Influence of Material Internal Stresses on the Coefficient of Friction

Didzis Rags[1], Andris Kamols[2], Oskars Lininsh[3], [1-3] *Riga Technical University, Institute of Mechanical Engineering*

Abstract **– The aim of this paper is to do an analytical research of internal stresses, which occur in contact of two rough surfaces. Research mainly has been done analytically and is based on the formulas described by von Mises yield criteria. As we can see from the obtained results, it's possible to find a coefficient of friction by using von Mises yield criteria. To get more precise values of the coefficient of friction, it is necessary to take into account also surface geometry and adhesion forces between the contacting surfaces. This will be done in further research works.**

Keywords **– coefficient of friction, tangential stress.**

I. Introduction

If we speak about friction force, we generally speak about material surface deformation in the direction of motion. The deformation arises already in the static contact, when there is no any movement, because two surfaces have the surface roughnesses, which are deformed when the contact takes place. When the motion starts, these surface roughnesses get deformed again. Deformation nature is really complex, because it can be just elastic or occur in combination with plastic deformations. Anyway, the amount of deformation is closely related to the properties of material and particularly depends on internal stresses, which are persistent under surface. In a simple case we can find the coefficient of friction by simply dividing tangential stresses by normal ones. By doing so, we will get the relation between both stresses and also their relation to the coefficient of friction.

II. Analytical Research

When doing research on friction forces between surfaces, it is always necessary to research internal stress fields, which originate from relative movement of two rough contacts.

There are many researches available, where the authors have described the states of internal stresses, when just normal forces are applied. Such cases are really simple ones, but in case with friction there are not only normal forces, but also tangential forces involved.

As allocation of surface roughness heights has a random nature, exact internal stress calculations are complex and we can find only approximate values. Even when we can find approximate values, these values can show us a relation between tangential and normal stresses.

To be able to determine the relation of stresses, firstly we must find stresses which arise during the contact of two spherical surfaces.

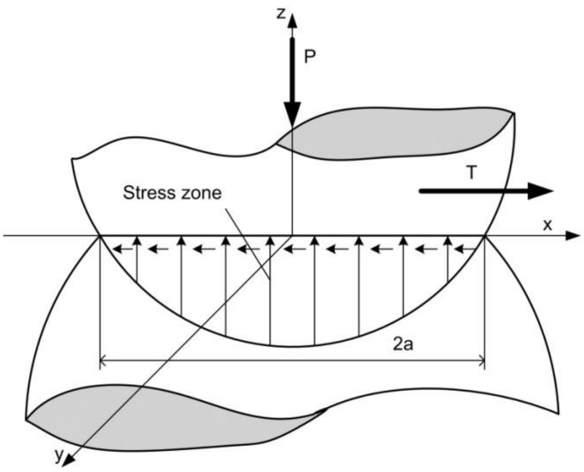

Fig.1. Stress zone allocation when two spherical surface heights are in contact.

As it is shown in Fig. 1., total normal force and total frictional force $T = f \cdot P$ in line of X axis are applied to contact height and a is radius of stress zone. Such stress zone allocation scheme can be seen besides each surface height. Stress components for such case are:

$$\sigma_i = \sigma_i^P + \sigma_j^T \quad (1)$$

$$\tau_i = \tau_i^P + \tau_j^T \quad (2)$$

where the first relation is for normal forces *P(x,y)*, and the second – for tangential forces *T(x,y)*. To assist stress calculations we treat contact as contact of two spheres and they are making a contact area, which has a predefined radius a.

As stated in A. Dinniks work [1], we can describe stresses in surface contact at plane when z = 0, in direction of X, Y and Z axis with the following formulas and with relation to normal forces:

$$\sigma_x^P = -\frac{3P}{2\pi a^2}\left\{\frac{a^2(1-2\mu)(1-(1-\frac{x^2}{a^2})^{3/2})}{3x^2} + 2\mu\sqrt{1-\frac{x^2}{a^2}}\right\} \quad (3)$$

$$\sigma_y^P = \frac{3P}{2\pi a^2}\left\{\frac{a^2(1-2\mu)(1-(1-\frac{x^2}{a^2})^{3/2})}{3x^2} - \sqrt{1-\frac{x^2}{a^2}}\right\} \quad (4)$$

$$\sigma_z^P = -\frac{3P}{2\pi a^2}\sqrt{1-\frac{x^2}{a^2}} \quad (5)$$

$$\tau_{xy} = \tau_{yz} = \tau_{zx} = 0 \qquad (6)$$

where: P – load to contact, x – coordinate.

