RIGA TECHNICAL UNIVERSITY
Faculty of Transport and Mechanical Engineering
Institute of Mechanical Engineering Technologies

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Doctoral of Programme "Instrument Engineering"

MEASUREMENT FORCE AND SURFACE ROUGHNESS INFLUENCE ON MEASUREMENT PRECISION OF LINEAR DIMENSIONS OF COMPONENTS FROM HIGHLY ELASTIC MATERIALS

Summary of Doctoral Thesis

Branch: Machine Science
Subbranch: Measuring Instruments and Metrology

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DOCTORAL THESIS
SUBMITTED FOR OBTAINING A DOCTORAL DEGREE OF ENGINEERING SCIENCES AT RIGA TECHNICAL UNIVERSITY

The Doctoral thesis developed for obtaining a doctoral degree of engineering sciences is submitted for public defence on May 26, 2015 at the Faculty of Transport and Mechanical Engineering, Riga Technical University, 6 Ezermalas street, room 405.

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CONFIRMATION

Hereby I confirm that I have developed the given Doctoral Thesis that has been submitted for reviewing at the Riga Technical University. The Doctoral Thesis has not been submitted to any other university for receiving scientific degree.

Anita Avišāne..(signature)

Date: 11.05.2015

The Doctoral Thesis has been written in the Latvian language and comprises an Introduction, 7 chapters, Conclusion, Bibliography, 2 Appendixes, 35 pictures, 92 pages.

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GENERAL DESCRIPTION OF THE DOCTORAL THESIS

Topicality of the Subject

Nowadays ever increasing production of highly elastic materials, particularly polymers, and application of products made of polymers, including also in manufacturing industry causes a necessity to increase the precision and efficiency of their production. Fast and high precision control of component dimensions promotes increase of production process precision and efficiency.

Up to now no methods have been developed for the measurement of components of highly elastic materials and determination of measurement errors, which fact makes the choice of proper measuring instruments for definite materials mode difficult. Development of such methods and their introduction into practice allows to carry out precise and high quality component control reducing the necessity for ungrounded high product tolerance, because making use of mechanical properties of plastic, rubber, nylon and other highly elastic materials, often components made of these materials are being designed and made with high tolerance. In view of the fact that synthetic polymers are obtained from non-renewable natural resources, crucial is the question about the limitation of utilization of these resources. Reducing consumption of these materials one can prevent many ecological problems. Results obtained in the research ensure increase of measurement precision of components made of highly elastic, also polymer materials, and also growth of production process efficiency.

The Aim and Tasks of Thesis

The objective of the Doctoral Thesis „Measurement force and surface roughness influence on measurement precision of linear dimensions of components from highly elastic materials" is to investigate the contact measurement process of components made from highly elastic materials and develop methods for the determination of measurement error.

The following tasks are being solved for achieving the given objective:

1) Development of contact model of components made from highly elastic materials;
2) Investigation of surface 3D roughness parameters needed for the solution of surface contact tasks;
3) Development of measurement error calculation formula;
4) Experimental check of calculation results;
5) Development of methods for the measurement of components of highly elastic materials and for the choice of measuring instruments.

Research Methods

The following theories will be used in the investigation of surface roughness model of components from highly elastic materials and theoretical research: contact theory, particular chapters of probability theory „Random process and field theory". Experimental researches for surface roughness determination were carried out by profilograph-roughnes indicator Form Talysurf Intra 50 (Taylor Hobson, UK), contactless linear dimension measurements with 3D coordinate measuring instrument MarVision MS222 (Mahrs, Germany), for linear dimension contact measurements there are used: digital micrometer DM2020 (Digital Micrometers Ltd, UK), micrometer MK (Калибр, Russia), digital length measuring system TG30 (Compac, Switzerland). Computer simulation is carried out by softwares ANSYS and SolidWorks. Mathematical statistics methods and softwares MatCAD and Graph were used for the processing of experimental data.

Scientific Novelty

The Doctoral Thesis presents the following scientific novelty:

1. Measurement force and surface roughness effect on the investigation of measurement precision of linear dimensions of components from highly elastic materials;
2. Use of 3D surface parameters in the solution of contact tasks in contact measurements of components from highly elastic materials;
3. Methods for linear measurement of components from highly elastic materials and for the choice of measuring instruments.

Practical Application

Researches carried out in the given Doctoral Thesis „Measurement force and surface roughness influence on measurement precision of linear dimensions of components from highly elastic materials" have high practical meaning in the solution of metrological tasks in the measurement process of linear dimensions of components from highly elastic materials. When measuring components made of rubber, nylon, polyethylene and other highly elastic materials in the measurement laboratories of enterprises contact deformations must be taken into consideration for the determination of component dimensions.

Methods for the measurement of components from highly elastic materials and for the choice of measuring instruments developed as a result of the research allow choosing measuring instruments appropriate for the measurement precision of components from highly elastic materials.

The research results can be used in the metrological laboratories both in Latvian and foreign companies.

Theses to Be Defended by the Author:

1. Surface contact model of components from highly elastic materials;
2. Analytical relations for the determination of measurement precision of linear dimensions of components from highly elastic materials;
3. Surface roughness parameter probability relations needed for the solution of contact tasks;
4. Methods for the measurement of linear dimension of components from highly elastic materials and for the choice of measuring instruments.

Thesis Approbation

The basic Doctoral Thesis results have been reported at the following conferences and workshops and positive evaluation was received;

- 12th euspen (European society for precision engineering & nanotechnology) International Conference, June, Stockholm, Sweden;
- 8th International Conference Mechatronic Systems and Materials (MSM 2012). 8 - 13 July, 2012, Bialystok, Poland;
- Riga Technical University 53rd International Scientific Conference, October 10-12, 2012, Riga, Latvia;
- 3rd International Advances in Applied Physics and Materials Science Congress (APMAS 2013) Conference,24-28 April 2013 Antalya, Turkey;
- 9th International Conference Mechatronic Systems and Materials (MSM 2013), 01 – 03 July 2013, Vilnius, Lithuania;
- 24th DAAAM International Symposium on Intelligent Manufacturing and Automation, University of Zadar, 23 – 26 October 2013, Zadar, Croatia;

- 4th International Advances in Applied Physics and Materials Science Congress (APMAS 2014) Conference, 24 – 27 April 2014, Antalya, Turkey;
- RTU Mehānikas institūta un Latvijas Nacionālās Mehānikas Komitejas apvienotais seminārs, 18.03.2014, Latvija, Rīga.

Publications

The performed researches have been published in 15 scientific articles:

1. Avišāne, A. The Elastic Deformation of Machine Elements in Mechatronics Systems. Solid State Phenomena, 2015, Volume 220 – 221, pp. 177 – 181. ISSN 1662-9779.
2. Avisane, A., Rudzitis, J., Kumermanis, M. Studies of the 3D Surface Roughness Height. No: 3rd International Advances in Applied Physics and Materials Science Congress (APMAS 2013): AIP Conference Proceedings, Turcija, Antalya, 24.-28. aprīlis, 2013. New York: AIP Publishing LLC, 2013, 406.-409.lpp. ISBN 978-0-7354-1197-5. ISSN 1551-7616. e-ISSN 0094-243X. Pieejams: doi:10.1063/1.4849304
3. Avisane, A., Rudzitis, J., Springis, G. Research into the 3D roughness of a rough surface. Latvian Journal of Physics and Technical Sciences, 2014, Volume 51, Issue 1, 62.-73.lpp. ISSN 0868-8257. Pieejams: doi:10.2478/lpts-2014-0007 **(SCOPUS)**
4. Avišāne, A., Rudzītis, J., Upītis, G., Vilcāns, J. Influence of Flexible Body Contact Deformation on the Linear Dimension Measurement Precision. Diffusion and Defect Data. V. Measurement techniques, Pt.B: Solid State Phenomena, 2013, Vol.199, 321.-325.lpp. ISSN 1662-9779. Pieejams: doi:10.4028/www.scientific.net/SSP.199 **(SCOPUS)**
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8. Rudzītis, J., Avišāne, A., Avišāns, D. Investigation of Elastic Machine Element Measurement Error. No: Procedia Engineering, Horvātija, Zadar, 23.-26. oktobris, 2013. Amsterdam: Elsevier Ltd, 2013, 1033.-1037.lpp. ISSN 1877-7058. Pieejams: doi:10.1016/j.proeng.2014.03.087 **(SCOPUS)**
9. Rudzītis, J., Avišāne, A., Spriņģis, G. Statistics of Roughness Peak Height of Friction Surface. Production Engineering. Nr.35, 2013, 149.-152.lpp. ISSN 1407-8015. e-ISSN 2255-8721.
10. Rudzītis, J., Krizbergs, J., Avišāne, A., Spriņģis, G., Kumermanis, M., Lungevičs, J. Calculation of 3D Texture Parameters. Production Engineering. Nr.35, 2013, 113.-117.lpp. ISSN 1407-8015. e-ISSN 2255-8721.
11. Spriņģis, G., Rudzītis, J., Avišāne, A., Kumermanis, M., Lungevičs, J. Wear Calculation Possibility in Slide-Friction Pairs Using Surfaces with Nanocoatings. No: Mechanical Engineering and Mechanics. Rīga: 2012, 24.-28.lpp. ISBN 978-9984-9990-7-4.
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CONTENTS OF DOCTORAL THESIS

Used notations and terms

$[\Delta_{m\bar{e}r}]$	–	tolerated measurement error value;
u	–	deformation level of surface roughness peaks;
γ	–	standardized deformation level of surface roughness peaks;
σ	–	standard deviation of random field;
a	–	surface deformation;
N_A	–	number of all surface roughness peaks per unit of area;
$N(\gamma)$	–	number of surface roughness peaks above level γ;
e	–	eccentricity of contact area;
$K(e)$	–	first level complete elliptic integral;
$E(e)$	–	second level complete elliptic integral;
b_i	–	the longest semi-axis of surface roughness peak elliptic contact area;
a_i	–	the shortest semi-axis of surface roughness peak elliptic contact area;
θ	–	constant of elastic properties of material;
E	–	modulus of elasticity;
μ	–	Poisson's ratio;
H	–	average bending of surface roughness peaks;
k_1, k_2	–	basic bendings of surface roughness peaks;
$h_p(u)$	–	height of surface roughness peaks above level u;
$E\{\ \}$	–	mathematical expectation value;
q	–	pressure on contact area;
Aa	–	nominal contact area determined by area dimensions;
ξ_p	–	standardized value of surface roughness peak height;
$f(\xi_p)$	–	density of probability distribution of surface roughness peak height;
m	–	number of maximum values of surface roughness profile per a unit of length;
$n(0)$	–	number of zeroes of surface roughness profile per a unit of length;
c	–	anisotropy coefficient of surface roughness;
$erfc(x)$	–	error integral;
Sa	–	mean arithmetic deviation of surface roughness from mid-plane;
RSm	–	mean spacing of profile irregularities
St	–	total surface roughness height;
ζ	–	standardized value of surface deformation;
l	–	nominal size of measured component.

Highly elastic materials – materials with elasticity modulus E up to $200\ N/mm^2$.

