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USE OF CHAOTIC SEQUNCES FOR DATA TRANSMISSION SYSTEMS

Summary of the Doctoral Thesis

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DOCTORAL THESIS

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The Doctoral Thesis for the degree of a doctor of science in engineering will be presented on June 16, 2017, at the Faculty of Electronics and Telecommunications of Riga Technical University, Azenes street 12, in the lecture room No. 120.

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DECLARATION OF ACADEMIC INTEGRITY

I hereby declare that the Doctoral Thesis submitted for the review to Riga Technical University for the promotion to the scientific degree of Doctor of Engineering Sciences (Telecommunications) is my own. I confirm that this Doctoral Thesis had not been submitted to any other university for the promotion to a scientific degree.

Anna Litvinenko……………………………. (Signature)

Date: ………………………

The Doctoral Thesis is written in Latvian, contains an introduction, four chapters, conclusions, references, appendices, 73 figures, and 19 tables; 207 pages in total. The list of references consists of 180 titles.

CONTENTS

LIST OF ABBREVIATIONS

AWGN – Additive White Gaussian Noise
BER – Bit Error Ratio
BPSK – Binary Phase-Shift Keying
CDMA – Code-Division Multiple Access
CDSK – Correlation Differential Shift Keying
COOK – Chaos On-Off Keying
CP – Cyclic Prefix
CSK – Chaos Shift Keying
DCSK – Differential Chaos Shift Keying
DECT – Digital Enhanced Cordless Telecommunications
DFT – Discrete Fourier Transform
DS – Direct Sequence
DS-CDMA – Direct Sequence Code-Division Multiple Access
FDE – Frequency Domain Equalizer
FFT – Fast Fourier Transform
FH – Frequency Hopping
FM-DCSK – Frequency-Modulated Differential Chaos Shift Keying
GLONASS – Global Navigation Satellite System
GPS – Global Positioning System
ICI – Inter Carrier Interference
IDFT – Inverse Discrete Fourier Transform
IFFT – Inverse Fast Fourier Transform
ISI – Inter-Symbol Interference
LPF – Low Pass Filter
LP-OFDM – Linear Precoded OFDM
NRZ – Non-Return-to-Zero
OCT – Orthogonalized Chaotic Transform
OFDM – Orthogonal Frequency-Division Multiplexing
PAPR – Peak-to-Average Power Ratio
PN – Pseudo Noise
PSK – Phase-Shift Keying
P/S – Parallel-to-Serial
QAM – Quadrature Amplitude Modulation
RFPA – Radio Frequency Power Amplifier
SLM – Selected Level Mapping
SNR – Signal-to-Noise Ratio
S/P – Serial-to-Parallel
ZF – Zero Forcing
WHT – Walsh-Hadamard Transform

INTRODUCTION

Urgency of the subject matter

With the development of economy and technology, the volumes of transmitted information and the requirements on the quality have rapidly grown. New solutions for developing data transmission systems are being constantly searched. A promising search direction is studies that are focused on the use of the chaotic system and chaotic fluctuations for these systems. Chaotic fluctuations open new possibilities for many specific applications. Considering that chaotic signals are aperiodic, their value over a long interval of time is practically unpredictable, they have a wide spectrum and a narrow autocorrelation function, practically uncorrelated chaotic signal sets can be generated. These properties indicate the possible application areas: spread-spectrum multiple-access communication systems that are resistant to interception, jamming, fading, and multipath propagation.

Applications of chaotic binary and non-binary sequences are considered as potentially promising for spread-spectrum code-division systems and for improvement of quality parameters in multicarrier systems. The proposed Doctoral Thesis is dedicated to the study of these applications.

Objective and tasks of the work

The main objective of the Thesis is to study the use of chaotic sequences for code-division and multi-carrier transmission systems.

To achieve the objective, the following tasks and subtasks are defined.

1. Explore the correlation properties of the binary chaotic sequences generated by 1-D maps and develop algorithms for selecting the sequences with better correlation properties:
   a) explore chaotic sequence generation methods;
   b) generate balanced binary chaotic sequences;
   c) evaluate the statistical parameters of the maximum absolute values of periodic cross-correlation functions for the binary chaotic sequences and compare them with the relevant parameters of pseudo-noise sequence;
   d) identify the ways to generate and select the binary chaotic sequences with better correlation properties than pseudo-noise sequences and develop selection algorithms.
2. Explore multiple access interference (MAI) probability distribution in DS-CDMA systems based on binary chaotic sequences and elaborate recommendations for BER (Bit Error Ratio) analysis in case MAI does not belong to a normal distribution:
   a) evaluate the transmission mode, the length of the message, the number of users, the length of the sequence, and the influence of the selection of the initial values on the statistical distribution of MAI;
   b) identify the ways to apply the known analytical BER estimation approaches based on a Gaussian approximation of MAI.
3. Explore the performance of a DS-CDMA system based on chaotic spreading sequences and evaluate the influence of system and transmission parameters.
4. Explore the applications of chaotic sequences to improve the performance of OFDM (Orthogonal Frequency-Division Multiplexing) communication systems:

a) perform the comparative analysis of PAPR (Peak-to-Average Power Ratio) reduction methods for OFDM signals and elaborate a PAPR reduction method based on the use of chaotic sequences;
b) propose the use of chaotic sequences for the improvement of OFDM system performance in multipath propagation conditions and evaluate the advantages of the method;
c) analyze the possibilities of using the chaotic sequences for OFDM system synchronization.

The results and scientific novelty of the investigation

The following results have been obtained:

- the statistical parameters of the maximum absolute values of the periodic cross-correlation functions for binary chaotic sequences generated by different 1-D maps have been evaluated;
- two algorithms for the selection of binary chaotic sequences with the lowest correlation have been developed;
- the performance of a DS-CDMA system based on 15–127 chip long selected binary chaotic sequences for different numbers of users and transmission modes has been evaluated;
- the conditions under which the MAI distribution in a DS-CDMA system based on binary chaotic sequences does not belong to a Gaussian (normal) distribution have been found;
- a PAPR reduction method for OFDM signals based on scrambling with a binary chaotic sequence and clipping has been proposed;
- a comparative analysis of PAPR reduction methods for OFDM signals has been done;
- linear precoding based on the orthogonalized chaotic transform has been proposed for OFDM system performance improvement in multipath propagation environment;
- a MATLAB simulation program of the OFDM communication system to perform simulation for different transmission modes, including radio emulation, has been developed.

Theses to be defended

1. Using 1-D maps it is possible to generate a greater number of chaotic sequences with a lower periodic cross-correlation level, thus providing higher DS-CDMA performance and capacity than in the case of m-sequences.
2. For performance evaluation of the DS-CDMA system based on 15–127 chip long chaotic sequences, generated by 1-D maps, the methods which are not based on MAI approximation with the Gaussian (normal) distribution must be used.
3. In an OFDM communication system with a comb-type channel estimation, signal scrambling with a binary chaotic sequence and clipping efficiently reduce the PAPR without BER degradation under multipath propagation conditions, without transfer rate reduction and without the change of signal average power.
4. In the case of a high SNR (Signal-to-Noise Ratio), greater than 10dB, the OFDM signal linear precoding based on the orthogonalized chaotic transform increases the OFDM system performance in multipath propagation environment.

Research technique

The Thesis investigates the properties of chaotic sequences and the methods for their generation, as well as the characteristics and modeling parameters of the DS-CDMA and OFDM communication systems. The literature has been explored, and the computer simulation and analytical and numerical methods have been applied.

The following statistical analysis methods have been employed:

- chaotic sequence cross-correlation analysis;
- evaluation of the statistical parameters of correlation function peaks within chaotic sequences;
- MAI probability density estimation, and testing of hypotheses.

A computer simulation based on Monte Carlo methods has been performed with the help of MATLAB software and used for the following tasks:

- in DS-CDMA systems:
  - for MAI simulation in the correlation receiver after passband processing;
  - for BER simulation in a baseband channel with MAI and AWGN (Additive White Gaussian Noise);
- BER estimations in an OFDM system:
  - for the proposed PAPR reduction technique and radio channel model with two-beam propagation;
  - for the proposed precoding method in a baseband Rayleigh channel with AWGN.

