

**8. LATVIJAS MATEMĀTIKAS
KONFERENCE**

Valmiera, 9.–10. aprīlis, 2010

TĒZES

**8th LATVIAN MATHEMATICAL
CONFERENCE**

Valmiera, April 9–10, 2010

ABSTRACTS

**LATVIJAS MATEMĀTIKAS BIEDRĪBA
VIDZEMES AUGSTSKOLA
LATVIJAS UNIVERSITĀTE**

INVERSE LYAPUNOV THEOREM FOR STOCHASTIC FUNCTIONAL DIFFERENTIAL EQUATIONS

JEVĢENĪJS CARKOVŠ and KĀRLIS ŠADURSKIS

Rīga Technical University

Kaļķu iela 1, Rīga LV-1658, Latvia

E-mail: carkovs@latnet.lv, skarlis@latnet.lv

The paper is devoted to the Second Lyapunov method development for stochastic functional differential equations. As it has been shown in many papers this method is one of the most powerful tools for asymptotic Lyapunov stability analysis of dynamical systems with delay (see, for example, [2], and references there). It should be mentioned that success of the Lyapunov approach in many respects depends on happy choice of Lyapunov-Krasovskiy type functional. Guided by the ideas and results of [2] and [3] we have proved inverse Lyapunov theorem and succeeded in finding of convenient algorithm for the above functionals construction in a case of stochastic linear functional differential equations. Our approach is based on extension of the resolving semigroup, defined by linear deterministic part [2], to linear operator semigroup, acting in the partially ordered space of countable additive symmetric matrix-valued measures. The weak infinitesimal operator of this semigroup helps to find such a quadratic functional that gives the necessary and sufficient asymptotic stability condition for the equation defined by selected deterministic linear part of analyzing quasi-linear equation. And what is more: substituting the solution of the analyzed stochastic equation as an argument of this functional we have got a stochastic process which has stochastic differential. This property permits to derive an analogue of Itô formula for the above mentioned stochastic process and, applying the martingale technique in the same way as it has been done in [3], to discuss equilibrium local exponential p -stability conditions for initial stochastic nonlinear functional differential equation. It should be mentioned that the proposal approach is very convenient for mean square stability analysis of linear stochastic functional differential equation. As an example we have constructed quadratic functional which permits to derive necessary and sufficient stability condition for delayed stochastic exponent given by Itô type scalar equation $dx(t) = \alpha x(t-1)dt + \beta x(t)dw(t)$ in a form of inequality $\beta^2 < -\frac{2\alpha \cos \alpha}{1 + \sin \alpha}$. Some of the mentioned above results have been published in [1] and [4].

REFERENCES

- [1] Ye.F.Carkov: *Random Perturbations of Functional Differential Equations*. Zinatne, Riga, 1989. (in Russian).
- [2] J. Hale and M. Sjord: *Introduction to Functional Differential Equations*. Springer-Verlag, NY, Hong Kong, 1993.
- [3] R. Khasminsky: *Stochastic Stability of Differential Equations*. Kluwer Academic Pubs., Norwell, MA, 1980.
- [4] J.Carkovs, K.Šadurskis. Ito Formula for Quadratic Functionals. In: *Proceedings of 2nd International Conference APLIMAT'2003, Bratislava, Slovakia, 2003.*, Slovak University of Technology, M.Kovacova(Eds.), Bratislava/Slovakia., 2003, 67 – 72. ISBN 80-227-1813-0