Imitation Modelling of Computing Algorithms for Identification of Technical Objects

Genady Burov, *Riga Technical University*

Abstract **– Slowly changing low-dynamic character of signals is a prominent feature of technical objects working in modes of normal operation. The traditional algorithms based on recurrent computing procedures lose serviceability because of the occurrence of singular situations. Results of identification become unreliable because of unpredictable surges of errors. Therefore a new class of more noise-tolerant algorithms has been developed on the basis of a new mathematical approach of their symbolical imitation modelling. It has allowed obtaining a number of new theoretical results in the field of identification and creating efficient software for systems of automatic control and diagnosis of technical objects.**

Keywords – identification, imitation modelling, algorithm, object, dynamic process

I. Principles of Formation of Imitation Models of Identification Algorithms

The state of a technical object is described by the parameters of its transfer operator

$$W(p) = A(p) / \prod_{k}^{n} (p + b_k). \tag{1}$$

This operator is an equivalent of object's differential equation and represents its symbolical Laplace image. If the values of parameters of the operator (1) are within the allowed limits, then it is possible to draw a conclusion that the object is in working conditions and is suitable for operation. The problem is that it is impossible to directly measure these parameters. The information about them is contained in the object's output signal

$$\begin{aligned} W(p) \Rightarrow y(t) &= C_1 \cdot \exp(b_1 \cdot t) + C_2 \cdot \exp(b_2 \cdot t) + \cdots \\ &+ C_{m-1} \cdot \exp(b_{m-1} \cdot t) = \sum_{i=1}^{m} C_i \cdot \exp(b_i \cdot t). \end{aligned} \tag{2}$$

The relation between the operator (1) and the signal (2) has an indirect character. Thus, it is necessary to find an estimate of the operator (1) using measurements of the signal (2) at discrete moments of time

$$\begin{aligned} y(mT) &= \sum_{k=1}^{n} C_k \exp(-b_k T m) = \sum_{k=1}^{n} C_k {q_k}^m; \\ q_k &= \exp[(-b_k T) \cdot m)]. \end{aligned} \tag{3}$$

Formula (3) indicates the discrete character of signal processing, and this circumstance inevitably leads to difficulties in system identification [3], [5] that are mostly related to the necessity of using approximate mathematical relations that do not contain the analog operator (2), instead relying on the discrete operator

$$\begin{aligned} D(z) &= R(z) / \prod_{k=1}^{n} (z - q_k) \\ &\Rightarrow R(z) / (z^n + \xi_{n-1} z^{n-1} + \cdots + \xi_0). \end{aligned} \tag{4}$$

The vector of parameters of this operator $\bar{\xi}$ has to be found by solving the system of difference equations

$$Y \cdot \bar{\xi} = \bar{y}. \tag{5}$$

Its solution

$$\bar{\xi} = Y^{-1} \cdot \bar{y} \tag{6}$$

is calculated using a numerically unstable algorithm for inverting matrix *Y*. Its elements are formed according to the formula

$$\begin{aligned} Y_{i\,j} &= \sum_{k}^{m} C_k \cdot \exp(-b_k (i+j) \cdot T) = \sum_{k}^{m} C_k \cdot {q_k}^{(i+j)}; \\ q_k &= \exp(-b_k \cdot T). \end{aligned} \tag{7}$$

However, after finding the vector $\bar{\xi}$ (6), it should be decoded by transforming its parameters into estimates of parameters of the analog operator (1). It is necessary because the parameters of the discrete operator (4) have abstract contents and, in such form, are unsuitable for control and diagnosis of a technical object. This decoding is difficult since the relations between the analog and the discrete operators are essentially nonlinear. The abovementioned difficulties in identification of dynamic objects currently have not been solved, and it is an obstacle for realization of automated computer control and diagnosis of technical objects.

In this paper, the solutions of the following problems are considered:

1. Development of new mathematical approach for solving theoretical problems of identification of dynamic objects.
2. Development of noise-tolerant computing algorithms for identification of technical objects on the basis of this approach.
3. Development of imitation models of computing algorithms of identification that can be conveniently programmed.

4. Development of accompanying imitation models that provide high accuracy and performance of identification algorithms.

It is offered to create new accurate, high-performance, noise-tolerant algorithms for solving systems of difference equations of identification (6) on the basis of the basic classical formula (8)

$$Y^{-1} = \tilde{Y}^T \cdot \frac{1}{\det Y}, \qquad (8)$$

in which the inverse matrix Y^{-1} is found in the form of adjugate matrix made of values of minors of Y [7].