Chapter 1. LITERATURE OVERVIEW

The given paper includes researches on the effect of measuring force and surface roughness on the measurement precision of linear dimensions of components from highly elastic materials. In this paper highly elastic materials cover materials having elasticity module E up to $200\ N/mm^2$. We examine researches carried up until now on the following issues:

1) Measurement error and choice of measuring instruments;
2) Formation of contact deformation under the effect of measurement force;
3) Influence of surface roughness on measurement precision.

Measurement error and choice of measuring instruments

The efficiency of technical control depends on the proper choice of measuring instruments both from the metrological and economical point of view. The universal basic rule for the choice of measuring instruments can be worded as follows [8]:

1) The precision of measuring instruments as compared with the production precision of the article to be manufactured should be sufficiently high.

2) The labour productivity in control operations should be possibly higher, but expenses related to the control, manufacturing cost of article under control should constitute a possibly smaller proportion.

The tolerated measurement error $[\Delta_{m\bar{e}r}]$depends on the article production tolerance T_{izstr}, which in its turn is connected with the nominal size and precision quality. The tolerated measurement errors are assumed from 20% to 35% of tolerance value. Main constituents of measurement errors that should be taken into consideration when evaluating errors of measuring instruments under different application conditions are give in Fig.1.1.

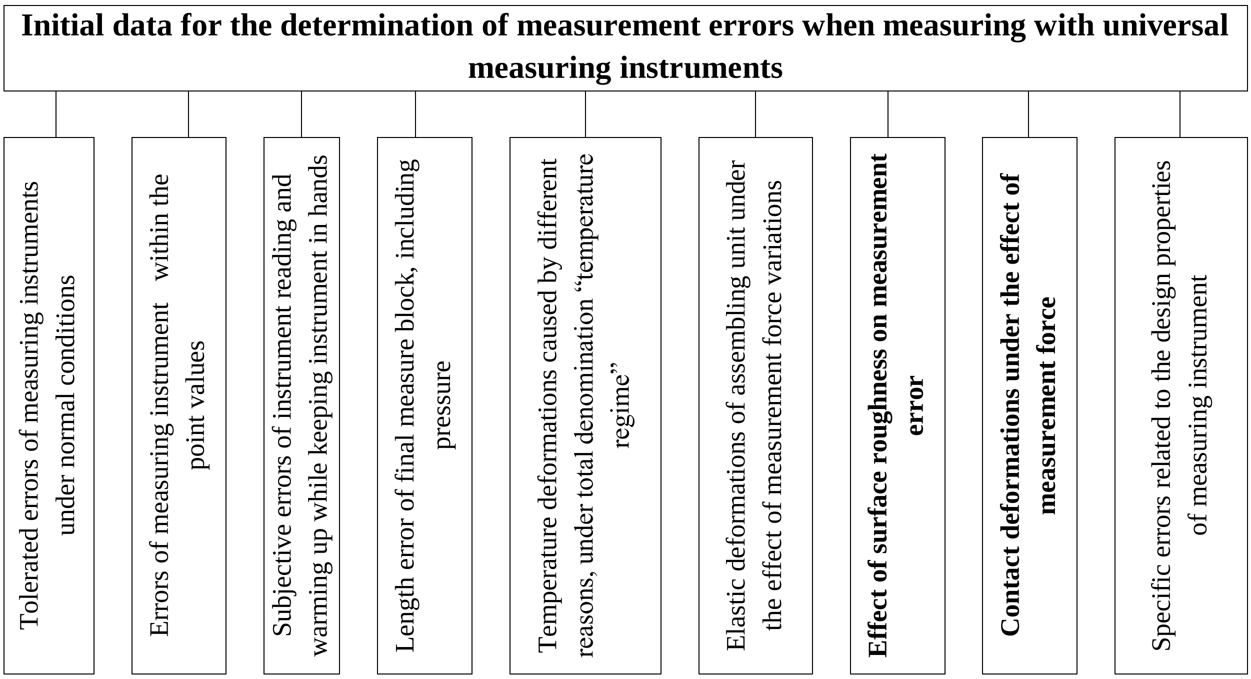


Fig.1.1.Main constituents of measurement error [6]

When choosing a definite measuring instrument Table 7.1 can be used, depending on the dimension to be measured, production tolerance and tolerated error according to Standard ГОСТ 8.051-1981 [12].

The given survey of literature shows that measurement error is affected by contact deformations, but if choosing measuring instruments in he described methods the size of contact deformation has not been taken into consideration.

Contact deformations under the effect of measuring force

Errors that are caused by measurement force can be divided into three groups: errors arising as a result of elastic deformation in the contact zone of measuring instrument's nozzle and component; errors arising as a result of deformations the contact zone and, errors arising as a result of elastic deformations of adjustment unit and components of measuring instrument [3]. To determine the measurement error caused by the effect of measurement force one should take into consideration the fact that in the place where the measuring instrument's nozzle and surface of the component to be measured come into contact elastic deformation – compression arises. Usually the maximum value of measurement force and its vibrations are being determined. However these data are insufficient for the evaluation of error caused by measurement force. The maximum measurement force should be taken into consideration when calculating contact deformations, which also depend on the material, shape and surface condition of the nozzle of the measuring instrument and the object to be measured. [6].

The given overview of literature shows that up to now in contact measuring the effect of measurement force has not been evaluated sufficiently.

Surface Roughness and Its Effect on Measurement Error

Theories concerning 2D parameters existing at present and used in practice are being and will be reconsidered, adjusting them to 3D parameters. With the growth of requirements and possibilities a new Standard characterising surfaces has emerged – ISO 25178, the full title being „Specifications of geometrical software products (GPS) Surface properties: area" [11]

A model of rough surface profile and of the surface itself can be regarded as an aggregate, which has to be described by a list of parameters characterising its geometrical form. Numerous calculation models are known deferring from each other with list of parameters and their number. The spherical model because of its simplicity is being most widely used. The character of roughness or asperity is explained by correlation function and density of probability distribution [9]. Recently researchers are using and the most complete view about surface roughness is given by a surface model described by the help of *random function*, because together with the height it considers also relations in longitudinal and cross-section direction [9]. The effect of article's surface roughness on the measurement error usually is being determined depending on the height of micro-peak, pitch and radius of the nozzle of measuring instrument. Measurement error because of surface micro-roughness is being regarded as insignificant not affecting the summary measurement error.

Until now the effect of surface roughness on the measurement error has not been evaluated, though practical experiments show that roughness influences the measurement precision.

Main Research Directions

The performed research and literature overview show that the issue regarding measurement of components from highly elastic ($E \leq 200N/mm^2$) materials, as well as effect of measurement force and surface roughness on measurement precision have not been studied sufficiently. At present there are no methods for the determination of measurement error for components from highly elastic materials and choice of measuring instruments.

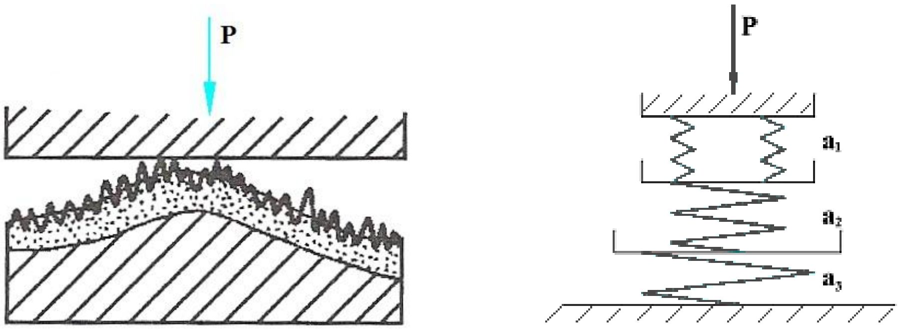
In view above the following main research directions are set forth:

1) Development of contact model of components fro, highly elastic materials;
2) Researches of surface roughness 3D parameters needed for the solution of surface contact tasks;
3) Obtaining the measurement error calculation formula;
4) Experimental checking of calculation results;
5) Development of methods for the measurement of components from highly elastic materials and choice of measuring instruments.

Chapter 2. ANALYSES OF SURFACE ROUGHNESS DEFORMATION

When measuring the linear dimensions of components it is essential to find out what are the expected surface deformations. Since the measuring process is characteristic of small loads, small contact areas tolerated are only elastic deformations.

In order to state precisely the total measurement error affected by the applied force surface deformation should be divided into three parts. The first part is deformation of surface roughness peaks a_1, the second part is settling of these peaks a_2, and the third part is deformation of basic material a_3. Schematically in the form of springs it is shown on Fig. 2.1.



a) b)

Fig. 2.1 Contact diagram

a) general; b) in the form of springs

To state elastic deformation of component surface peaks let us assume that contact takes place between a rough (measured component) surface and ideally smooth and perfectly hard (measuring instrument) surface. In this case under the effect of applied force P the ideal surface moves from position I – I to position II - II (Fig. 2.2), where balance sets in between the applied force and resistance of peak deformation. Let us assume that in this position the distance between the ideal and rough surface is equal to u.

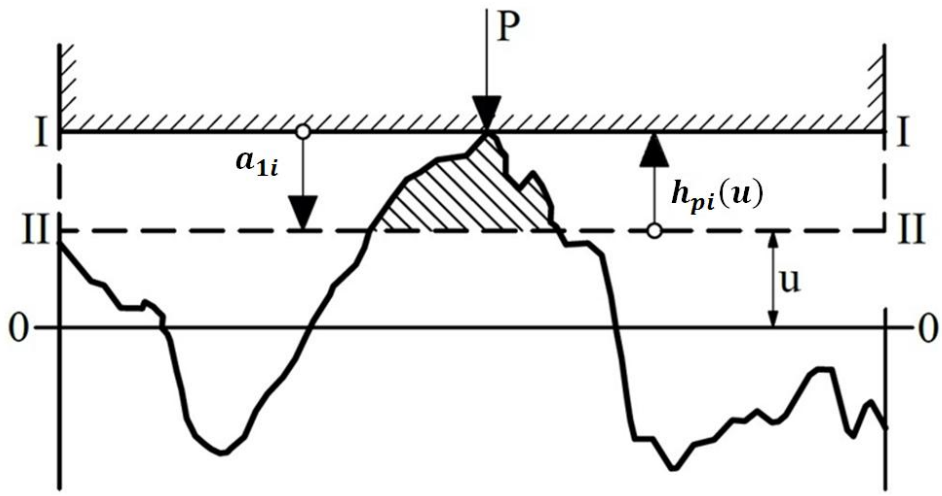


Fig. 2.2 Contact between rough and ideally smooth surface

2.1. Determination of Peak Deformation

Since the surface roughness peaks are situated on the surface on different heights have irregular form the elasticity theory does not give solution of such peak contact. If we consider contact of rough surface then in this contact theory usually it is being assumed that deformation level of surface roughness peaks stops above the mid-plane of surface roughness, which on the figure (Fig. 2.2) is shown with line 0 – 0, on level u. Those surface peaks that are situated above this deformation level are subjected to deformation. Further in calculations it is advantageous to assume standardized deformation level γ ($\gamma = u/\sigma$, where σ is standard deviation of random field).

