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Non-Destructive Technique for Determination of Elastic Material Properties

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Abstract. **An inverse technique, based on the planning of experiments and response surface methodology have been applied to determine elastic properties of unidirectional carbon fiber composite plate and carbon nanotubes reinforced aluminium plate.**

The nuances of application of investigated technique for orthotropic material properties determination are considered on the example of a composite plate. In turn, possibility of using of this technique on specimens with small dimensions (80x60x2 mm) is checked up and estimated on an example of an isotropic nanocomposite plate.

The results obtained were verified by comparing the experimentally measured eigenfrequencies with numerical ones obtained by FEM using determinated elastic material properties.

Keywords: **non-destructive technique, identification, elastic properties, composites, nanocomposites.**

I. Introduction

Due to the need for high-performance materials for a variety engineering applications such as aircraft, automobiles, civil engineering, sporting goods and electronics, material scientists are continuously developing new materials. Unfortunately the material data provided by manufacturers very often do not contain all necessary information to predict the behaviour of new materials using different analyses tools. So it is important to have possibility to repeat tests many times and under different conditions (temperature, humidity). Additionally, due to high costs of new materials, their experimental testing with conventional fracture methods suffers from high expenses too. On that reasons a number of various non-destructive evaluation techniques have been proposed for determining the elastic material properties. In the present study, attention is focused on the identification of the elastic properties of plate specimens using vibration test data.

Vibration testing of a modal is a rapid and inexpensive method to obtain data to identify of elastic properties [1]. There is a great deal of information in the literature on the identification of the elastic constants of laminated plates employing vibration test data [2]. The problem associated with vibration testing is converting the measured modal frequencies to elastic constants. A standard method for solving this problem is the use of a numerical-experimental model and optimization techniques [3]. The identification functional represents the gap between the numerical model response and the experimental one. This gap should be minimized, taking into account the side constraints on the design variables (elastic constants). The minimization problem is solved by using non-linear mathematical programming techniques and sensitivity analysis [2, 3]. A similar identification functional has been employed in [4], but the minimization method was different. Instead of the direct minimization of the functional, the experiment design and response surface approach are employed for approximation of the numerical (finite element) model. Such an approach can reduce the computational efforts significantly.

In order to reduce the computational efforts, methods based on the approximation concepts were used in the structural optimization for the first time [5]. The development of approximation functions has become a separate problem in optimum structural design. Approximating models can be built in different ways. To construct a more general model of the original function, the methods of experiment design [6] and approximate model building [7] can be used. A simplified model, called a meta-model [8], is elaborated using results of the numerical experiment at a sample point of the experiment design. The response analysis using the simplified model is computationally much cheaper than the solution employing the original model.

II. Numerical-experimental method

In the present work identification of elastic constants of plate's materials is performed employing numerical-experimental method based on experiment design method and response surface approximations. In this work, instead of physical experiment, numerical vibration data were used to determine the elastic constants.

The numerical-experimental method proposed in the present investigation consists of the experimental modal analysis, the numerical model and material parameters identification procedure (Figure 1).

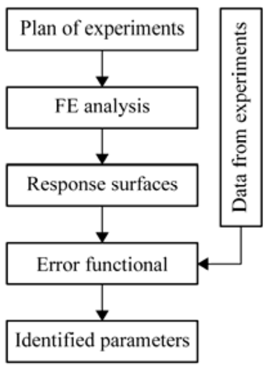


Fig. 1. The numerical-experimental method.

A criterion for the elaboration of the plans of the experiment is independent of the mathematical model of thedesigning object [9]. The initial information for development of the plan is the number of factors n and the number of experiments k. The points of experiments in the domain of factors are distributed as regularly as possible (Figures 7 and 8). For this reason the following criterion is used:

$$\Phi = \sum_{i=1}^{k} \sum_{j=i+1}^{k} \left(\frac{1}{l_{ij}} \right)^2 \Rightarrow \min \quad (1)$$

where l_{ij} is a distance between the points having numbers i and j ($i \neq j$). Physically it is equal to the minimum of potential energy of repulsive forces for the points with unity mass if the magnitude of these repulsive forces is inversely proportional to the distance between the points.