Its determinant $\det Y$ is calculated separately using a direct algorithm without the use of recurrent procedures. It is offered to use the basic expression

$$\tilde{Y}_{ji} = \sum_P (-1)^m \cdot (Y_{1\alpha_1} \cdot Y_{2\alpha_2} \cdot, \ldots \cdot Y_{n\alpha_n}) \qquad (9)$$

as the basic computing algorithm for finding the determinant $\det Y$ and the determinants of sub-matrices of Y.

Such algorithm has a number of significant advantages in comparison with traditional algorithms [1], [3]. It does not rely on the use of recurrent computing procedures that are based on the method of elimination, in which singular situations can arise in an unpredictable way. These situations are the principal cause of interruption of computing process and occurrence of large errors in the vector (6). It is not always possible to prevent such situations by using the known method of pivoting as all elements can be so small, that they are comparable to round-off noise [2], [4]. Algorithm (9) uses operation of summation, which can guarantee the creation of algorithms suitable for realization in parallel modes of processing of signals (7) [2], [5]. It allows increasing the performance of algorithms by using software methods instead of perfection of hardware, the possibilities of which are practically already exhausted.

The abovementioned problems can be solved by application of principles of imitation modeling of algorithms based on the basic expression (9). Until now, such algorithms have not been developed because they cannot be realized using traditional methods. For this purpose, it would be necessary to generate the full set of all possible permutation from expressions (7), which is practically impossible. For this reason, it is also impossible to create software for realization of algorithms of this class. Since they have a number of essential advantages in comparison with traditional algorithms, interest for their realization remains. This problem can be solved in a symbolical form by using imitation models (IMs). In such models, sets of permutations are simulated by permutations created from the coordinates of elements of the matrix of system of difference equations (7), which are offered to create from components of ordered numerical sequences [8], [9]. It allows satisfying the main condition of the basic formula (9) – avoiding inclusion of repeating components in the set. IM generated from coordinates of sub-matrices, for which the minors (determinants with signs) are calculated in (8), can be written in lexicographic form from indices of rows and columns in which the sub-matrix is located in the matrix Y (5):

$$\varphi Dpv^{(n)} * Y \tilde{\Rightarrow} \begin{bmatrix} \tilde{r} : r_1 \; r_2 \; r_3 \cdots n \\ \tilde{L} : \varphi Perm * (\overline{1.n}) \end{bmatrix}. \qquad (10)$$

Here the operator $\varphi Dpv*$ is introduced, which forms the matrix of the model. Its rows consist of permutations of indices of columns attached to the component of row indices [7]. These permutations are created, in a non-relative form, from elements of an interval of natural number sequence $\overline{1.n}$, where n is the order of the sub-matrix. For example, for $n = 3$, the model (10) has the form of an index matrix grid *ims*

$$\varphi Dpv^{(3)} * Y \Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ \underline{r_1} & \underline{r_2} & \underline{r_3} \\ +[L_1 & L_2 & L_3] \\ -[L_1 & L_3 & L_2] \\ -[L_2 & L_1 & L_3] \\ +[L_2 & L_3 & L_1] \\ +[L_3 & L_1 & L_2] \\ -[L_3 & L_2 & L_1] \end{bmatrix} \Rightarrow \begin{bmatrix} & & \\ \underline{1} & \underline{2} & \underline{3} \\ +[1 & 2 & 3] \\ -[1 & 3 & 2] \\ -[2 & 1 & 3] \\ +[2 & 3 & 1] \\ +[3 & 1 & 2] \\ -[3 & 2 & 1] \end{bmatrix}, \qquad (11)$$

so that it is possible to express the model (10) as

$$\varphi Dpv^{(3)} * Y \Rightarrow \left\lfloor ims^{(3)} = \tilde{r} \times \circ (\varphi Perm * \tilde{L}) \right\rfloor. \qquad (12)$$

From such *ims*, used for calculation of determinants in the matrix of minors (8), the description of the real sub-matrix can always be restored, and its features, which can be used for optimization of computing algorithm [3], [6], can be determined. Below such feature is used for application of Vandermonde formula, which is used for finding the determinant.