Research matter

The research matter of the present Thesis is to study opportunities for the deployment of chaos applications in communication systems:

- in a DS-CDMA system:
  - implement asynchronous and synchronous transfer modes,
  - implement simple signal processing,
  - improve security aspects;
- in an OFDM system to improve functioning:
  - reduce PAPR of OFDM signals,
  - improve performance in multipath propagation environment,
  - implement chaotic synchronization,
  - improve security aspects.

Practical application of the results

The obtained results can be applied for the creation of a chaotic sequence-based DS-CDMA system. The proposed system will have the following properties: increased security, increased number of users compared to m-sequence-based systems, simple signal processing, and ability to provide synchronous and asynchronous transfer modes. Such systems can be used, for example, in sensor networks where a relatively low data rate is required. The main research direction of the Institute of Radioelectronics in Riga Technical University is to study the sensor network nodes. Therefore, the obtained results of the Thesis are planned to be used in further research projects and prototype development.

The proposed improvements of OFDM systems can be used for the development of baseband modems for highly-secure wireless networks.

The developed models and programs will be used in educational work with students.

Approbation

The results of the Thesis are reflected in ten scientific papers.

1. E. Bekeris, A. Litvinenko, Correlation Properties of Binary Spreading Sequences Generated by Chaotic Logistic Map. *Telekomunikācijas un elektronika.* RTU Zinātniskie raksti, No. 9, Latvia, 2009, pp. 39–42.
2. A. Litvinenko, E. Bekeris, Probability Distribution of Multiple-access Interference in Chaotic Spreading Codes Based on DS-CDMA Communication System. *Electronics and Electrical Engineering*, Vol. 123, No. 7, Lithuania, 2012, pp. 87–90.
3. A. Zelenkovs, A. Litvinenko, OFDM PAPR Reduction by Pre-Scrambling and Clipping. *The 13th Biennial Baltic Electronics Conference (BEC 2012): Proceedings*, Estonia, Tallinn, 3–5 October, 2012, pp. 141–144.
4. A. Litvinenko, OFDM Signal PAPR Reduction Method by Pre-Scrambling and Clipping for Frequency Domain Comb-Type Channel Estimation Case. *23rd International Conference "Radioelektronika 2013": Proceedings*, Czech Republic, Pardubice, 16–18 April, 2013, pp. 273–277.
5. A. Litvinenko, A. Aboltins, Chaos Based Linear Precoding for OFDM. *Advances in Wireless and Optical Communications (RTUWO 2015): Proceedings,* Latvia, Riga, 5–6 November, 2015, pp. 13–17.
6. A. Litvinenko, A. Aboltins, On Modified Chua's Circuit Based Linear Precoding for OFDM Systems. *1st International IEEE Conference on Advances in Wireless and Optical Communications 2015*, Latvia, Riga, 5–6 November, 2015. Riga: RTU Press, pp. 23–40.
7. A. Aboltins, A. Litvinenko, P. Misans, Parametric Linear Precoding for OFDM using Generalized Unitary Rotation. *The 15th Biennial Conference on Electronics and Embedded Systems: Proceedings,* Estonia, Tallinn, 3–5 October, 2016, pp. 131–134.
8. A. Litvinenko, A. Aboltins, Selection and Performance Analysis of Chaotic Spreading Sequences for DS-CDMA Systems. *Advances in Wireless and Optical Communications (RTUWO 2016): Proceedings,* Latvia, Riga, 2016, pp. 38–45.
9. A. Litvinenko, A. Aboltins, Use of Cross-Correlation Minimization for Performance Enhancement of Chaotic Spreading Sequence Based Asynchronous DS-CDMA System. *The 4th IEEE Workshop on Advances in Information, Electronic and Electrical Engineering (AIEEE 2016): Proceedings,* Lithuania, Vilnius, 2016, pp. 1–6.
10. A. Litvinenko, E. Bekeris, Statistical Analysis of Multiple Access Interference in Chaotic Spreading Sequence Based DS-CDMA Systems. *Electronics Journal.* Faculty of Electrical Engineering Banja Luka, Bosnia and Herzegovina. Submitted in May 2016. Accepted for publication in June 2017.

Seven articles have been published in international conference proceedings, two articles have been published or accepted for publishing in international scientific journals (Electronics and Electrical Engineering, Electronics Journal), and one article has been published in "RTU Zinātniskie raksti" ("Scientific Proceedings of RTU"). Currently, seven publications are indexed in IEEE/SCOPUS databases, and it is expected that eight publications will be soon indexed in SCOPUS database.

The author has presented the results of her research in 12 conferences.

1. RTU international conference "Riga Technical University 50th Scientific Conference (Section "Electronics, Telecommunications and eSociety")", Latvia, Riga, 15 October, 2009.
2. Innovation and new technology conference "RTU Inovāciju un jauno tehnoloģiju konference", Latvia, Riga, 16 March, 2011.
3. RTU international conference "Riga Technical University 52nd Scientific Conference (Section "Electronics, Telecommunications and eSociety")", Latvia, Riga, 13 October, 2011.
4. International scientific conference "International Conference ELECTRONICS' 2012", Lithuania, Palanga, 18–20 June, 2012.
5. International scientific conference "Biennial Baltic Electronics Conference BEC2012", Estonia, Tallinn, 3–5 October, 2012.
6. International scientific conference "Microwave and Radio Electronics Week 2013 MAREW2013", Czech Republic, Pardubice, 16–18 April, 2013.
7. International student scientific conference "The 1st IEEE Workshop on Advances in Information, Electronic and Electrical Engineering AIEEE'13", Latvia, Riga, 25–26 November, 2013.
8. RTU international conference "Riga Technical University 56th International Scientific Conference (Section "Electronics")", Latvia, Riga, 16 October, 2015.
9. International scientific conference "Advances in Wireless and Optical Communications RTUWO2015", Latvia, Riga, 5–6 November, 2015.
10. RTU international conference "Riga Technical University 57th International Scientific Conference (Section Electronics)", Latvia, Riga, 14 October, 2016.
11. International scientific conference "Advances in Wireless and Optical Communications RTUWO2016", Latvia, Riga, 3–4 November, 2016.
12. International student scientific conference "The 4th Workshop on Advances in Information, Electronic and Electrical Engineering AIEEE'16", Lithuania, Vilnius, 10–-12 November, 2016.

In addition, at the conference "The 4th Workshop on Advances in Information, Electronic and Electrical Engineering AIEEE'16", the author was awarded for the best paper presentation.

Part of the results presented in the Thesis were obtained accomplishing the research project

- No. KC/2.1.2.1.1./10/01/005, "Research and experimental development on time division duplexing packet transmission in microwave environment", Ltd. "LEO Research Center", 2011–2012.

Structure of the Thesis

The Thesis consists of an introduction, four chapters, conclusions, and appendices.

The Introduction presents the urgency of research, defines the main objective and formulates the tasks, identifies the Thesis statements, describes scientific novelty and main results of the research results, proposes their practical application, and reports on the Thesis' approbation.

The first chapter defines the concept of chaos and its generation methods, as well as provides the analysis of the literature on the application of chaos in communication systems. It shows that the generation of chaotic spreading sequences for DS-CDMA systems is a promising application and identifies the main concerns.

The second chapter is devoted to the chaotic sequence generation algorithms, which are based on 1-D maps and Chua circuit applications. Besides, the second chapter presents a study of chaotic sequence correlation properties and provides a comparison with pseudo-noise sequences.

The third chapter is devoted to the application of chaotic sequences in DS-CDMA communication systems. The probability distribution of multiple access interference in such systems has been explored. Chaotic sequence selection algorithms that decrease the sequence correlation level and improve the interference immunity of DS-CDMA system have been proposed. The comparison of proposed algorithms is done. The BER performance dependence on various factors in a DS-CDMA system based on chaotic sequences has been explored.

