PROCESS PARAMETER AND MEASUREMENT ACCURACY AND PRECISION

PROCESA PARAMETRU UN MĒRĪJUMU PAREIZĪBA UN PRECIZITĀTE

J.Mazais, J.Miķelsons, N.Salenieks

Key words: accuracy, measurement, parameter, precision, process, stability

Summary

Technology management, process control and performance improvement methodology is based on the-state-of-the-practice process parameter measurement, accuracy, precision and stability analysis, statistical estimation of past and current track of the process forecast of its possible run in the future, prediction of undesirable future changes and identification of the causes and effect of such changes.

Measurement Accuracy and Precision

Accuracy, precision, and related indices are evaluated as real-time functions using sample function probability distributions and such characteristics as mean, variance, and correlation functions. The real-time estimation of accuracy, repeatability (precision), and reproducibility of measurements and stability in general is based on analysis of values measured at consecutive time moments or on analysis of measurement errors – deviations from the actual value x0.
In general, the object of analysis is the sample function

$$^{(i)}\Delta X(t) = {}^{(i)}X(t) - x_0 = \overline{{}^{i}\Delta x} + {}^{(i)}h(t) + {}^{(i)}c(t)\,, \qquad (1)$$

where $\overline{{}^{i}\Delta x} = {}^{i}Q - x_0$ is the expected initial value of the error.
A sample function, which describes series of measurements taken at consecutive time intervals is marked $^{(i)}X(t)$ traditionally. In this article, the structure type $^{(i)}X(t) = {}^{i}Q + {}^{(i)}h(t) + {}^{(i)}c(t)$ is used, but, in general, the presence of a deterministic component $^{i}z(t)$ (deterministic fluctuations) is possible as well.

Statistical estimates are obtained for probability characteristics of sample function components [1] and on this basis for accuracy, precision, and related indices (Fig.1).
Accuracy characterizes the level of adequacy between the average of measurements and the actual value of the parameter. In general, it is a real-time function

$$\overline{{}^{i}\Delta x(t)} = \overline{{}^{i}x(t)} - x_0 = \overline{{}^{i}\Delta x} + {}^{i}h(t) \qquad (2)$$

and if the trend is linear, then

$$\overline{{}^{i}\Delta x(t)} = \overline{{}^{i}\Delta x} + {}^{i}ht. \qquad (3)$$

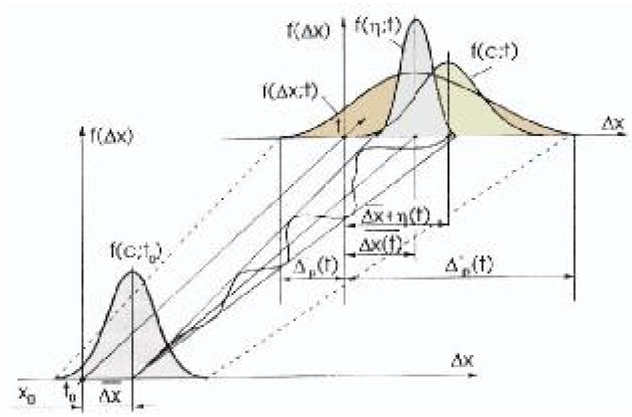


Figure 1. Measurement accuracy, precision, and stability evaluation at each moment of time (the index ***i*** is omitted for clarity)

Variation is obtained as a standard deviation of the point distribution:

- for a deterministic trend

$$ {}^{i}s_{Dx}(t) = {}^{i}s_{c}(t), \tag{4}$$

- for a random trend

$$ {}^{i}s_{Dx}(t) = \sqrt{{}^{i}s_{c}^{2}(t) + {}^{i}s_{h}^{2}(t)}\,, \tag{5}$$

- for a linear random trend

$$ {}^{i}s_{Dx}(t) = \sqrt{{}^{i}s_{c}^{2}(t) + {}^{i}dt}\,. \tag{6}$$

Precision (repeatability) is estimated by the natural spread of the measurement error

$$ {}^{i}Dp(t) = {}^{i}\Delta_{p}^{+}(t) - {}^{i}\Delta_{p}^{-}(t)\,. \tag{7}$$

Error bounds [${}^{i}\Delta^{+}(t)$, the upper one; ${}^{i}\Delta^{-}(t)$, the lower one] are roots of the equations