To find stresses, which are caused by tangential forces, we can use formulas, which are described by M. Korovcinski [2] in relation to tangential stresses in direction to X and Y axis:

$$\sigma_x^T = -\frac{3P}{2\pi a^2}\frac{\pi x f(\mu+4)}{8a} \qquad (7)$$

$$\sigma_y^T = -\frac{3P}{2\pi a^2}\frac{2\pi f \mu x}{8a} \qquad (8)$$

$$\tau_{yz}^T = -\frac{3P}{2\pi a^2}\sqrt{1-\frac{x^2}{a^2}} \qquad (9)$$

$$\sigma_z^T = \tau_{yz} = \tau_{zx} = 0 \qquad (10)$$

When we speak about material capability to resist against deformations and about long service life, quite often we refer to the theory of Huber – Mises – Henky, which takes into account three main stresses and also gives us the relations between the yield strength at tensile, shearing or compressive loadings. As stated in this theory, hypothetically plastic deformations arise only when potential energy of the shaped body reaches certain critical boundary, which can be determined for each material.

In general form we write this relation as equivalent stress in the following way:

$$\sigma_{ekv} = \frac{1}{\sqrt{2}}\sqrt{\left(\sigma_x-\sigma_y\right)^2+\left(\sigma_y-\sigma_z\right)^2+ \\ +(\sigma_z-\sigma_x)^2+6\tau_{yz}^2} \qquad (11)$$

And the sum of stresses on each axis can be found with with help of formula (1). For Z axis according to formula (10) the stress will consist only from component, which is raised from normal forces. Graphically these stresses can be seen in Fig. 2.:

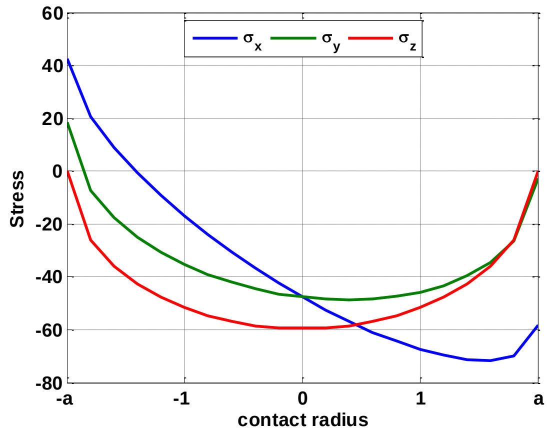

Fig. 2. Comparison of stresses for each axis and coefficient of friction is taken as f =0.5. 500 N force is used as load P.

As we want to look on stresses general without load, we will divide the equivalent stress by the load in the center of surface roughness height:

$$\sigma_{ekv}/p_0 = \frac{2\pi a^2\sqrt{\left(\sigma_x-\sigma_y\right)^2+\left(\sigma_y-\sigma_z\right)^2+(\sigma_z-\sigma_x)^2}}{3\sqrt{2}P} \qquad (12)$$

Thus we are getting a relative equivalent stress, whose numerical value is not affected by the amount of applied load, which we are using for calculations. The obtained results graph can be seen in Fig. 3.:

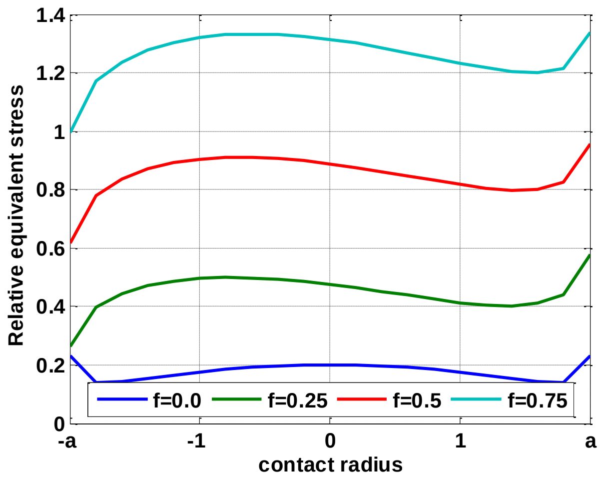

Fig. 3. Relative from load independent equivalent stress.