The fourth chapter is devoted to the application of chaotic sequences in OFDM communication systems. A comparative analysis of PAPR reduction methods has been done. A PAPR reduction method for OFDM signals based on prescrambling with a chaotic sequence and clipping has been proposed. Furthermore, a linear precoding method based on the orthogonalized chaotic transform has been proposed for OFDM system performance improvement in multipath propagation environment.

In the conclusions, a comprehensive summary of the obtained research results is presented.

1. REVIEW OF CHAOS APPLICATIONS IN COMMUNICATION SYSTEMS

This chapter defines the concept of chaos, lists chaotic signal generation methods, describes the chaotic signal application-based communication systems, including multi-carrier communication systems, and identifies the main concerns related to the use of chaotic sequences in code-division systems.

1.1. Chaos concept

Chaos can be defined as "an aperiodic long-term behavior of a determinate system, with a high sensitivity to initial conditions" [1].

1.2. Chaos generation methods

Chaotic fluctuations are possible in a nonlinear dynamic autonomous system with at least three state variables. Such system behavior can be described by the equation

$$\frac{dx_i}{dt} = F_i(x_i), \tag{1.1.}$$

where x_i – state variable;
t – time;
F_i – function which shows the rate of change for state variable;
i – indexes of state variables $i = \overline{1, N}; \ N \geq 3$.

The most known and studied chaos generator is Chua circuit [2].

Chaotic oscillations can also be obtained in various popular self-oscillation generators: Wien bridge based RC chaos generator [3], [4] Colpitts oscillator [4], chaotic

LC generators based on tunnel diodes [4], [5]. Since the development of a memristor in 2007, a number of simple memristor-based chaotic circuits have been proposed [6], [7].

To generate discrete chaotic sequences, discrete maps are used. The simplest model of chaotic sequence generation is a one-dimensional map:

$$x(n+1) = f(x(n)), n \geq 0 \tag{1.2.}$$

where *x(n)* – values of chaotic signal in discrete time moments;
f – function which shows the dependence of sequence value on the previous value.

Most popular and frequently used one-dimensional maps (logistic, Bernoulli, Gauss, sine-circle, cubic, Pincher) and a comparison rule for sequences conversion into a binary form are presented in the second chapter.

1.3. Chaos application in communication systems

The principles of chaos-application based communication systems are similar to the principles of spread-spectrum communication systems: the spectrum of information signal is spread at the transmitter side and the inverse operation is done at the receiver side, where signal detection and demodulation are performed. Chaos-application based communication systems are resistant to multipath propagation and provide opportunities for spectrum reuse.

1.3.1. Chaos masking

At the transmitter, a chaotic signal is added to the low-level information signal [8], [9]; whereas, at the receiver side, chaos generator synchronization is done using the received signal. The detected signal is formed as the difference between the received signal and the signal of local synchronized chaos generator. The proposed transmission technology efficiently works in a noiseless channel and if the chaotic signal power is higher than the information signal power by 35–65 dB [10].

1.3.2. Chaotic carrier

In communication systems, chaotic oscillations (carriers) can be used for information transfer. There are several transmission technologies and their modifications which use chaotic carriers: COOK, CSK, DCSK, FM-DCSK, and CDSK.

Chaos On-Off Keying – **COOK** [11].

At the transmitter side, the chaos generator is turned on for “1” bit transmission and turned off for the “0” bit transmission. At the receiver side, the bit energy is evaluated and, in the case of “1” bit transmission, it is higher. This method efficiently works in multipath environments [12]–[15].

Chaos Shift Keying – **CSK** [16]–[18].

The transmitter contains two or more chaos generators with different parameters. Generators are switched on/off in accordance with the transmitted information signal. Detection depends on the occurrence of the synchronization in the receiver generators. A significant disadvantage of this method is the switching time and processes occurring during it, leading to a reduction of transmission rate [9].

Differential Chaos Shift Keying – DCSK [19].

In this modulation scheme, each transmitter symbol consists of two chaotic signal parts [19], [20]. The first part of the symbol is the support signal, while the second part contains information. For example, if the "1" bit is transmitted, then both parts of the symbol are equal, but in the "0" bit transmission case, the second part is the inverted first part of the symbol. At the receiver, the correlation between the first and second parts of the symbol is evaluated, thus "1" or "0" bit is reconstructed. Several multi-level modulation schemes have also been proposed [21]–[23].

To generate chaotic broadband signals with a constant power, FM-DCSK has been proposed [24]. The chaotic signal is used to perform frequency modulation. This scheme has shown high resistance to multipath propagation [25]–[27].

Chaotic multiple access communication system solutions based on DCSK method have been proposed in [27]–[30].

Correlation Differential Shift Keying – CDSK [30][31]**.**

A transmitted signal is a sum of chaotic signal and its delayed copy multiplied with the information signal ("1" and "−1"). This scheme realizes faster data transmission comparing with DCSK, because no switching is required. A combination of FM-DCSK and CDSK transmission methods has been proposed [31].

1.3.3. Chaotic sequences in Code-division Multiple Access system

Chaotic sequences have been proposed for spectrum spreading in direct sequence code-division multiple access (DS-CDMA) systems [32], [33].

The chaotic spreading sequences have been studied from different points of view. Two main approaches of using them have been identified. In the first case, for information bit transmission, chaotic signal values are used [34]–[40], while in the second approach – chaotic sequences are quantized, also into binary form [41]–[45]. The correlation properties of chaotic signals have been studied and analyzed [37], [39]–[41], [44]. An advanced sequence generation method has been proposed with the aim to improve transmission quality and to reduce BER level [36]–[39], [46].

Chaotic communication systems and chaotic sequences have also been analyzed from the cryptographic point of view. Data coding and encryption are another application of chaos theory. Several works are devoted to these research areas, including audio-video signal compression and data cryptography [47]–[51], and channel coding [52].

In the present Thesis, applications of chaotic digital signals for code-division multiple access (CDMA) and OFDM communication systems are explored. Therefore, the next section summarizes the research results concerning chaotic digital signal generation methods and their properties, and the applications of chaotic digital signals in communication systems.

1.4. Chaotic sequences generation, properties and applications for communication systems

Chaotic sequence as an alternative of pseudo-noise (PN) sequences for spread-spectrum communication systems were proposed in the early 1990s [34], [35]. The applications of chaotic non-binary [34]–[40], [46] and binary sequences [41]–[45] were considered. Chaotic sequences generation algorithms, statistical properties, and chaotic sequence based DS-CDMA system performance were studied [32]–[40], [53], [54]. As literature analysis shows, there exist two main approaches for BER performance estimation: through computer simulations [36], [40], or through theoretical analysis, including MAI approximation with a Gaussian process [33], [53], [54]. However, it is unclear whether MAI distribution corresponds to a Gaussian distribution [40], [55].

It is possible to generate a higher number of chaotic sequences compared to PN sequences (m-sequences and Gold codes) of given length [34], [35], [40]. But the presented results demonstrate higher BER levels in the case of chaotic sequence based spread-spectrum systems, even if the sequence length is 7–127 chips. These results are contrary to the results in [32], [37], which show that an asynchronous and synchronous chaotic sequences based DS-CDMA system has higher performance compared to a DS-CDMA system that uses m-sequences, Gold codes, or Kasami sequences [56].

The studies of chaotic sequence selection criteria have been done with the aim to increase the noise immunity and performance of chaos communication systems [37]–[39], [46].

Moreover, chaotic multi-carrier DS-CDMA systems have been proposed [57]–[59].

1.5. Summary of the review of chaos applications in communication systems

The literature analysis reveals that one of the most promising applications of chaotic sequences is spectrum spreading in DS-CDMA systems.