In a general case, the function of expansion of determinant along any of the rows or columns can be used. In this particular case, the rule of expansion along the first row is used. This expression can be considered as the original of symbolical IM (11):

$$\begin{aligned} \varphi Det * Dvp^{(3)} \Rightarrow\; & Y_{11} \cdot Y_{22} \cdot Y_{33} - Y_{11} \cdot Y_{23} \cdot Y_{32} - Y_{12} \cdot Y_{21} \cdot Y_{33} \\ & - Y_{12} \cdot Y_{23} \cdot Y_{31} + Y_{13} \cdot Y_{21} \cdot Y_{32} - Y_{13} \cdot Y_{22} \cdot Y_{31}. \end{aligned} \qquad (13)$$

It can also be expressed in the lexicographic form:

$$\begin{aligned} & \left\{ \tilde{r} \circ \times (\varphi Perm * \tilde{L}) \right\} \Rightarrow \left\{ U = \sum_{j=1}^{n!} Zn_j \prod_{r=1}^{n} Y_{r,L} \right\}; \\ & Zn_j = (-1)^{Kj}; \; K_j = f(j). \end{aligned} \qquad (14)$$

Here, mapping the IM into the area of originals is realized by replacement of indices of *ims* (12) by the values of the process (7), which have been stored in the elements of *Y* (13). Using such approach, IMs for realization of parallel computing algorithms can be created. For example, the IM of algorithm for calculation of determinant of 6-th order matrix *Y* can be generated from IM (11) if the *ims* of the sub-matrices extracted from the 1st and 2nd bands, into which the initial matrix is partitioned, is designated in the following form:

$$(r_1; r_2; r_3) \otimes \circ B_1\ ;\ (r_4; r_5; r_6) \otimes \circ B_2\,. \tag{15}$$

Here the components of row indices $\tilde{r}_1 = (1, 2, 3)$ and $\tilde{r}_2 = (4, 5, 6)$ are used. Components of column indices are represented by arrays B_1 and B_2, the columns of which are formed according to the conditions of Laplace theorem [10]:

$$B_1^{\,j} \bigcup B_2^{\,j} = \overline{1.6};\quad B_1^{\,j} \bigcap B_2^{\,j} = \varnothing;\quad j \in \overline{1.20}\,. \tag{16}$$

For the given example, they are

$$B_1 = \begin{bmatrix} 1 & 1 & & 2 & & 4 \\ 2 & 2 & \cdots & 3 & \cdots & 5 \\ 3 & 4 & & 4 & & 6 \end{bmatrix};$$
$$B_2 = \begin{bmatrix} 4 & 3 & & 1 & & 1 \\ 5 & 5 & \cdots & 5 & \cdots & 2 \\ 6 & 6 & & 6 & & 3 \end{bmatrix}. \tag{17}$$

The vectors of values of determinants for two bands can be determined by applying formulas (13) and (14):

$$\overline{\psi}_1 \Rightarrow \varphi Det * \left[(r_1; r_2; r_3) \otimes \circ B_1\right];$$
$$\overline{\psi}_2 \Rightarrow \varphi Det * \left[(r_4; r_5; r_6) \otimes \circ B_1\right]. \tag{18}$$

The vector of signs is introduced:

$$Zn_i \Rightarrow (-1)^P;\quad p = (r_{11} + r_{12} + r_{13}) + \sum_{i=1}^{3} \left(B_1\right)_{i,j}\,. \tag{19}$$

The determinant of the initial matrix is found as the product of (18) and (19), with the use of operation of vectorization:

$$\varphi Det * Y \Rightarrow \sum \left(\overline{\psi}_1 \otimes \overline{\psi}_2 \otimes \overline{Zn}\right). \tag{20}$$

In [1], [3], [7], such method has been applied for any variant of partitioning of the matrix. As a result, the IM as a graph in the form of a branching tree was obtained, the sections of which are generated from *ims* of local sub-matrices [5], [7], [9]. They are acted upon by standard function (13) or (14) for finding determinants of local sub-matrices, which is designated by introduction of operator φDet in graph sections:

$$\begin{aligned} Minr_{i\,j} &\Rightarrow \varphi Det * \varphi Gr * (n, \overline{\nu}) \Rightarrow \left[\varphi Det * \varphi KC(\nu_1) * (\overline{1.n})\right] \\ &\times \circ \left[\varphi Det * \varphi KC(\nu_1) * (\overline{1.n}) / \tilde{a}_1\right] \times \circ \\ &\times \circ \left[\varphi Det * \varphi KC(\nu_1) * (\overline{1.n}) / \tilde{a}_{12}\right] \ldots \\ &\times \circ \left[\varphi Det * \varphi KC(\nu_1) * (\overline{1.n}) / \tilde{a}_{12.....n-1}\right]; \\ \overline{\nu}^T &= \left[\nu_1\, \nu_2 \ldots \nu_k\right];\quad \sum_{i=1}^{k} \nu_i = n\,. \end{aligned} \tag{21}$$