$$ P\{{}^{i}\Delta X \le \Delta_{p}^{+}\}\frac{1+p}{2}\,, \tag{8}$$

$$ P\{{}^{i}\Delta X \le \Delta_{p}^{+}\}\frac{1-p}{2}\,, \tag{9}$$

where p *is* the probability that the error will stay within the error bounds. For the normally distributed error, ${}^{i}DX(t) \sim N\,[\,{}^{i}\overline{\Delta x(t)}\,, {}^{i}s_{Dx}(t)]$,

$$ {}^{i}D^{+}(t) = {}^{i}m_{D}(t) + w_{(1+p)/2}\,{}^{i}s_{Dx}(t) \tag{10}$$

$$ {}^{i}D^{-}(t) = {}^{i}m_{D}(t) - w_{(1+p)/2}\,{}^{i}s_{Dx}(t) \tag{11}$$

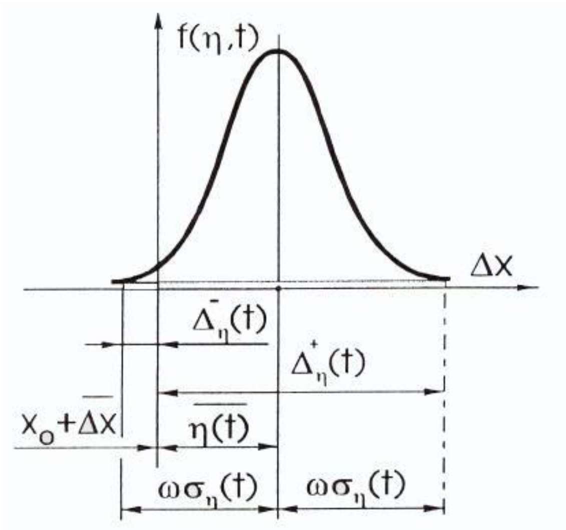
and, accordingly,

$$ {}^{i}D_{p}(t) = 2w\,{}^{i}s_{Dx}(t), \tag{12}$$

where $w = w_{(1+p)/2} = |w_{(1-p)/2}|$ is the argument of the normal distribution function for the probability (1 + p)/2. For example, ω = 2 for p = 0.9545, ω = 3 for p = 0.9973, and ω = 6 for p = 0.999999998.

The point index of precision (error or measurement range) traditionally expressed as

$$R = \Delta X_{max} - \Delta X_{min} = X_{max} - X_{min} \quad (13)$$



is not convenient for evaluation of the plausibility of the obtained indices estimates.

Stability is characterized by the deviation of the grouping center from its initial position. Essentially, it is a trend (Fig.1). For the deterministic model, stability is characterized by trend values ${}^{i}h(t)$; in the case where the trend is linear,

$${}^{i}h(t) = {}^{i}ht \quad (14)$$

Figure 2. Probability distribution, characteristics of random trend, and stability indices at a moment *t* of time (the index ***i*** is omitted for clarity).

For a random model (Fig.2) stability is characterized by the following:

- The expected value $\overline{{}^{i}h(t)}$ and variation ${}^{i}s_{h}(t)$ of the trend; in the case where the trend is linear,

  $$\overline{{}^{i}h(t)} = {}^{i}ht. \quad (15)$$

  $${}^{i}s_{h}(t) = \sqrt{dt} \quad (16)$$

- The trend's natural bounds ${}^{i}\Delta^{+}_{h}(t)$ and ${}^{i}\Delta^{-}_{h}(t)$ in the case where the trend is linear

  $${}^{i}\Delta^{+}_{h} = {}^{i}ht + w\ {}^{i}s_{h}(t) \quad (17)$$

  $${}^{i}\Delta^{-}_{h} = {}^{i}ht - w\ {}^{i}s_{h}(t) \quad (18)$$

Use of the traditionally most usual estimate of stability - deviation of the grouping center from its initial position - is acceptable only when the model is deterministic. The point density ${}^{i}f(h, t)$ of a distribution for a random trend in this case is ignored and ${}^{i}f(c, t)$ is considered to be the distribution density for measurement error; whereas, in fact, the essential distribution density is ${}^{i}f(Dx, t)$ which is obtained (Fig.1) as a composition of distribution densities ${}^{i}f(c, t)$ and ${}^{i}f(h, t)$.
The precision estimates discussed are based on time series probability characteristics and therefore are effective and statistically easy to use. For instance, in models with a linear trend only three characteristics $\overline{{}^{i}\Delta x}$, ${}^{i}h$, and ${}^{i}d$ need to be estimated. Fluctuations can usually be considered as stationary or quasi-stationary and then only one or two characteristics need to be estimated.
Precision and plausibility of statistical estimates are obtained by processing all measurements at

all time moments. The efficiency of these statistical estimates is therefore essentially improved.