The plan of the experiment is characterized by the matrix of plan B_{ij}. The domain of the experiments is determined as $x_j \in [x_j^{min}, x_j^{max}]$ and the points of the experiments are calculated by the following expression:

$$x_j^{(i)} = x_j^{\min} + \frac{1}{k-1}\left(x_j^{\max} - x_j^{\min}\right)\left(B_{ij} - 1\right) \quad (2)$$

Here: $i = 1, 2, \ldots k$ and $j = 1, 2, \ldots n$.

The finite element analysis implies numerical modelling of the specimens and their modal analysis. Dimensions and density of the model correspond to the parameters of the specimens, but other material properties variates according to the plan of the experiments.

The response surface method is used for approximation of the numerical experiments results. The form of the regression equation is a priori unknown. There are two requirements for the regression equation: accuracy and reliability. Accuracy is characterised by the minimum of the standard deviation of the table data from the values given by the regression equation. By increasing the number of terms in this equation, it is possible to obtain complete agreement between the table of data and the values obtained from the regression equation. However, it is necessary to note that the predictions in the intervals between the points of the table could be not so good. To improve the accuracy of the predictions, it is necessary to decrease the distance between the points of experiments by either increasing the number of experiments or decreasing the domain of factors. The reliability of the regression equation can be characterised by assuming that the standard deviations for the points of the table and for other points are approximately the same. Obviously, the reliability is greater for a smaller number of terms in the regression equation.

The regression equation can be written in the following form:

$$y = \sum_{i=1}^{p} A_i f_i(x_j) \quad (3)$$

where A_i are the coefficients of the regression equation; $f_i(x_j)$ are the functions from the pool of simple functions θ_1, θ_2, … θ_m which are assumed to satisfy:

$$\theta_m\left(x_j\right) = \prod_{i=1}^{s} x_j^{\xi_{mi}} \quad (4)$$

here ξ_{mi} is a positive or negative integer, including zero.

The derivation of the equation from the pool or simple function is carried out in two steps: the selection of the perspective function from the pool and step-by-step elimination of the selected functions. In the first step, all variants are tested using a least square method and the function that leads to a minimum of the sum of the deviations, is chosen for each variant. In the second step, the elimination is carried out using the standard deviation defined as:

$$\sigma_0 = \sqrt{\frac{S}{k-p+1}}, \quad \sigma = \sqrt{\frac{1}{k-1}\sum_{i=1}^{k}\left(y_i - \frac{1}{k}\sum_{j=1}^{k} y_j\right)^2} \quad (5)$$

or the correlation coefficient given by:

$$c = \left(1 - \frac{\sigma}{\sigma_0}\right) \times 100\% \quad (6)$$

where: k is the number of experimental points; p is the number of selected prospective functions; S is the minimum of the sum of the deviations.

It is more convenient to characterise the accuracy of the regression equation using the correlation coefficient (Figure 2) since it is particularly sensitive to the choice of selected functions. If insignificant functions are eliminated from the regression equation, the reduction of the correlation coefficient is negligible. If only significant functions are present in the regression equation, eliminating one function leads to an important decrease of the correlation coefficient.

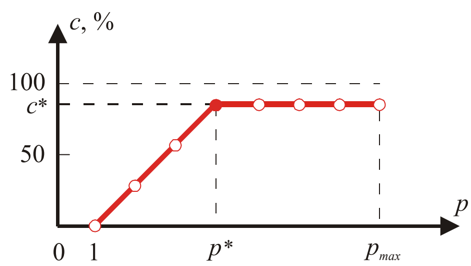


Fig. 2. Diagram of elimination for the correlation coefficient.

