RIGA TECHNICAL UNIVERSITY

Jurijs Svabs

THE METHODS OF SPATIAL RUBBER TECHNICAL PRODUCTS OPTIMAL SYNTHESIS PROBLEMS SOLUTION

Summary of Thesis

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RIGA TECHNICAL UNIVERSITY
Faculty of Transport and Mechanical Engineering
Institute of Mechanics

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Summary of Thesis

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CONFIRMATION

I confirm that I have developed this doctoral thesis that has been submitted for reviewing to the Riga Technical University in order to obtain the engineering sciences doctor's degree. The doctoral thesis has not been submitted to any other university with the goal of obtaining a scientific degree.

Jurijs Svabs …………………………….(Signature)

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The doctoral thesis has been prepared in the Latvian language, it contains an introduction, 4 sections, conclusions, 3 annexe, list of references, 100 drawings and illustrations, in total 186 pages. The list of references includes 122 titles.

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GENERAL DESCRIPTION OF THESIS

The topicality of the theme:

Rubber technical products, due to its unique properties - mechanical, technological, etc. - are widely used in various fields of modern technique as flexible articulated joints and supports, shock absorbers and vibro - protective devices, different kinds of deformation compensators, etc. Many parameters – construction simplicity, safety, dimensions, cost, etc. – of elastomeric elements exceed the traditional systems of the same application. The topicality of the problem is obvious.

For successful application and optimal construction of rubber technical products it is necessary to develop the available methodologies, how to obtain analytical solutions for real possible sizes of elastomeric products and schemes of the static load. In the publications there are practically no analytical solutions that completely take into consideration all geometric features of the construction of elastomeric products and mechanical characteristics of the elastomer.

Because of the elastomers weak compressibility the difficulties appeared. This problem, although it is practically solved for simple shape of rubber technical products, however, there is no complete analysis and methodological recommendations on the application of these analytical solutions, which are obtained, using the hypothesis of the elastomer non-compressibility, depending on the geometric characteristics of products and real values of the elastomer Poisson's ratio.

Most of the analytical solutions are obtained by variation of the methodologies if the condition of the elastomer`s non-compressibility executed. In addition, the hydrostatic pressure function, which is included in equations of incompressible materials elasticity theory, is missed the solution, which does not allow executing the analysis of compressed state in elastomer`s product, and to solve the issues of durability and stability.

There is practically no complex configuration in the methodologies of rubber technical products calculation with and without compliance of the elastomer`s weak compressibility.

There is no complete methodology of the thin layer of rubber technical products calculation, which take into consideration the non-linearity of the deformation of reinforcement non-elastomer`s layers and mechanical characteristics of elastomer in the case of large specific axle load.

It is necessary to pay attention to the fact that after the obtaining of the approximated analytical solution in the publications there is practically no methodology, which allows estimating the accuracy of the obtained approximated solution. The absence of the verified experimental data does not allow to test the obtained solutions and unequivocally to recommend them for the analysis and construction of elastomeric products.

Therefore, the topicality of the discussed theme is obvious. Also the solutions of the above mentioned problems will allow extending the application area of existing rubber technical products, and to make more efficient the optimal construction of elastomeric products, executing the conditions of the durability and stability, and providing the necessary operating modes that are proposed for the torpidity characteristics of construction products.

The objectives of the thesis:

• the development of rubber technical products calculation new methodologies and the improving of existing in the case of the static load in the small deformation area, in accordance with elastomer`s properties and the construction features of these products;
• the development of rubber technical products hardness characteristics, "force -displacement", the approximated solution in the area of small deformations accuracy assessment methodology;
• the analysis of the constructive scheme of compensatory rubber - technical products, which provides the characteristics of given stiffness "force - displacement" and the development of these constructions calculation methodology.

Research tasks:

• the development of rubber - technical products compressible state calculation methodology in static loading in the area of small deformations, without the elastomer compressibility, using the principle of the deformation complete potential energy minimum;

• rubber - technical products approximated solutions, which are obtained, using the hypothesis of the elastomer non-compressibility, the determination of the application area, taking into consideration the factor of the product form and real value of the Poisson's ratio;

• the development of thin layer of rubber technical products calculation analytical methodology in the case of static load in small deformation area, taking into consideration the elastomer`s weak compressibility and the deformation of the non-elastomer layers;

• the development of thin layer of rubber technical products calculation methodology, taking into consideration the elastomer`s physical non-linearity;

• the development of thin layer of rubber technical products calculation analytical methodology in the case of static load in small deformation area, taking into consideration the elastomer`s weak compressibility and the deformation of the non-elastomer layers;

• the development of thin layer of rubber technical products calculation methodology, taking into consideration the elastomer`s physical non-linearity;

• the development of rubber technical products calculation analytical methodology in the case of static load in average deformation area, taking into consideration the elastomer`s weak compressibility, using the Delta-method;

• the development of rubber technical products with composite construction form calculation methodology, using the variation methods for discontinuous forses and movements;

• the development of rubber technical products hardness characteristics, "force – displacement ", the development of the analytical solution accuracy assessment methodology.

• the analysis of compensatory rubber - technical products constructive schemes, which allows realizing the given stiffness characteristics "force - displacement". The development of these constructions calculation methodology.

Scientific novelty of the thesis:

• the calculation methodology of developed rubber - technical products compressed state in the case of static load in small deformation area, do not taking into consideration the elastomer`s compressibility, using the principle of the deformation complete potential energy minimum;

• rubber - technical products approximated solutions, which are obtained, using the hypothesis of the elastomer non-compressibility, the determination of the application area, taking into consideration the factor of the product form and real value of the Poisson's ratio;

• the development of thin layer of rubber technical products calculation analytical methodology in the case of static load in small deformation area, taking into consideration the elastomer`s weak compressibility and the deformation of the non-elastomer layers;

• the development of thin layer of rubber technical products calculation methodology, taking into consideration the elastomer`s physical non-linearity;

• the development of rubber technical products calculation analytical methodology in the case of static load in average deformation area, taking into consideration the elastomer`s weak compressibility, using the Delta - method;

• the development of rubber technical products with composite construction form calculation methodology, using the variation methods for discontinuous forses and movements;

• the development of rubber technical products hardness characteristics, "force – displacement", the development of the analytical solution accuracy assessment methodology, which is obtained, taking into consideration the principle of complete potential energy minimum;

• three new compensatory constructions of rubber - technical products are proposed, which provide the necessary stiffness characteristics "force - displacement". The calculation methodology of mentioned compensatory rubber - technical products is developed.

Research results determined in the thesis:

• the methodology of rubber - technical products approximated solutions, which are obtained, using the hypothesis of the elastomer non-compressibility, the application area determination, taking into consideration the factor of the product form, the real value of the Poisson's coefficient and the necessary accuracy. The results are shown in the graphs and tables. In the example the rectangular and circular rubber -technical products are considered;

• the calculation methodology of rubber - technical products compressed state in the case of static load in small deformation area, do not taking into consideration the elastomer`s compressibility, and using the principle of the deformation complete potential energy minimum;

• the calculation methodology of thin layer of rubber technical products in the case of static load in small deformation area, taking into consideration the elastomer`s weak compressibility and the deformation of the non-elastomer layers;

• the calculation methodology of thin layer of rubber technical products, taking into consideration the elastomer`s physical non-linearity;

• the analytical methodologies of rubber technical products calculation in the case of static load in average deformation area, taking into consideration the elastomer`s weak compressibility, using the Delta - method;

• the calculation methodology of rubber technical products with composite construction form, using the variation methods for discontinuous powers and movements;

• rubber technical products hardness characteristics, "force – displacement", the analytical solution accuracy assessment methodology, which is obtained, taking into consideration the principle of complete potential energy minimum;

• three new compensatory constructions of rubber - technical products are proposed, which provide the necessary (including non-linear) stiffness characteristics "force - displacement" and the calculation methodology of mentioned compensatory rubber - technical products.

