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Minimax Decision for the Reliability of Aircraft Fleet With and Without Information Exchange

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***Abstract* – The scientific article addresses the dependence of aircraft fleet safety on the human factor. The article demonstrates the significance of information exchange concerning the open fatigue cracks, which is necessary to bring a new type of aircraft into operation. The article provides numerical examples obtained by means of the Monte Carlo method and considers the dependences of failure probability on various factors.**

***Keywords* – Inspection programmes, Monte Carlo, safety.**

I. Introduction

Metal fatigue problems in aviation are described in a number of sources, as well as in [1]. The inspection programmes for significant structural components are used widely. The content of the programme is based on aircraft component fatigue tests and theoretical calculations.

II. Statement of the Problem

It is possible to make a conclusion on the aircraft safety [1] using the information about the development of fatigue cracks owing to full-scale aircraft fatigue tests. In terms of aircraft fleet, the information exchange related to open fatigue cracks on the aircraft of the same type is very important. Such a situation is topical when a new type of aircraft is being brought into operation and when the aircraft fleet is small. This problem is discussed in [2]. The general problems of system reliability are considered in a great number of publications (see, for example, [3]–[5]). Each airplane of the fleet is serviced independently of the others without the exchange of information and the fleet safety is calculated as follows:

$$1 - p_{\mathrm{f}1N} = (1 - p_{\mathrm{f}1})^N, \tag{1}$$

where N is a number of airplanes in the fleet and $p_{\mathrm{f}1N}$ is at least one aircraft failure in the fleet. Operational life of each airplane is limited by t_{SL} – a specified safe life, after which an airplane is taken out of service.

This article also takes into consideration the impact of the human factor on the fleet safety. We assume that during the inspection the fatigue crack will be discovered with probability w, $w \le 1$, even if at the moment of inspection it has a detectable size. Value w is determined by the house rules of the airline, as well as the applied detection methods. Significant structural components of the aircraft will be characterized by vector (T_{D}, T_{C}), where T_{D} is service time when the fatigue crack has already been detected and reached the size of a_{d}. T_{C} is service time when the lasting strength of the significant structural component is reduced to the minimum permissible limit because of the crack; it means that the crack reached its critical size a_{c}. The aircraft fatigue failure may occur in case when $T_{\mathrm{C}} < t_{\mathrm{SL}}$ and its probability is equal to $p_{\mathrm{f}1w} = (1 - w)^r$, where r is a number of inspections in interval (T_{D}, T_{C}).

The fatigue crack development in time may be shown with $a(t) = a_0 e^{Qt}$, where a_0 and Q are the crack development parameters discussed in detail in [1]. Q is crack development speed on a logarithmic scale; in this article we assume that Q is a random variable with lognormal distribution.

a_0 is the initial crack size, and in this article we assume that it is a constant. Then T_{D} and T_{C} are log-normally distributed. If we assume that

$$T_{\mathrm{D}} = (\ln a_{\mathrm{d}} - \ln a_0)/Q = C_{\mathrm{d}}/Q \text{ and } T_{\mathrm{C}} = (\ln a_{\mathrm{c}} - \ln a_0)/Q = C_{\mathrm{c}}/Q \tag{2}$$

values are known [6], [8], it becomes possible to calculate the failure probability for different fleets and inspection programmes.

Information about open cracks on the aircraft of the same type helps to prevent the fatigue on the other aircraft of the same type in the fleet. This means that it is sufficient to find at least one crack prior to any aircraft failure in the fleet. For the fleet, each i-th airplane begins operation in the calendar time $t_1 < t_2 < \cdots < t_N, i = 1, \dots, N$, then each airplane will have ${T_{\mathrm{d}_i}}^+ = t_i + T_{\mathrm{d}_i}$ and ${T_{\mathrm{c}_i}}^+ = t_i + T_{\mathrm{c}_i}$. Then