However, for programming the algorithm, it is more preferable to use compressed form of IM, as the operator $\varphi Det *$ is formalized and it can be entered into any necessary place of graph IM:

$$\begin{aligned} \varphi Gr * (n, \overline{\nu}) &\Rightarrow \left[\varphi KC\,(\nu_1) * (\overline{1.n})\right] \\ &\times \circ \left[\varphi KC\,(\nu_1) * (\overline{1.n}) / \tilde{a}_1\right] \\ &\times \circ \left[\varphi KC\,(\nu_1) * (\overline{1.n}) / \tilde{a}_{12}\right] \ldots \\ &\times \circ \left[\varphi KC\,(\nu_1) * (\overline{1.n}) / \tilde{a}_{12.....n-1}\right]; \\ \overline{\nu}^T &= \left[\nu_1\, \nu_2 \ldots \nu_k\right];\quad \sum_{i=1}^{k} \nu_i = n. \end{aligned} \tag{22}$$

By analyzing (15) and (16), it can be determined that arrays B_1 and B_2 are formed from components of ordered numerical sequences of combinative type that are generated by the operator $\varphi KC(\nu) * \overline{1.m}$ (combinations of *m* elements from *v*). Thus, graph (21) can be considered as an IM generalizing the Laplace theorem for any variant of partitioning of the initial matrix into a set of independent fragments – local sub-matrices, for which the values of determinants can be calculated independently from each other. It means that such IM can be used for realization of parallel modes of identification of technical objects. The IM (22) can be used for creation of software that can increase the performance of algorithms of identification [5], [7], [9] without resorting to the use of expensive hardware.

II. Optimization of Imitation Models of Computing Algorithms

It is necessary to take into account that in the area of an IM, in general, arithmetic operations should not be performed, as its symbols are indices *ims* – the indices of rows and columns of the initial matrix. However, in some kinds of IMs, this restriction can be lifted, and the IM can be optimized by using elementary arithmetic operations. For example, its information redundancy can be eliminated, and its compressed form can be created, which is more convenient for creation of a computer program. Actually, such method of optimization is based on the principle of information monitoring, when there is an exchange of mathematical objects between the space of symbolical images of algorithms and the space of their originals, where arithmetic operations used in the real algorithm are specified. During such exchange, there is a recurrent improvement of algorithm and the corresponding computer program.

Such approach can be used to create an auxiliary IM, accompanying the basic IM that is expressed in coordinates of *ims*. It is done to increase the accuracy of identification using software methods instead of using expensive hardware. From the analysis of basic model (11) and model (21), it generally follows that the most critical operation in relation to the loss of accuracy is the operation of calculation of products in rows of the matrix (11) and in the branches of graph (21). In these operations, there can be numbers with similar absolute values, but opposite signs. For such operations, it is preferable to organize computing based on accompanying IM that prevents the loss of the most valuable information because of least significant digits leaving the limits of computer precision during number multiplication. Since increasing the computer precision is costly and unproductive, it is preferable to solve this problem using software methods. For formation of the corresponding program, its accompanying IM can be created. Additionally, a method for deriving analytical descriptions of computing algorithms of identification can be developed on its basis [1], [2].

The products of expressions of the dynamic process (7), stored in elements of matrix Y_{ij} in a symbolical form, can be expressed using components of numerical sequences created by combinative operators Y_{ij} (22). Storing $\underline{Y_{ij}}$ in the elements of an interval of natural number sequence $1.m$, where m is the number of partial components of the dynamic process (7), is used:

$$Y_{i\,j} \Rightarrow \bigcup_{i=1}^{m} a_i\,;\ a_i \in \overline{1.m}\,. \tag{22}$$

Here, the symbol of set union represents lexicographic sum of powers of discrete poles $q_i = \exp(-b_i \cdot T)$ expressed by the elements of intervals of the sequence $\overline{1.m}$. Elements q_i are the result of nonlinear transformation of analog poles of technical object's transfer operator (1). Such transformation leads to strong information compression of the whole model of identification; therefore, the noise tolerance of algorithm can significantly decrease. Such compression is an inevitable consequence of computer processing of analog signals because of its discrete character. The analog poles of the operator (1), located in the left infinite complex semi-plane at large enough distances from each other and, consequently, well distinct at the presence of noise, move to a narrow area in the right unit semi-circle of the complex plane because of the discrete processing (stable instead of diverging dynamic processes are considered). For this reason, especially if the sampling frequency of process (7) $f = 1/T$ is increased, the distances between discrete poles can be so small, that their set is perceived as one multiple pole of operator (4), and, hence, of the operator (1). It is obvious that there is a distortion of the aprioristic information about the object, as the real technical object cannot be made of only resonant frequencies. Thus, in computing procedures of identification, there is an algorithmic uncertainty and the obtained estimates of parameters are unreliable and can be used neither for diagnosis nor for control. Excessive overloading of identification model by introduction of unnecessary operators on subjective assumptions leads to proliferation of superfluous poles, and the placement of object's poles on the complex plane becomes even more uncertain and the algorithmic uncertainty increases.