If deformation of rough surface peaks is a_1 (Fig. 2.1) it can be defined according to relation [10]:

$$a_1 = (\gamma_0 - \gamma) \cdot \sigma \,, \tag{2.1}$$

where

$\gamma_0 = St/\sigma$ – standardized in itial deformation level;

St – total height of surface.

Standardized deformation level γdepends on the peak height. The height of rough surface peaks is being deducted from the mid-line (in case of profile) or mid-plane (in case of 3D surface). First it is necessary to analyse deformations of one surface roughness peak.

2.2. Analyses of Deformation of One Surface Roughness Peak

In our case there will be used a peak model - in the form of an elliptic paraboloid lately used in contact theory. This model allows to consider the peak form in the surface longitudional and cross-section direction. In such surface contact N.M Beliayev's solution can be used - for ellipsoid surface contact where we assume one body as an ideal hard plane, then one randomly chosen deformation $a_{1i}$of surface roughness peak can be determined as follows [4]:

$$a_{1i} = \frac{3}{2} K(e) \frac{P_i \cdot \theta}{b_i} \,, \tag{2.2}$$

where

$P_i = \frac{P}{N(\gamma)}$ – force applied to surface roughness peak;

P – force applied to surface;

$N(\gamma)$ – number of surface roughness peaks above level γ;

$K(e)$ – first level total elliptic integral;

$e = \sqrt{1 - \frac{{a_i}^2}{{b_i}^2}}$ – eccentricity of contact area;

b_i – the longest semi-axis of elliptic contact area of surface roughness peak (Fig. 2.3).

a_i – the shortest semi-axis of elliptic contact area of surface roughness peak (Fig. 2.3).

$\theta = \frac{1-\mu^2}{\pi \cdot E}$ – constant of elastic properties of material (E – elasticity module, μ - Poisson's ratio).

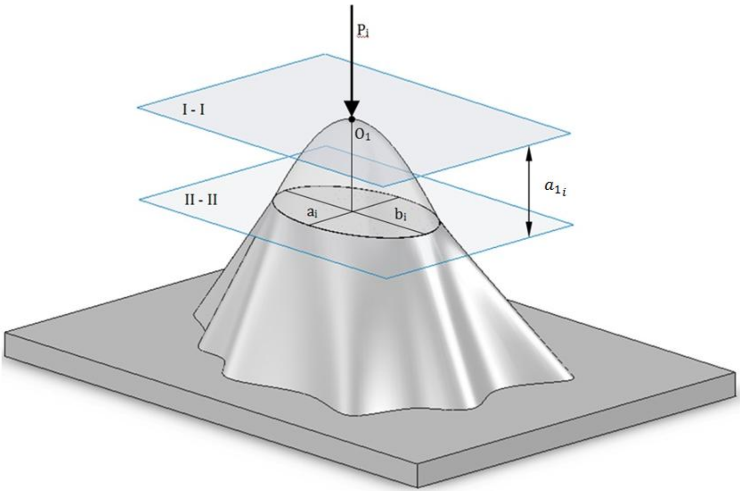


Fig. 2.3 Deformation of one surface roughness peak

The diagram of contact of ideal plane and surface roughness peak (Fig.2.2) shows that deformation value a_{1i} is equal to the height of surface roughness peak above level u,marked with $h_{pi}(u)$. Thus from the equation (2.2) we can obtain an expression for the determination of force applied to one surface roughness peak, inserting there value of the longest semi-axis $b_i$of the basis of elliptic parabolic, which can be determined according to Herz's formula [4]:

$$P_i = \frac{2}{3 \cdot \theta} \cdot k'_q \cdot \frac{[h_{pi}(u)]^{3/2}}{H_i^{1/2}}, \tag{2.3}$$

where

$H_i = (k_{1_i} + k_{2i})/2$ – mean peak bending;

k_{1i}, k_{2i} – main peak bendings [1];

$k'_q = \frac{1}{K(e)} \cdot \left[\frac{E(e)}{K(e)(1-e^2)}\right]^{1/2}$ – coefficient depending on eccentricity e of surface roughness peak.

Using expression (2.3) it is possible further on to obtain an expression for one surface roughness peak to determine roughness deformation for the whole surface.

2.3. Determination of Surface Roughness Peak Deformation

In order to obtain expression for the determination of force Papplied to contact area it is necessary to transform the expression (2.3) replacing variables of surface roughness peaks with respective variables along the contact area. Since the surface roughness peaks along the surface are situated at different heights and because bendings of these peaks are different we will use the mathematical expectation values of these parameters above level γ:

$$E\{P\} \approx \frac{2}{3 \cdot \theta} \cdot k_q{}' \cdot \frac{E\{\xi_{p\gamma}\}^{3/2}}{E\{H\}^{1/2}} \cdot E\{N(\gamma)\}, \tag{2.4}$$

where

$E\{P\}$ – mathematical expecation value applied to the surface;

$E\{\xi_{p\gamma}\}$ – mathematical expectation value of surface roughness peak top heigh [Chapter 0

$E\{H\}$ – mathematical expectation value of peak mean bending.

In order to determine the surface roughness deformation it is useful to assume that the applied force has been distributed evenly along the contact area, in this case for making calculations it is better to use pressure on the contact area:

$$q \approx \frac{2}{3 \cdot \theta} \cdot {k_q}' \cdot \frac{E\{\xi_{p\gamma}\}^{3/2}}{E\{H\}^{1/2}} \cdot E\{N_A(\gamma)\}\,, \tag{2.5}$$

where

$q = \frac{P}{Aa}$ – pressure on contact area , Aa – rated contact area determined by area dimensions;

$N_A(\gamma) = \frac{N(\gamma)}{Aa}$ – number of surface roughness peaks per area unit.

Formula (2.5) shows that for the determination of surface roughness deformation one must know the height $\xi_{p\gamma}$of surface roughness peaks, the mean bending of peaks $H$and the number of surface roughness peaks per an area unit $N_A(\gamma)$. Further on we will consider each of these parameters separately.

Chapter 3. RESEARCHES ON SURFACE ROUGHNESS PEAK HEIGHT

In order to determine peak deformation one of the crucial parameters is the surface peak height. The Paper considers and compares three different formulas for the calculation of surface peak height distribution density and determination of mathematical expectation value of surface peak height for those peaks that are situated above the set deformation level γ. In contact theory surface is being modelled as a normal random field. For such normal random field the peak height distribution density was obtained by P.R. Nayak, yet this expression practically is inapplicable for the solution of engineering tasks, therefore this Paper states that the above formula can be substituted by a simpler distribution law.

3.1. Distribution Density of Surface Peak Height

The surface peak height distribution law for 3D irregular form surface (mathematically – for normal random field) has been studiet in Nayak's work [2] where surface peak height distribution density $f_1(\xi_p)$ can be found by dividing the number of surface peaks, situated on level $[\gamma, \gamma + d\gamma]$ by the number of all surface peak tops above level γ:

$$f_{1_\gamma}(\xi_p) = \frac{E\{n_p(\gamma)\}}{E\{M_p(\gamma)\}}, \tag{3.1}$$

where

ξ_p	–	standardized value of peak height – $\xi_p = \frac{h_p}{\sigma}$;
$E\{n_p(\gamma)\}$	–	mathematical expectation value of the number of peak tops number on level $[\gamma, \gamma + d\gamma]$;
$E\{M_p(\gamma)\}$	–	mathematical expectation value of the number of all surface tops above level γ.

By inserting into relation (3.1) $E\{n_p(\gamma)\}$ and $E\{M_p(\gamma)\}$ calculation formulas we get the density of probability distribution of surface peak height $f_{1_\gamma}(\xi_p)$ for peaks situated above level γ:

$$f_{1_\gamma}(\xi_p) = \varkappa_1 \cdot \frac{\sqrt{3}}{2\pi} \left\{ \frac{\lambda}{4}\sqrt{3(8-3\lambda^2)} \cdot \gamma \cdot e^{-\frac{4}{8-3\lambda^2}\gamma^2} + \frac{3\sqrt{2\pi}}{4}\lambda^2(\gamma^2-1)e^{-\frac{\gamma^2}{2}}\phi\left(\sqrt{\frac{3\lambda^2}{8-3\lambda^2}}\gamma\right) + 4\sqrt{\frac{2\pi}{3(4-\lambda^2)}}\, e^{-\frac{2}{4-\lambda^2}\gamma^2}\phi\left(\sqrt{\frac{4\lambda^2}{(4-\lambda^2)(8-3\lambda^2)}}\gamma\right) \right\}, \tag{3.2}$$

where:

λ	–	non-dimensional parameter $\lambda = \frac{E\{n_1(0)\}}{E\{m_1\}}$; $\lambda = \frac{E\{n_1(0)\}}{E\{m_1\}}$; $E\{n(0)\}$ - zero number mathematical expectation value per a unit of length;
$\varkappa_1$	–	function of roughness parameter $\lambda$and level γ;
$\phi(\dots)$	–	Laplacian function: $\phi(x) = \frac{1}{\sqrt{2\pi}}\int\limits_{-\infty}^{x} e^{-\frac{t^2}{2}dt}$, Its numerical values can be found [1].

Since the formula offered by P.R. Nayak is complicated for engineering calculations it is essential to find a simpler distribution law which might substitute the precise formula. The Paper studies two most often used probability distribution laws: normal distribution (Gauss) law and Rayleigh law. Equations have been obtained for the determination of the density of probability distribution of surface peak height $f(\xi_p)$ according to these laws.

One-sided normal distribution law:

$$f_{2\gamma} = \frac{1}{erfc\left(\frac{\gamma}{\sqrt{2}}\right)} \cdot \frac{2}{\sqrt{2\pi}} \cdot e^{-\frac{1}{2}\cdot{\xi_p}^2}, \qquad \xi_p \geq 0 \qquad (3.3)$$

kur

$erfc(x)$ – Error integral $\quad erfc(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-t^2} \cdot dt$

Rayleigh law:

$$f_{3\gamma}(\xi_p) = e^{\frac{1}{2}\gamma^2} \cdot \xi_p \cdot e^{-\frac{1}{2}{\xi_p}^2} = \xi_p \cdot e^{\frac{1}{2}(\gamma^2 - {\xi_p}^2)}. \qquad \xi_p \geq 0 \qquad (3.4)$$

The distribution density curves are given on Fig. 3.1. The figure shows that starting from value $\xi > 1$ the distribution density expressions are coming nearer, and closer to the precise distribution density is Rayleigh distribution density. Thus Rayleigh density distribution for range $\xi > 1$can be used in the solution of engineering tasks.