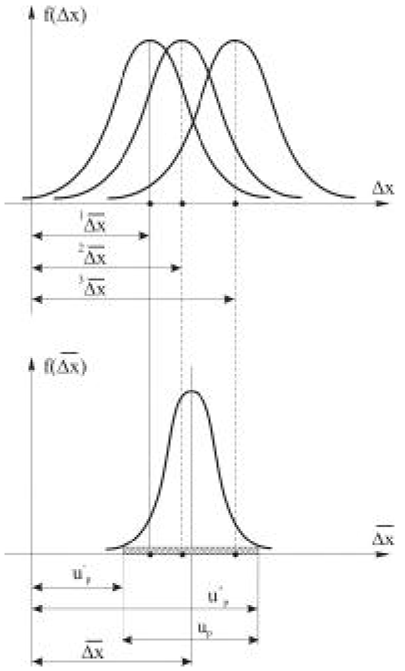
To compare the precision of several measurement series ${}^{i}X(t),\ i = 1, \dots, l,$ appraisers, equipment, or processes, similar precision characteristics are used.

Total accuracy for arrays of measurement errors ${}^{i}DX(t),\ i = 1, \dots, l,$ is evaluated as the mean value $\overline{{}^{i}\Delta x(t)}$ $i = 1, \dots, l,$ of position errors for all grouping centers at every time moment

$$\overline{\overline{\Delta x(t))}} = \frac{1}{l}\sum_{i=1}^{l} \overline{{}^{i}\Delta x(t)} \qquad (19)$$

Reproducibility (Fig.3) is estimated as a natural spread for all $\overline{{}^{i}\Delta x(t)}$ $i = 1, \dots, l,$

$$u_p = u_p^+ + u_p^-, \qquad (20)$$



where u_p^+ is the upper bound and u_p^- is the lower bound of grouping center position errors, which are obtained as roots of the equations

$$P\{\overline{\Delta x(t)} \le u_p^+(t)\} = \frac{1+p}{2} \qquad (21)$$

$$P\{\overline{\Delta x(t)} < u_p^-(t)\} = \frac{1+p}{2} \qquad (22)$$

For normally distributed grouping centers position errors,

$$\overline{\Delta x(t)} \sim N[\overline{\overline{\Delta x(t)}}, s_{\overline{\Delta x}}(t)]$$

calibration bias is evaluated as

$$u_p(t) = [\overline{\overline{\Delta x(t)}} + w_{(1+p)/2} s_{\overline{\Delta x}}(t)] - [\overline{\overline{\Delta x(t)}} - w_{(1+p)/2} s_{\overline{\Delta x}}(t)] = 2ws_{\overline{\Delta x}}(t) \qquad (23)$$

Figure 3. Reproducibility estimates: range *r* and precision of average, u_p.

The traditional index of grouping center positions at every time moment with range

$$r = \overline{\Delta x_{\max}} - \overline{\Delta x_{\min}} \qquad (24)$$

can be used only in approximate accuracy and precision analysis according to the obtained plausibility of the index's estimates.

Parameter Accuracy

The parameter of equipment, process, and product sequence is considered as a random sample function ${}^{(i)}z(t)$, which exhibits the real-time values and changes in time.
Error,

$${}^{(i)}Dz(t) = {}^{(i)}z(t) - z_0 \qquad (25)$$

is the characteristic of parameter accuracy, where z_0 is a specified target value.

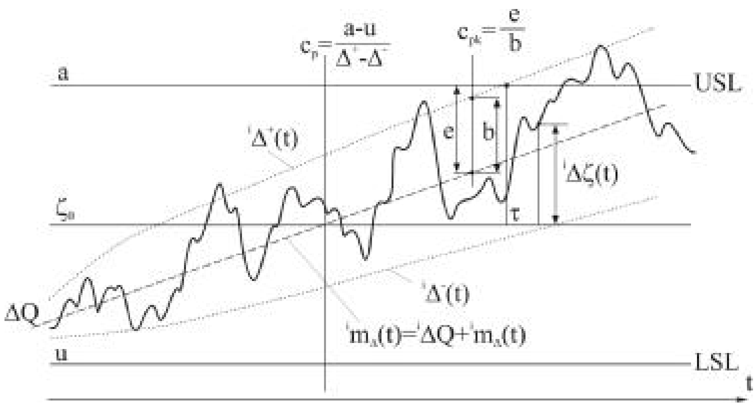


Figure 4. Parameter accuracy indices, potential, and capability.