Practical importance:

Proposed rubber technical products calculation methodologies can be successfully used in engineering calculations, as well as in the construction design of new elastomers that provides the necessary (constant or variable) stiffness characteristics in static load. The accuracy assessment methodology of proposed and obtained solution allows assessing the accuracy of obtained solution, which is important for the calculation of the constructions durability. Proposed compensatory rubber technical products constructions with variable stiffness can be successfully used in different engineering areas.

Approbation:

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- Gonca V., Švabs J., Kobriņecs R. ***„Rigidity of Rubber-Metal Elements with Thin Layers at Compression".*** 7th International Scientific and Practical Conference Environment. Technology. Resources. Latvia, Rēzekne, 25.-27. June, 2009
- Gonca V., Shvab J. ***„Variāciju metodes elastomēra amortizatora ar saliktu konfigurāciju aprēķināšanai"*** RTU 50. Starptautiskā zinatniskā conference, Rīga, 12.-16. Oktobris, 2008
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• Gonca V., Švabs J. ***„Spēks–nosēde” veida tuvinātu risinājumu kļūdas noteikšana gumijas tehniskajiem izstrādājumiem”*** RTU 51. Starptautiskā zinatniskā conference, Rīga, 2010. 11.-15. Oktobris

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• Gonca V., Švabs J. ***“Calculation of Rubber Shock Absorbers at Compression at Middle Deformations Taking into Account Compressibility of Elastomeric Layer”*** 16th International Conference Mechanika – 2011, 7.-8. April , Kaunas, Lithaunia.

• Gonca V., Švabs J. ***“Projecting Elastomeric Shock Absorbers with Adjustable Stiffness”*** 7th International Scientific Conference „Transbaltica 2011”, Lithuania, Vilnius, 5.-6. May, 2011

• Gonca V., Švabs J. ***“Definition of Poisson's Ratio of Elastomers”*** 10th International Scientific Conference “Engineering for Rural Development" Latvia, Jelgava, 26.-27. May, 2011

• Gonca V., Švabs J., Mačanovskis A., Zaharevskis V. ***“Stiffness Characteristics of Shock Absorber of the Type "Force - Settlement" at Presence of the Liquid Including”*** The 10th International Conference (ICOVP 2011), Czech Republic, Praga, 3.-10. September, 2011

• Gonca V., Švabs J. ***“Design of Elastomeric Shock Absorbers with a “Soft” Stiffness Characteristics of Type "Force-Settlement””*** 10th INTERNATIONAL CONFERENCE „VIBROENGINEERING - 2011” *October 13-14, 2011, Kaunas, Lithaunia.*

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1. Vība J., Gonca V., Švabs J., Kobriņecs R., Kruusmaa M., Fontaine J., Megill W., Fiorini P. ***Жесткость тонкослойных резинометаллических элементов при сжатии*** // Динамика виброударных (сильно нелинейных) систем “DYVIS-2009”: Сборник трудов 16 симпозиума, Krievija, Zvenigoroda, Maskavas apgabals, 24.-30. maijs, 2009. - 97.-103. lpp.

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3. Gonca V., Švabs J. ***Desing of Elastomeric Shock Absorbers with Variable Stiffness*** // Journal of Vibroengineering. - Vol.12, Issue 3. (2010) pp 347-354.

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16. Y. Shvab, V.Gonca. ***Thin rubber – metal element's stiffness at compression*** // Scienfitic Works of Riga Technical University.Mechanical Engineering. Nanotechnology. Composite and Rubber Materials.-2012.- pp 82-86

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18. V. Gonca, S. Polukoshko, J. Shvab, A. Boiko. ***Multilayer spherical control joint-hinge stiffness characteristics optimization***// Journal of Vibroengineering (submit by publication)

19. Gonca V., Švabs J. ***Ritca metodes precizitātes novērtēšana gumijas tehnisko izstrādājumu aprēķinos*** // RTU zinātniskie raksti. 6. sēr., Mašīnzinātne un transports. - 34. Sēj. (submit by publication)

20. Gonca V., Švabs J.***„Spēks–nosēde" veida tuvinātu risinājumu kļūdas noteikšana gumijas tehniskajiem izstrādājumiem"*** // RTU zinātniskie raksti. 6. sēr., Mašīnzinātne un transports. - 34. Sēj. (submit by publication)

CONTENT OF THE THESIS

The first chapter presents a literature review on analytical calculation methods of technical rubber products. It follows from the literature review on rubber technical product calculations such as static and dynamic load, small, middle and large deformation; service life determination, determination of viscous-elastic and various other temperature factors including hysteresis loss calculation and many other tasks that during the initial stage of all the above-listed problems, the calculation of rubber technical products under various conditions of use commence with determination of the stress-deformed state, provided that the static load in the small deformation area has been determined. Thus the matter of developing new approximated methods and improvement of existing elastomer part estimation methods in terms of small deformation, both while taking and not taking into consideration the flat compressibility of elastomers, remains of importance.

In order to enable an effective use and optimum construction of rubber technical products, one must have suitable methods to obtain analytic solutions for realistically possible elastomer product geometries and static load patterns. Up-to-date publications do not offer any analytical solutions that would fully account for all geometric specifics of elastomer product structures and elastomer mechanical descriptions.

The weak compressibility quality of elastomer adds further complexity. This issue, even though it is practically solved in simple shapes of technical rubber products, does not have a full analysis and a set of methodology recommendations regarding the use of such analytical solutions that were obtained via a hypothesis on the non-compressibility of elastomer, depending on the geometrical parameters of the product and the actual values of elastomer's Poisson ratio.

Most of the analytical solutions have been obtained via a means of variation methods provided that the conditions for elastomer non-compressibility are satisfied. Furthermore, the hydrostatic pressure function included in the boundary problem equations of material elasticity theory, is discarded from the solution in the above case, thus preventing to conduct a tensioned state analysis of an elastomer product and subsequently – resolve the matters of stiffness and stability.

Generally speaking, to this date there is no methodology available to tackle complex configuration rubber technical product calculations with or without taking into account the weak compressibility feature of elastomer.

Also there is no comprehensive thin-layer rubber technical product calculation methodology that would account for threaded non-elastomer layer deformation and the non-linearity of elastomer technical description upon severe specific axial load.

One must pay attention to the fact that in terms of acquisition of approximated analytic solutions, the present reference literature does not offer any methodology that would allow for precision assessment of the obtained approximated solutions. The lack of verified experimental data prevents one from testing the obtained results to be able to unequivocally recommend such solutions as effective means of analysing and designing elastomer products.

The second chapter of the paper presents a set of developed rubber technical product calculation methods.

The solution of a specific elasticity theory exercise is attained by determining tension and displacement satisfying the tension or displacement equation system and other relevant exercise conditions i.e. boundary conditions [18], [19].

The mathematical model of linear elasticity theory boundary problem for a weak compressibility material is as follows:

- equations of equilibrium:

$$G\left[\nabla^2 u_i + \frac{3}{2(1+\mu)} s_i\right] + f_i = 0, \tag{1}$$

-bulk deformation relation:

$$u_{j,j} = \frac{3(1-2\mu)}{2(1+\mu)} s, \tag{2}$$

-deformation components ε_{ij}:

$$\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right), \tag{3}$$

-stress components σ_{ij} :

$$\sigma_{ij} = G\left(2\varepsilon_{ij} + \frac{3\,\mu}{1+\mu} s\delta_{ij}\right), \tag{4}$$

-force boundary conditions:

$$\sigma_{ij} n_j = p_i \quad \text{on } F_{\sigma}, \tag{5}$$

-geometrical boundary conditions:

$$u_i = u_{oi} \text{ on } F_u, \tag{6}$$

where f_i – force components

u_i – displacement components

G – rubber shear modulus;

σ_{ij} – stress components;

s – hydrostatic pressure function;

$\delta = \begin{cases} 1, \, at \; i = j \\ 0, \, at \; i \neq j \end{cases}$;

i;j = 1,2,3 or x, y, z;
μ – Poisson's ratio

If the weak compressibility feature of elastomer may be disregarded then μ = 0.5. In this instance, the following equation system (1) – (6) with μ being 0.5 reflects a boundary problem of a non-compressible material [6], [15], [20].