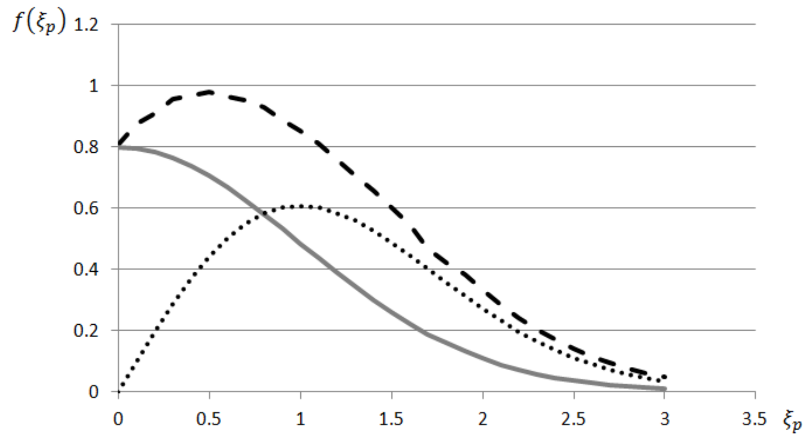


Fig. 3.1 Density of probability distribution of surface peak height

- - - P.R. Nayak's law
—— One-sided normal distribution law
·········· Rayleigh distribution law

3.2. Mathematical Expectation Value of Peak Height

In order to determine peak deformations and thus also measurement error it is essential to determine the mathematical expectation value of surface peak height. Like previously in the Paper the mathematical expectation value of surface peak height also has been determined by the solution offered by P.R Nayak, according to one-sided distribution law and according to Rayleigh law.

P.R.Nayak [2] in his research on random processes of rough surfaces determines the mathematical expectation value of surface peak top height according to formula:

$$E_{1_\gamma} = C_1 \cdot \frac{\sqrt{3}}{2\pi} \left\{ \frac{3\lambda^2\sqrt{2\pi}}{4}(\gamma^2+1) \cdot e^{-\frac{\gamma^2}{2}} \cdot \phi(\beta_6\gamma) + \frac{4\sqrt{\pi}\lambda}{\sqrt{3}} \cdot [1 - \phi(\beta_7\gamma)] + \frac{\sqrt{6}\lambda}{\beta_7} \cdot \gamma e^{-\frac{{\beta_7}^2\gamma^2}{2}} + 2 \cdot \sqrt{\frac{\pi}{6}(4-\lambda^2)} \cdot e^{-\frac{2\gamma^2}{4-\lambda^2}} \cdot \phi(\beta_8\gamma) \right\}, \qquad (3.5)$$

where:

C_1 – standardized multiplier;

$$\beta_6 = \sqrt{\frac{3 \cdot \lambda^2}{8 - 3\lambda^2}} \quad ; \quad \beta_7 = \sqrt{\frac{8}{8 - 3\lambda^2}} \quad ; \quad \beta_8 = \sqrt{\frac{4\lambda^2}{(4 - \lambda^2)(8 - 3\lambda^2)}} \quad .$$

The mathematical expectation value of surface peak top height according to one-sided normal distribution law is determined according to the following expression:

$$E_2\left\{\xi_{p_\gamma}\right\} = \frac{2 \cdot e^{-\frac{1}{2}\gamma^2}}{\sqrt{2\pi} \cdot erfc\left(\frac{\gamma}{2}\right)} . \tag{3.6}$$

Mathematical expectation value of surface peak top height according to Rayleigh law:

$$E_3\left\{\xi_{p_\gamma}\right\} = e^{\frac{1}{2}\gamma^2} \cdot \left[\sqrt{\frac{\pi}{2}} \cdot erfc\left(\frac{\gamma}{\sqrt{2}}\right) + \gamma \cdot e^{-\frac{1}{2}\gamma^2}\right] = e^{\frac{1}{2}\gamma^2} \cdot \sqrt{\frac{\pi}{2}} \cdot erfc\left(\frac{\gamma}{\sqrt{2}}\right) + \gamma . \tag{3.7}$$

Diagrams of mathematical expectation value of surface peak height is given on Fig. 3.2. The figure shows that starting from value $\gamma > 1$ mathematical expectation values of surface peak height are coming nearer, and closer to the precise value is mathematical expectation value according to Rayleigh law. Thus Raileigh law can be used in the solution of engineering tasks for range $\gamma > 1$.

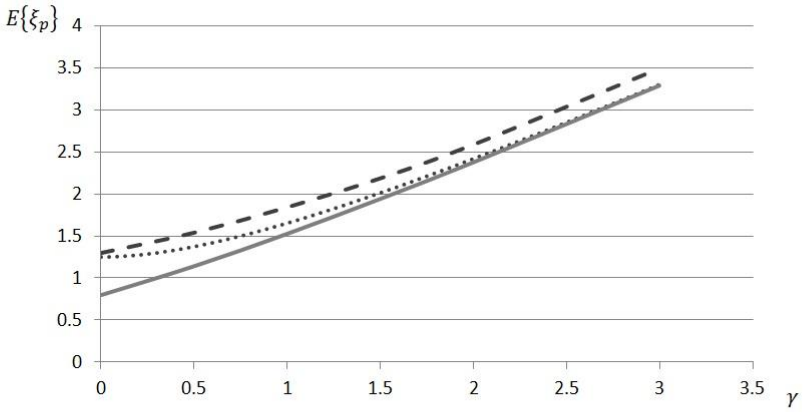


Fig. 3.2 Mathematical expectation value of surface peak

P.R. Nayak's formula

One-sided normal distribution law

Raileigh distribution law

3.3. Asymptota of Mathematical Expectation Value of Peak Height According to Rayleigh's Law

As we have stated previously the closest to the precise but complicated P.R Nayak's formula for the determination of mathematical expectation value of peak height is Raileigh's distribution law. The graphically represented mathematical expectation values on Fig. 3.2 of surface peak height show that Rayleigh's law is coming closer to the precise law at high levels ($\gamma > 1.5$) and we get an asymptotic formula for the mathematical expectation value of peak height.

$$E_4\left\{\xi_{p_\gamma}\right\} \sim \gamma + \frac{1}{\gamma} . \tag{3.8}$$

All previously considered asymptotic numerical values (peak height mathematical expectations) of distribution law, precise formulas and Rayleigh's law asymtota, at different deformation levels γ are given in Table 3.1.

Table 3.1.

Comparison of mathematic expectation values of peak height

γ	**Precise formula**	**On-sided distribution law**		**Rayleigh's law**		**Asymptota of Rayleigh's law**	
	$E_{1\gamma}\{\xi_p\}$	$E_{2\gamma}\{\xi_p\}$	**Deviation %**	$E_{3\gamma}\{\xi_p\}$	**Deviation %**	$E_{4\gamma}\{\xi_p\}$	**Deviation %**
0	1,3032	0,7979	39%	1,2533	4%	-	-
0,5	1,5445	1,1411	26%	1,3764	11%	-	-
1,0	1,8254	1,5251	16%	1,6557	9%	-	-
1,5	2,1893	1,9387	11%	2,0158	8%	2,1667	1%
2,0	2,5897	2,3732	8%	2,4214	6%	2,5000	3%
2,5	2,6424	2,8227	7%	2,8543	8%	2,9000	10%
3,0	3,4922	3,2831	6%	3,3046	5%	3,3333	5%

3.4. Conclusions

From the obtained data we can conclude that the precise formula for determination of surface peak height density of probability distribution and mathematical expectation value of peak height can be replaced by a simpler one. The most appropriate for replacing the precise formula is Rayleigh's law, but at high levels ($\gamma > 1.5$) asymptota of Rayleigh's law can be used.

Chapter 4. BENDING OF SURFACE ROUGHNESS PEAKS AND THEIR NUMBER

4.1. Bending of Surface Roughness Peaks

Rough surface peak bending is an important parameter of contact surface. It characterises the surface properties under friction and wearing, hardness of contact, etc. As is knows from differential geometry the mean surface bending H is being determined as the mean value of main bendings. The sum of main bendings is equal to the sum of bendings determined into two mutually perpendicular directions [1]. The peak top bending into $x$and y directions is determined:

$$H_x = \frac{\delta^2 h(x,y)}{\delta x^2}, \qquad H_y = \frac{\delta^2 h(x,y)}{\delta y^2}. \tag{4.1}$$

The mean value of main dimensions on Gauss random field maximum points $\gamma \to \infty$ are:

$$E\{H_{1,2}\} = -\frac{\left[k_{22} + k_{33} \pm \sqrt{(k_{22} - k_{33} + 4k_{23})^2}\right] \cdot u}{2k_{11}}, \tag{4.2}$$

kur

$k_{11}, k_{22}, k_{33}, k_{23}$ – elements of correlation matrix of random field derivatives [10];

u – level above which field maximum values are to be considered.

Making transformations according to investigations by J. Rudzītis [10] we get that the mathematical expectation value of bending is:

$$E\{H\} = \frac{1}{2}[E\{H_1\} + E\{H_2\}] = \frac{1}{2}\pi^2\sigma E^2\{n_1(0)\} \cdot (1 + c^2)\gamma; \tag{4.3}$$

For isotropic component surface (homogeneous) at $c = 1$:

$$E\{H_{vid}\} = \pi^2 E^2\{n_1(0)\}\sigma\gamma.$$

4.2. Number of Surface Roughness Peaks

The number of surface $h(x,y)$ peaks above level $u$is understood as that part of rough surface crossing the plane on level u.The top view on the crossing plane is given on Fig. 4.1. Crossing areas are shaded in.

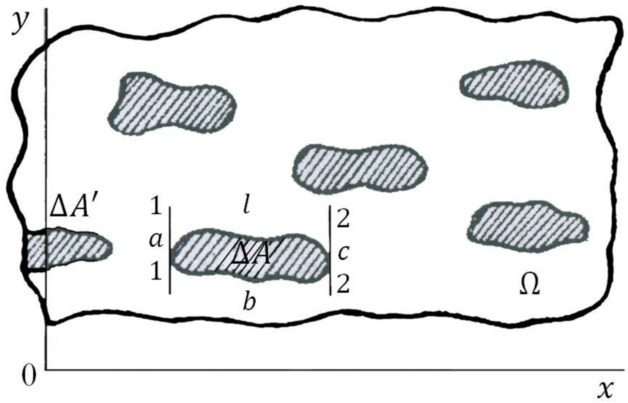


Fig. 4.1 Diagram for the calculation of number of surface roughness peaks [10]

The number of peaks (section fields) $N(\gamma)$ in the researched area Ω can be determined using differences between the number of total maximum values and minimum values situated on curves similar to the curve $a\ b\ c$, since the number of maximum values exceeds by one the number of minimum values. Then the mean number of peaks per an area unit is [10]:

$$E\{N(\gamma)\} \approx E\{m_i(\gamma)\} - E\{m_a(\gamma)\}, \tag{4.4}$$

where

$E\{m_i(\gamma)\}, E\{m_a(\gamma)\}$ – respectively the average number of minimum and maximum values at the bottom part of curve l on level γ, which has been deducted from the mid-plane.

A relatively simple result can be obtained for the standardized distribution law, and the number of surface peaks per an area unit can be calculated according to the following expression:

$$E\{N_A(\gamma)\} \approx \frac{\pi c E^2\{n_1(0)\}}{2\sqrt{2\pi}} \cdot \gamma e^{-\frac{\gamma^2}{2}}, \tag{4.5}$$

where

$N_A(\gamma) = \frac{N(\gamma)}{Aa}$ – number of surface roughness peaks per an area unit;

Aa – nominal contact area determined by area dimensions.