The parameter accuracy indices (Fig.4) are as follows:

- Mean deviation

$${}^{i}m_{D}(t) = {}^{i}DQ + {}^{i}ht \quad (26)$$

where ${}^{i}DQ = {}^{i}Q - z_0$

- Variance

$${}^{i}s^{2}(t) = {}^{i}dt + {}^{i}S_{Z}^{2}(t) + {}^{i}s_{c}^{2}(t) \quad (27)$$

where ${}^{i}s(t)$ is a standard deviation (variation)

- Error bounds [${}^{i}\Delta^{+}(t)$, the upper one; ${}^{i}\Delta^{-}(t)$ the lower one] are roots of the equation

$$P\{{}^{i}D - {}^{i}D^{-}(t) < {}^{(i)}Dz(t) < {}^{i}D^{+}(t)\} = p, \quad (28)$$

where p is the probability of a nondefective parameter value. For a normally distributed error, ${}^{(i)}Dz(t) \sim N[{}^{i}m_{D}(t), {}^{i}s(t)]$ expression (28) can be written as

$${}^{i}\Delta^{+}(t) = {}^{i}m_{\Delta}(t) + w_{(1+p)/2}\,{}^{i}s(t)$$

$${}^{i}\Delta^{-}(t) = {}^{i}m_{\Delta}(t) + w_{(1-p)/2}\,{}^{i}s(t) \quad (29)$$

Conformity monitoring

Accuracy, precision, and related indices are evaluated as real-time functions using sample function Conformity monitoring is based on verification of performance by indices of parameter accuracy, precision, stability, potential, capability and efficiency and their evolution in the previous period of time and forecasting of probable future changes.

In order to secure the correctness of control signals and statistical estimates of measurements, and to provide a high level of their credibility, a system of measurement process conformity monitoring is required. It is necessary to create an effective computer aided measurement confirmation system – filigree calibration, verification, validation, stability and traceability ensurance methodologies, supporting super-high level (one defect per million observations and less) process / technology control and management.

The *six sigma* process control is based on the selected performance measurement methodology for each pattern of the process / technology and includes a careful sample function analysis for the main parameters of the process / technology comprising such activities as estimation of process / technology probability characteristics, the forecast of future run, diagnostics of the causes of undesirable changes, and development of the most efficient control measures.

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Jānis Mazais
Associate Professor of the Institute for Quality Engineering at Riga Technical University
1 Kalku Street, Riga, LV 1658
Phone: +371 708 9787
e-mail: janis.mazais@inbox.lv

Jānis Miķelsons
Associate Professor of the Institute for Quality Engineering at Riga Technical University
1 Kalku Street, Riga, LV 1658
e-mail: lsq@latnet.lv

Narimants Salenieks
Professor and Director of the Institute for Quality Engineering at Riga Technical University
1 Kalku Street, Riga, LV 1658
e-mail: lsq@latnet.lv

J.Mazais, J.Miķelsons, N.Salenieks. Procesa parametru un mērījumu pareizība un precizitāte.
Veikta procesu parametru mērījumu raksturlielumu – pareizības, precizitātes, stabilitātes un atbilstības novērtējumu statistiskā analīze tiešlaika mērīšanas apstākļos. Dotas matemātiskās izteiksmes raksturlielumu novērtējumu iegūšanai gadījuma funkcijas komponenšu dažāda veida izmaiņām laikā. Aplūkotās metodes atvieglo procesa parametru nevēlamu izmaiņu prognozēšanu un to cēloņu identificēšanu.

J.Mazais, J.Miķelsons, N.Salenieks. Process parameter and measurement accuracy and precision.
Process parameter statistical analysis of just-in-time measurement characteristics – measurement accuracy, precision, stability, and conformity assessment are analyzed and mathematical relationships for assessment of the above mentioned characteristics for different patterns of a random sample function components are provided. Developed methods simplify forecasting of undesirable process parameter future changes and facilitate identification of the root causes of such changes.

Я.Мазайс, Я.Микелсонс, Н.Салениекс. Точность и правильность параметров процессов измерений.
Рассмотрены вопросы статистического анализа характеристик измерений параметров процессов – точности, правильности, стабильности и оценок соответствия в условиях измерения в реальном времени. Представлены математические соотношения для получения оценок характеристик составляющих случайной функции при различном характере изменений. Рассмотренные методы облегчают прогнозирование нежелательных изменений параметров процесса и идентифицирование их причин.