As we are interested in relation of stresses to the coefficient of friction, it is necessary to calculate tangential and normal stresses separately. For such calculations we will use formulas of stresses which were formulated by N. Belajev [3]. Total tangential stress in shear can be found in the following way:

$$\tau_n = \frac{1}{3}\sqrt{\left(\sigma_x-\sigma_y\right)^2+\left(\sigma_y-\sigma_z\right)^2+(\sigma_z-\sigma_x)^2} \qquad (13)$$

Total normal stress can be determined by:

$$\sigma_n = \frac{1}{3}\left(\sigma_x+\sigma_y+\sigma_z\right) \qquad (14)$$

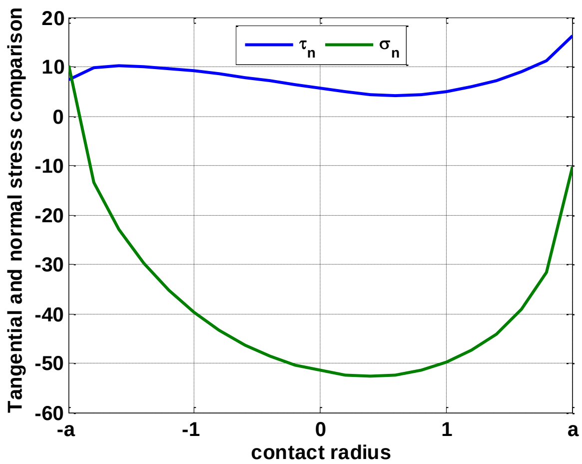

Fig. 4. Comparison of tangential and normal stress when the coefficient of friction is taken as f =0.25. 500 N force is used as load P.

As we can see in the graphs from Fig. 3, normal stress has negative values, which can be explained by the applied load which acts towards inside of material. If we speak about tangential stress graph, we can see that this stress changes merely in comparison to the normal stress within contact area in relation to radius *a*.

To make a comparison for both stresses and to understand the nature of these stresses it is usefull to use relative coefficient, which we can obtain by dividing tangential stress by normal stress as follows:

$$k_1 = \frac{\tau_n}{\sigma_n} \quad (15)$$

Actually this relative coefficient theoretically is equal to the coefficient of friction *f*, but as we have already used the coefficient of friction in stress calculations, we can't call it a coefficient of friction. Represantive graph of this coefficient is shown in Fig. 5, where absolute stress values are used for calculations.

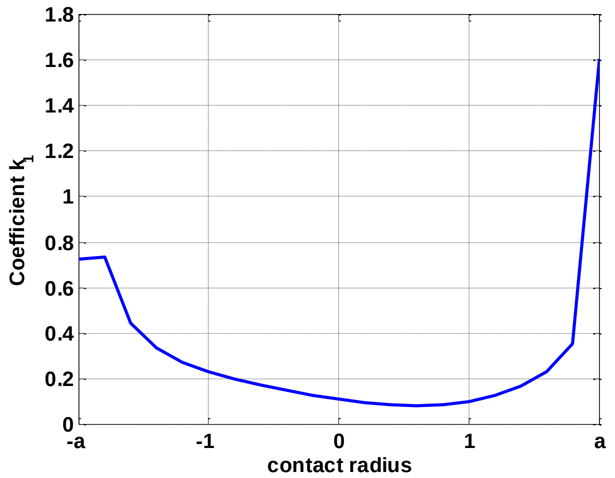

Fig. 5. Tangential and normal stress relation coefficient k_1. Calculations were done at f = 0.25.

With the help of integration we can find the mean values of coefficient k_1 in relation to the coefficient of friction. Mean values are shown in Table 1.

TABLE I

MEAN VALUES OF COEFFICIENT *K1*

f	$\overline{k_1}$
0.00	-2.88243
0.25	-1.45298
0.50	-1.83028

As we can see from the mean values, there is no linear correlation between both tangential and normal stresses and the coefficient of friction, because k_1 values did not change linearly according to the linear changes of the coefficient of friction. The reason for this could be simplified calculation of total normal stress (see formula 14). To check this, we will divide tangential stress by von Mises defined equivalent stress (see formula 11) as follows:

$$k_2 = \frac{\tau_n}{\sigma_{ekv}} \quad (16)$$

The results for the newly defined coefficient k_2 can be seen graphically in Fig. 6:

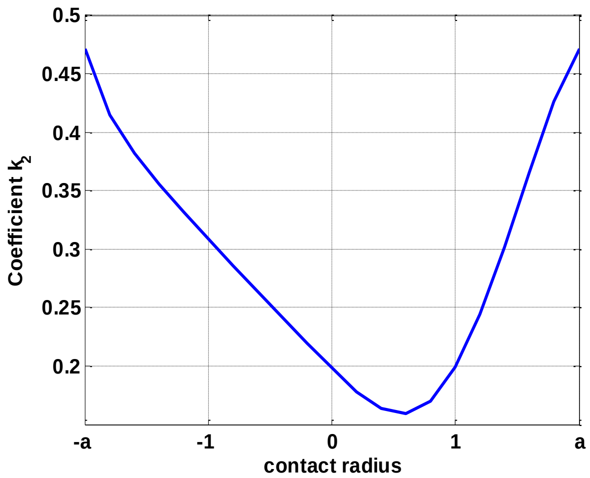

Fig. 6. Tangential and equivalent stress relation coefficient k_2. Calculations were done at f = 0.25

The graph shows us a clearly asymmetric nature of k_2 which can be explained in the following way: when contact is in sliding friction one side of the contact will experience tensile loads, when the opposite side will have compressing loads.

Again by using integration we found the mean values, which are shown in Table 2:

TABLE II

MEAN VALUES OF COEFFICIENT K_2

f	$\overline{k_2}$
0.00	0.334594
0.25	0.325911
0.50	0.315672

As we can see from mean values of k_2, these values are quite similar and probably it is because the shape of the contact hasn't changed, so these mean values now mainly are changing by changing materials or contact geometry.

III. CONCLUSIONS

In this paper we took a look on calculations of internal stresses within the contact area, by using different values of coefficient of friction. As we saw from the results, there is a certain relation between the coefficient of friction and internal stresses. This means that there is a possibility to calculate the coefficient of friction by taking into account contact geometry and material internal stresses. But for exact calculation of the coefficient of friction we need to use different calculation formulas, because in the current ones the coefficient of friction itself is used as input data.

As we can see from graphs, all curves are asymmetrical, which can be explained by material properties (in this case we used steel in analysis) to resist against tensile stresses. Also these asymmetric results conform to experimental results, which were obtained by Hamilton – Goodman [4].

Most of the used formulas are intended for calculations of internal stresses not vice versa, so for analytical analysis we had to define coefficient of friction by ourselves. After analytical research of stress relations, and with the help of two newly defined coefficients k_1 and k_2 we can now proceed with further research on influence of internal stresses on the coefficient of friction and find different ways to calculate tangential forces by using probability theory .

REFERENCES

1. **Динник А.Н.,** *Удар и зажатие упругих тел*, Киев, УССР, 1952.- 150
2. **Коровчинский М.В.,** *Распределение напряжений в окрестности локального контакта упругих тел при одновременном действии нормальных и касательных усилий в контакте*, Машиностроение, 1967, №6, 85 - 96.
3. **Беляев Н.И.** *Труды по теории упругости пластичности*. М., Гостехтеориздат, 1975, 629.
4. **Гамилтон, Гудмен.** *Поле напряжений, создаваемое круговым контактом скольжения.* Прикл. Механика, 1966, №2. 125 - 131.
5. **E.P. Bowden**, D. Tabor, *The Friction and Lubrication of Solids*, vol. 1, Clarendon Press, Oxford, 1950-1954.
6. **Greenwood J.A., Williamson B.P.** *Contact of nominally flat surfaces*, Proc. Soc. London. - 1966. - Vol. 295, N1442. - P. 300-317.

Didzis Rags graduated from Riga Technical University in 2005 as the production engineer, and then afterwards in 2007 he got a Master Degree in the field of process control technology, which was obtained at Kaunas Technical University. Now he has finished his doctoral studies at Riga Technical University and is developing the Doctoral Thesis to obtain a Doctoral degree in the field of producing technology.

Currently he is working as an automation engineer and the Head of Technical Support in the family owned company Festo. He started to work in the company as apprentice during his engineering studies, but now he has worked there already for 10 years as an automation engineer. His main interests include production and manufacturing of goods and research in the field of production technology.