An accurate solution of the differential equation system (1) – (6) is only possible for a limited range of exercises when both the shape of the product and boundary conditions are fairly simple. Too often one must accept only an approximate solution. For this reason, approximation methods have gained substantial popularity within the field among them the most effective being one based on the principle of full potential energy minimum of the system.

Full potential energy $J(u_i, s)$ is as follows:

$$J\,(u_i,\, s) = U - A, \tag{7}$$

where U – deformation potential energy;
A – external force energy loss.

$$U = G\int_V \left[\frac{1}{2}\left(u_{i,j}u_{j,i} + u_{i,j}u_{j,i}\right) + \frac{3\mu}{1+\mu} s u_{i,i} - \frac{9(1-2\mu)}{4(1+\mu)^2} s^2\right] dV\,, \tag{8}$$

where i; j = 1, 2, 3 or x, y, z – coordinate system components.

A is the work of external force p_I and body forces f_i:

$$A = \int_V f_i u_i dV + \int_{F_\sigma} p_i u_i dF\ . \tag{9}$$

It may be verified if minimum conditions constitute the equivalent of boundary problem exercise (1) – (6), taking into account that on F_u $\delta u_i = 0$.

Solution method algorithm:
-displacement components u_i and hydrostatic pressure functions s:

$$u_i = a_{ji}\, \varphi_j(\bar{x}),\ s = b_k \psi_k(\bar{x})\,, \tag{10}$$

where $\varphi_j(x)$ - mandatory geometrical boundary conditions (6);
$\psi_\kappa(x)$ – may be selected freely;
a_{ij}, b_k – unknown constants;
-it follows from the full potential energy minimum condition of the system:

$$\frac{\partial J(u_i, s)}{\partial(a_{ij}, b_k)} = 0\,,$$

the constants of a_{ij}, b_k are set.

-the components of deformation and stress are calculated in accordance to the methods of (3) and (4).

Stiffness characteristic curves [7],[15],[20],[21] have been obtained for a substantial class of rubber technical products, provided that it is possible to disregard the compressibility of elastomer (this is true for products with specific geometrical parameters). If the geometrical shape or the value of Poisson's ratio μ for rubber already restricts from disregarding the weak compressibility of elastomer then the functional of (7) – (8) must be used in order to determine the relation of "force – displacement", of which u_i and s are unequivocal components. In this case, the exercise grows in complexity.

There may be a necessity to assess the influence of weak compressibility on the solution obtained with the use of non-compressibility condition. Here, in order to avoid having to solve the complex exercise with the functional of (7) – (8), it is recommended to use approximation method to account for weak compressibility. It is of particular convenience if one must determine the stiffness characteristic curve of "force – displacement" relation. In the following, we shall review a rectangular shape rubber technical product with a flat rubber layer in a coordinate system. It is loaded with an axial force of compression P (along z- axis, perpendicular to the reviewed flat layer):

$$\int_F \sigma_{zz}\, dF = -P\,, \tag{11}$$

where F – cross-section of the rubber layer.

Since a compressible material:

$$\sigma_{zz} = G\left(2\frac{\partial \omega}{\partial z} + \frac{3\mu}{1+\mu} s \right), \tag{12}$$

where ω - displacement parallel to compression P.

Then from (2), (11) and (12):

$$2G\int_F \frac{\partial \omega}{\partial z}\, dF + \frac{3\mu G}{1+\mu}\int_F s\, dF = -P\,. \tag{13}$$

Since displacement function ω has already by found via approximation, then (13) may be used to find the approximated value of hydrostatic pressure function s. By placing s into (2) and integrating its value, we obtained the approximate value of volume deformation resultant of weak compressibility:

$$\Delta V_c - \frac{1-2\mu}{2\mu}\left(\frac{P}{G} h + 2\int_V \frac{\partial \omega}{\partial z}\, dV \right), \tag{14}$$

where h - the thickness of rubber layer.

Since $\Delta V_c \approx - \Delta_c F$ then from equation of the hydrostatic pressure function s taking into account the known displacement ϖ, we obtained an additional displacement value of Δ_c, resultant of weak compressibility:

$$\Delta_c = \frac{1-2\mu}{2\mu F}\left(\frac{P}{G}h + 2\int_V \frac{\partial \varpi}{\partial z}\, dV\right). \tag{15}$$

In this instance, the aggregate displacement of the rubber technical product is as follows:

$$\Delta_\Sigma = \Delta + \Delta_c. \tag{16}$$

By comparing Δ_c with Δ, for each geometry of rubber technical product and each type of elastomer, it may be approximately determined if it is necessary to specify the contribution of compressibility in shock absorbers displacement by solving the present exercise with a use of a more precise method.

The use of a full potential energy functional (7) leads to a weak relative algebraic equation system with associated constants of a_{in} and c_{in}, since the functions of u_i and s approach (7) incorrectly: u_i – quadratic, s – linear. Thus we obtained an incorrect variation exercise for a functional (7) [24] requiring a development of new specialised methods for acquisition of approximate solutions for u_i and s. The method proposed herein enables obtained a stable approximated solution and was developed on the basis of various methods for an incorrectly given boundary problem of a linear theory. The basis of the proposed solution is formed by the regularisation of the basic functional. Also a number of requirements for such regularisation functional and parameter were set forth. The proposed method may be used for acquisition of analytical approximated solutions or numeric approximated solutions.

A number of experiments with rubber – metallic shock absorbers with thin elastomer layers [7], [9], [13] for which $\rho = b_{min}/h \geq 40 \div 50$ (where b, h – the width and thickness of rubber layer respectively) at axial compression have indicated towards a significant non-linearity in the relation of “force – displacement” already at small deformation not exceeding 3 – 5%. The dependency of “force – displacement” relation on the geometric factor $\rho = b_{min}/h$ is displayed in Figure 1.

Generally accepted methods of rubber technical product calculation [17], [20] do not allow one to describe the characteristic curves obtained in compression experiment [10], [15] (Black lines in Figure 1) provided an exercise in which geometrical and physical qualities are of linear mature (Blue lines in Figure 1).

Most of the researchers explain this phenomenon with the physical non-linearity of elastomer. It could be argued that in a sufficiently thin, compressed layer of elastomer results in significant hydrostatic pressure which affects the mechanical qualities of elastomer. Upon characterising force the acquisition of approximated solutions may be significantly simplified if one assumes that a physically non-linear solution does not contain anything unpredictable when compared to physically linear solutions. In this instance, approximated method may be used for the acquisition of a analytic solution in a physically non-linear exercise.

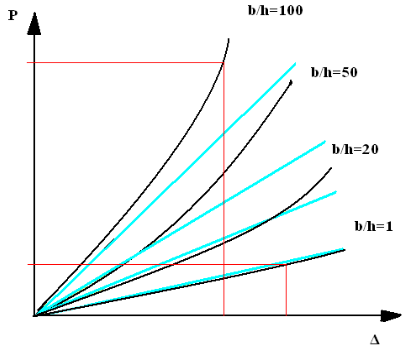


Figure 1. A qualitative graph of „force – displacement" relation, Δ/ h ≤0.05

Thus, the calculation of compensators with extremely thin layers of elastomer consists of two stages:
1) Solution of the exercise for a non-compressible material;
2) Solution of the exercise which takes into account a type of bulk compression assuming that the function of hydrostatic pressure is constant across the entire volume of elastomer.