$$I_{\mathrm{SL}} = \{i: T_{ci} < t_{\mathrm{SL}}, i = 1, \dots, N\} \tag{3}$$

is a sequence of failure time, if fleet is not inspected. Thus, the first failure time will be $T_{\mathrm{f}}^* = \min\{T_{ci}^*: i \in I_{\mathrm{SL}}\}$. While inspecting, $R = \sum_{i \in I_{\mathrm{SL}}} R_i$ is the total number of inspections before the first fleet failure, where $R_i = \max\left(\left\{\left[\frac{T_{\mathrm{f}i}^+ - t_i}{D}\right] - \left[\frac{T_{\mathrm{d}i}^+ - t_i}{D}\right]\right\}, 0\right), i \in I_{\mathrm{SL}}$ is the i-th number of inspections of the aircraft for an interval between inspections D (each airplane in the fleet has its own inspection calendar $t_i + D, t_i + 2D, \dots$; i = 1, …, N). Then for a fleet of N aircraft with the exchange of information:

$$p_{\mathrm{f}NNw}(D) = \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} \left((1-w)^{r(q)}\right) \mathrm{d}F_{Q_1}(q_1) \dots \mathrm{d}F_{QN}(q_n), \tag{4}$$

where $r(q), q = (q_1, \dots, q_n)$ are random values of R realization and $F_Q(q)$ is cumulative distribution function, $Q_i, i = 1, \dots, N$. The above-mentioned equations make it possible to calculate function $p_{\mathrm{f}NNw}(n)$, where $n = ([t_{\mathrm{SL}}/D] - 1)$ is a number of inspections, and to choose the required inspection programme, which corresponds to some maximum permissible failure probability.

Log-normally distributed value Q has distribution parameters Th_0 and Th_1. Parameter value Th_1 is assumed to be 0.346, but parameter value Th_0 is unknown, then you have to use the minimal calculation in order to obtain function $p_{\mathrm{f}NNw}(Th_0, n)$, where the dependence of failure probability on parameter Th_0 will be visible. Now the probability of failure will be presented by the function of Th. We suppose that $p_{\mathrm{f}}(Th, n)$ is such that $\lim_{n \to \infty} \left(p_{\mathrm{f}}(\theta, n)\right) = 0$ for all Th, and for every small value P_{des} there is minimal inspection number $n(Th, P_{\mathrm{des}})$ such that $p_{\mathrm{f}}(\theta, n) \leq P_{\mathrm{des}}$ for all $n \leq n(\theta, P_{\mathrm{des}})$, but a true value of θ is unknown. Thus, $n = n(Th, P_{\mathrm{des}})$ and $p_{\mathrm{f}} = p_{\mathrm{f}}(Th, n)$ are random variables. It means that during the acceptance trials the aircraft may appear to be “weak” if $Th_0 > Th_{\mathrm{max}}$, which is not accepted and sent to redesign, or to be “strong” if $Th_0 < Th_{\mathrm{min}}$, when the fatigue cracks develop very slowly. Then there will be some value ${Th_0}^*$ when there will be maximum failure probability. Then, using the minimum value, it is possible to obtain function $p_{\mathrm{max}}(p_{\mathrm{des}})$, which allows to receive the required probability p_{des} making it possible for maximum probability p_{max} not to exceed maximum permissible probability p_{all}.

III. Numerical Example

By this numerical example we assume that $t_{\mathrm{SL}} = 40\,000$ h, $w = 0.9$, $\alpha_0 = 0.28613258$ mm, $\alpha_{\mathrm{d}} = 20$ mm, $\alpha_{\mathrm{c}} = 237$ mm and the maximum number of inspections is 20. Two aircraft fleets with 10 airplanes in each were taken for comparison. We can consider that for the fleet without the exchange of information all the airplanes start operating at the same time because they are serviced independently (see Fig. 1).

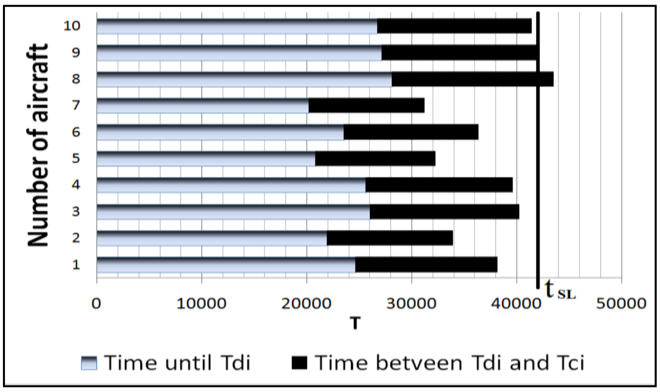

Fig. 1. Example of Monte Carlo [7] modelling of fatigue crack development for the fleet without the exchange of information.