Andris Kamols graduated from Riga Polytechnic Institute (now RTU) in 1972, where he got an engineer degree in machine building. He got a PhD science engineering degree in the field of production of friction and wear in machines at RTU in 1992.

Scientific work started at Riga Polytechnic Institute in 1976. From 1976 the author started his pedagogical work at Riga Technical University and since 1994 he continued as an assistant professor. Since 2001 he is an associate professor at Riga Technical University, the Institute of Machine Building.

The main scientific interests: contact of sliding friction pair and wear.

Oskars Lininsh graduated from Riga Polytechnic Institute (now RTU) in 1965, where he got an engineering degree in production technology. He obtained a PhD Engineering Degree in the field of friction and wear in machines at RTU in 1992.

His scientific work started in 1966, when he worked as an assistant at Riga Polytechnic Institute. From 1970 the author started his pedagogical work and since 1973 he continued as a professor assistant in Riga Technical University. In 2001 he was elected associate professor and in 2008 - a professor at RTU Institute of Machine Building. At the same time he worked as the Secretary of Science in the Commission of Production Technology and currently he is the Secretary of the Council of the Faculty of Transport and Mechanical Engineering

The main research fields are calculations of friction and wear in machines and design of apparatuses.

Didzis Rāgs, Andris Kamols, Oskars Liniņš. Materiāla iekšējo spriegumu ietekme uz berzes koeficientu
Šī pētījuma mērķis ir izpētīt materiāla iekšējo spriegumu ietekmi uz berzes koeficientu, lai pēc tam varētu iegūt berzes koeficienta aprēķina formulas, ar kuru palīdzību varētu aprēķināt berzes koeficientu, izejot no materiāla īpašībām un tās virsmas ģeometrijas parametriem. Spriegumu ietekme uz berzes koeficientu tika pētīta, aprēķinot iekšējos spriegumus uz katru asi un pēc tam izrēķinot summāros tangenciālos un normālos spriegumus. Aprēķinu pamatā tika izmantotas labi zināmās Mizesa spriegumu sakarības un spriegumu tenzors. Veicot šo analītisko pētījumu, tika iegūts apstiprinājums spriegumu ietekmei uz berzes koeficientu, kā arī iegūts aptuvena tangenciālo un normālo spriegumu attiecība kontaktā. Rezultātu grafiskie attēlojumi principā sakrīt ar citu autoru eksperimentālajiem pētījumiem, kuros ir izteikts spriegumu lauku nesimetrisks novietojums. Lai gan pētījumu laikā analītiski tika noteiktas materiāla iekšējo spriegumu attiecības un to vērtības, tomēr ir jāņem vērā fakts, ka aprēķinos tika pieņemtas berzes koeficienta vērtības, kas nozīmē to, ka pēc šīm sakarībām, berzes koeficientu nevar rēķināt, jo izejas datos ir tas pats berzes koeficients. Tādēļ turpmākajā pētījuma gaitā ir ieteicams lietot citas sakarības, kuras ļautu aprēķināt berzes koeficientu tikai no materiāla un tā virsmas ģeometrijas parametriem.

Дидзис Рагс, Андрис Камолс, Оскарс Лининьш, Влияние внутренних напряжений материала на коэффициент трения.
Это исследование направлено на изучение влияние внутренних напряжений материала на коэффициент трения для того, чтобы получить формулы расчета коэффициента трения в зависимости от свойства материала и геометрических параметров шероховатости поверхности. Влияние внутренних напряжений было изучена расшитая внутренних напряжений на каждую ось и потом расшитая суммарных касательных и нормальных напряжений.
Исследование основаны на хорошо известной теории соотношения Мизеса. В этом аналитическом исследовании было подтверждено влияние внутренних напряжений на коэффициент трения, а также получены приблизительные соотношения касательных и нормальных напряжений в контакте. В графическом варианте результат совпадает с результатами, полученными другими авторами экспериментально, в которых поля напряжений имеют ассиметричное распределение. Хотя во время аналитических исследований были определены соотношения внутренних напряжений, однако мы должны принимать во внимание тот факт, что в расчетах были приняты значения коэффициента трения. Это означает то, что в этих соотношениях коэффициент трения не может быть вычислен, поскольку в качестве входных данных принят такой же коэффициент трения.
Поэтому, в дальнейших исследованиях рекомендуется использовать другие формулы соотношениях, которые позволяют вычислить коэффициент трения из параметров шероховатости поверхности.