Therefore, the statement that using such models with unnecessary operators it is possible to obtain additional information about the object is unreasonable. In this relation, in [1], [3], [4], the dangers of introduction of such operators in stochastic models of identification are noted. It does not achieve the stated goal; instead it makes the operation of decoding the vector (6) impossible.

In this case, additional algorithmic uncertainty can arise already at the level of formulation of imitation model, if the a priori structure of the operator of object (1) is determined incorrectly, so that the dimension of the set

$$Y_{i\,j} \Rightarrow \bigcup_{i=1}^{m} a_i\,;\ a_i \in \overline{1.m} \tag{23}$$

does not correspond to the structure of the operator of object (1). In this case, there will be a distortion of filtering properties of imitation model, and it will be reflected in the accuracy of identification algorithm. From here it is possible to draw a conclusion about the importance of optimization of imitation model directly in the symbolical area before its mapping into the area of originals [7], [8].

Here, the problem of formation of auxiliary IM for algorithm of multiplication of values of dynamic process in the rows of the matrix of model (11) is considered. Such model can then be applied in the branches of the graph IM (21).

The multipliers of Y_{ij} in the rows of the matrix of model (11) have identical structure because their sums in all rows contain the same number of addends, which is equal to m. They differ by the indices of powers of these addends, which can be expressed by operation of allocation of these powers:

$$\varphi Arang(St_{i\,j}) * q_k\,;\ q_k \in \tilde{q}^{(n)}; St_{i\,j} = i + j\,. \tag{24}$$

According to (7), the powers directly depend on the coordinates *ims* of elements of matrix (23) $Y_{i\,j}$: $St_{i\,j} = i + j$. It provides a simple way to map the model (11)

$$\varphi Dpv^{(3)} * Y \Rightarrow \begin{bmatrix} \underline{1} & \underline{2} & \underline{3} \\ +[Y_{11} & Y_{22} & Y_{33}] \\ -[Y_{11} & Y_{23} & Y_{32}] \\ -[Y_{12} & Y_{21} & Y_{33}] \\ +[Y_{12} & Y_{23} & Y_{31}] \\ +[Y_{13} & Y_{2,1} & Y_{32}] \\ -[Y_{13} & Y_{22} & Y_{31}] \end{bmatrix}. \tag{25}$$

into its accompanying IM by summation of coordinates *ims* in the model:

$$\varphi Dpv^{(3)} * Y \Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ \underline{r_1} & \underline{r_2} & \underline{r_3} \\ +[L_1 & L_2 & L_3] \\ -[L_1 & L_3 & L_2] \\ -[L_2 & L_1 & L_3] \\ +[L_2 & L_3 & L_1] \\ +[L_3 & L_1 & L_2] \\ -[L_3 & L_2 & L_1] \end{bmatrix}. \quad (26)$$

In model (25), analytical expressions are substituted:

$$y(kT) = C_1 q_1{}^k + C_2 q_2{}^k + C_3 q_3{}^k, \quad (27)$$

where C_i are the weight factors representing the coefficients of expansion of the operator of the object into the sum of partial fractions generated by the operator (1):

$$W(p) = \frac{C_1}{p+b_1} + \frac{C_2}{p+b_2} + \frac{C_3}{p+b_3} \Rightarrow \left\{ y(kT) = \sum_i^3 C_i \exp(-b_i \cdot (kT) \right\}. \quad (28)$$

Here, the problem is considered for the case, when they are equal to one:

$$Y_{r,L} = (q_1{}^{r+L} + q_2{}^{r+L} + ... + q_k{}^{r+L} + ... + q_m{}^{r+L}). \quad (29)$$

The products in rows (25) can be expressed in the form of a set of components of product of the sums of powers of discrete poles:

$$\Omega_{,} = Y(r_1, L_1) \cdot Y(r_2, L_2) \cdot Y(r_3, L_3) = \left(\sum_{k=1}^m \cdot q_k{}^{r_1+L_1} \right) \cdot \left(\sum_{k=1}^m \cdot q_k{}^{r_2+L_2} \right) \cdot \left(\sum_{k=1}^m \cdot q_k{}^{r_3+L_3} \right). \quad (30)$$