Thin layer rubber – metal elements are calculated with the use of classic solutions [17], [20]. The calculation of "force – displacement" shows that there is a sufficiently significant difference between the calculated value and data obtained during experiment [11], [16]. Upon analysing analytical and experimental data, it may be concluded that as Poisson's ratio approaches 0.5 and the thinner the layer of rubber, the more substantial the aforementioned discrepancy. A difference between analytical and experimental data may be explained with the fact that during compression both elastomer and non-elastomer layer are subjected to the force of compression. The present paper describes a method allowing to calculate the stiffness description of "force – displacement" for thin rubber – metallic layers. Proposed method accounts for the weak compressibility of and non-elastomer (metallic) layer deformation during axial compression. The solution for the determination of displacement (17) is obtained via the use of Ritz method and the principle of full potential energy minimum

$$\Delta = \frac{P\, h_e n}{2{,}5\, G_e ab} \frac{1 + 1.25 \dfrac{B_1 B_2}{\chi (B_1 + B_2)}}{1 + \dfrac{B_1\, B_2}{B_1 + B_2 + \dfrac{1-2\mu}{\mu} B_1\, B_2}}, \tag{17}$$

where $B_1 = 1 + \dfrac{5\,\alpha^2}{12}$; $B_2 = 1 + \dfrac{5\,\beta^2}{12}$; $\alpha = \dfrac{a}{h_e}$, $\beta = \dfrac{b}{h_e}$, $\chi = \dfrac{G_m h_m}{G_e h_e}$

a, b, h_e, h_m – geometric parameters for the layers of rubber and non-elastomer;

G_e, G_m – shear module for each layer;

n – layer number.

If the layers of rubber and non-elastomer are of different sizes, rubber layer being “thick”, then it saves us having to account for the weak compressibility of rubber, and if the geometrical dimensions and physical – mechanical features of the non-elastomer layer is ($h_e < h_m$, $G_e << G_m$, i.e. parameter ($\chi \rightarrow \infty$), then we also do not have to account for interlayer deformation.

Complex configuration parts or/and parts consisting of contact area components produced of various materials and the deformation of which must be accounted for during calculation, the use of $\Pi(u_i,s)$ (7) functional becomes complex due to problems relating the selection of coordinate functions for displacements u_i and hydrostatic pressure s, which are continuous across the entire volume of the complex product. One of the feasible approaches involves dividing the complex area in question into a number of sub-areas. However, in this case, one must account for the displacement u_v, its derivative $u_{i;j}$ and the continuity condition of hydrostatic pressure s, crossing the boundaries of divided areas. It effectively leads to a substantial amount of calculation or even to impossibility to select the sought functions. The general provisions relating direct calculations with the use of functions that allow to mitigatecontinuity requirements are comprehensively reviewed in the paper by V. Prager [22].

With the use of V. Prager’s procedure [22], basing the full potential energy minimum principle on functional $\Pi(u_i,s)$ (7), varying only displacements u_i and the hydrostatic pressure function of s, it is possible to obtain a number of functionals that may be used for non-continuous displacement and force functions because the volume in question is seen as a sum of simpler volumes. The following four versions have been reviewed in the course of the present paper:

Version No. 1

- Displacement u_i functions are continuous in each of the sub-regions V_n of the division and satisfy the continuity conditions of $u_i^n = u_i^{n+1}$ on division surfaces

$\Gamma_{n,}$ as well as fulfil geometrical boundary conditions on surfaces F_u^n;
- Hydrostatic pressure function s is continuous in each sub-area V_n of the division.

Version No. 2

- Displacement functions are continuous in each of the sub-regions V_n of the division and satisfy all mandatory geometrical boundary conditions on surface F_u^n, but do not satisfy the continuity condition for displacement on surfaces Γ_n of the division.
- Hydrostatic pressure function s is continuous in each sub-area V_n of the division.
- in line with the continuity condition for forces on surfaces Γ_n of the division, functions u_i and s satisfy the continuity conditions on division surfaces $\Gamma_{n.}$

Version No. 3

- displacement functions are continuous in each sub-area V_n of the division and satisfy the mandatory geometrical boundary conditions on surfaces F_u^n and satisfy displacement conditions $u_i^n = u_i^{n+1}$ on some surfaces Γ_n of the division;
- hydrostatic pressure functions are continuous in each sub-area V_n of the division;
- functions u_i and s satisfy continuity conditions for forces on the surfaces Γ_n of the division, on which displacement continuity conditions have not been met;

Version No. 4

- displacement functions are continuous in each sub-area V_n of the division and satisfy the mandatory geometrical boundary conditions on surface F_{un}, but does not satisfy continuity conditions for displacements $u_i^n = u_i^{n+1}$ on surfaces Γ_n of the division;
- hydrostatic pressure functions are continuous in each sub-area V_n of the division;
- functions u_i and s do not satisfy the continuity conditions for forces on surfaces Γ_n of the division;

The idea to divide area V into a number of sub-area and in the meantime do not or partially satisfy displacement and force continuity conditions on surfaces Γ_n of the division, allow to form a general method for the calculation of complex products. Such methods would obsolete the necessity to use complex functions while the precision of the solutions would be obtained by using a sufficient number of sub-area.

Often enough rubber technical products, shock absorbers of various configurations, in particular, are used not only in small deformation applications, but also in applications with middle deformation (40 – 50%). This chapter shall present an approximated calculation method for rubber technical products in the case of middle elastomer deformation, titled – delta-method.

Delta-method uses a sequential small deformation layering while accounting for the sequence of the loads and the resultant changes in configuration via deformation. Thus, by setting deformation $\delta\varepsilon_{ij}{}^{k}$ and the increase of tension $\delta\sigma_{ij}{}^{k}$ in each stage of deformed elastomer load, one may use linear physical relation of (1) – (6): Stages of calculation in the general case according to delta-method:

- assume that there is no previous load, does it not affect the changes of product configurations under the following instances of load:
- the given load range of the product is divided in n steps;
- the size of a step in the respective load sequence (according to displacements and loads) is selected in such a manner that elastomer deformation during each stage of load could be considered small when looking at the configuration of the product.

The first step ($k = 1$) involves a solution of an exercise of small deformation in a previously non-deformed (load free) product and the deformed configuration is thus set. Each following step ($1 < k \leq n$) also involves solving linear exercises on small deformation with geometrical configuration set by the load in the preceding step. The calculation is concluded when sequence of load (displacement and load) reaches fixed final value. This algorithm requires that the configuration of the rubber technical product is recalculated during each stage which severely complicates the acquisition of the solution in the analytical approach. Nevertheless, a number of rubber technical products allow a transition from having to add up all results from the stages of load to integration by obtaining an analytical expression for the description of "force – displacement" stiffness. In particular it holds true for the most of compensators by acquiring a dependency on "force – displacement" $P(\Delta)$. Each load stage K must obtain a recurring dependency of "force – displacement":

$$P_k = \delta\,\varphi_k(\beta, \alpha_k), \qquad \delta = \frac{\Delta}{n},\; P_k = \frac{P}{n}, \tag{17}$$

where n – number of calculation steps;

φ_k – a function that included the geometrical parameters of the β not subjected to change during stages of load, and geometrical parameters α_k, subjected to change during each stage of load.

Parameters α_k and functions φ_k (β, α_k) constitute the solution in each stage. If the dependencies of (17) allow substitute adding up with integration then from

$$P = \sum_{k=1}^{n} P_k = \int_0^{\Delta} dP = \int_0^{\Delta} \overline{\varphi}(\beta, \overline{\alpha}) d\delta \qquad (18)$$

we obtain an analytical "force – displacement" for the case of middle shock absorber deformation.

Same as it was observed in the case with small deformation also in the instance of middle deformation the sufficiently thin elastomer layers (or massive rubber technical parts with relatively small elastomer mass free side surfaces) must take into consideration the weak compressibility of elastomer. The assessment of elastomer low compressibility effect on compensator's description "force – displacement" is possible with the following two approximation approaches.

In the first version of [4], [20] if the solution and displacement Δ_o^k is found without taking into account the weak compressibility of elastomer layer, then from the physical relation between deformations and stress for bulk deformation due to weak compressibility of elastomer, as well as by taking into consideration (2) and (3), we obtained the following relation:

$$\int_{V_1} u_{i,i}{}^k V_1 = -\frac{1-2\mu}{\mu} \int_0^h \left(\frac{P}{2G} + \int_{F_\sigma} \frac{\partial u_z{}^k}{\partial z} dF_\sigma \right) dz, \qquad (19)$$

where F_σ^k- area of surface subjected to external load
h^{k-} elastomer layer thickness;
u_z^k – displacement function along z-axis (towards compression force), not taking into consideration the weak compressibility of elastomer layer.