Although there are many publications on chaotic sequences, it is useful to research the following issues further:

- correlation properties of chaotic sequences generated by different algorithms and their comparison with those of PN sequences;
- efficient chaotic sequence selection algorithms for the improvement of correlation properties and for the performance enhancement of chaotic sequence based DS-CDMA systems;
- conditions under which MAI in a chaotic sequence based DS-CDMA system does not belong to a Gaussian distribution and analytical BER estimation methods based on this approximation cannot be used;
- applications of chaotic sequences for the improvement of modern multi-carrier communication system parameters.

2. CHAOTIC SEQUENCES GENERATION AND STATISTICAL PROPERTIES

Pseudo-noise sequences are widely used in data transmission systems (for spectrum spreading, multiple access implementation). The properties of pseudo-noise sequences are thoroughly explored. At the same time, other aperiodic sequences are searched for to provide a lower cross-channel interference. Chaotic sequences are considered as a promising solution, because of the possibility to generate a large number of low correlated sequences [34], [60]. CDMA system capacity is mainly limited by interference signals from other users. Therefore, it is essential to use spreading sequences with a possibly lower cross-correlation level.

This chapter is devoted to the generation algorithms and statistical properties of chaotic sequences.

2.1. Chaotic processes in nonlinear dynamic systems

Nonlinear systems can realize deterministic chaos mode where undamped aperiodic oscillations are generated.

The behavior of an autonomous dynamic system is described by differential equations (1.1.). The phase space is used to analyze these systems, and the state variables are the orthogonal coordinates of this n-dimensional space. The system behavior is characterized by X point coordinates x_1, x_2, ..., x_n in the phase space. During evolution process, the point moves from starting (initial) condition $x(t_0)$ along the phase trajectory. In dissipative systems, a wide variety of starting conditions of the system tend to evolve to a point or a set of points in the phase space – attractors. In first- and second-order systems ($n \leq 2$), there exist two types of attractors: fixed point and limit cycle.

System determinism defines that phase trajectories in the phase space cannot be crossed. If the number of dimensions (state variables) is high enough, then trajectories can twist in space around each other without crossing. The behavior of such systems can be quite complex, and “strange” (or “chaotic”) attractors can be obtained in these systems. A strange (or chaotic) attractor is an area of attraction in the phase space, which is neither a fixed point nor a limit cycle and which is characterized by an exponential increase of small differences between two respective trajectories during time.

2.2. Chaos generators

Chaotic oscillations can be obtained by Chua circuit. In this work, one of the Chua circuit modifications is used for the generation of a chaotic sequence. This modification is a fourth-order chaotic oscillator [61], which is described by the system of equations:

$$\begin{aligned} \frac{dp_1}{dt} &= -g(p_1 - p_3) - p_2, \\ \frac{dp_2}{dt} &= p_1 + \gamma p_2, \\ \frac{dp_3}{dt} &= \theta(g(p_1 - p_3) - p_4), \\ \frac{dp_4}{dt} &= \sigma p_3, \end{aligned} \tag{2.1.}$$

where $g(p_1 - p_3)$ is a nonlinear function

$$g(p_1 - p_3) = \begin{cases} c(p_1 - p_3 - d) & (p_1 - p_3) \leq 1 \\ 0 & (p_1 - p_3) > 1 \end{cases} \tag{2.2.}$$

and θ, σ, γ, c, d are real numbers.

Then, for the generation of chaotic signal χ (sequence), the sum of differential equations and nonlinear function is used:

$$\chi = \left(\frac{dp_1}{dt} + \frac{dp_2}{dt} + \frac{dp_3}{dt} + \frac{dp_4}{dt}\right) + g\,(p_1 - p_3). \tag{2.3.}$$

The chaotic signal is simulated using MathWorks Simulink software and is used for precoding and synchronization tasks in the fourth chapter of this work. The proposed chaotic sequence is a discrete chaotic signal.

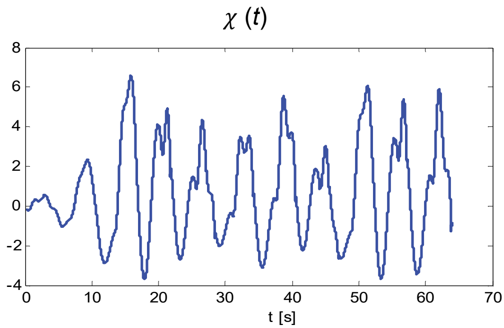

Fig. 2.1. The chaotic signal χ (t) generated by modified Chua circuit (2.3.) if Δt = 0.01 s, t = 64 s.

Table 2.1. Chua circuit parameters for chaotic signal generation

Initial conditions				Coefficient of differential equation				
p_1	p_2	p_3	p_4	γ	θ	σ	c	d
0.05	0.06	0.07	0.08	0.5	10	1.5	3	1

The parameters of the differential equation (2.1.) used for the simulation of a chaotic system and generation of a chaotic sequence are presented in Table 2.1, while Fig. 2.1. illustrates the waveform of the obtained chaotic signal χ (t).

2.3. One-dimensional maps

Maps also are used to describe dynamic systems. Maps show the dependence of the system state on the previous state (1.2.). The simplest model for chaotic sequence generation is a one-dimensional map.

Logistic map

$$x(n+1) = rx(n)(1 - x(n)), \tag{2.4.}$$

where n – index of the generation step (an integer number);
$x(n)$ – sequence value in the n-th generation step;
$x(n+1)$ – sequence value in the n+1 generation step;
r – parameter which describes the system's dynamics, and its value [3.57; 4] corresponds to the behavior of a chaotic system. In this work, $r = 4$, thus $x \in (0, 1)$.

Bernoulli map

$$x(n+1) = mod(\rho x(n), 1), \tag{2.5.}$$

where n – index of the generation step (an integer number);
$x(n)$ – sequence value in the n-th generation step;
$x(n+1)$ – sequence value in the n+1 generation step;
p – parameter which describes the system's dynamics. In this work, $p = 1.999$, thus $x \in (0, 1)$.

Tent map

$$x(n+1) = \begin{cases} \mu(1 - x(n)), & x(n) \geq 0.5 \\ \mu x(n), & x(n) < 0.5 \end{cases}, \quad (2.6.)$$

where n – index of the generation step (an integer number);
$x(n)$ – sequence value in the n-th generation step;
$x(n+1)$ – sequence value in the n+1 generation step;
μ – parameter which describes the system's dynamics. In this work, $\mu = 1.999$, thus $x \in (0, 1)$.

Gauss map

$$x(n+1) = e^{-\alpha} x(n)^2 + \beta, \quad (2.7.)$$

where n – index of the generation step (an integer number);
$x(n)$ – sequence value in the n-th generation step;
$x(n+1)$ – sequence value in the n+1 generation step;
α – parameter which describes the system's dynamics. In this work, $\alpha = 9.75$;
β – parameter which describes the system's dynamics. In this work, $\beta = -0.53$.

Sine-circle map

$$x(n+1) = mod\left(x(n) + \Omega - \frac{K}{2\pi} sin(2\pi x(n)),\ 2\pi\right), \quad (2.8.)$$

where n – index of the generation step (an integer number);
$x(n)$ – sequence value in the n-th generation step;
$x(n+1)$ – sequence value in the n+1 generation step;
Ω – parameter which describes the system's dynamics. In this work, $\Omega = 0.2$;
K – parameter which describes the system's dynamics. In this work, $K = 12$.

Cubic map

$$x(n+1) = Ax(n)(1 - x(n)^2), \quad (2.9.)$$

where n – index of the generation step (an integer number);
$x(n)$ – sequence value in the n-th generation step;
$x(n+1)$ – sequence value in the n+1 generation step;
A – parameter which describes the system's dynamics. In this work, $A = 3$, thus $x \in (-1.1547, 1.1547)$.

Pincher map

$$x(n+1) = |\tanh s\,(x(n) - C)|, \quad (2.10.)$$

where n – index of the generation step (an integer number);
$x(n)$ – sequence value in the n-th generation step;
$x(n+1)$ – sequence value in the $n+1$ generation step;
s – parameter which describes the system's dynamics. In this work, $s = 2$;
C – parameter which describes the system's dynamics. In this work, $C = 0.5$.