In the case of the identification of elastic material properties, the error functional, which describes the difference between the experimental and numerical parameters of structural responses, can be written as:

$$\Phi(x) = \sum_{i=1}^{I} \left| \frac{f_i^{EXP} - f_i^{FEM}\left(x\right)}{f_i^{EXP}} \right| \Rightarrow \min \quad (7)$$

where: f_i^{EXP} are experimentally measured resonant frequencies; f_i^{FEM} are the regression equations in terms of unknown factors for each resonant frequencies obtained from numerical calculations and response surface method; I denoted number of resonant frequencies used in the identification procedure.

To minimise the error functional, the following constrained nonlinear optimisation problems must be solved:

$$\begin{gathered} \min \Phi(x),\ H_i\left(x\right) \geq 0,\ G_j(x) = 0 \\ i = 1,\ 2,\ \ldots L;\ j = 1,\ 2,\ \ldots J \end{gathered} \tag{8}$$

where: L and J are the numbers of inequality constants.

This problem is replaced by an unconstrained minimisation problem, in which the constraints are taken into account with the penalty functions. Random search method is used to solve the formulated optimisation problem.

Experimental modal analysis of specimens helps describe their behaviour using dynamic descriptors (modal parameters) such as modal frequency and mode shapes. Developments and advances done in electronic instrumentations and computer resources allow making the both experimental and numerical modal analyses very effectively nowadays. The design of lighter, more flexible and less damped structures, which are highly prone to the action of dynamic forces, brings a need for more accurate and sophisticated measurements to be performed. On the other hand, highly damped materials, such as sandwich structures, are unsusceptible to dynamic forces and a special approach to perform vibration tests is needed. The Experimental Modal Analysis (EMA) is the most widely used procedure for such investigations [10]. It provides the frequency response functions (FRFs) of the system using known input excitation forces and the corresponding measured output vibrational responses. Typically, artificial excitation forces are applied at a number of locations and the corresponding excitation force signals (inputs) as well as the vibration responses at all locations (outputs) are measured [11]. From these data, the modal parameters are extracted using system identification methods. Mechanical reciprocity makes that it is not necessary to excite all inputs as long as all outputs have been measured.

III. NUMERICAL-EXPERIMENTAL PROCEDURE SETUP

A. *Materials and specimens*

Two types of materials were tested: unidirectional reinforces laminated composite and aluminium alloy LM24 with carbon nanotubes (CNT) volume content of 1.0 %.

Composite plate dimensions are shown in Figure 3. The parameters to be identified are the elastic constants of orthotropic composite plate:

- two Young's modulus: E_1, $E_2 = E_3$;
- shear modulus: $G23$; $G_12 = G_{13}$;
- Poisson's ratio: v_{13}.

Since, for a composite plate, some elastic constants are less sensitive to frequencies; the one independent elastic constant is fixed [12]:

- Poisson's ratio: $v_{23} = 0.3$.

The density of a panel, as measured by hydrostatic weighing, is $\rho = 1579$ (kg/m^3).

CNT reinforced aluminium plate manufacturing procedure and dimensions are shown in Figure 4. Nanocomposite aluminium alloy have been made by adding the CNT's to molten aluminium alloy and gravity casting into a mould. There is a gradient of distribution of CNT throughout the aluminium alloy, therefore specimen from interlayer of the billet is used in the testing.

The distribution of CNT throughout the specimen is assumed as chaotic, therefore the material is considered as isotropic. There are 3 elastic constants to be identified:

- Young's modulus E;
- shear modulus G;
- Poisson's ratio v.

Since between elastic constants of isotropic material exists dependence (9), there are only two independent constants.

$$\nu = \frac{E}{2G} - 1 \tag{9}$$

The density of a plate, as measured by hydrostatic weighing, is $\rho = 2720$ (kg/m^3).