Chapter 5. DETERMINATION OF COMPONENT SURFACE DEFORMATIONS

5.1. Deformations of Surface Roughness Peaks

In order to determine surface roughness peak deformations we will use values obtained in the previous chapter. For the determination of mathematical expectation values of surface roughness peak height let us use asymptote of Rayleigh's law (3.8). For the determination of mathematical expectation value of surface bending we will use equation (4.3), number of surface roughness peaks per an area unit above level γ formula (4.5). Inserting in the equation (2.5) these expressions and carrying out shortening we get an equation for the calculation of pressure on the contact area.

$$q = \frac{\sigma \cdot E\{n_1(0)\}}{3\sqrt{\pi} \cdot \theta} \cdot \frac{k_q{}' \cdot c}{\sqrt{(1 + c^2)}} \cdot \sqrt{\left(\gamma + \frac{1}{\gamma}\right)^3 \cdot \gamma \cdot e^{-\gamma^2}}\,. \tag{5.1}$$

To make the equation (5.1) applicable for the solution of engineering tasks we will replace theoretical parameters σ and $n_1(0)$ with standard parameters Sa(mean arithmetic deviation of surface roughness from mid-plane) and RSm(mean pitch of surface roughness) used in the practice. We replace the equation (5.1) part $\sqrt{\left(\gamma + \frac{1}{\gamma}\right)^3 \cdot \gamma \cdot e^{-\gamma^2}}$ with function $t(\gamma)$:

$$t(\gamma) \approx 2 - \frac{1}{2} \cdot \gamma\,. \tag{5.2}$$

$(\gamma = 1.0 - 4.0)$

We replace the equation (5.1) part $\frac{k_q{}' \cdot c}{\sqrt{(1+c^2)}}$ with coefficientk_c, which depends on surface anisotropy. The dependence of this coefficient on surface anisotropy is shown on Fig. 6.1.

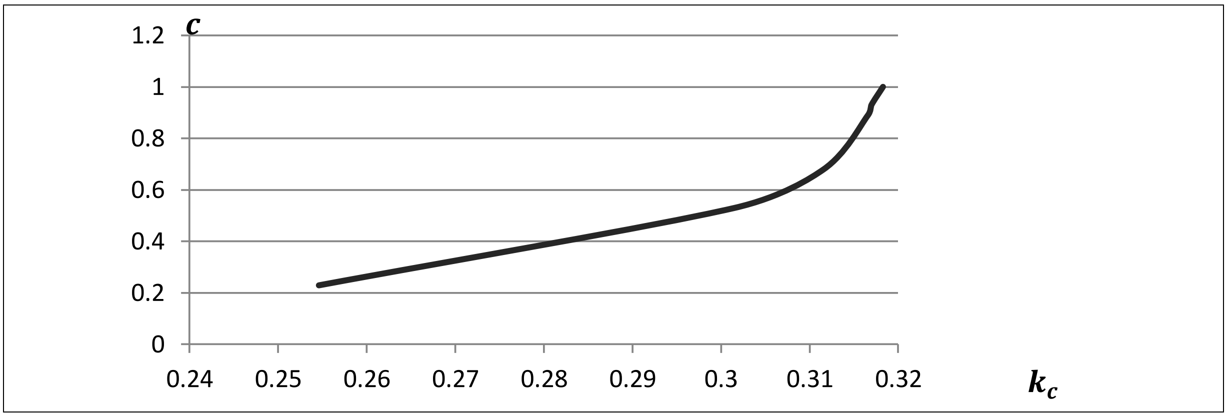


Fig. 5.1 Dependence of coefficient k_c on surface anisotropy c

Thus we can write the equation (5.1) as follows:

$$q = \frac{\sqrt{2}}{3 \cdot \theta} \cdot \frac{Sa}{RSm} \cdot k_c \cdot \left(2 - \frac{1}{2} \cdot \gamma\right). \tag{5.3}$$

The equation (5.3) allows to obtain the variable value γ and, inserting it into equation (2.1) we get the formula for the determination of surface roughness deformation a_1, and simplifying this formula for the solution of engineering tasks we can write:

$$a_1 \approx St - 5Sa \cdot \left(1 - \frac{q}{E} \cdot \frac{RSm}{Sa}\right). \tag{5.4}$$

5.2. Settlement of Surface Roughness Peaks

On contact of two surfaces peaks, especially the highest of them under the applied force not only are deformed but there also take place vertical movement or settlement (Fig. 5.2). The size of settlement a_2 likewise peak deformation depends on the size of applied force and physical and mechanical properties of material.

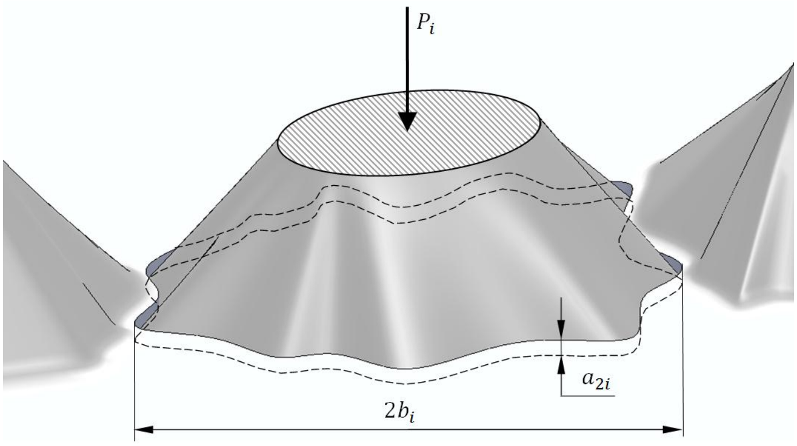


Fig. 5.2 Settlement of peak

According to researches carried out by L.A. Galyin [5] settlement of these peaks can be determined by the help of the following formula:

$$a_{2i} = \frac{K(e) \cdot \theta \cdot P_i}{b_{2i}} \tag{5.5}$$

Parameter b_{2i} characterises the distance between the peak hollows and thus it can be determined also as the distance between the peak tops. In this case we are interested only in high peaks situated above level $\gamma = 2$, then $E\{b_{2i}\}$ is approximately similar to RS_2, where RS_2 is a pitch along the line on level $\gamma = 2$ (see. Fig.5.3).

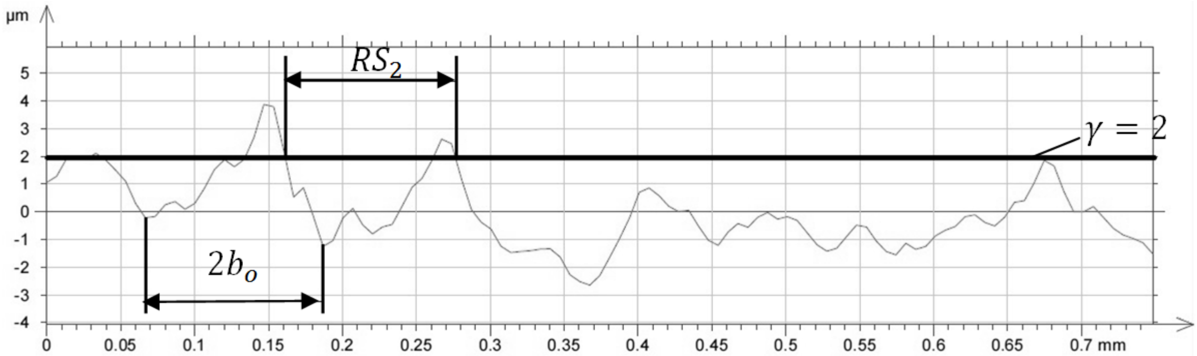


Fig. 5.3 Pitch of surface roughness peaks along line RS_2 lon level $\gamma = 2$

Thus settlement of one surface peak can be calculated as follows:

$$a_{2i} = \frac{K(e) \cdot \theta \cdot P_i \cdot e^{-2}}{RSm} \tag{5.6}$$

When calculating settlement of one peak the contact area covers the whole peak, but calculating peak settlement in the contact area between the nozzle of measuring instrument and measured component in the formula there should be taken into consideration not only the applied force but also the contact area. Therefore in the calculations it is useful to use the unit pressure q Thus in order to determine settlement of roughnesses in the whole area of contact between component and nozzle of measuring instrument we transform the obtained equation of one peak settlement as follows:

$$a_2 = \frac{K(e) \cdot \theta \cdot e^{-2} \cdot q}{RSm \cdot N_A(\gamma)}, \tag{5.7}$$

$q = \frac{P}{Aa}$ – unit pressure, Aa – nominal contact area;

$N_A(\gamma) = \frac{N(\gamma)}{Aa}$ – number of surface peaks per an area unit according to equation (4.5)

Inserting the equation (4.5) into the relation (5.7) we get a simple formula suited for engineering calculations for determining roughness settlement values:

$$a_2 = \frac{q \cdot \theta \cdot RSm}{2\sqrt{2\pi}} \cdot k_{2c}, \tag{5.8}$$

where

$k_{2c} = \frac{K(e)}{c}$ – coefficient depending on the eccentricity of surface peaks.

Values of coefficient k_{2c} at different anisotropy coefficient c values are given on Fig. 5.4.

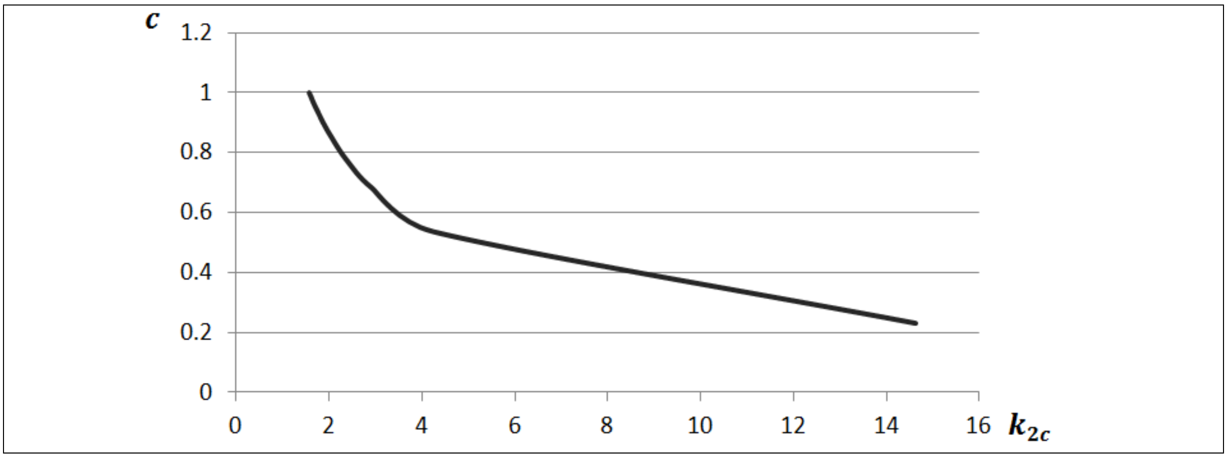


Fig. 5.4 Dependence of coefficient k_{2c} on surface anisotropy c

In the case described in this thesis – measurement of components from highly elastic material we assume that anisotropy coefficient $c = 1$ and coefficient (Poisson's ratio) of lateral deformation of material $\mu = 0$. By simplifying this formula for the solution of engineering tasks we can write the equation (5.8) as follows:

$$a_2 \approx 0.1 \cdot RSm \cdot \frac{q}{E}. \tag{5.9}$$

5.3. Determination of Basic Material Deformation

Measurement of linear dimensions of components is characterised by small loads, small contact areas and only elastic deformations are allowed, thus this Paper considers only such cases when contact is elastic. Based on the classical theory of elasticity relations have been established linking measurement force and component surface deformation.

