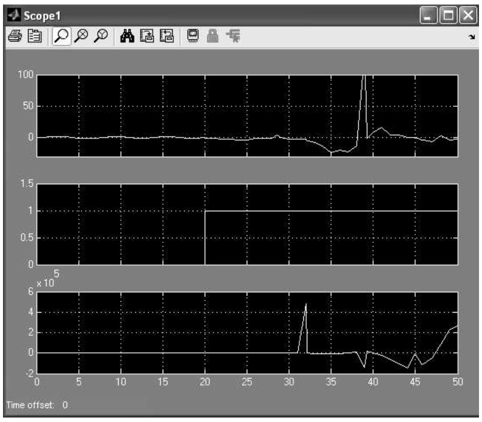
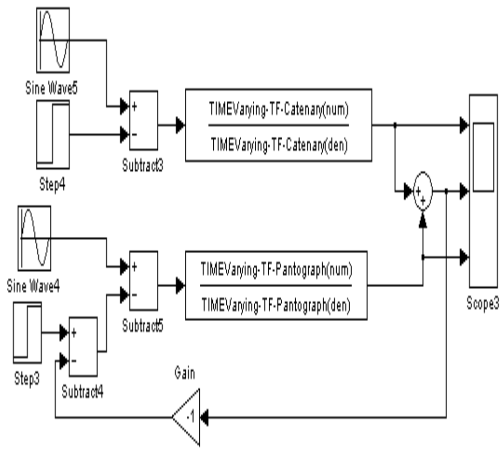


Figure 1a. Schemes of the pantograph and catenary models

Figure1b.Transient response of the catenary (upper), transient response of the pantograph (bottom)

Similarly, dynamic model of passive pantograph with time-varying parameters was developed (Figure 2);



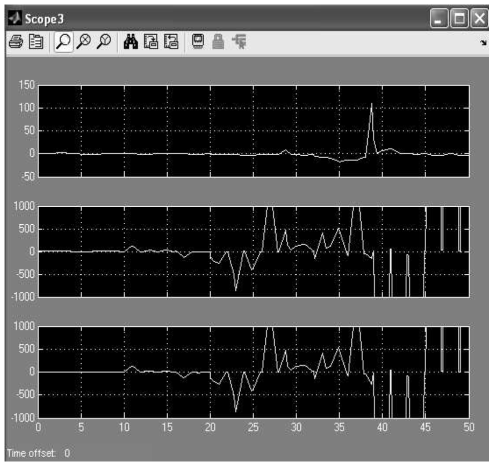
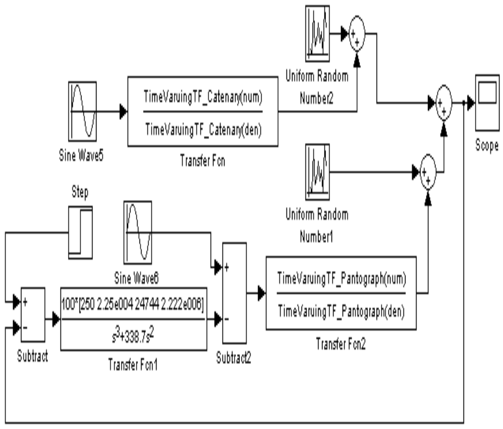


Figure 2.Scheme of the pantograph-catenary system

Figure 2b.Transient response of the system (midlle), of the catenary (upper), of the pantograph (bottom)

The transient response of the catenary (upper photo in the Figure 2b) and the transient response of the pantograph (bottom photo in the Figure 2b) are represented, would be conclude that this system is unstable too.

Dynamic model of the pantograph-catenary system with time-varying parameters with controller (active pantograph) is shown in Figure 3a. The transient response of this system with controller (Transfer Fun1) without changing of its parameters is shown in Figure 3b. This system is also unstable.



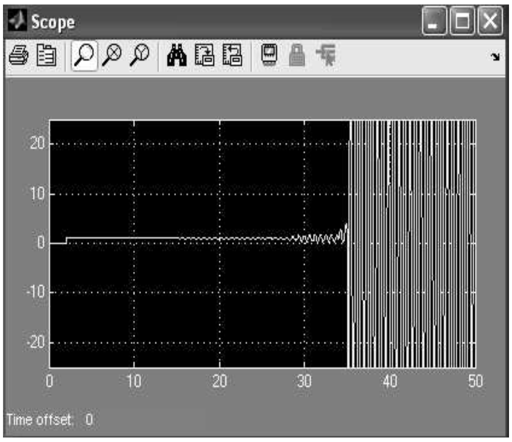


Figure 3a. Scheme of the system with controller (active pantograph)

Figure 3b. Transient response of the system with controller with constant parameters

Plots of the transient responses of the system with the three types of changing rate of the parameters as function of time are shown in Figure 4a, b, c. With a low rate of change of parameters the system loses its steady state already at 15 seconds (Figure 4a). With the increased rate of change of the parameters of the loss of stability occurs more rapidly (Figure 4b, at 7 sec.). At even higher rates of parameter change buckling occurs at 3 seconds (Figure 4c). At certain ratios of time-varying system parameters in it the resonant vibration processes are discovered.