The charge ΔV_k* of elastomer layer volume may be written down as follows:

$$\int_{V_1} u_{i,i}{}^k dV_1 \approx \Delta V_k{}^* = -\Delta_k{}^* \chi_k F_k, \qquad (20)$$

where Δ_k* - compensator deformation observed the bulk deformation of elastomer layer by taking into account the weak compressibility feature of elastomer;
χ_k - a ratio dependent of the geometry of elastomer layer;
F_k – cross section surface area of an elastomer layer.

Deformation of a compensator which observes the bulk deformation of elastomer layer by taking into account may be calculated according to the following formula:

$$\Delta_k{}^* = \frac{1-2\mu}{\mu F_k} \int_0^h \left(\frac{P}{2G} + \int_P \frac{\partial u_z}{\partial z} dF \right) dz \qquad (21)$$

The solution of the second version uses a solution for a non-compressible elastomer. Subsequently the expression for displacement Δ_k*, may be found from equation (19) taking into consideration the weak compressibility of elastomer:

$$\Delta_k{}^* = -\frac{3(1-2\mu)}{2F(1+\mu)} \int_{V_{k=1}} s_k \; dV_{k=1} \qquad (22)$$

The expression of (22) includes the function of hydrostatic pressure *s* which remains unknown due to the fact that is not a part of the functional if displacement function is calculated under the condition of non-compressibility. In this instance, provided that a substantial effect by weak compressibility is observed only in thin layers of elastomer, an approximated function *s* may obtained for the balance condition of cross section of an elastomer layer towards which force is being exerted.

If the effect of weak compressibility is taken into consideration then during step *k* of the load in delta function, the sag may be determined with the help of the following formula:

$$\Delta_{k\Sigma} = \Delta_k + \Delta_k{}^* \qquad (23)$$

All analytical solution for rubber products were conducted with the use of approximated methods. There are no precise analytical methods for the calculation of rubber product. When using approximated methods, it is important to know the extent of the marginal error for the solution found and if such solution is larger or smaller than the actual value. The present paper offers an approach enabling to determine the precision of the solution with the help of Ritz method. Two approximated solutions of a single exercise were obtained during the study: the first solution was obtained with the help of Ritz's method for a rubber buffer with displacement discrepancy, second –displacement surplus with the use of a functional developed by Slobodjanskiy [23]. An accurate result lies between the two. The principal advantage of the method is the possibility to assess the extent of error present in yielded approximated solutions in the meantime not disregarding Ritz's method.

The essence of the method is as follows: Let us assume that we have a rubber compensator loaded with a force of P and its displacement is $\Delta = \Delta^*(P)$. By employing the principle of potential energy minimum, we obtain the following displacement value with discrepancy:

$$\Delta^*(P) \le \Delta_p(P) \ , \qquad (24)$$

where Δ*- displacement calculated with the use of Ritz's method

Δ_p - actual displacement value and accurate solution.

Say that we were able to come up with a functional in $J_1(u_1, u)$ such a manner that it possesses minimum features and

$$J_1(u_1,u) \le J(u)\,, \tag{25}$$

where $J(u)$ - an expression for potential energy

With the solution found, *J(u)* may be written down with *Δ* and *P* without much trouble $J = J(\Delta, P)$, which reaches its minimum when $\Delta = \Delta_p(P)$. If $J_1(u_1,u)$ shall be also expressed with *Δ* and *P*, then in line with (25),

$$J_1(\Delta, P) \le J(\Delta, P) \tag{26}$$

at any value of *Δ*. Considering the quadratic nature of functionals J_1 and *J*, “the fork” for a specific value of $\Delta_T(P)$ may be found from (24) and (25), not knowing the expression of $J(\Delta, P)$. The “fork” is obtained from the root of the following equation:

$$J[\Delta^*(P)] = J_1(\Delta, P) \tag{27}$$

We shall denote these roots as Δ_1 and Δ_2. Then

$$\Delta_1 < \Delta^*(P) \le \Delta_p(P) < \Delta_2 \tag{28}$$

Provided that $J_1(\Delta, P)$ and $J(\Delta, P)$ are quadratic functions in relation to, then a narrow “fork” is obtained from (25) and (26):

$$\Delta^* \le \Delta_p \le \frac{\Delta^* + \Delta_2}{2}\,. \tag{29}$$

Third chapters presents three new structures of compensating rubber technical products that would ensure the satisfaction of the required stiffness description of “force – displacement”.

The first such solution of how to form a compensator with a varying stiffness upon compression is presented in Figure 2.

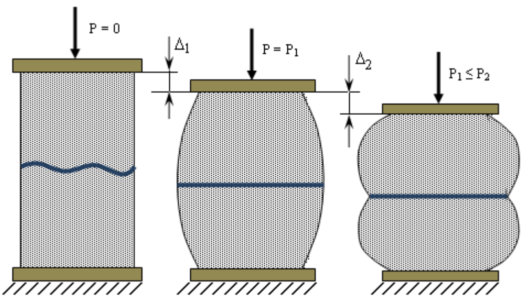


Figure 2. A cylindrical shape compensator with varying stiffness.

The compensator presented in Figure 2 consists of a rubber layer with a integrated thin layer of frilled non-elastomer layer (metal, textile, polymer ect.

materials). Say that the layer of non-elastomer is sufficiently thin to allow the resistance to bending forces would be close to 0.

The operation of the proposed compensator consists of two stages:

First stage: if force exerted is $0 \leq P \leq P_1$, it acts like a single layer type compensator by gradually straightening the thin non-elastomer layer.

Second stage: If force exerted is $P_1 \geq P \geq P_2$, then it starts to act as a double-layer type rubber compensator, because due to vulcanisation effect, elastomer interlayer does not allow the attached rubber to deform at the area of attachment. The general nature of "force – displacement" nature for a stifness compensator is presented in Figure 3.

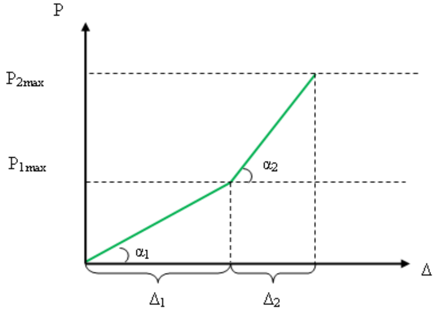
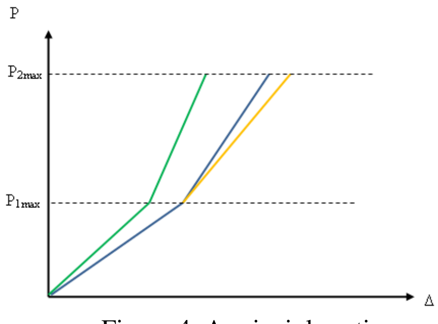


Figure 3. Principal "force – displacement" relation

During the conducted calculation rubber was viewed as a uniform mass without hollow areas and pores, being isotropous, flexible, the relations between rigidity σ_{ij} and ε_{ij} deformation may be denoted with the linear equation of (1) – (6), rubber layer deformation may be considered as being small $0 < \varepsilon <$ 15%, it is assumed that the compensator is subject to a static load, the layer of non-elastomer is sufficiently thin not to take into consideration its resistance to the forces of bending and stress, the upper and lower supports of compensator are attached to the rubber, being solid and non-deforming.



where

a solution for non-compressible materials without interlayer deformation;

a solution for compressible materials without interlayer deformation;

a solution for compressible materials, including interlayer deformation.

Figure 4. A principle action graph of a compensator calculated with the help of various solutions

All calculations were based on the principle of full potential energy minimum [20]. As a result of which, three solutions for the specific type of compensator were obtained.