For the fleet with the exchange of information, we assume that the interval at which the aircraft is put into operation is $t_{i+1} - t_i = 500$ h.

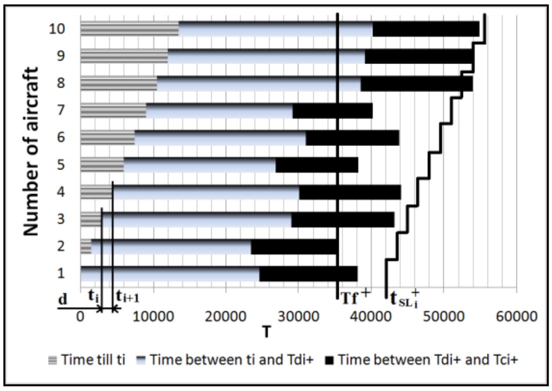

Fig. 2. Example of Monte Carlo modelling of fatigue crack development for the fleet with the exchange of information.

The dependences of both fleet failure probability on the number of inspections are shown in Fig. 3.

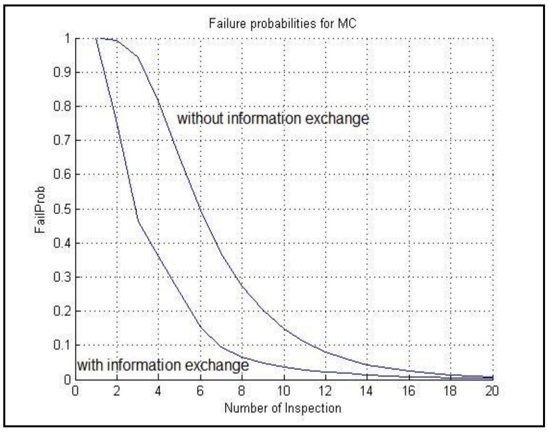

Fig. 3. Dependence of failure probability on the number of inspections for the fleet with or without the exchange of information.

It is obvious that the exchange of information is very significant because it requires less frequent inspections. This example provides an analysis of one of the real fatigue test cracks, and the calculated parameter of this crack is $Th_0 = -8.5885$. If it is necessary to ensure that $P_{\text{all}} = 0.05$, it will be enough with 14 inspections for each airplane during the operating time for the fleet without the exchange of information, corresponding to the interval between inspections $D = 2857$ h. In order to ensure the same $P_{\text{all}} = 0.05$, 9 inspections will be necessary for each airplane during the operating time for the fleet with the exchange of information, corresponding to the interval between inspections $D = 4444$ h.

These inspection intervals are selected on the basis of data of a specific crack. The average crack parameter Th_0 in operation may vary; therefore, let us check the possible maximum failure probability of the selected aircraft packages with the help of a minimax if we assume that $P_{\text{des}} = 0.05$.

Figure 4 shows that the maximum failure probability is much higher: $P_{\max} > P_{\text{all}}$.

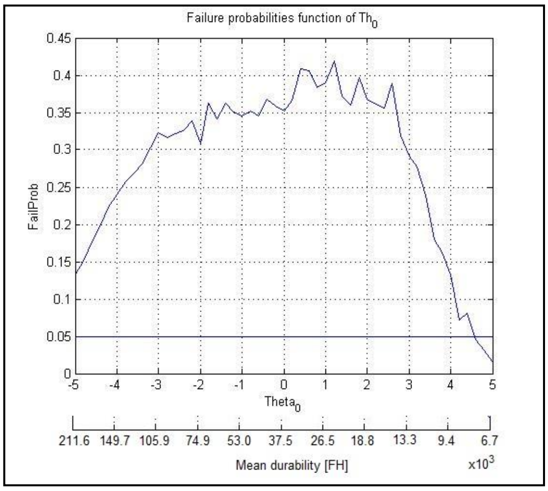

Fig. 4. Dependence of failure probability on parameter Th_0 for the fleet without the exchange of information, $P_{\text{des}} = 0.05$.