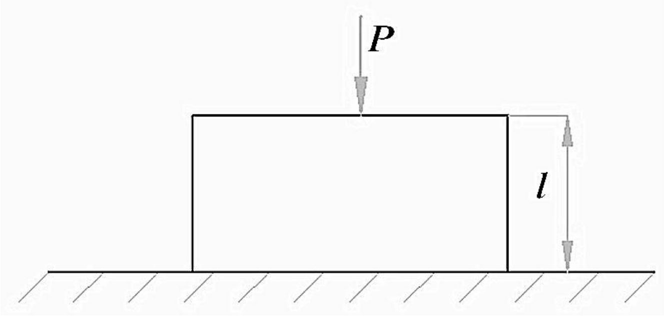


Fig. 5.5 Compression load diagram

For the calculation of deformation of basic material one can use classical formulas of elasticity theory. Let us assume that in our case only normal stress is functioning (tangential is not), in such case for the calculation of deformation a_3 we can use Hooke's law. In order to determine deformation of basic material it is useful to assume that the applied force has been distributed uniformly along the contact area, thus for performing calculations it is easier to use pressure on contact area q. The Hooke's law for the simplest form (Fig.5.5) body can be written as follows

$$a_3 = \frac{q}{E} \cdot l \ , \tag{5.10}$$

where:

l – initial height of body;

E – elasticity modulus or Young's modulus for basic material;

$q = \frac{P}{Aa}$ – pressure on area unit, Aa – nominal contact area.

5.4. Determination of Rough Surface Deformation of Components from Highly Elastic Materials

Taking into consideration that surface deformations are divided into three parts (surface roughness peak deformation a_1, settlement of these peaks a_2 and deformations of basic material a_3), total rough surface deformation $a$can be calculated according to the following formula:

$$a = a_1 + a_2 + a_3, \tag{5.11}$$

where

a_1– surface roughness peak deformation;

a_2 –settlement 0f surface roughness peaks;

a_3 – deformation of basic material.

Thus we can use the obtained calculation equations of separate parts of total deformation suited for engineering calculations and carrying out simplification we get an equation for the calculation of total surface deformation:

$$a \approx \frac{q}{E} \cdot \left[l + 1.1 \cdot RSm + \frac{E}{q}(St - 5 \cdot Sa) \right]. \tag{5.12}$$

In making engineering calculations and for practical application it is more convenient to use the standardized deformation size ζthat is normalized by surface deformation $a$with nominal size l of the measured component:

$$\zeta = \frac{a}{l}, \tag{5.13}$$

where

a – surface deformation (5.12);

l – nominal size of measured component.

The numerical values of standardized deformation size ζ for different measuring instruments and different elasticity modules of component material $E$are given in Table 7.4.

Chapter 6. EXPERIMENTAL CHECKING OF ANALYTICAL FORMULAS

Let us carry out experiments in order to check the suitability of surface deformation calculation formula gained in Chapter 8 for the evaluation of deformation of components from highly elastic material.

For experimental checking 10 rubber samples with elasticity modulus $E = 5\ N/mm^2$ were prepared. Minimum sizes of samples: 28x20x9 mm.

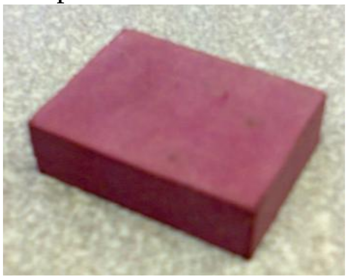


Fig. 6.1 Prepared sample

Prior to measuring linear dimensions using contact methods micro-topography of samples was made using profilograph –roughness indicator *Hobson Form Talysurf Intra 50* and linear dimensions were stated using 3D coordinate measuring device *MahrVision* (Fig. 6.4) by means of non-contact measurement method.

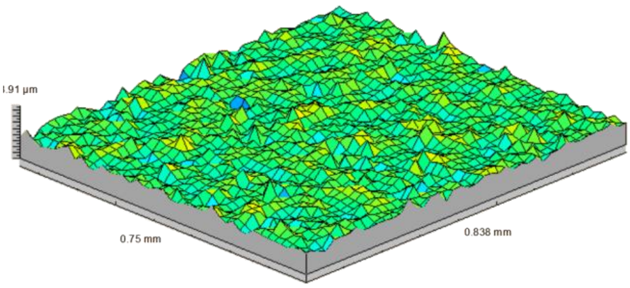
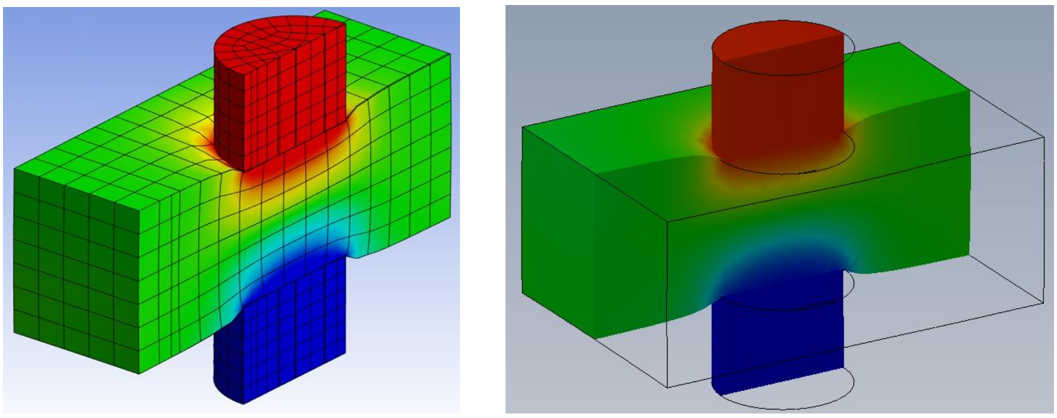


Fig. 6.2.Microtopography of experimental sample

For surface deformation evaluation samples were measured by three different micrometres, indicator clip and digital length measurement system TESA TG 30. One of the used methods for checking experimental data was computer simulation. ANSYS and SolidWork softwares were used for this purpose.



a B

Fig. 6.3 Measuring model created software ANSYS (a) and SolidWorks (b)

Measuring process in computer simulation was simulated with digital micrometer.

6.1. Measurement of Linear Dimensions of Experimental Samples

For carrying out experiments ten samples of rubber with elasticity modulus $E =$

$5\ N/mm^2$ were chosen. Prior to taking linear dimensions surface roughness parameters and profile parameters for these samples were taken. In order to evaluate as precisely as possible deformations arising as a result of measurement force linear dimensions of samples were measured in the following sequence:

1. all samples were measured by non-contact method,
2. surface roughness parameters of samples were stated,
3. each sample was measured by three different contact measurement methods.

Between each contact measurement method samples once more were measured with non-contact method and their surface roughness was stated to make sure that no plastic deformations have occurred in samples.

For non-contact measurement of linear dimensions of experimental samples 3D coordinate measurement device *MahrVision MS222* was used (Fig. 6.4). The applied measurement method allows to establish precisely (point value 0.001 mm) the sample thickness.

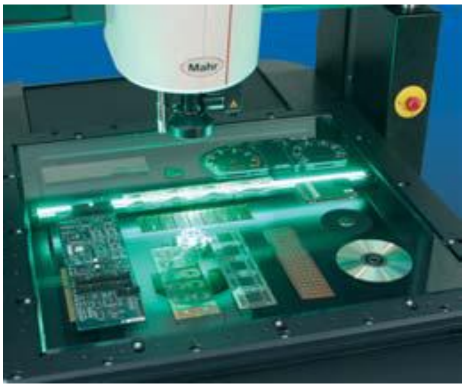


Fig. 6.4. MahrVision MS222

In the course of experiment each sample was measured 10 times, measuring the sample along its whole length at 10 points.

Table 6.1

Mean values of measurements of non-contact measurement device

Parauga Nr.	Gumijas paraugu biezums [mm]
1	8.861
2	9.009
3	8.852
4	8.876
5	8.679
6	8.911
7	9.460
8	8.647
9	8.885
10	8.454

Experimental measurement using contact measuring methods, as stated before, were carried out with smooth micrometer (measurement force 7N), digital micrometer system TESA TG30 (measurement force 3N) and indicator clip (measurement force 9N). The obtained mean values are summarised in the table (Table 6.2).

Table 6.2

Mean values of results of measurements of experimental samples using contact measurement methods

Sample	MEASURING INSTRUMENTS			
	Micrometer	Digital micrometer	TESA TG-30	Indicator clip
	Point value (mm)			
	0.01	**0.001**	**0.0001**	**0.01**
Number				
1	8.66	8.676	8.7722	8.62
2	8.84	8.845	8.9346	8.79
3	8.76	8.713	8.8033	8.71
4	8.61	8.646	8.6766	8.57
5	8.34	8.319	8.5085	8.30
6	8.67	8.751	8.7879	8.63
7	9.25	9.248	9.3922	9.21
8	8.40	8.484	8.5923	8.35
9	8.66	8.664	8.7792	8.61
10	8.25	8.215	8.3693	8.19

6.2. Computer Simulation of Measuring Process

As mentioned before in this Paper for the checking of analytical calculations of deformation computer simulation was also used. There were chosen two at present most often in the world used softwares suited for engineering calculations ANSYS and SolidWorks. Computer simulation was carried out for the case (according to diagram given on Fig. 6.3) when a rubber component with elasticity modulus $E = 5\ N/mm^2$ and sizes obtained by means of coordinate measurement device *MahrVision MS222* was pressed with cylindrical steel bars corresponding to parameters of digital micrometer. The obtained results if deformation size are summarised in Table 6.3.

6.3. Analyses of Analytical and Experimental Results

Carrying out analytical calculations there were used respective surface roughness values obtained prior to and after making contact measurements. As was stated previously surface roughness of each sample was measured before and after making contact measurements. Having summarized all obtained experimental results and analytically calculated deformations in one table (Table 6.3) deviation between actual (experimentally obtained) deformation and analytically or by computer simulation gained value was also calculated in percents.

Table 6.3

Comparison of data of analytical calculation and experimental measurements

No	MarVision MS 222	Digital micrometer	Deformation	Calculated deformation	Deviation	ANSYS	Deviation	Solid-Works	Deviation
1	8.861	8.656	0.205	0.202	1%	0.190	8%	0.187	9%
2	9.009	8.799	0.210	0.206	2%	0.192	8%	0.189	10%
3	8.852	8.646	0.206	0.202	2%	0.189	8%	0.186	10%
4	8.876	8.671	0.205	0.203	1%	0.188	8%	0.185	10%
5	8.679	8.476	0.203	0.199	2%	0.190	7%	0.187	8%
6	8.911	8.702	0.209	0.204	3%	0.190	9%	0.187	10%
7	9.46	9.240	0.220	0.215	2%	0.202	8%	0.199	10%
8	8.647	8.442	0.205	0.198	3%	0.193	6%	0.190	7%
9	8.885	8.678	0.207	0.203	2%	0.191	8%	0.188	9%
10	8.454	8.256	0.198	0.194	2%	0.181	8%	0.178	10%

6.4. Conclusions

As a result of performed experimental, analytical and computer simulation researches we can conclude that:

1) Deviation between actual deformation of experimentally measured samples and analytically calculated predicted deformation is within the range of 1% to 3%, thus it can be concluded that theory and experiments basically coincides.
2) Deviation between actual deformation of samples measured experimentally and final deformation of element models developed by computer simulation softwares ANSYS and SolidWorks is within the range 7% to 10%, thus it is concluded that final element models developed by simulation softwares ANSYS and SolidWorks and experiments basically coincide.
3) Analytical formula of deformation determination and experimental measurements are compared with final element models developed by computer simulation softwares ANSYS and SolidWorks. It was stated that values of deformation models by computer simulation are smaller than values obtained experimentally and analytically. This is connected with the fact that in computer simulation surface roughness deformations have not been considered. Formulas gained for analytical calculations in principle are correct and close to the model of final elements developed in computer simulation softwares.

