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ABSTRACTS

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ON STABILITY ANALYSIS OF LINEAR DIFFERENTIAL EQUATION WITH DIFFUSION COEFFICIENTS

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The paper deals with linear differential equation in $\mathbb{R}^n$

$$\frac{dx}{dt} = G(y(t))\,x(t) \tag{1}$$

where matrix-function $G(y(t)) := A + y(t)B$ is dependent on diffusion Markov process $\{y(t)\}$ defined by a stochastic differential Ito equation in $\mathbb{R}$ of Ornstein-Ulenbeck type with weak infinitesimal operator

$$Q := -\nu y \frac{d}{dy} + \sigma^2 \frac{d^2}{dy^2}, \ \ \nu > 0 \tag{2}$$

and invariant distribution $p(y)$. Using Cauchy matrix-family $\{X(t+s,s,y),\ s \geq 0,\ t \geq 0\}$ under condition $y(s) = y$ for (1) we introduce the linear continuous operators

$$(\mathbf{T}(t)q)(y) := \mathbf{E}_y^{(s)}\{X^T(t+s,s,y)q(y(t+s))X(t+s,s,y)\}, \tag{3}$$

in the space $\mathbb{M}_2$ of continuous symmetric $n \times n$-matrix-functions $\mathrm{q} := \{q(y), y \in \mathbb{Y}\}$ satisfying condition $\int_{\mathbb{R}} ||q(y)||p(y)dy < \infty$. Initially we have proved that the above family $\{\mathbf{T}(t), t \geq 0\}$ is continuous operator semigroup with infinitesimal operator $\mathbf{L} = \mathbf{A} + \mathbf{B}$ where $(\mathbf{A}\mathrm{q})(y) = A^T q(y) + q(y)A$, and $(\mathbf{B}\mathrm{q})(y) = y(B^T q(y) + q(y)B)$ is $\mathbf{A}$-relatively bounded operator [1]. Besides for any $t > 0$ operator $\mathbf{T}$ leaves as invariant cone $\mathbb{K} \subset \mathbb{M}_2$ of positive defined matrices. These assertions permit to apply well known Krein-Ruthman theorem [2] jointly with Kato perturbation theory [1] to mean square Lyapunov stability analysis of (1) by both the first and the second Lyapunov methods. Our approach is most effective for small random perturbation analysis if matrix $G(y)$ in (1) has a form $G(y,\varepsilon) := A(\varepsilon) + \varepsilon y B(\varepsilon)$. In this case one can construct quadratic Lyapunov function $(q_\varepsilon(y)x, x)$ and find mean square Lyapunov index applying Laurent series decomposition algorithm for spectral projector of operator $\mathbf{L}_\varepsilon$. The algorithm proposed has been illustrated by mean square stability analysis of second order differential equation. The results of our presentation are partly published in [3].

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