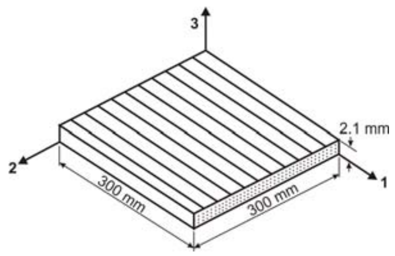


Fig. 3. Unidirectionally reinforced laminated composite plate.

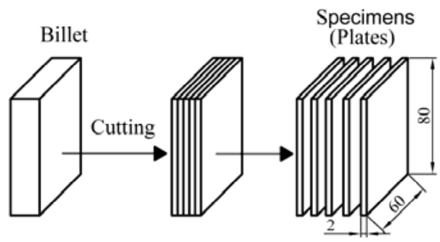


Fig. 4. Manufacturing of CNT reinforced aluminium plates.

B. *Physical modal experiment*

The experimental setup developed with Scanning Laser Vibrometer Polytec PSV-400-B is applied for the vibration testing. Free-free boundary conditions are simulated by suspending hanging the specimens on thin threads attached to the upper corners of the specimens. (Figure 5).

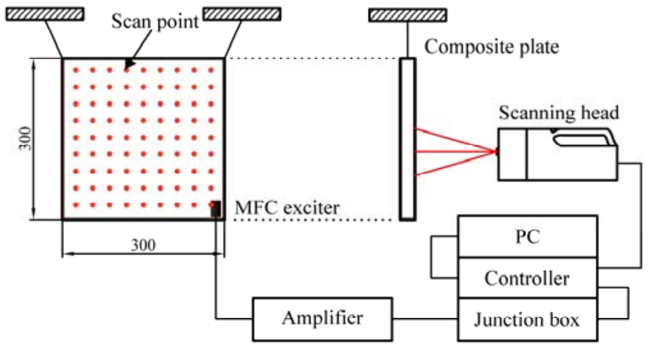


Fig. 5. Experimental modal analysis setup: composite plate.

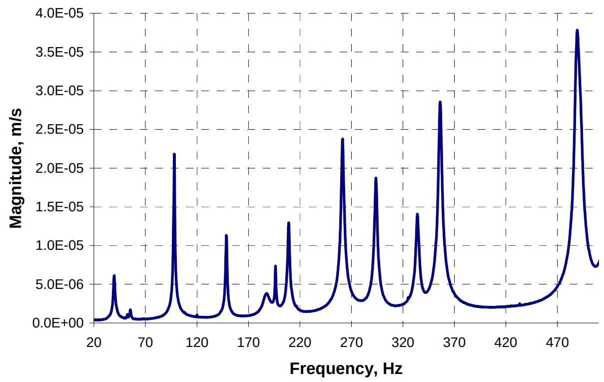


Fig. 6. Frequency responding function for composite plate.

The composite plate was excited using Macro Fiber Composite (MFC) actuator attached to the surface of the plate at the left corner of the bottom edge.

The CNT reinforces aluminium plate was excited by non-contact (acoustic) method, using the loudspeaker.

After the measurements have been performed at the each specified measurement point, the average of the both input and output signals are converted inside a signal analyser into the frequency domain using a fast Fourier transformation giving the frequency responding function (Figure 6) used for estimation of structural eigenfrequencies and corresponding mode shapes.

C. *Planning of the experiments*

The plan of experiments for the CNT reinforced aluminium plate has been produced for 2 design parameters and 38 experiments. The plan of experiments has been created .or E and v parameters (Table 1 and Figure 7).

The plan of experiments for the composite plate is formulated for 5 design parameters and 101 experiments (Figure 8). The limits of the search region are given in Table 2.