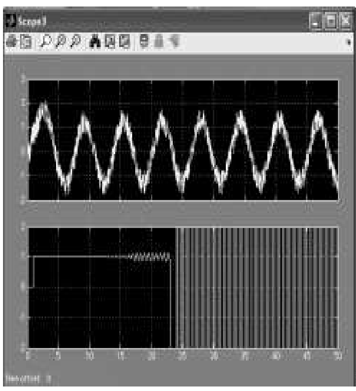


Figure 4a

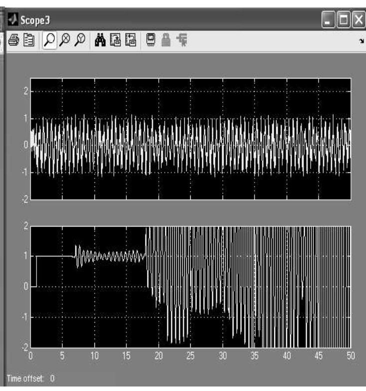


Figure 4b

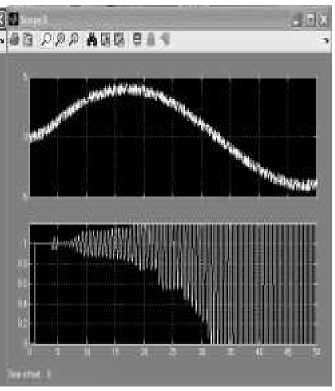


Figure 4c

3 Pantograph-catenary system based on parametric identification and adaptive control

Simulation results of pantograph-catenary system with time-varying parameters which contains controller with constant parameters in the control loop confirm that this system does not have the required stability.

To provide required dynamic characteristics of the system in its real driving conditions in electric systems adaptive controller algorithms were developed based on parametric identification methods to estimate current values of the system parameters. Based on the model estimated parameters and current values of state vector, controller coefficients are calculated, which guarantee the required dynamic characteristics of the system in each input and/or output time interval.

Block diagram of the pantograph-catenary model with time-varying parameters containing adaptive controller is shown in Figure 5.

Blocks that calculate adaptive controller coefficients are presented in the lower part of the scheme:

1) memory1, memory2 – to check system input and output signals;
2) Matrix Concatenate – to calculate system matrices in state space and matrices of the Riccati equation;
3) Logical Operator1 – to calculate matrices in state space based on pantograph state vector;
4) $K_{\mathbf{lqr}}$ – to calculate vector of current values of the system adaptive controller;
5) Environment Controller – to prepare adaptive controller algorithm.

To evaluate dynamic characteristics of the pantograph-catenary system, which block diagram is shown in Figure 5, special program "ADAPPANCAT" in MATLAB was developed, which implements all above mentioned algorithms for the system with time-varying parameters where various external (shock) perturbations action on the system simultaneously. (Footnotes explain using functions and parameters).

The following conclusions can be made based on the studying of different examples of pantograph-catenary model with time-varying parameters:

1) Simulation results show effectiveness of the controller adaptive settings ($K_{\mathbf{lqr}}$) along with random variations of the pantograph-catenary model parameters as well as random external (shock) perturbations acting in the system.
2) Transient response of pantograph-catenary system as well as system response to external shock perturbation (shown in Figure 6) confirms high efficiency of pantograph-catenary adaptive system.
3) BODE diagram of pantograph-catenary adaptive system represents that the system operates steadily in time changing random parameters (Figure 7).
4) Adaptive controller effective response to random variations of system parameters and external perturbations should be mentioned also.