The second solution to the acquisition of a compensator with a varying stiffnes during compression is offered by an elastomer compensator with liquid inclusion. The cylindrical shape elastomer compensator in question with liquid inclusions have two different stages of stiffness ($c_2 > c_1$): 1st Stage – deformation 0 - β% and stiffness C_1. 2nd stage – deformation –β% ÷ 15% and stiffness C_2 (where: β% < 10%÷ 15%) The presented compensator structure is presented in Figure 5. With the use of such compensator and by changing the level within the internal chamber it is possible to affect the value of displacement at which the increase in rigidity takes place (Figure 6).

The action of the compensator consists of the two following stages:

First stage: by subjecting a cylindrical compensator to a load of $0 \leq P < P_,$ the elastomer chamber is being compressed thus acting as a regular ring-like cylindrical elastomer compensator. As the chamber becomes compressed the air in it is pushed out. When entire amount of air has been pushed out, the valve closes and the first stage of action is concluded.

Second stage: by subjecting the cylindrical elastomer compensator with a force of compression $P_2 \geq P_1$, the stiffness of compensator severely increase, due to the absence of air in it, the valve closes and the internal liquid counteracts the compression process and thus affects the internal walls of the chamber.

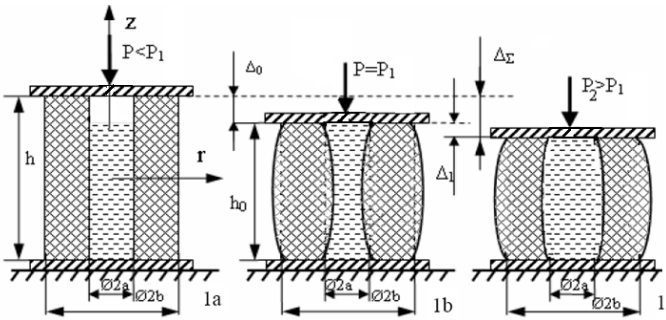


Figure 5. Action scheme of an elastomer compensator with liquid inclusions

Definitions:

1a. – Load scheme including air and liquid;

1b. – Liquid filled the entire internal volume.

1c. – The process of compression is affected by the presence of liquid.

where h – the height of elastomer layer; h_o – height of elastomer layer when liquid has filled the entire internal liquid; Δ_0 – deformation at which air is being pushed out; Δ_1 – deformation resultant of continuous compression after all the air has been extracted; $\Delta_\Sigma = \Delta_0 + \Delta_1$– total deformation; P – force exerted; P_1 – force exerted at which all the air is pushed out; P_2 – force exerted at which the compression process begins to affected by the liquid.

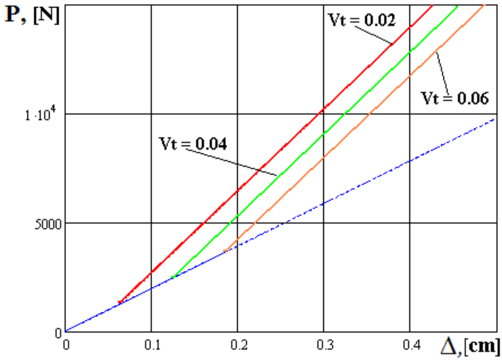


Figure 6. Relation "force – displacement" for an elastomer ring-like cylindrical compensator with liquid inclusions at various liquid levels in the internal chamber

Within calculations, rubber was described as a uniform material without pores, hollow areas, isotropous, relations between tensions σ_{ij} and deformations ε_{ij} may be denoted with linear equations of (1) – (6), the deformation of rubber layer were fairly small $0 < \varepsilon < 10\% \div 15\%$. Assume that the compensator is subjected to a static compression, upper and lower compensator supports are attached to the rubber, being completely solid and non-deforming. All calculations are based on the principle of full potential energy minimum.

The dependency of compensator displacement values on the force exerted may be calculated with the use of the following formula:

$$\Delta_\Sigma = \Delta_0 + \Delta_0^* + \Delta_I + \Delta_I^* \tag{30}$$

where Δ_0 - compensator displacement in the first stage not taking into account the weak compressibility of rubber.

$$\Delta_0 = \frac{Ph}{G\pi(b^2 - a^2)\cdot c}$$

Where c constant is dependent on the geometrical dimensions of the compensator.

Δ_0^* - compensator's displacement during first stage due to weak compressibility.

$$\Delta_I^* = \frac{3Ph(1-2\mu)}{\pi(b^2 - a^2)2(1+\mu)G}$$

Δ_1- compensator displacement in the second stage not taking into account the weak compressibility of rubber.

$$\Delta_0 = \frac{Ph}{G\pi(b^2 - a^2)\cdot D}$$

Where D constant is dependent on the geometrical dimensions of the compensator.

Δ_1^* - compensator's displacement during second stage due to weak compressibility.

$$\Delta^* = \frac{3Ph(1-2\mu)}{\pi \cdot b^2 \cdot 2(1+\mu)G}$$

Over the recent years, the demand has grown for use of shock absorbers, in which stiffness reduces upon increased load. For instance, such shock absorbers are required for fastening of aircraft engines [1]. Due to ecological norms, parameters of aircraft engines are subjected to change thus there is a necessity to suppress low frequency vibrations caused by the engine of an aircraft. Figure 7 demonstrates the stiffness characteristic curve of the required shock absorber.

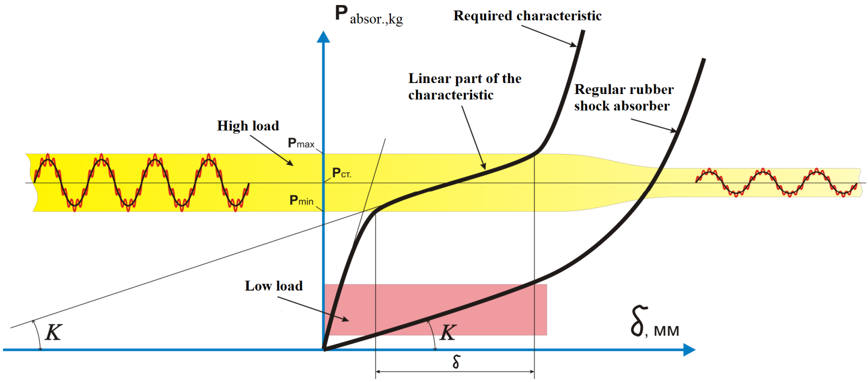


Figure 7. Stiffness characteristic curve of an aircraft engines [1]

Rubber compensator with varying height, non-deformable side support (Figure 8) ensures the features of the above stiffness characteristic curves. The use of varying height side supports one to substantially improve the types of "force – displacement" relation for rubber compensators, operating with axial compression. Varying height side supports enable both increasing and decreasing the thickness of free rubber layer. While together with the thickness of the free rubber layer, stiffness characteristics of the compensators also change i.e. if the thickness of rubber layer increases – the stiffness of the compensator decreases while if the former decreases – the said stiffness increases. The principal advantage and difference of a compensator of such structure from compensators with varying stiffness (form examples, ones with fixed side stop) that by moving the side stop, we can not only increase stiffness but also reduce it.

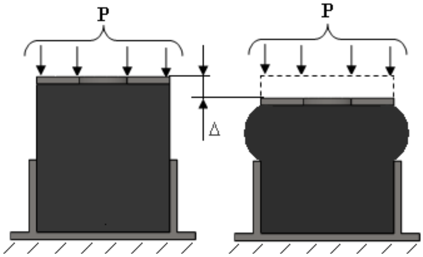


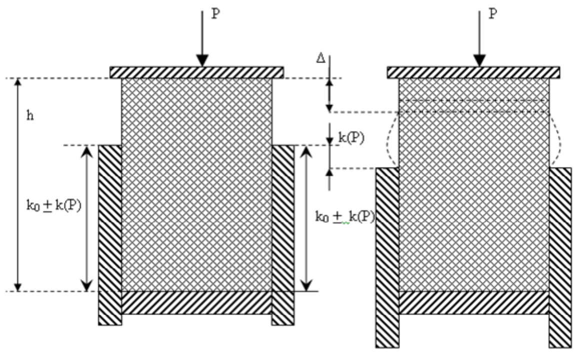
Figure 8. Rubber Compensator with a varying height side stop

The type of the rubber compensator "force – displacement" characteristic curve shall be dependent on the fact according to which of the curves the height of our side stop is changing. There are three following types of side support height variations.