TABLE I

RANGES OF IDENTIFIABLE PARAMETERS FOR CNT REINFORCES ALUMINIUM PLATE MATERIAL

Parameters of identification	Minimum value	Maximum value
Young's modulus E, GPa	65	75
Poisson's ratio v	0.30	0.35

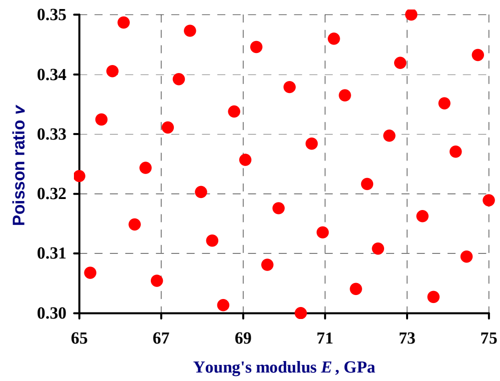


Fig. 7. Plan explication using E and v for Al-CNT plate.

TABLE II

RANGES OF IDENTIFIABLE PARAMETERS FOR COMPOSITE PLATE MATERIAL

Parameters of identification	Minimum value	Maximum value
Young's modulus E_1, GPa	90	120
Young's modulus $E_2 = E_3$, GPa	7	10
Shear modulus G_{23}, GPa	3	5
Shear modulus $G_{12} = G_{13}$, GPa	3	5
Poisson's ratio $v_{12} = v_{13}$	0.2	0.4

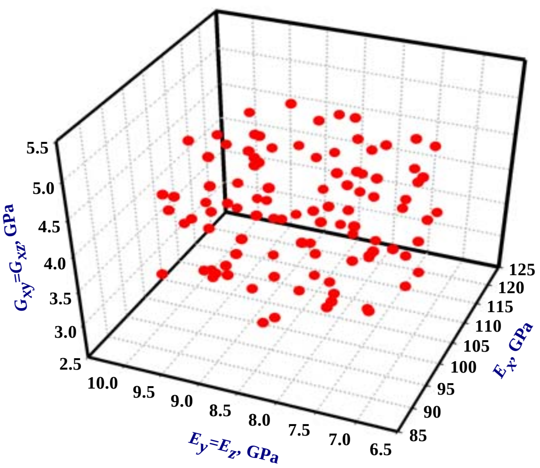


Fig. 8. Plan of experiment for composite plate: 3D view.

D. *Finite element analysis*

In the presented inverse technique, the finite element method (FEM) is used for modelling and dynamic analysis of plate specimens.

The commercial finite element code ANSYS 11.0 is used for the modelling and solution of free undamped vibrations of plates with free-free boundary conditions. The numerical model of plates are built with the linear layered structural shell elements SHELL99 (Figure 9). The modal analysis with bock Lanczos mode-excitation method is applied to determine eigenfrequencies and corresponding mode shapes of the specimens.

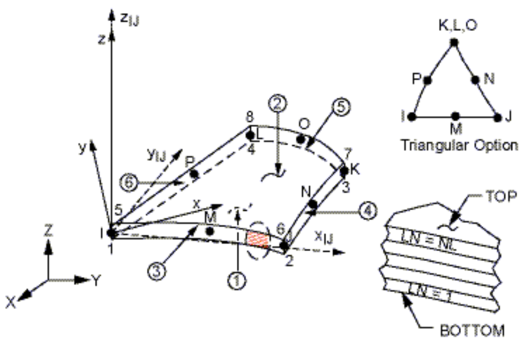
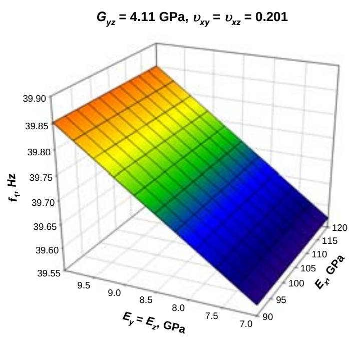


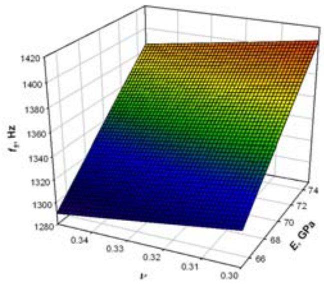
Fig. 9. Shell99 element geometry.