They are calculated using formulas (13), (14):

$$\left\{ \tilde{r} \circ \times (\varphi Perm * \tilde{L}) \right\} \Rightarrow \left\{ U = \sum_{j=1}^{n!} Zn_j \prod_{r=1}^{n} Y_{r,L(r,j)} \right\}; \quad Zn_j = (-1)^{Kj}; K_j = f(j). \quad (31)$$

Since the indices of positions of multipliers (30) determine the values of powers of discrete poles, the positional principle of formation of auxiliary IM can be used. Powers in elements of Y_{ij} can be found by summation of indices in the model (26)

$$St_{r\,L} \Rightarrow r + L, \quad (32)$$

and the IM of powers can be found by mapping

$$\varphi Dpv^{(3)} * Y = \begin{bmatrix} 1 & 2 & 3 \\ \underline{r_1} & \underline{r_2} & \underline{r_3} \\ +[L_1 & L_2 & L_3] \\ -[L_1 & L_3 & L_2] \\ -[L_2 & L_1 & L_3] \\ +[L_2 & L_3 & L_1] \\ +[L_3 & L_1 & L_2] \\ -[L_3 & L_2 & L_1] \end{bmatrix} \Rightarrow St = \begin{bmatrix} \underline{\tilde{r}:1} & \underline{2} & \underline{3} \\ +[2 & 4 & 6] \\ -[2 & 5 & 2] \\ -[3 & 3 & 6] \\ +[3 & 5 & 4] \\ +[4 & 3 & 5] \\ -[4 & 4 & 4] \end{bmatrix}. \quad (33)$$

The signs in that are preserved. This mapping can be written in a shorthand form as

$$\varphi Dpv^{(3)} * Y \Rightarrow \varphi Arang(St) * \tilde{q}^{(m)}. \quad (34)$$

Here, the model of powers (33) is placed over every component of discrete poles, formed from elements of interval of number sequence $\overline{1.m}$. They coincide with the indices of poles in the sums (30). Their allocation is designated by the operator

$$\varphi Arang(k) * \overline{1.m} \Rightarrow (q_1{}^k + q_2{}^k + ... + q_m{}^k). \quad (35)$$

The set of components obtained as the Kronecker product of the sets specified in (35) is designated as

$$\Omega_{,} = Y(r_1, L_1) \cdot Y(r_2, L_2) \cdot Y(r_3, L_3) = \left(\sum_{k=1}^m \cdot q_k{}^{r_1+L_1} \right) \cdot \left(\sum_{k=1}^m \cdot q_k{}^{r_2+L_2} \right) \cdot \left(\sum_{k=1}^m \cdot q_k{}^{r_3+L_3} \right), \quad (36)$$

or in an expanded form as

$$\left[\sum_{i=1}^m q_i * Arang(r_1 + L_1) \right] \times \left[\sum_{i=1}^m q_i * Arang(r_2 + L_2) \right] \times \left[\sum_{i=1}^m q_i * Arang(r_m + L_m) \right]. \quad (37)$$

The algorithm for formation of Ω is examined in [7], [8]. It is taken into account that the structure (37) in all rows of the matrix (33) is the same. The set of components (36) is found using the IM for n-ary Cartesian power of interval $(\overline{1.m})$, over the elements of which the indices of powers are placed with the observance of the positional principle. The carrier set (36) consists of sets of components formed by operators $\varphi KC(\nu_i)$ from an interval of natural number sequence $\overline{1.n}$:

$$\Omega \Rightarrow \varphi KC(\nu_i) * 1.m; \nu_i \in \overline{1.n}. \quad (38)$$

For example, for n = 3, the result is

$$\varphi KC(1)*\overline{1.3} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}; \quad \varphi KC(2)*\overline{1.3} = \begin{bmatrix} 12 \\ 13 \\ 23 \end{bmatrix}; \qquad (39)$$

$$\varphi KC(3)*\overline{1.3} = (123).$$

The set (38) is formed according to the condition that the number of elements in all its components is the same and is equal to *n*. In [1], [4], [9], it is proved that to comply with this condition for formation of components (38), it is necessary to use an algorithm with application of operator of partitioning $\varphi Part(\nu_i)*n$ of number n into ν_i parts, and it is necessary to use permutations of components of these partitions:

$$\Omega \Rightarrow (\overline{1.m})^n = \sum_{i=1} \varphi Arang(\theta_i)*\left[\varphi KC(\nu_i)*(\overline{1.m})\right];$$
$$\tilde{\theta}_i = \varphi Perm*\left[\varphi Part(\nu_i)*n\right]. \qquad (40)$$

Here ν_i is the argument of the operator $\varphi KC(\nu_i)$.