2.4. Chaotic binary sequences

To obtain the binary chaotic sequence $c(n)$, the first step is to generate a non-binary chaotic sequence $x(n)$, and for this purpose 1-D chaotic maps are used: Logistic (2.4), Bernoulli (2.5), Tent (2.6), Gauss (2.7), Sine-circle (2.8), Cubic (2.9), and Pinchers (2.10).

According to [62], when dividing the state space of a 1-D map into two parts and using different initial conditions, completely different binary sequences are generated.

Table 2.2. Threshold value *Th* for 1-D chaotic maps

Map	Logistic	Bernoulli	Tent	Gauss	Sine-circle	Cubic	Pinchers
Threshold value *Th*	0.5	0.5	0.5	0.01	3.25	0	0.38

The generated non-binary chaotic sequence *x(n)* is converted into the binary NRZ (Non-Return-to-Zero) chaotic sequence *c(n)* using the comparison rule:

$$c(n) = \begin{cases} 1, x(n) \geq Th \\ -1, x(n) < Th \end{cases}, \tag{2.11.}$$

where *Th* is a defined threshold, which is equal to the mean value of *x* definition interval, and the value of the threshold for each observed map is presented in Table 2.2.

2.5. Cross-correlation properties of binary chaotic sequences

In this section, the distribution of maximum absolute values of cross-correlation among binary chaotic sequences is studied and compared with the parameters of PN sequences.

To analyze cross-correlation properties, 100 binary chaotic sequences with the same length (for four different lengths L_c = 15, 31, 63, 127) have been generated using each of the seven previously discussed maps. The distribution of the maximum absolute values of the periodic cross-correlation $|R_{lk}|$ is analyzed for all possible pair combinations in 100 sequences of the same map and same length. Table 2.4 presents statistical parameters: minimal, maximal, mean values and standard deviation of the maximum absolute values of the periodic cross-correlation for the generated chaotic sequences. These statistical parameters of chaotic sequences are compared with the corresponding maximum values of the periodic cross-correlation for PN sequences [63].

Table 2.3. Statistical parameters of the cross-correlation peaks
of binary chaotic sequences generated by Logistic map

L_c	**15**	**31**	**63**	**127**
Binary chaotic sequences				
Mean value	0.447	0.37	0.297	0.22
Standard deviation	0.104	0.072	0.049	0.031
Maximal value	0.8	0.581	0.422	0.336
Maximal absolute values of the cross-correlation for PN sequences				
M-sequences	0.6	0.35	0.36	0.32
Gold codes	0.6	0.29	0.27	0.13

Table 2.4. Statistical parameters of the periodic cross-correlation peaks of binary chaotic sequences

L_c	15	31	63	127
Logistic map				
Minimal value	0.12	0.18	0.18	0.15
Mean value	0.52	0.41	0.31	0.24
Maximal value	1	0.7	0.58	0.37
Standard deviation	0.12	0.07	0.05	0.03
Bernoulli map				
Minimal value	0.2	0.23	0.21	0.16
Mean value	0.52	0.41	0.32	0.25
Maximal value	1	0.74	0.59	0.45
Standard deviation	0.12	0.08	0.05	0.03
Tent map				
Minimal value	0.2	0.23	0.17	0.16
Mean value	0.53	0.41	0.32	0.25
Maximal value	1	0.87	0.62	0.39
Standard deviation	0.12	0.07	0.05	0.03
Gauss map				
Minimal value	0.2	0.16	0.17	0.16
Mean value	0.54	0.43	0.34	0.25
Maximal value	1	0.87	0.59	0.45
Standard deviation	0.12	0.08	0.06	0.04
Sine-circle map				
Minimal value	0.07	0.23	0.2	0.16
Mean value	0.56	0.45	0.36	0.28
Maximal value	1	0.8	0.62	0.48
Standard deviation	0.14	0.09	0.06	0.04
Cubic map				
Minimal value	0.07	0.16	0.17	0.16
Mean value	0.54	0.44	0.35	0.27
Maximal value	1	0.87	0.62	0.54
Standard deviation	0.15	0.09	0.06	0.04
Pinchers map				
Minimal value	0.07	0.16	0.17	0.19
Mean value	0.67	0.55	0.45	0.35
Maximal value	1	0.94	0.8	0.46
Standard deviation	0.17	0.13	0.09	0.05
Maximum values of periodic cross-correlation $\mid R_{lk} \mid$ for PN sequences				
M-sequences	0.6	0.35	0.36	0.32
Gold codes	0.6	0.29	0.27	0.13
Kasami sequences	0.33	–	0.14	–

The obtained results show that it is possible to generate binary chaotic sequences with the same or better cross-correlation properties than in the case of Gold codes (for a sequence length of 15 chips) or in the case of m-sequences (for a sequence length of 15, 63, or 127 chips). Obviously, the number of generated chaotic sequences can be increased, whereas it is impossible in the case of conventional PN sequences.

To prove this hypothesis, a selection of sequences generated using the Logistic map is explored. Chaotic sequences with better correlation properties are selected and compared with the characteristics of PN sequences. For this purpose, 100 cross-correlation functions with different initial conditions and sequence lengths have been generated. Table 2.3 shows statistical parameters: mean, maximal values and standard deviation of the maximum absolute values of the cross-correlation for the generated chaotic sequences, and the maximal absolute values of the cross-correlation for PN sequences.

Next, four sequences with lower levels of cross-correlation have been selected from the generated binary chaotic sequences. Table 2.6 shows the cross-correlation levels of the selected sequences, while Table 2.5 presents the initial conditions.

Thus, the hypothesis that it is possible to generate and select binary chaotic sequences with better correlation properties than in the case of conventional PN sequences is successfully proved.

Table 2.5. Initial values for the generation of four selected binary chaotic sequences

N_{x0}	15	31	63	127
x_0^1	0.393845	0.393845	0.595053	0.339301
x_0^2	0.945375	0.878122	0.959039	0.887737
x_0^3	0.339301	0.509951	0.216938	0.407252
x_0^4	0.20211	0.989607	0.524964	0.988737

Table 2.6. Statistical parameters of cross-correlation peaks of four selected binary chaotic sequences

L_c	**15**	**31**	**63**	**127**
Cross-correlation properties of selected binary chaotic sequences				
Minimal value	0.2	0.2258	0.1746	0.1811
Mean value	0.333	0.2796	0.2487	0.2126
Standard deviation	0.0843	0.0754	0.0384	0.0199
Maximal value	**0.4667**	**0.4194**	**0.2698**	**0.2441**

2.6. Summary of chaotic sequences generation and statistical properties

The correlation parameters of 15–127 chip long binary chaotic sequences generated using seven different algorithms are estimated. Statistical characteristics of maximum absolute cross-correlation values for binary chaotic sequences are described. A comparison of correlation levels between binary chaotic sequences and PN sequences is done. It is shown that it is possible to generate and select a higher number of chaotic sequences with a lower cross-correlation level than in the case of PN sequences.

3. APPLICATION OF CHAOTIC SEQUENCES FOR DS-CDMA SYSTEMS

This chapter explores the application of binary chaotic sequences for DS-CDMA systems. The distribution of multiple access interference in DS-CDMA systems based on chaotic sequences is analyzed. Chaotic sequence selection algorithms are proposed to reduce cross-correlation and improve performance in such DS-CDMA systems.

3.1. Principles of spread-spectrum communication systems

Spectrum spreading is used in different data transfer modes and communication systems, such as satellite navigation systems GPS and GLONASS, Digital Enhanced Cordless Telecommunications (DECT) radiotelephone standard, wireless networks Bluetooth, IEEE 802.11b Wi-Fi, IEEE 802.15.4 ZigBee standards, and in many other related areas. The design and operating principles of these systems are different, but all of them use the spread spectrum.

Various methods for spectrum spreading are known, but in the present Thesis, the code-division multiple access systems (CDMA) are discussed. Two types of spectrum spreading are used in CDMA systems: frequency hopping (FH) and direct sequence (DS) techniques.