It is possible to see from Fig. 4 that the maximum value of failure probability is in the middle. P_{f} value is reduced when Th_0value is reduced, because mean durability is increased. This means that the speed of fatigue crack development is too low and the aircraft are taken from service before the cracks reach their critical size. On the other hand, when Th_0 is increased, P_{f} value is decreased. This happens because mean durability is too low and it increases the number of airplanes sent to redesign.

The same inspection will be carried out for the fleet with the exchange of information.

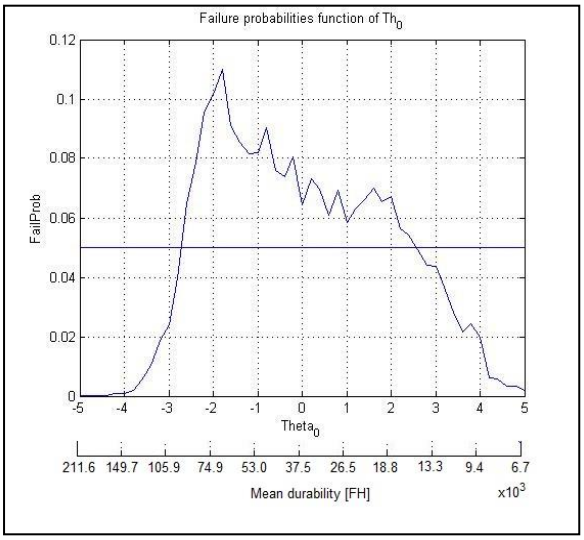

Fig. 5. Dependence of failure probability on parameter Th_0 for the fleet with the exchange of information, $P_{\text{des}} = 0.05$.

Figure 5 shows the same situation. It means that the maximum failure probability may be higher than the planned one during the trials. This means that it is necessary to reduce the value of P_{des}.

Some different curves for the fleet without the information exchange are presented in Fig. 6.

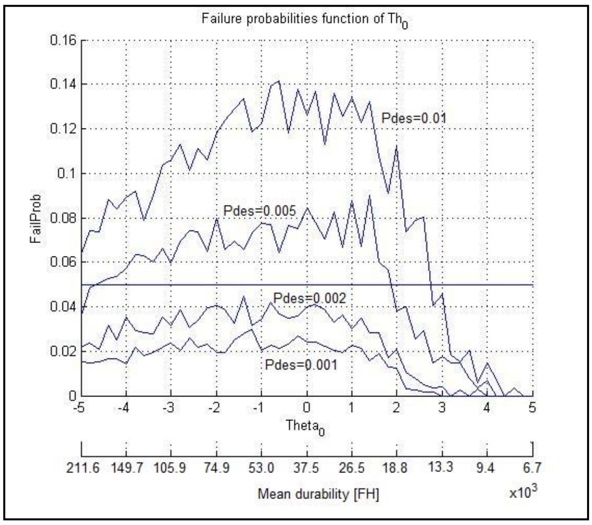

Fig. 6. Dependence of failure probability on parameter Th_0 for the fleet without the exchange of information for different P_{des} values.

The method of minimax will enable us to obtain function $P_{\mathrm{max}}(P_{\mathrm{des}})$ in order to select such P_{des}, the maximum possible failure probability of which will not exceed the maximum permissible one. This function is shown in Fig. 7.

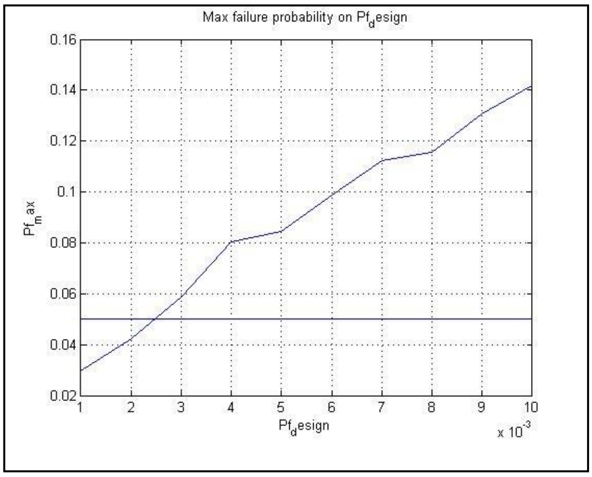

Fig. 7. Dependence of maximum failure probability on the design failure probability for the fleet without the exchange of information.