The full set Ω, obtained using the model (40), can be represented by the following matrix:

$$A := \begin{pmatrix} 1 & 2 & 3 & 1 & 1 & 2 & 2 & 2 & 3 & 3 & 3 & 1 & 1 & 1 & 2 & 2 & 3 & 3 & 3 & 1 & 1 & 1 & 2 & 2 & 2 & 3 & 3 \\ 1 & 2 & 3 & 2 & 3 & 1 & 2 & 3 & 1 & 2 & 3 & 1 & 2 & 3 & 1 & 3 & 1 & 2 & 3 & 1 & 2 & 3 & 1 & 2 & 3 & 1 & 2 \\ 1 & 2 & 3 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 \end{pmatrix} \qquad (41)$$

The auxiliary IM of powers (33) can be used for optimization of computing algorithm, and also for deriving its analytical description. Here, its application for one of the components of the set (40)

$$\Omega_i \Rightarrow \varphi Arang(St)*(q_1 q_2 q_3) \qquad (42)$$

is considered.

Symbolical model (33) allows application of such arithmetic operations to it, which can be used for optimization of computing algorithm. For example, from its matrix, the common lexicographic multiplier can be extracted:

$$St \Rightarrow (2\,3\,4)\circ \begin{bmatrix} +[0 & 1 & 2] \\ -[0 & 2 & 1] \\ -[1 & 0 & 2] \\ +[1 & 2 & 0] \\ +[2 & 0 & 1] \\ -[2 & 2 & 0] \end{bmatrix}, \qquad (43)$$

which in the area of originals will also provide an adequate common multiplier of powers of discrete poles:

$$St \Rightarrow (2\,3\,4)\circ \begin{bmatrix} +[0 & 1 & 2] \\ -[0 & 2 & 1] \\ -[1 & 0 & 2] \\ +[1 & 2 & 0] \\ +[2 & 0 & 1] \\ -[2 & 2 & 0] \end{bmatrix} \Rightarrow \left[\varphi Arang(2\,3\,4)*(q_1 q_2 q_3)\right]$$
$$\cdot \varphi Arang\left\{ \begin{bmatrix} +[0 & 1 & 2] \\ -[0 & 2 & 1] \\ -[1 & 0 & 2] \\ +[1 & 2 & 0] \\ +[2 & 0 & 1] \\ -[2 & 2 & 0] \end{bmatrix} \right\} *(q_1 q_2 q_3). \qquad (44)$$

The initial model becomes simpler and is transformed into the model (43). It can be checked that, in the field of originals, it represents an algorithm of finding the determinant of the matrix consisting of powers of discrete poles:

$$Wnd^{(3)} = \begin{bmatrix} 1 & 1 & 1 \\ q_1 & q_2 & q_3 \\ q_1^{\,2} & q_2^{\,2} & q_3^{\,2} \end{bmatrix}. \qquad (45)$$

The determinant is calculated using the known Vandermonde formula and is equal to the product of differences of discrete poles (45), which are designated as the function

$$\varphi Det*Wnd^{(3)} = FG(\tilde{q}^{(3)});$$
$$FG(\tilde{q}^{(3)}) = (q_1 - q_2)\cdot(q_1 - q_3)\cdot(q_2 - q_3). \qquad (46)$$

This result can be expressed as

$$\varphi Arang\left\{ \begin{bmatrix} +[0 & 1 & 2] \\ -[0 & 2 & 1] \\ -[1 & 0 & 2] \\ +[1 & 2 & 0] \\ +[2 & 0 & 1] \\ -[2 & 2 & 0] \end{bmatrix} \right\} *(q_1 q_2 q_3) \qquad (47)$$
$$\Rightarrow FG(q_1, q_1, q_3) = (q_1 - q_2)\cdot(q_1 - q_3)\cdot(q_2 - q_3).$$

As a result,

$$\varphi Arang(St)*\tilde{q}^{(3)} \Rightarrow (q_1^{\,2} q_2^{\,3} q_3^{\,4})\cdot FG(\tilde{q}^{(3)})$$
$$= (q_1^{\,2} q_2^{\,3} q_3^{\,4})\cdot(q_1 - q_2)\cdot(q_1 - q_3)\cdot(q_2 - q_3). \qquad (48)$$