In the Thesis, the DS technique, where information bits are replaced by spreading sequence chips, is used for spectrum spreading.

The main requirements for spreading sequences are:

1) information bits are encoded with the codes of equal length;
2) the value of the periodic autocorrelation function at zero shift is significantly higher than the values at nonzero shifts;
3) for MAI reduction, cross-correlation values of spreading sequences must be as low as possible;
4) each coding sequence consists of −1 and 1 chips, and the number of unequal chips must be approximately the same to maintain the balance of the sequence.

PN sequences, such as m-sequences, Gold codes or Kasami sequences, are usually used for spectrum spreading in DS-CDMA systems.

These sequences have few shortcomings:

1) a low number of sequences with the same length;
2) the length of sequences is strongly fixed;
3) the complexity of the generation algorithm leads to the loss of sequence balance.

In order to realize multiple access, orthogonal codes, such as Walsh-Hadamard sequences, are also used. However, orthogonal codes have high oscillations in the autocorrelation function. Besides, the number of sequences with equal length is strictly limited.

Over the past 30 years, spreading chaotic sequences have been intensively investigated [32]–[46], [64]. These sequences have low values of cross-correlation functions, possibility to generate any length, and do not have a strict limit on the number of sequences with the same length.

3.2. Statistical characteristic of MAI in chaotic DS-CDMA systems

Considering that short chaotic sequences (length $L_c \leq 127$) have better cross-correlation properties than m-sequences, the MAI distribution in a DS-CDMA system based on such sequences should be explored. This section describes the results of the MAI distribution analysis realized by means of MATLAB simulation of a DS-CDMA communication system based on short chaotic spreading sequences. The evaluation of the MAI probability density has been done by simulation of a chaotic DS-CDMA system.

3.2.1. Simulation of MAI in chaotic DS-CDMA systems

For the evaluation of the MAI probability distribution in a DS-CDMA communication system based on chaotic spreading sequences, a model with K transmitters and one receiver is used and a block diagram of this model is presented in Fig. 3.1. An

NRZ sequence of information bits $b_i(t)$ is sent at the input of each transmitter. Then information bits are spread by a chaotic NRZ sequence $c_i(t)$, which is unique for each transmitter. After this, the signal is transferred on the carrier frequency $U_m \cos(\omega_0 t)$. Each transmitted signal has a corresponding time delay τ observed by the receiver. Since the subject of this research is the probability distribution of the multiple-access interference, the channel noise during simulation is neglected.

The correlation receiver is matched to the (K+1)-th transmitter's spreading sequence c_{K+1}. The input signal of the receiver $r(t)$ causes multiple-access interference:

$$r(t) = \sum_{i=1}^{K} b_i(t) \cdot c_i(t) \cdot U_m \cos(\omega_0 t + \omega_0 \tau_i) \tag{3.1.}$$

where $r(t)$ – input signal of the receiver;
$b_i(t)$ – NRZ information bits of the i-th transmitter;
$c_i(t)$ –NRZ spreading chaotic sequences of the i-th transmitter;
τ_i – time delay of the i-th transmitter signal;
$\omega_0 \tau_i$ – influence of the initial phase on the transmitted signal;
i – transmitter index;
K – number of transmitters;
U_m – amplitude of the carrier;
ω_0 – frequency of the carrier.

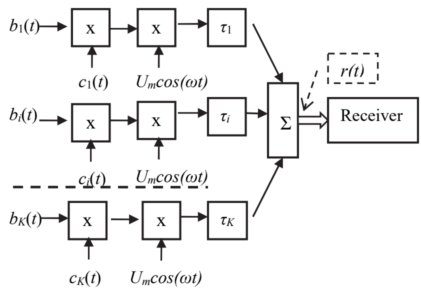

Fig. 3.1. Block diagram of a DS-CDMA system.

The following parameters have been selected for simulation:

1) $b_i(t)$ – NRZ information signal of duration $B_i T_b$;
2) T_b – duration of one bit, which is equal to the duration of spreading sequence c_i;
3) B_i – number of transmitted bits, which is randomly selected from the set (0,300) with an equal probability;
4) T_c – duration of spreading sequence chip;
5) $c_i(t)$ – binary NRZ spreading chaotic sequence;
6) L_c – length of spreading sequence, which can have one of four different values L_c = 15, 31, 63, 127 chips;
7) Δt_i – transmission starting moment for the i-th transmitter, $\Delta t_i = nT_c$, where n is randomly selected from the set $[0, L_c \cdot (B_{max} - B_i)]$ with an equal probability;
8) K – number of transmitters causing multiple-access interference for the correlation receiver, K = 6, 18, 33, 65, 129, 259.

At time moments jT_b, the following values will be obtained at the output of the correlation receiver:

$$U_j = A \int_{(j-1)T_b}^{jT_b} [\sum_{i=1}^{K} b_i(t) c_i(t) \cos(\omega_o \tau_i)]\, c_{K+1}(t) dt, \quad (3.2.)$$

where U_j – output signal of the receiver at time moments jT_b;
A – coefficient which depends on the parameters of the correlation receiver;
$b_i(t)$ – NRZ information bits of the i-th transmitter;
$c_i(t)$ – NRZ spreading chaotic sequence of the i-th transmitter;
τ_i – time delay of the i-th transmitter signal;
$\omega_0 \tau_i$ – influence of the initial phase on the transmitted signal;
i – transmitter index;
K – number of transmitters;
T_b – duration of one bit.

3.2.2. The results of MAI simulation

The main purpose of the simulation is to test the hypothesis that, in the case of DS-CDMA system based on relatively short chaotic spreading sequences, MAI samples calculated according to (3.2) come from a normally distributed population. Two types of DS-CDMA system model have been examined during simulation:

- synchronous system – all time delays τ_i are equal;
- asynchronous system – time delays τ_i observed by the receiver are different, the delay impact $\omega_0\tau_i$ is randomly selected from the set $(0, 2\pi)$.

Considering that the expected value and variance of the distribution are not specified in advance, the Lilliefors test has been used for MAI distribution analysis. The null hypothesis of the test is as follows: MAI samples come from a normally distributed population. If the result is "0", it means that the hypothesis of normality cannot be excluded. If the result is "1", it means that the hypothesis of normality can be rejected with the probability 0.95 (significance level 0.05).

The following results of MAI distribution have been obtained for six different 1-D chaotic maps (Bernoulli, Tent, Gauss, Sine-Circle, Cubic, and Pinchers).

- The probability that the MAI does not come from a normal distribution is 25% higher in the case of a synchronous DS-CDMA system than in the case of an asynchronous DS-CDMA communication system.
- The difference between the results of MAI distribution tests for different 1-D chaotic maps can be up to 30%. For example, in the case of Bernoulli and Gauss maps, the probability that the MAI does not come from a normal distribution is 30% higher than in the case of Cubic map for all transmission modes.
- The length of chaotic spreading sequences also has influence on MAI distribution – if the length is 15 chips, then in 50% cases of synchronous and 35% cases of asynchronous transfer mode the MAI does not come from a normal distribution. For other tested lengths (31, 63, 127), the MAI does not come from a normal distribution in 40% cases of synchronous and 25% cases of asynchronous transfer mode.
- The most important factor is the number of transmitters causing multiple-access interference. For all transfer modes and all explored maps, the MAI does not come from a normal distribution if the number of users is less than or equal to 6. In more than half of the cases of a synchronous multi-user DS-CDMA communication system, the MAI does not come from a normal distribution if the number of users is less than 65. In the case of an asynchronous multi-user DS-CDMA communication system, if the number of transmitters is 18–259, then in 25% cases the MAI does not come from a normal distribution.