Chapter 7. METHODS FOR THE MEASUREMENT OF COMPONENTS FROM HIGHLY ELASTIC MATERIALS AND CHOICE OF MEASURING INSTRUMENTS

The task of the given methods is to evaluate the measurement error of components from highly elastic materials using three surface deformation calculation methods.

In the Standard (ГОСТ 8.051-1981) [12] there are 15 maximum tolerated series of mistakes that need not be calculated. The permissible values of measurement errors are assumed to be 20% to 35% of tolerated value. Tolerated values of measurement errors are given in Table 10.3 of these methods.

7.1. General Provisions

1. When measuring component dimensions with universal measuring instruments one should consider also the surface roughness of measured component. In some cases the measurement error is to be equated to roughness peak deformation.
2. Measurement of precision components where manufacturing tolerated values are some micrometers the tolerated measurement error is 20 -35% of manufacturer's tolerance. In such cases one should take into consideration deviations resulting from surface peak deformation.
3. The methods have been envisaged for the determination of measurement error in the measurement of components from highly elastic ($E = 200N/mm^2$) materials in manufacturing enterprises and measurement laboratories.
4. The methods determine the sequence of measurement of linear sizes of components from highly elastic materials and of determination of measurement error.
5. These methods can serve as a basis for the development of methods for the measurements of components from definite highly elastic materials or determination of measurement error.
6. These methods may be supplemented with other measuring instruments and materials according to the specific character of manufacturing enterprise and material and technical basis.

7.2. Choice of Measuring Instruments

The correct choice of measuring instruments determine the efficiency of technical control both from metrological and economic point of view. Knowing the nominal dimensions of the component and manufacturer's tolerance the sequence of choice of measuring instrument is as follows:

1) Using Table 7.3 find the tolerated measurement threshold error $[\Delta_{m\bar{e}r}]$;
2) From Table 7.2 choose measuring instruments the measuring error of which Δ_{instr} fall within the limits of tolerated measurement error:

$$\Delta_{instr} < [\Delta_{m\bar{e}r}]. \tag{7.1}$$

Although several instruments correspond to the established measurement limit value one should take into consideration that there should be chosen such measurement method and measurement instruments, which ensure the required control precision, not increasing significantly the production costs because of too complicated or extended control.

7.3. Determination of the Measurement Error

Full calculation of measurement error

In order to determine the measurement error of components from highly elastic materials using the surface contact deformation calculation formula (7.2), after the determination of physical and mechanical parameters of surface roughness and measured component that are needed for calculations, the following tasks are to be carried out:

1) The measuring instrument is chosen according to instructions of the given methods 7.2 and from the table 7.2 to get pressure and surface (q);
2) Total surface deformation is calculated under the effect of measurement force, using the expression:

$$a \approx \frac{q}{E} \cdot \left[l + 1.1 \cdot RSm + \frac{E}{q}(St - 5 \cdot Sa) \right]. \quad (7.2)$$

3) The obtained deformation value a is summed with the measurement error for the chosen measuring instrument:

$$\Delta_{m\bar{e}r} = a + \Delta_{instr} \quad (7.3)$$

4) Check whether the obtained value does not exceed the tolerated limit error $[\Delta_{m\bar{e}r}]$:

$$\Delta_{m\bar{e}r} \leq [\Delta_{m\bar{e}r}]$$

Although several instruments correspond to the established measurement limit value one should take into consideration that there should be chosen such measurement method and measurement instruments, which ensure the required control precision, not increasing significantly the production costs because of too complicated or extended control.

Graphical determination of measurement error

In order to determine measurement error of components from highly elastic materials under the effect of measurement force and surface roughness, using diagram (Fig. 7.1) the following tasks have to carried out:

1) Choose measuring instrument according to the 7.2 directions given in these methods;
2) According to graph (Fig.7.1) determine deformations caused by the chosen measuring instrument;
3) Sum up the obtained deformation $a$with the measurement error for the chosen measuring instrument (7.3):

$$\Delta_{m\bar{e}r} = a + \Delta_{instr}$$

4) Check whether the value of the obtained measurement error does not exceed the tolerated measurement limit value $[\Delta_{m\bar{e}r}]$:

$$\Delta_{m\bar{e}r} \leq [\Delta_{m\bar{e}r}]$$

Table 7.1

Elasticity modulus(E) of highly elastic materials

Material	**Elasticity modulus** (E) $[N/mm^2]$
Soft rubber	1.1 – 1.5
Teflon	3
Vulcanized rubber	5 – 10
Polyethylene	7
Caoutchouc	7.9
Polypropylene	13
Cellulose	14
Nylon	24
Acrylic	30
PVC	34
Polystyrene	34
Melamine	70
Filled with carbamide formaldehyde, cellulose	70 – 170
Acetal	70
Carbamide formaldehyde	100
Epoxide resin	200
Phenol formaldehyde	170 – 350

Table 7.2

Parameters of measuring instruments

Title of measuring instrument	**Limiting error of measuring instrument** Δ_{instr} [μm]	**Pressure on surface** (q) $[N/mm^2]$
МН type minimeter	0.5	0.14
ИЧМ type indicator	2	0.13
ИЧ type indicator	8	0.13
Indicator clip	10	0.32
Indicator internal size	15	0.03
Lever-type clamp	2	0.35
Level micrometer	4	0.14
Electrocontact (signal devices	1	0.06
Horizontal and vertical telescope calipter	0.3	0.13
Length meter	1.4	0.20
Micrometer K - 6 type		
For sizes <1 mm	0.5	0.02
For making ordinary measurements	7	0.07
Digital micrometer DM2020		
For sizes 0 – 25	4	0.11
25 – 50	10	0.11
125 – 150	20	0.11
Micrometer		
For sizes 0 – 25	5	0.25
25 – 50	15	0.25
125 – 150	30	0.25
Lever micrometer ЧМ		
For sizes 0 – 25	5	0.11
25 – 50	3	0.18
125 – 150	7	0.32
Vertical length meter	1	0.31
Digital measuring system TESA TG 30	1	0.06
Diameter and circumference length meter Altia	1.5	0.01
Roundness meter	3	0.04
Diameter and circumference length meter Z_CAL	3	0.05
Height meter Mestra	1.4	0.01
Height meter Digimar	1.8	0.02
Micrometer Micromar		
For sizes 0 – 25	3.5	0.02
25 – 50 ... 450 – 475	4	0.04
475 – 500	7	0.07

Table 7.3

Permissible measurement limitting errors according to ГОСТ 8.051-1981[12]

Size intervals [mm]													**Error % of tolerance**	**Quality**
līdz 3	3 ... 6	6 ... 10	10 ... 18	18 ... 30	30 ... 50	50 ... 80	80 ... 120	120 ... 180	180 ... 250	250 ... 315	315 ... 400	400 ... 500		
Measurement limitting error [µm] (±) **Controlled tolerance** [µm]														
2.0 6	2.5 8	3.0 9	3.5 11	4.0 13	5.0 16	6.0 19	7.0 22	7.5 25	7.0 29	8.0 32	10.0 36	10.0 40	30	6
3.0 9	3.5 12	4.0 14	5.5 18	6.0 21	7.5 25	9.0 30	10.0 35	12.0 40	12.0 46	14.0 52	16.0 57	18.0 63	30	7
3.5 14	4.5 18	5.0 22	7.0 27	8.0 33	10.0 39	11.0 46	14.0 54	16.0 63	18.0 72	20.0 81	24.0 89	26.0 97	25	8
5.0 20	6.0 25	7.0 30	9.0 35	11.0 45	12.0 50	15.0 60	18.0 70	20.0 80	30.0 115	30.0 130	40.0 140	40.0 155	25	9
7.0 33	8.0 40	10.0 50	12.0 60	14.0 70	17.0 85	20.0 100	23.0 115	27.0 135	40.0 185	50.0 210	50.0 230	50.0 250	20	10
8.0 40	10.0 48	12.0 58	14.0 70	17.0 84	20.0 100	24.0 120	28.0 140	32.0 160	60.0 290	70.0 320	80.0 360	80.0 400	20	11
12.0 60	16.0 80	20.0 100	24.0 120	28.0 140	34.0 170	40.0 200	45.0 230	52.0 260	100.0 460	120.0 520	120.0 570	140.0 630	20	12
24.0 120	32.0 160	40.0 200	48.0 240	55.0 280	70.0 340	80.0 400	90.0 460	105.0 530	160.0 720	180.0 810	190.0 890	200.0 970	20	13
50.0 250	60.0 300	70.0 360	85.0 430	105.0 520	125.0 620	150.0 740	170.0 870	200.0 1000	240.0 1150	260.0 1300	280.0 1400	320.0 1550	20	14
80.0 400	95.0 480	120.0 580	140.0 700	160.0 840	200.0 1000	240.0 1200	280.0 1400	320.0 1600	380.0 1850	440.0 2100	450.0 2300	500.0 2500	20	15
120.0 600	150.0 750	180.0 900	220.0 1100	260.0 1300	320.0 1600	380.0 1900	420.0 2200	500.0 2500	600.0 2900	700.0 3200	800.0 3600	800.0 4000	20	16
200.0 1000	240.0 1200	300.0 1500	380.0 1800	440.0 2100	500.0 2500	600.0 3000	700.0 3500	800.0 4000	1000.0 4600	1100.0 5200	1200.0 5700	1400.0 6300	20	17