The operation of application of the model (33) can be realized for all components of the set (40). This result can be expressed in an expanded form using the formula (13):

$$\Omega \Rightarrow (q_1{}^2+q_2{}^2+q_3{}^2)\cdot(q_1{}^4+q_2{}^4+q_3{}^4)\cdot(q_1{}^6+q_2{}^6+q_3{}^6)$$
$$-(q_1{}^2+q_2{}^2+q_3{}^2)\cdot(q_1{}^5+q_2{}^5+q_3{}^5)\cdot(q_1{}^5+q_2{}^5+q_3{}^5)$$
$$-(q_1{}^3+q_2{}^3+q_3{}^3)\cdot(q_1{}^3+q_2{}^3+q_3{}^3)\cdot(q_1{}^6+q_2{}^6+q_3{}^6)$$
$$-(q_1{}^3+q_2{}^3+q_3{}^3)\cdot(q_1{}^5+q_2{}^5+q_3{}^5)\cdot(q_1{}^4+q_2{}^4+q_3{}^4)$$
$$+(q_1{}^4+q_2{}^4+q_3{}^4)\cdot(q_1{}^3+q_2{}^3+q_3{}^3)(q_1{}^5+q_2{}^5+q_3{}^5)$$
$$-(q_1{}^4+q_2{}^4+q_3{}^4)\cdot(q_1{}^4+q_2{}^4+q_3{}^4)\cdot(q_1{}^4+q_2{}^4+q_3{}^4). \quad (49)$$

The application of model (33) to components with identical elements, for example, in relation to permutations formed from the component $(q_1q_1q_2)$, produces

$$\varphi Perm*(q_1q_1q_2) = (q_1q_1q_2\,; q_1q_2q_1\,; q_2q_1q_1) \in \tilde{\Omega}\,. \quad (50)$$

Over the sum of components of permutations

$$\tilde{\alpha} = \bigcup_i \Omega_i = \varphi Perm*(q_1q_1q_2);$$
$$\tilde{\alpha} = (q_1q_1q_2\,; q_1q_2q_1\,; q_2q_1q_1). \quad (51)$$

the matrix of powers is placed, which gives the following result:

$$\tilde{\alpha} * \varphi Arang(St) \Rightarrow \left[(q_1q_1q_2) * \varphi Arang(St)\right]$$
$$\bigcup\left[(q_1q_2q_1) * \varphi Arang(St)\right]\bigcup\left[(q_1q_1q_2) * \varphi Arang(St)\right]. \quad (52)$$

The powers of identical elements can be summed, and this operation is allowed to be made in the space of symbolical IMs. In this case, it is necessary to sum indices of columns of the model (33) that stand in the positions of identical elements. Indices of such positions of repeating elements, in which it is necessary to sum indices, can be determined using the operator $\varphi KC(\nu = 2) * \overline{1.3} = (1\,2; 1\,3; 2\,3)$. As a result, three transformed IMs of powers are obtained. In them, the signs related to the parity of permutation, given in the basic formula (9) – the basis of the considered IMs, are preserved:

$$St_1(q_1q_1q_2) \Rightarrow \begin{bmatrix} +[4 & 5] \\ -[5 & 4] \\ -[4 & 5] \\ +[6 & 3] \\ +[5 & 4] \\ [6 & 3] \end{bmatrix};\ St_1(q_1q_2q_1) \Rightarrow \begin{bmatrix} +[6 & 3] \\ -[5 & 4] \\ -[7 & 2] \\ +[5 & 4] \\ +[7 & 2] \\ -[6 & 3] \end{bmatrix};$$
$$St_1(q_2q_1q_1) \Rightarrow \begin{bmatrix} +[1 & 8] \\ -[1 & 8] \\ -[2 & 7] \\ +[2 & 7] \\ +[3 & 6] \\ -[3 & 6] \end{bmatrix}. \quad (53)$$

From these expressions follows that summation of products of values Y_{ij} in rows of the matrix of model (33) in the field of originals using the formula (13) will lead to zero result. In general, for an arbitrary number p of identical elements in components of (40) and n-th order matrix Y, the number n of positions in model (33), in which it is necessary to sum indices, can be determined by components formed by the operator $\varphi KC(p) * (\overline{1.n})$. Thus, the studies on the basis of auxiliary IMs of powers lead to a conclusion that an IM based on expression (40) possesses the properties of filtration, which can be used for optimization of the basic IM of the computing algorithm created on the basis of the basic formula (9).

Physically such filtration means that the IM of the algorithm automatically excludes all permutations of coordinates *ims* in (9) corresponding to singular matrices.

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Genady Burov was born in 1937 in Saint Petersburg, Russia. He received the doctoral degree from Riga Higher Engineering School in 1966.

Currently Genady Burov is a Leading Researcher at the Environment Modelling Centre of Riga Technical University.