Moreover, in the case of Logistic map, a different set of simulation parameters has been chosen for the MAI distribution study with the purpose to check the influence of these parameters on test results. The study includes three different initial condition sets for the generation of chaotic spreading sequences, two different rules for the choice of initial conditions, an increased number of tests, as well as another maximal message duration, the same four sequence lengths, and a different number of transmitters causing multiple-access interference. During simulation, the following results have been obtained:

- the applied tests (chi-square, Kolmogorov, Lilliefors) provide generally similar results;
- the MAI distribution is most affected by the number of transmitters and sequence lengths;
- the maximal number of transmitted bits affects the test results by 36% in synchronous mode and by 30% in asynchronous mode;
- The initial conditions and the method of their selection (with or without iterations) also affect the test results, and the difference between the results is about 20%.

If the length of chaotic spreading sequences is relatively short and the number of transmitters causing MAI is low, then in many cases the MAI does not belong to a normal Gaussian distribution. Synchronous transfer mode also increases the probability that the MAI does not come from a normal distribution. Therefore, BER evaluation methods based on MAI approximation with the normal distribution cannot be used in the case of a DS-CDMA system based on chaotic spreading sequences with the length up to 127 chips. In such cases, for example, Monte Carlo simulations can be used.

3.3. Performance estimation of chaotic DS-CDMA system

In previous sections, the possibility to generate short-length binary chaotic sequences with the same or better cross-correlation properties compared to PN-sequences (m-sequences, Gold codes), which are widely used in DS-CDMA systems, has been presented. For this reason, relatively short (15–127 chip long) chaotic sequences are examined and used as spreading sequences in a DS-CDMA system for the BER performance study. The detection of a DS-CDMA signal is based on the principles of correlation receiver; therefore, the MAI level in the receiver depends on the value of cross-correlation function at a particular time moment. Taking into consideration the principles of correlation receiver, the cross-correlation of spreading sequences must be minimized and it is useful to explore BER performance of a DS-CDMA system based on selected chaotic sequences with a possibly lower cross-correlation level. The proposed BER performance study is based on a Monte Carlo simulation of a DS-CDMA system using MATLAB software.

3.3.1. Algorithms for the selection of chaotic spreading sequences

3.3.1.1. Minimization of the maximum absolute values of periodic cross-correlation

This section is devoted to selection of chaotic spreading sequences N_{sel} with the lowest periodic cross-correlation level from a several times larger pool of chaotic sequences N_{all}. For this reason, the minimum sum of periodic cross-correlation maximum absolute values for all possible pair combinations from N_{sel} sequences is chosen as selection criterion Φ_{sel}. In that way, N_{sel} sequences with a lower selection criterion Φ_{sel} value are selected from N_{all} generated chaotic sequences. As an example, Algorithm 3.1 presents the selection process for $N_{sel} = 4$ sequences from N_{all}, where, for example, $R_{1,2}$ is the cross-correlation function for sequences c_1 and c_2, and N_{1final}, N_{2final}, N_{3final}, N_{4final} are indexes of selected sequences.

The following parameters are used for generation and selection of chaotic spreading sequences:

1) seven chaotic sequence generation algorithms (2.4.–2.10.) and the comparison rule (2.11) for conversion into binary form;
2) sequence length L_c = 15, 31, 63, 127;
3) number of selected sequences N_{sel} = 4, 6, 8, 10;
4) number of generated sequences N_{all}. This parameter is adaptive and changes according to the number of selected sequences:

$$N_{all} = W \cdot N_{sel} , \tag{3.3.}$$

where N_{all} – number of generated sequences;
W – selection ratio W = 1, 3, 5, 10, 100;
N_{sel} – number of selected sequences, which is equal to the number of users in the system.

The selection ratio W defines the number of generated chaotic sequences N_{all} and the total number of tested combinations:

$$\binom{N_{all}}{N_{sel}} = \frac{N_{all}!}{N_{sel}!\,(N_{all} - N_{sel})!} , \tag{3.4.}$$

where N_{sel} – number of selected sequences;
N_{all} – number of generated sequences;
$\binom{N_{all}}{N_{sel}}$ – number of tested combinations.

Data: Periodic cross-correlation functions for all possible generated chaotic sequences
$[R_{1,2}, R_{1,3}, \ldots R_{1,Nall}, \ldots, R_{Nall-1,Nall}]$
Data: Number of generated sequences N_{al}
Result: Indexes of selected sequences N_{1final}, N_{2final}, N_{3final}, N_{4final}
initialize the value of selection criterion $\Phi_{sel} = \infty$;

```
for N1 = 1 : Nall - 3 do
  for N2 = N1+1 : Nall - 2 do
      for N3 = N2+1 : Nall - 1 do
          for N4 = N3+1 : Nall do
              Φ = max|RN1,N2|+ max|RN1,N3| + max|RN1,N4|+ max|RN2,N3| + max|RN2,N4|+
                  max|RN3,N4| ;
              if Φsel > Φ then
                      N1final = N1 ;
                      N2final = N2 ;
                      N3final = N3 ;
                      N4final = N4 ;
              end
            end
          end
      end
  end
```

Algorithm 3.1. Four chaotic spreading sequences selection algorithm.

Since the number of generated sequences affects the computational complexity as well as the chances to select sequences with the lowest cross-correlation level, its impact (that is, the impact of selection ratio) on selection criterion has been explored.

Impact of the selection ratio on the selection criterion

For four selected sequences and four different sequence lengths, the dependence of selection criterion Φ_{sel} on the number of generated binary chaotic sequences has been investigated. Fig. 3.2 presents the results for 15-chip long chaotic sequences. The selection criterion – the minimum sum of the maximum absolute values of normalized periodic cross-correlation for all pair combinations of N_{sel} sequences – decreases when the number of generated sequences N_{all} increases. The proposed selection algorithm provides the best minimization of selection criterion for the sequences of shortest length, because the number of all possible binary combinations of this length is lower. Thus, in case of shorter sequences, the amount of generated sequences compared to the number of all possible combinations for these sequences length is relatively higher.

The distribution of the maximum absolute values ("peaks") of periodic cross-correlation for all possible pair combinations from 40 and 400 generated sequences and for four selected sequences of the same map and same length is explored, and statistical parameters – maximal, mean, minimal values and standard deviation – are compared. Selecting 4 from 40 and 400 generated sequences gives the following results:

1. selecting 10% of generated sequences decreases the diversity of peak values of periodic cross-correlation function among selected sequences compared to all generated sequences. Besides:
   1.1. for all lengths and maps (except Pinchers map), maximums of the peak values of periodic cross-correlation function for selected sequences decrease by 37–47% and are close to the mean values of peaks for all generated sequences;
   1.2. minimums of periodic cross-correlation peaks for all generated and selected sequences are close in both cases;
   1.3. for all maps except Pinchers, standard deviation values of periodic cross-correlation peaks for selected sequences decrease by 29–57% compared to all generated sequences.

Table 3.1. presents the distribution of the maximum absolute values of periodic cross-correlation parameters, averaged among all maps (except for Pinchers map).

2. Selecting 1% of generated sequences decreases the diversity of peak values of periodic cross-correlation function between selected sequences compared to the 10% case. Besides:
   2.1. distribution of the maximum absolute values of periodic cross-correlation function for selected sequences of the same length significantly differs among 1-D maps, therefore, averaging among maps is not informative;
   2.2. increasing the number of generated sequences decreases the maximal peak of periodic cross-correlation for selected sequences, but this reduction depends on the length of sequences L_c: if $L_c = 15$, the reduction is by 2.35 times; if $L_c = 31$, the reduction is by 1.24 times; if $L_c = 127$, the reduction is only by 1.015 times compared to selecting 10% of sequences.
3. Impact of the selection on the maximum absolute values of the periodic cross-correlation function is higher for shorter sequences in both cases.

During the selection of binary chaotic sequences, a huge number of combinations must be compared. To overcome computational complexity, the sequence selection programs have been developed in MATLAB and then ported into C++ using MATLAB Coder®. The actual sequence selection has been performed by programs launched on a supercomputer, which can search in hundreds of combinations of chaotic map, sequence length and number of sequences in parallel. But this approach requires also a large volume of calculations, which limits the possible value of the selection ratio and the number of users. Therefore, a modified selection algorithm is proposed, which allows increasing the number of users and reducing the computational complexity.