E. *Identification procedure and verification of results*

The program RESINT was used to approximate results of numerical experiment. The equations of regressions for each eigenfrequency have been obtained as a result of approximation procedure. Employing these numerical values, the response surfaces for all eigenfrequencies were obtained with correlation coefficients *c* since 80% until 99 % for composite plate and 100% for CNT reinforced aluminium plate. (Examples of the responding surfaces are shown in Figure 10).



a.



b.

Fig. 10. The 1st frequency dependency on the identification parameters. a. – for composite plate. b. – for CNT reinforced aluminium plate.

The elastic constants of the specimens were obtained by minimizing the error functional (7). For the CNT reinforced aluminium plate it is:

$$\Phi(E,\nu)=\frac{1331-f_1^{FEM}(E,\nu)}{1331}+\frac{1625-f_2^{FEM}(E,\nu)}{1625}+\frac{3063-f_3^{FEM}(E,\nu)}{3063}+$$
$$+\frac{3184-f_4^{FEM}(E,\nu)}{3184}+\frac{3895-f_5^{FEM}(E,\nu)}{3895}+\frac{4812-f_6^{FEM}(E,\nu)}{4812}+$$
$$+\frac{6105-f_7^{FEM}(E,\nu)}{6105}+\frac{6496-f_8^{FEM}(E,\nu)}{6496}+\frac{8296-f_9^{FEM}(E,\nu)}{8296}+$$
$$+\frac{9179-f_{10}^{FEM}(E,\nu)}{9179}+\frac{9252-f_{11}^{FEM}(E,\nu)}{9252}+\frac{9971-f_{12}^{FEM}(E,\nu)}{9971}$$

This equation has minimal value then E = 69.3 GPa and ν = 0.34:

$$\Phi(69.3,0.34)=\Phi(E,\nu)_{\min}=0.000306$$

Identified elastic constants of the CNT reinforced aluminium plate are shown in Table 3. (Shear modulus G is calculated by equation (9)).

TABLE III

ELASTIC CONSTANTS OF THE AL-CNT PLATE

Parameters of identification	Value
Young's modulus E, GPa	69.3
Shear modulus G, GPa	25.9
Poisson's ratio ν	0.34

Elastic constants of the composite plate were identified in the same way. The results of the identification procedure a shown in Table 4.

TABLE IV

ELASTIC CONSTANTS OF THE COMPOSITE PLATE

Parameters of identification	Value
Young's modulus E_1, GPa	100.9
Young's modulus $E_2 = E_3$, GPa	8.4
Shear modulus G_{23}, GPa	4.9
Shear modulus $G_{12} = G_{13}$, GPa	4.1
Poisson's ratio $\nu_{12} = \nu_{13}$	0.20
Poisson's ratio ν_{23}	0.30

The results obtained were verified by comparing the experimentally measured eigengrequencies with the numerical ones obtained by FEM at the point of optima (using the identified elastic properties). The residuals Δ_i are calculated be the expression:

$$\Delta_i = \frac{\left| f_i^{FEM} - f_i^{EXP} \right|}{f_i^{EXP}} \times 100 \quad (10)$$

It is seen from comparison of the results presented in Tables 5 and 6 that frequencies calculated by the FEM using the elastic properties obtained by identifications are in good agreement with the experiments. Mostly residuals do not exceed 3% for composite plate and 0.5% for CNT reinforced aluminium plate.