1. The height of side support changes when the compensator is not subjected to a load.

2. The height of side support changes when the function of the exerted force changes (displacement value.).

3. The third version of side support height change provides a smooth stiffness characteristic curve "force – displacement" – either "soft" or "hard. Similar to the above example, the compensator shall be "soft", if the height of the side stop changes with curve $k = k_0 - \Delta k(P)$, " hard" - $k = k_0 + \Delta k(P)$.



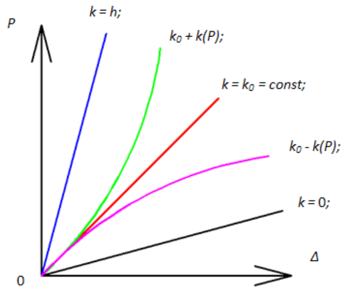


Figure 9. Stiffness characteristic curves of a rubber compensators with a varying side stop height. The height of side stop changes as a function of the amount of force exerted

Third version of side stop hide regulation offers the broadest range of opportunities to regulate the stiffness of the compensator. However, from the structural point of view the latter is significantly more complex to implement than the former two.

We shall review an analytical solution of "force – displacement" relation force compensators of the aforesaid structure. The compensator shall be first divided in two (Figure 10).

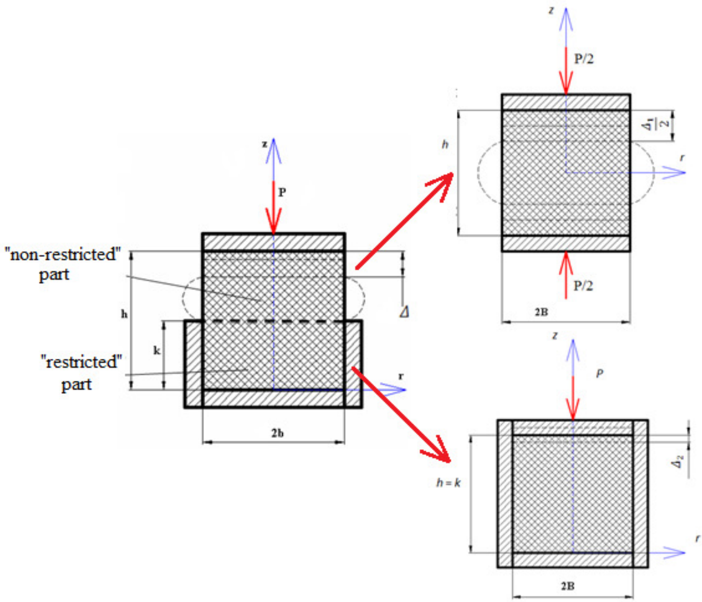


Figure 10. A calculation model for a rubber compensator with a varying height side stop

First part (non-restricted part) – (h – k(P)) ≤ z ≤ h is a compression symmetrical towards the axis while the second part (restricted part) 0 ≤ z ≤ (h – k(P)) is bulk compressible.

The total displacement of the rubber compensator in question shall be

$$\Delta = \Delta_I + \Delta_{II}$$

$$\Delta = \frac{(h-k(P))}{\pi b^2 G}\left[2{,}4 + \frac{3\,D_1}{5\left(1+\dfrac{1-2\,\mu}{\mu}\,D_1\right)}\right]^{-1} + \frac{3P\cdot k(P)}{2\pi b^2 G}\cdot\frac{(1-2\mu)}{(1+\mu)} \tag{31}$$

The fourth chapter presents natural rubber compensators compression experiments.

Testing thin layer rubber – metal compensators.

Rubber – metal compensators of various geometrical dimensions were produced and tested (three samples per kind were produced. All compensators had three layers of one and the same rubber. Experimental samples were compressed with the help of ZWICK/Roell Z-150 testing equipment (See Figure 11).

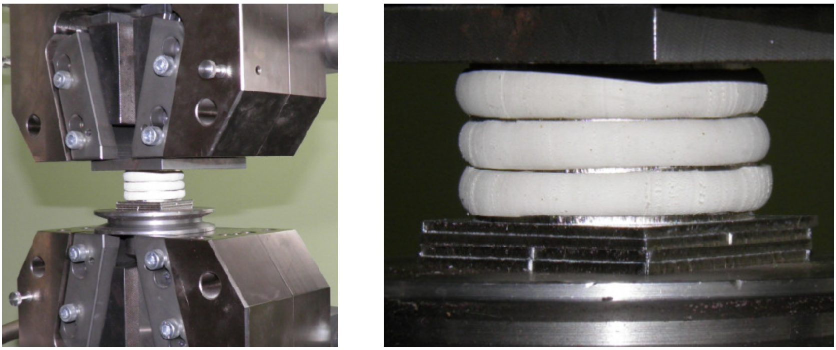


Figure 11. Compression of thin layer rubber – metal compensator in a ZWICK/Roell Z-150 testing equipment

All analytical calculations were conducted within MathCad. A set of two analytical solutions were obtained for each size of the compensator. First solutions (solution – “a”) does not account for the weak compressibility of rubber (μ = 0.5). While the second solution (solution – “b”) accounts for the weak compressibility of rubber (μ ≠ 0.5)

Compensator modelling was conducted with the help of final experiment software SolidWorks. Since all rubber layers in a compensator were equal and could be divided into equal parts along the symmetrical axis, thus simplifying the solutions, a model was created only for a quarter of the entire rubber layer.

Displacement limitations were placed at the place of rubber and steel attachment i.e. layers cannot displace towards one another at the place of attachment. Sides were applied with symmetry condition (limitation). Upper interlayer of steel was subjected to distributed load.

The results of the experiment, analytical calculation and modelling are summarised in Table 1. Upon analysing the obtained data, one may conclude that the analytical calculation results that accounted for the weak compressibility feature correspond to the experimental data the best.

A increase of the error margin could have been observed for results that did not account for weak compressibility upon decreasing thickness of the rubber layer. Such findings correspond the theory. Modelling in SolidWorks yielded sufficient result. The results may be further improved by improving the model in SolidWorks. Such modelling is advantageous in the fact that one may not only view the vertical displacement of the compensator but also review displacements along axis along with tensions and deformations (in both elastomer and non-elastomer layers).

Table 1

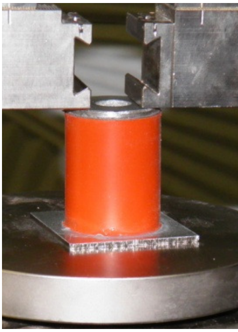
The results of rubber – metal compensator testing, calculation and modelling

Sample No.	Force, kN	Rubber compensator displacement, mm				Rubber compensator displacment, error %		
		Experiment	Analytical solution„a”	Analytical solution„b”	Modelling in SolidWorks	Analytical solution „a”	Analytical solution „b”	Modelling in SolidWorks
1	16	0.515	0.125	0.500	0.444	2.9	75.7	7.1
2	16	0.429	0.072	0.426	0.349	0.7	83.2	18.6
3	16	1.567	0.937	1.650	1.477	5.3	40.0	9.0
4	16	1.181	0.560	1.141	1.151	5.4	52.3	4.7
5	0.8	2.339	1.843	2.455	2.209	5.0	21.2	5.5
6	1	1.832	1.120	1.670	1.646	8.8	39.1	10.0
7	0.8	3.584	3.204	3.500	3.396	2.3	10.6	5.2
8	0.8	2.606	1.978	2.481	2.702	3.9	24.1	3.7

Testing of rubber compensators with varying height side stop

A total of 4 samples were produced for the purpose of the experiment – rubber cylinders with the following dimensions – h = 40 mm and d = 36 mm (Figure 12). Two steel bases were attached to the rubber cylinder. All samples were produced of the same rubber and in the same size with the use of the same technology.