Figure 7 shows that the fleet consisting of 10 airplanes requires to reduce the failure probability by modifying the inspection programme to $P_{\mathrm{des}} = 0.002$ to make the maximum failure probability during operation not exceed $P_{\mathrm{all}} = 0.05$.

For the fleet with the information exchange, it is also necessary to reduce the value of P_{des}.

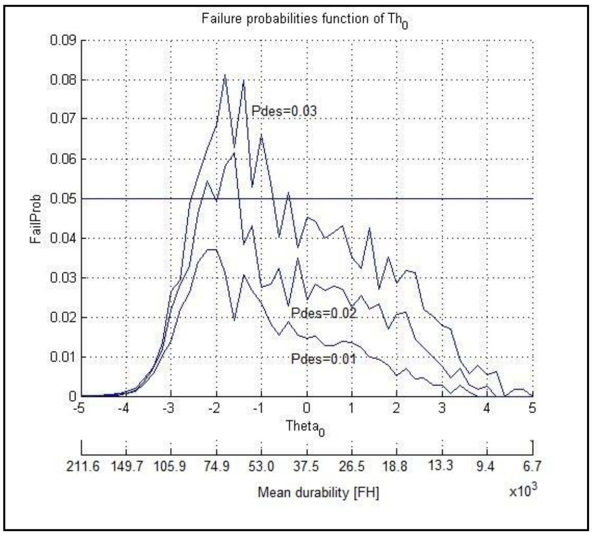

Fig. 8. Dependence of failure probability on parameter Th_0 for the fleet with the exchange of information for different values of P_{des}.

The same function $P_{\max}(P_{\mathrm{des}})$ will be found for the fleet with the exchange of information in Fig. 9.

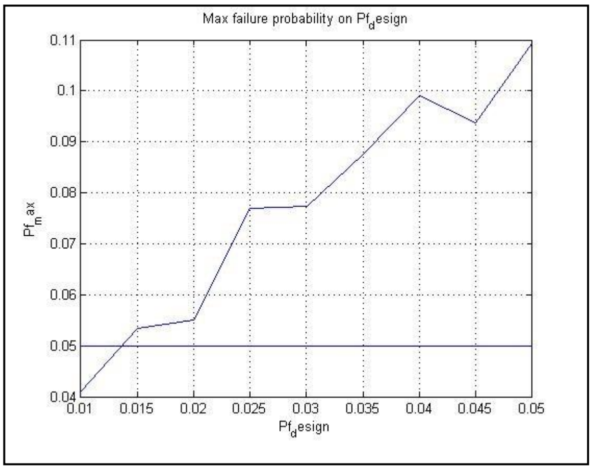

Fig. 9. Dependence of maximum failure probability on the design failure probability for the fleet with the exchange of information.

In this case, the probability should also be reduced, but not so much $P_{\mathrm{des}} = 0.01$, which reaffirms the higher safety level for the fleet with the exchange of information.

Now, being aware of probability P_{des} , it is possible to determine the inspection programmes in accordance with a random crack so that the maximum fleet failure probability does not exceed the value of $P_{\mathrm{all}} = 0.05$. In this example, we will assume that the random crack is presented by the dependence of failure probability on a number of inspections, as shown in Fig. 3. Reducing the probabilities while selecting the programme, more than 20 inspections will be necessary during the operating time for the fleet without the exchange of information, but for the fleet with the exchange of information it will be enough with 16 inspections, corresponding to the interval between inspections $D = 2600$ h.

IV. CONCLUSION

The comparison of fleets with and without the information exchange in terms of open crack changes makes it obvious that, in case of equally permissible failures, fleets with the information exchange can be inspected less frequently than fleets without the information exchange in order to ensure the probability. It should be noted that the maximum failure probability in operation may be higher than in trials; therefore, the minimax method should be used for the creation of the programme. The human factor is significant as well, and it will be discussed in another article.

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