TABLE V

FREQUENCIES AND RESIDUALS FOR COMPOSITE PLATE

Mode	f_i^{EXP} (Hz)	f_i^{FEM} (Hz)	Δ (%)
1	39.75	39.70	0.1
2	55.25	55.27	0.00
3	98.00	97.97	0.00
4	148.50	152.31	2.6
5	187.75	191.76	2.1
6	196.25	194.18	1.1
7	209.00	207.56	0.7
8	261.50	256.82	1.8
9	293.75	299.05	1.8
10	334.00	337.68	1.1
11	356.25	345.82	2.9
12	-	480.04	-
13	489.25*	494.02	1.0

* the frequency not used in identification

TABLE VI

FREQUENCIES AND RESIDUALS FOR AL-CNT PLATE

Mode	f_i^{EXP} (Hz)	f_i^{FEM} (Hz)	Δ (%)
1	1331	1337	0.45
2	1625	1632	0.45
3	3063	3067	0.12
4	3184	3180	0.13
5	3895	3893	0.04
6	4812	4815	0.07
7	6105	6096	0.14
8	6496	6470	0.40
9	8296	8296	0.00
10	9179	9184	0.05
11	9252	9246	0.06
12	9971	9934	0.37

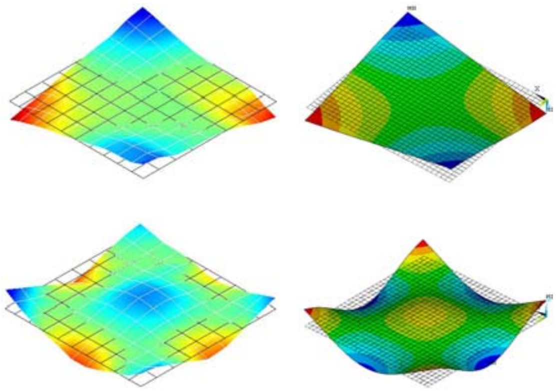


Fig. 11. Mode shapes for the 1st and 8th frequency of composite plate. Left – experimentally measured, right – numerically calculated.

Among comparison of numerical values of eigenfrequencies mode shapes of experimentally measured and numerically calculated eigenfrequencies were compared to verify identified values of elastic properties. Some typical vibration modes of both the experimentally measured and respective numerically calculated eigenfrequencies of the specimens are presented in Figures 11 and 12.

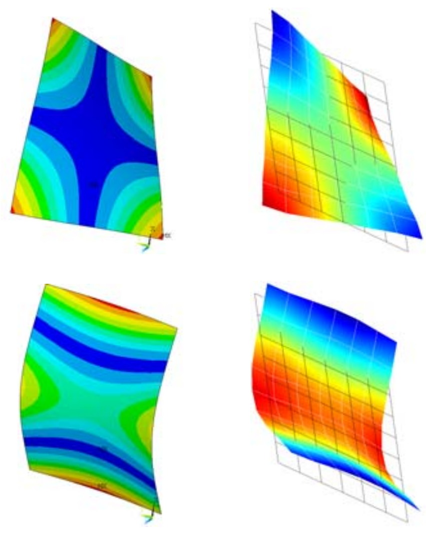


Fig. 12. Mode shapes for the 1st and 2nd frequency of Al-CNT plate. Left – experimentally measured, right – numerically calculated

IV. CONCLUSIONS

The identification of the actual material properties was performed on a unidirectional carbon fiber composite plate and plate made of aluminium alloy LM24 with carbon nanotubes (volume content 1.0%).

The results obtained for the composite plate are sufficiently accurate for the in-plane longitudinal Young's modulus and for the in-plane shear modulus. The transverse Young's modulus differs significantly from the nominal constants of carbon-fibre-reinforced composites. These discrepancies are explained by the fact that the parameters of the real structure differ from the nominal values (layer thickness, layer angles, the material density is not so homogeneous in all parts of the panel, etc.) of laminated composites. These differences should be taken into account in designing the real structures by choosing the safety factors and calculating the limit and collapse loads of composite stiffened structures.

The results obtained for the CNT reinforced aluminium plate do not show visible improvement of the elastic properties of aluminium alloy with addition of CNT (Elastic properties of nanocomposites aluminium alloy obtained by identification procedure meet corresponding standard properties of aluminium alloy LM24). These could be explained by imperfections of the technological process (nanocomposite alloy have been made by adding the CNT's to molten aluminium alloy and gravity casting into a mould).

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