A number of different height side supports were placed on the cylinder during experiment. The compression of rubber compensator was conducted with ZWICK/Roell Z150 testing equipment and testXpert II computer software.


a)

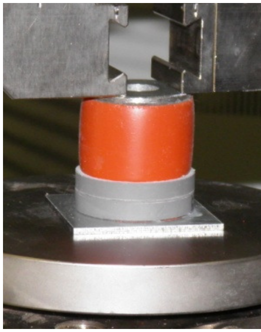

b)

Figure 12. Experimental sample a) k = 0; b = k 13mm

First, the relation of "force – displacement" is calculated for the compensator based on the height of the side stops. In first calculation (a) it is assumed that rubber is non-compressible ($\mu = 0.5$), while the second calculation (b) takes into consideration the weak compressibility of rubber ($\mu = 0.493$).

SolidWorks software was used for the purposes of modelling rubber compensator. SolidWorks conducted the necessary calculations via definite elements method. Static non-linear analysis was used for the purposes of compensator modelling. Two models of rubber were created in SolidWorks.

1) Rubber is defined as a linear, flexible, isotropous material;

2) Rubber is defined as a linear as a very flexible Mooney Rivlin material.

Then, for each model a total of six different models with side stop height varying from 0 to 37 mm were produced.

The analytical results and the results yielded by SolidWorks were then compared with results obtained within the experiment. More specifically, we compared displacement characteristics within the compensator while subjecting it to a force of 150N.

Results of the analytical calculation b, which takes into account the weak compressibility of the rubber correspond the best to the experimental data. The analytical calculation that did not account for weak compressibility yielded a high marginal error at increased side support heights. In his case, the total displacement of the compensator is severely affected by the weak compressibility due to the fact the free surface area of the object is fairly small. Whereas, SolidWorks yielded the most consistent results when rubber was described as a very flexible Mooney Rivlin material, however, it was not better than the analytical result. If a model of rubber being a linear, isotropous material is being used then the margin is sufficiently large, however, consistent.

Error was positive for each height of the side support (rigidity was larger) and it was determined to be within the boundaries of 23% - 51%. It would be very useful for an isotropous model to use a seemingly flexible model in order to achieve lower rates of compensator stiffness.

Testing of rubber compensator with varying stiffness

Since it is impossible to produce a varying stiffness compensator of a cylindrical shape (Figure 2) without the use of specialty equipment (cut the rubber in a manner that it would match the shape of the frilled interlayer), a simplified version of compensator construction was used for the research while the essence of the said compensator is persevered. It was substituted with a cylindrical compensator the interlayer of which was substituted with the steel ring (Figure 13). The internal diameter of the ring was 1 m larger than the external diameter of the cylinder compensator.

A total of 3 samples were prepared for experimental purposes. All samples were produced of the same rubber and in the same size with the use of the same technology. The compression of the rubber compensator was conducted in ZWICK/Roell Z150. Rubber compensator was loaded with a force of 270 N at loading velocity of 10mm/min.

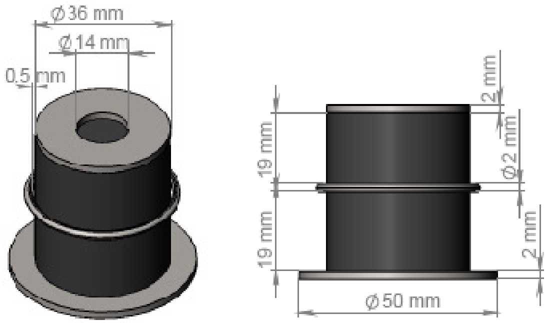


Figure 13. Rubber compensator with ring-like stop

The first calculation assumed that rubber is non-compressible, the second – rubber is compressible, while the assumed that rubber is both compressible as well as that the interlayer deforms. The latter model corresponds to the observations made within the experiment, due to the fact that during experiment deformation was only limited to external surface and due the rubber deformation in the contact area with the right. The same is observed upon interlayer deformation.

SolidWorks software was used for the purposes of modelling rubber compensator. SolidWorks conducted the necessary calculations via definite elements method. Static non-linear analysis was used for the purposes of compensator modelling. Compensator material (rubber) is of the same material than the compensator with varying height side support, thus Mooney Rivlin rubber model was deemed to be the most suitable. The middle of rubber layer was applied with displacement limiter along it radius (allowing to displace until a certain point).

Since rubber shock absorbers are mostly operate under dynamic load then this part shall clearly show how the obtained solutions in static may be used for the study of dynamic under low frequency of initiating force.

We shall review the following case – rubber compensator with a varying stiffness is subjected to static load of $P_0 = m_0g$ and a varying force of $P\cos(\omega t)$ (Figure 14).

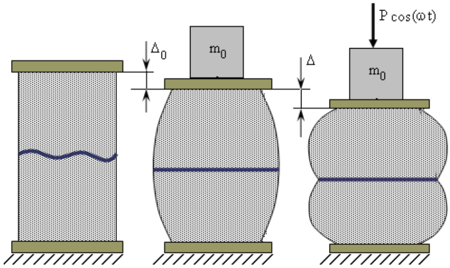


Figure 14. Rubber compensator with varying stiffness and dynamic load

The sag of rubber compensator under static load of Δ_0 is lower than a sag of such rubber compensator upon an increase of stiffness Δ_1. The varying force is selected in a manner that during fluctuation process, the compensator would fluctuate with coercion ($\Delta_0 + \Delta > 0$).

Feugt's model [2] (Figure 15) was used for purposes of studying compensator's dynamic, which consists of one flexible and one viscous element.

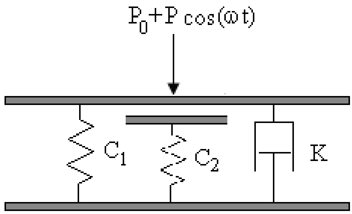


Figure 15. A model to study the rubber compensator's dynamic with a varying stiffness

We shall analyse the dynamics and along with system behaviour in NLO computer system. This software enables the analysis of bilinear system.

The following conclusions may be drawn upon conducting the analysis on rubber compensator dynamics:

-A single stable period mode is observed for a dynamic system.

-When the frequency of the initiator force is ω = 4.7, one mode with two periods is present.

-Dynamic system resonance at ω = 7.3.

- No unstable modes were observed for the dynamic system.

CONCLUSIONS

1. The calculation methodology of developed rubber - technical products compressed state in the case of static load in small deformation area, do not taking into consideration the elastomer`s compressibility, and using the principle of the deformation complete potential energy minimum;

2. The determination of rubber - technical products approximated solutions, which are obtained, using the hypothesis of the elastomer non-compressibility, the application area determination, taking into consideration the factor of the product form, the real value of the Poisson's ratio;

3. The analytical methodologies of developed thin layer of rubber technical products calculation in the case of static load in small deformation area, taking into consideration the elastomer`s weak compressibility and the deformation of the non-elastomer layers;

4. The calculation methodology of developed thin layer of rubber technical products, taking into consideration the elastomer`s physical non-linearity;

5. The analytical methodology of developed rubber technical products calculation in the case of static load in average deformation area, taking into consideration the elastomer`s weak compressibility, using the Delta-method;

6. The calculation methodology of developed rubber technical products with composite construction form, using the variation methods for discontinuous powers and movements;

7. Developed rubber technical products hardness characteristics, "force – displacement", the analytical solution accuracy assessment methodology, which is obtained, taking into consideration the principle of complete potential energy minimum;

8. Three new compensatory constructions of rubber - technical products are proposed, which provide the necessary stiffness characteristics "force - displacement". The calculation methodology of mentioned compensatory rubber - technical products is developed.

9. The accuracy of obtained formula is verified in the experiment.

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