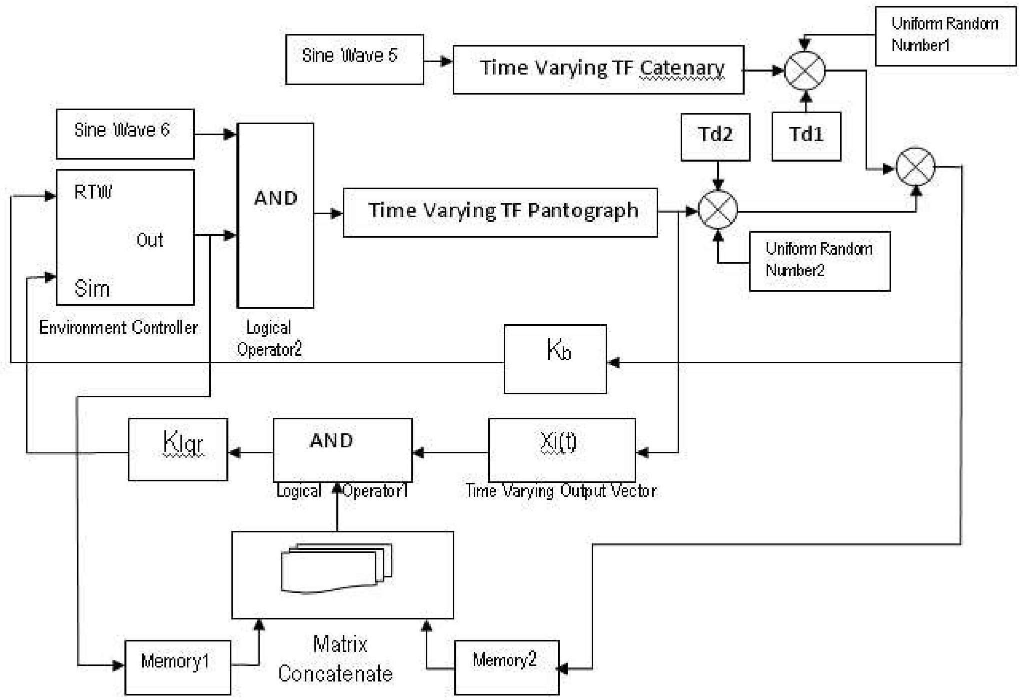


Figure 5. Scheme of the pantograph-catenary model with adaptive controller

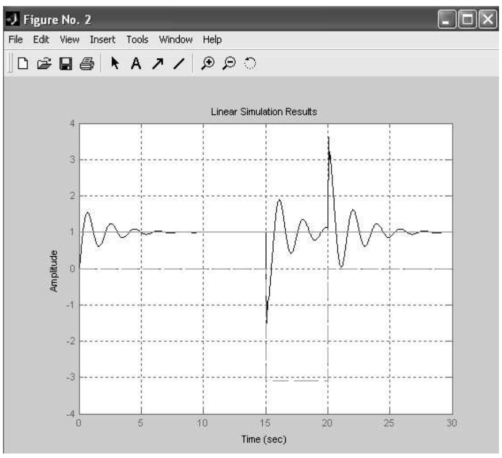


Figure 6. plot(t,u,'r',t,Td1,'r'); lsim(cl_lqr,u,t)

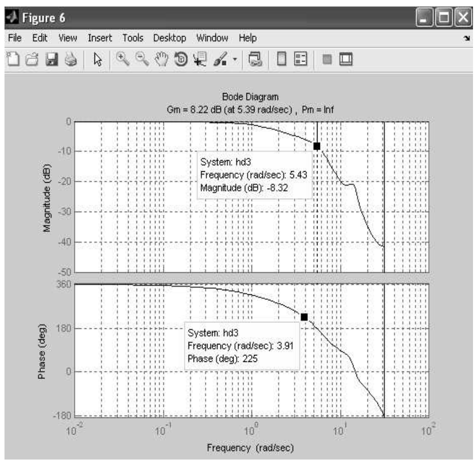


Figure 7. margin lsim(cl_lqr,u,t)

Table 1 shows controller coefficients for 10 consecutive cycles of random system parameters and random shock perturbations. (External pressure is specified in Newtons at a nominal value of Td1 = 3.1N).

Round	Function	Coefficient's values in i-th round of calculations	Td1
1	K_lqr1	[44.7214 3.6817 1.7127 9.7904 25.6275 100.4836 137.3634]	-12.3566
2	K_lqr2	[44.7214 3.6797 1.9140 9.9172 25.3894 48.3065 144.4489]	-8.9054
3	K_lqr3	[44.7214 3.6587 2.1938 10.0973 29.5113 79.2822 106.2715]	-36.0669
4	K_lqr4	[44.7214 3.7032 1.9056 9.9774 27.3275 120.9058 146.8093]	-39.2068
5	K_lqr5	[44.7214 3.7114 1.9486 10.0315 30.5245 84.5308 140.1249]	-56.4442
6	K_lqr6	[44.7214 1.8109 1.3642 5.2435 16.6799 107.1087 75.7185]	-22.7472
7	K_lqr7	[44.7214 3.6760 2.2341 10.1712 29.3525 78.9103 178.3005]	-76.4752
8	K_lqr8	[44.7214 3.7241 2.4193 10.4142 36.1082 121.4122 172.8915]	-29.6197
9	K_lqr9	[44.7214 3.8097 2.5170 10.6846 34.1508 98.1926 103.3616]	-43.9041
10	K_lqr10	[44.7214 3.5404 1.6466 5.4825 22.5220 99.6028 70.1200]	-89.3966

Table 1. Adaptive adjustment of the controller coefficients with varying system parameters and initial forcing Td1

4 Conclusions

Mathematical models of pantograph-catenary adaptive system whose parameters vary randomly in time and the system is acted by external shock disturbance led to the following conclusions:

a) the model adequately reflects dynamic characteristics of real pantograph-catenary autonomous systems used in modern electric trains moving at high speeds (200 km/h and more);

b) evaluation of time-varying system parameters should be done in real time based on measurement data of system input and output and further implementation in parameter identification algorithms (using pantograph computing device);

c) based on estimates of current values of system parameters (obtained by solving the parametric identification problem) optimal values of controller coefficients are calculated using Riccati equation;

d) system adaptive controller provides the necessary dynamic and accuracy characteristics of the system with random changes in system parameters and external shock impacts during the operation;

e) elaborated programs, which implement basic algorithms of measurement data, parametric identification and adaptive corrector, do not impose special requirements for pantograph computer in train.

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