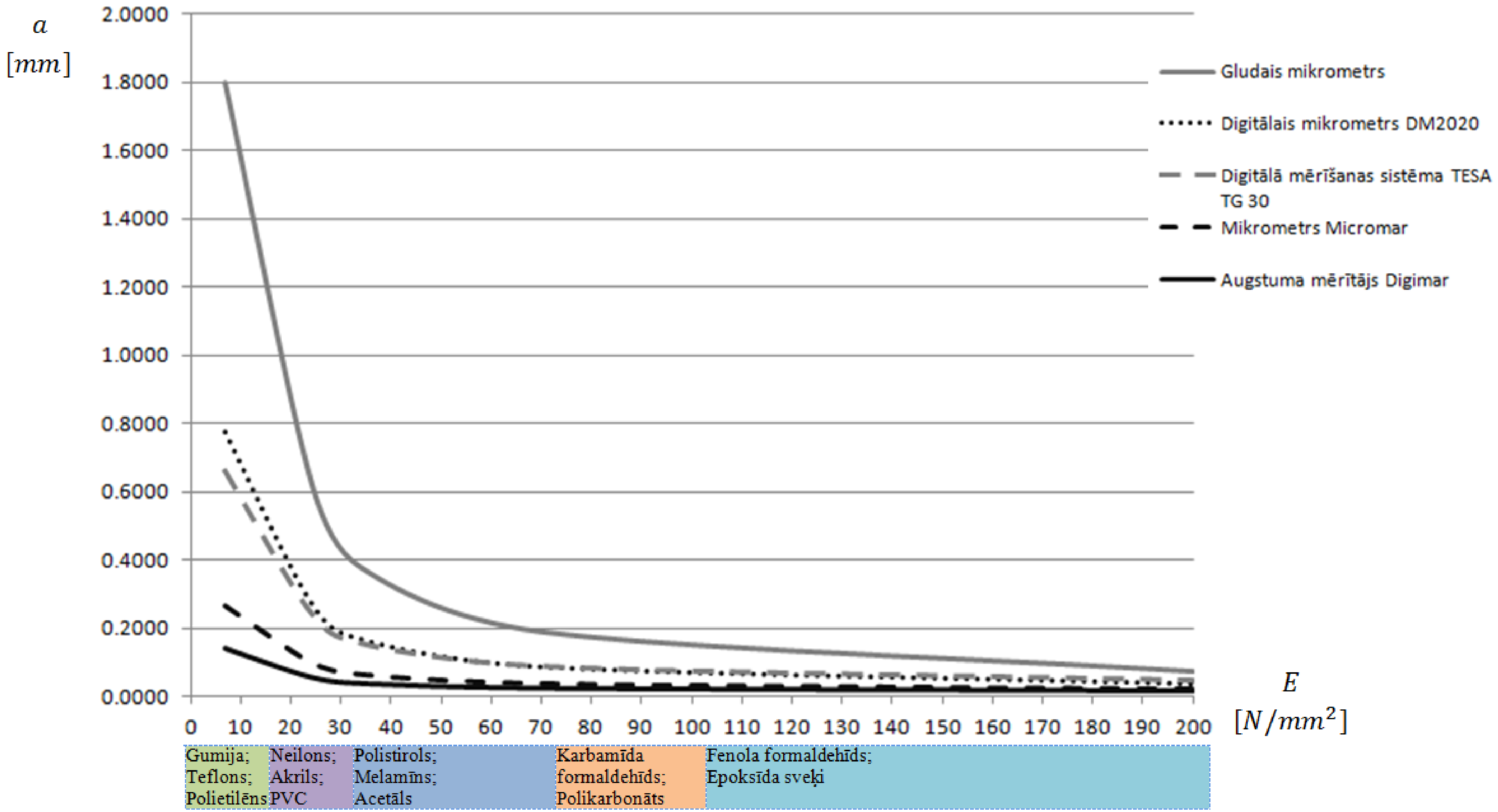


Fig. 7.1 Deformations of components from different materials (with nominal size 50 mm) measuring with different instruments

Simplified Calculation of Measurement Error

In these methods the relative size ζ formed by dividing the total deformation a with nominal size l of measured component is being assumed as standardized deformation:

$$\zeta = \frac{a}{l}, \tag{7.4}$$

where

l –nominal size of measured component;

ζ – standardized deformation size.

Thus in order to determine the error of measurement of components from highly elastic materials under the effect of measurement force and surface roughness using a simplified calculation of measurement error the following tasks should be carried out:

1) Choose a measuring instrument according to instruction 7.2 of these methods;
2) Depending on the elasticity modulus of the material of measured component (Table 7.1) find in table (Table 7.2) the standardized size ζ of deformation for nominal size of measured component;
3) Insert the standardized deformation size ζ into the equation (7.4) and calculate deformation a of measured component.
4) The obtained value of deformation is summed with the measurement error for the chosen measuring instrument:

$$\Delta_{m\bar{e}r} = a + \Delta_{instr} .$$

5) Check whether the obtained value does not exceed the permissible measurement limit of threshold error $[\Delta_{m\bar{e}r}]$:

$$\Delta_{m\bar{e}r} \leq [\Delta_{m\bar{e}r}].$$

Table 7.4

Standardized value ζ of maximum deformation of components of soft rubber (E=7 N/mm^2)

Instrument	**līdz 3**	**3-6**	**6-10**	**10-18**	**18-30**	**30-50**	**50-80**	**80-120**	**120-180**	**180-250**	**250-315**	**315-400**	**400-500**
ИЧ tipa indikators	0.0260	0.0220	0.0204	0.0193	0.0188	0.0184	0.0183	0.0182	0.0181	0.0181	0.0180	0.0180	0.0180
Indikatora skava	0.0256	0.0216	0.0200	0.0189	0.0184	0.0181	0.0179	0.0178	0.0178	0.0177	0.0177	0.0177	0.0177
Indikatora iekšmērs	0.0118	0.0091	0.0080	0.0072	0.0068	0.0066	0.0065	0.0064	0.0064	0.0064	0.0063	0.0063	0.0063
Sviru skava	0.0256	0.0216	0.0200	0.0189	0.0184	0.0181	0.0179	0.0178	0.0178	0.0177	0.0177	0.0177	0.0177
Elektrokontakta (luksafora) aparāti	0.0103	0.0077	0.0066	0.0059	0.0056	0.0054	0.0052	0.0052	0.0051	0.0051	0.0051	0.0051	0.0051
Horizontālais un vertikālais optimetrs	0.0260	0.0220	0.0204	0.0193	0.0188	0.0184	0.0183	0.0182	0.0181	0.0181	0.0180	0.0180	0.0180
Digitālais mikrometrs DM2020													
Izmēriem 0-25	0.0042	0.0021	0.0013	0.0007	0.0004								
25-75					0.0158	0.0158	0.0158						
75-150							0.0155	0.0155	0.0155				
Gludais mikrometrs													
Izmēriem 0-25	0.0469	0.0411	0.0387	0.0372	0.0364								
25-50					0.0364	0.0359							
50-150							0.0354	0.0354	0.0354				
Vertikālais garuma mērītājs	0.0180	0.0147	0.0134	0.0125	0.0121	0.0118	0.0117	0.0116	0.0115	0.0115	0.0115	0.0115	0.0115
Digitālā mērīšanas sistēma TESA TG 30	0.0067	0.0052	0.0046	0.0045	0.0045	0.0039	0.0039	0.0038	0.0038	0.0038	0.0038	0.0038	0.0038
Diametra un riņķa līnijas garuma mērītājs Altia	0.0066	0.0043	0.0034	0.0028	0.0025	0.0023	0.0022	0.0021	0.0021	0.0021	0.0021	0.0020	0.0020
Apaļuma mērītājs	0.0118	0.0090	0.0079	0.0072	0.0068	0.0066	0.0065	0.0064	0.0064	0.0064	0.0063	0.0063	0.0063
Diametra un riņķa līnijas garuma mērītājs Z_CAL	0.0130	0.0101	0.0090	0.0082	0.0078	0.0076	0.0075	0.0074	0.0074	0.0073	0.0073	0.0073	0.0073
Augstuma mērītājs Mestra	0.0057	0.0035	0.0026	0.0020	0.0017	0.0015	0.0014	0.0014	0.0013	0.0013	0.0013	0.0013	0.0013
Augstuma mērītājs Digimar	0.0072	0.0049	0.0039	0.0033	0.0030	0.0028	0.0027	0.0026	0.0026	0.0026	0.0026	0.0026	0.0025
Mikrometrs Micromar													
izmēriem no 0 -25	0.0164	0.0132	0.0120	0.0111									
25 - 50					0.0056	0.0054							
50 - 125							0.0052	0.0520					
125-500									0.0510	0.0051	0.0051	0.0051	0.0051

SUMMARY

After fulfilment of work we can make the following conclusions:

1) Upon studying literature it was stated that in the process of work development in the available sources of information there have been no researches on the effect of measurement force and surface roughness on the precision of measurements of linear sizes of components from elastic materials.
2) The Paper for the first time studies the effect of surface roughness on the precision of measurements of components from highly elastic materials, using 3D surface basic principles. The surface is being described as two-dimension random field with normal height distribution function and continuous correlation function.
3) Choice of 3D surface parameters and surface model in case of elastic contact calculation has bee substantiated. In the formation of surface model two roughness height parameters (St – total surface height, Sa – mean arithmetic deviation of surface form mid-plane) and roughness step parameter (RSm – roughness step along the mid-line) have been used.
4) The Paper proves that total surface deformation consists of surface roughness peak deformation, their settlement and deformation of basic material. The Paper has stated formulas for all three deformation constituents.
5) The Paper studies a possibility to replace a theoretically precise determination formula of parameters of surface roughness peak height (Nayak's formula) with simpler probability determination laws. Gauss and Rayleigh's probability distribution laws have been examined and is proved that the closest analogue is Rayleigh's distribution law. The equation for the determination mathematical expectation value of surface roughness peak height can be used also for other types of surface deformations, for example, for the calculation of wear, friction coefficient, etc.
6) Relation for the determination of surface roughness height deformation $a_1$has been established comprising 3D surface parameters (St – common surface height, Sa – mean arithmetic deviation of surface from mid-plane, RSm_1 – roughness step along mid-line and c – coefficient of anisotropy), characteristic physical mechanical values of measured component (E – elasticity modulus, μ – Poisson's coefficient) and pressure on contact area.
7) Relation for the determination of surface roughness settlement value $a_1$has been established comprising 3D surface parameters (RSm_1 – roughness step along mid-line and c – coefficient of anisotropy), characteristic physical mechanical values of measured component (E – elasticity modulus, μ – Poisson's coefficient) and pressure on contact area.
8) Relation for the determination of total surface deformation $a$has been established, formed as a sum of three deformations (a_1-surface roughness peak deformations, surface roughness peak settlement value, $a_3$deformation of basic material), to determine, for the determination of which one must know 3D surface parameters (St, Sa, RSm, c), physical and mechanical properties of components (E, μ) and pressure on contact area.
9) Measurements of linear sizes of components from highly elastic materials have been made to check the theory. 10 samples were measured using contact and non-contact

measuring instruments. For non-contact measurements 3D coordinate measuring equipment MarVision MS222 (Mahrs, Germany) was used. For contact measurements there were used: digital micrometer DM2020 (Digital Micrometers Ltd, UK), smooth micrometer MK (Калибр, Russia), digital length measuring system TG30 (TESA COMPAC, Switzerland). As a result of the performed experiments it was concluded that there is a principal correspondence of theory and experiments. Deviation of values calculated analytically from experimentally obtained results do not exceed 5%.

10) The analytical deformation determination formula and experimental measurements were compared with final element models developed by computer simulation softwares ANSYS and SolidWorks, where contact deformations are being established disregarding the effect of surface roughness. It was stated that the results of computer simulation models are smaller than experimentally obtained results by about 10%. It proves the correctness of formulas obtained for analytical calculations.
11) Universal methods for the choice of measuring instruments have been developed for the measurement of components from highly elastic materials allowing to predict possible component deformation and measurement error.

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