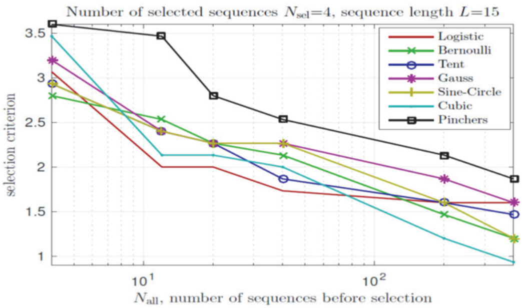

Fig. 3.2. Dependence of the selection criterion on the number of generated binary chaotic sequences – for four selected sequences of 15-chip length.

Table 3.1. Averaged statistics of the maximum absolute values of periodic cross-correlation – for four selected sequences (10% of generated chaotic sequences)

L_c	Averaged statistical parameters of periodic cross-correlation maximums											
	Maximal value			Mean value			Minimal value			Standard deviation		
	N_{all}	N_{sel}	Reduction, %	N_{all}	N_{sel}	Reduction, %	N_{all}	N_{sel}	Reduction, %	N_{all}	N_{sel}	Reduction, %
15	0.89	0.47	47	0.53	0.37	31	0.24	0.27	-9	0.126	0.089	29
31	0.70	0.40	43	0.43	0.33	24	0.25	0.28	-11	0.082	0.050	39
63	0.52	0.30	42	0.33	0.27	19	0.23	0.23	0	0.052	0.029	44
127	0.38	0.24	37	0.26	0.22	16	0.19	0.20	-6	0.037	0.016	57

3.3.1.2. Adaptive limitation of the maximum absolute values of periodic cross-correlation

The proposed chaotic sequence selection algorithm significantly reduces the computational complexity. Only sequence pairs with cross-correlation peaks which value is less than a defined threshold value ξ are employed by the selection algorithm. The threshold value ξ is adaptive and depends on the sequence generation algorithm and the length and the number of selected sequences. For sequence selection, the above-described Algorithm 3.1 is used, but since the number of tested pairs of chaotic sequences is significantly reduced, the number of compared combinations and the amount of calculations are also significantly reduced. To explore the effectiveness of the proposed selection approach, the following parameters have been used in sequence generation and selection:

1) three chaotic sequence generation algorithms (Logistic (2.4), Gauss (2.7), Cubic (2.9)) and the comparison rule (2.11) for conversion into binary form;
2) sequence length L_c = 15, 31, 63, 127;
3) number of selected sequences N_{sel} = 8, 10, 12, 14, 16, 18, 20;
4) number of generated sequences N_{all} =100.

For example, the number of possible combinations of 8 out of 100 sequences is more than $18 \cdot 10^{10}$ combinations, whereas, if the number of selected sequences N_{sel} increases to 20, then more than $5 \cdot 10^{20}$ combinations exist. Therefore, in order to reduce the computational complexity, the adaptive algorithm of cross-correlation levels limitation has been proposed.

The threshold estimation method is explained in Algorithm 3.2. The threshold interval is equal to the interval of normalized periodic cross-correlation function: $\xi \in [0, 1]$. The threshold is defined as the average between the maximum ξ_{max} and minimum ξ_{min} values. The threshold level is iteratively changed. A combination of sequences with a periodic cross-correlation level lower than the threshold ξ is searched. If such combination is found, then the test is stopped and the maximum threshold value ξ_{max} is updated and becomes equal to the verified threshold level ξ. But if the required combination of sequences is not found, the minimum threshold value ξ_{min} is updated. The threshold is changed and testing is repeated until the difference between the minimum and maximum threshold level becomes equal to or less than the accuracy level Δ. When this condition is met, the threshold becomes equal to the current maximal value ξ_{max}. If the accuracy $\Delta = 10^{-12}$, the number of required iterations for threshold estimation is more than 20.

Fig. 3.3 presents the dependence of estimated threshold levels on the number of selected sequences N_{sel} and the length L_c for three 1-D maps. The graph shows that, in the case of equal sequence length, the estimated threshold is close enough or the same for all maps. Initial conditions for selected sequences are presented in the appendices of the Thesis work.

In most cases, the proposed chaotic spreading sequence selection method reduces the number of possible combinations by more than 10^8 times (in some cases even 10^{19}), leading to the possibility of sequence selection in reasonable time.

```
Data: accuracy level Δ
Result: threshold ξ_final for periodic cross-correlation level
initialize threshold interval from ξ_min = 0 to ξ_max = 1
    while |(ξ_max - ξ_min)| > Δ do
            set threshold at the middle ξ = (ξ_max + ξ_min)/2
            search for a set of sequences having all periodic cross-correlation values lower
            than the threshold ξ
            if success then
                    ξ_max = ξ;
                    ξ_final = ξ;
            else
                    ξ_min = ξ;
            end
    end
    end
```

Algorithm 3.2. Cross-correlation threshold adaptation algorithm.

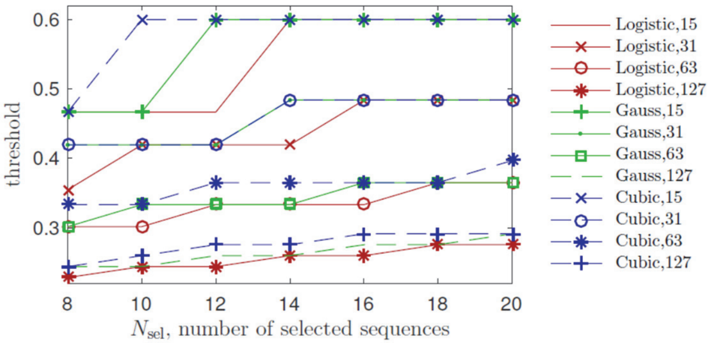

Fig. 3.3. Adaptive threshold levels for 15–127 chip long chaos spreading sequences generated by Logistic, Gauss, and Cubic maps.

3.3.2. Simulation of chaotic DS-CDMA system performance

For the validation of the proposed sequence selection approaches, a computer baseband model of a DS-CDMA communication system under the assumption of perfect chip synchronization and an AWGN channel has been created. The block diagram of the baseband model, consisting of *K* transmitters and *K* receivers connected through a common baseband communication channel, is shown in Fig. 3.4. Selected binary chaotic sequences *c* are used to spread information signals. The number of selected sequences N_{sel} is equal to the number of active users *K* in the system. Asynchronous and synchronous transfer modes are analyzed. In the case of synchronous transfer mode, the delay is equal to zero for all transmitters, whereas, in the case of asynchronous mode, each transmitter has its own transmission delay. The input signal *r(t)* of each receiver is the sum of the signals from all transmitters and a white Gaussian noise $\eta(t)$:

$$r(t) = \left[\sum_{i=1}^{K}(\tau_i + b_i(t) \cdot c_i(t))\right] + \eta(t)\,, \tag{3.5.}$$

where *r(t)* – input signal of the receiver;
$b_i(t)$ – NRZ information bits of the *i*-th transmitter;
$c_i(t)$ – NRZ spreading chaotic sequence of the *i*-th transmitter;
τ_i – time delay of the *i*-th transmitter signal;
i – index of the transmitter;
K – number of transmitter-receiver pairs in the DS-CDMA system;
$\eta(t)$ – white Gausian noise AWGN.

The simulation represents a communication scenario with a high MAI, where all received signals are equal and consist of the transmitted signals of equal power. The following values are obtained at the output of the receiver *n* and at the input of the threshold detector:

$$U_n = A\int_{jT_b - T_b}^{jT_b}\left[\left(\sum_{i=1}^{K}(\tau_i + b_i(t) \cdot c_i(t))\right) + \eta(t)\right] c_n(t)\,\mathrm{dt}\,, \tag{3.6.}$$

where U_n – output signal of the receiver in time moments jT_b;
A – coefficient which depends on the parameters of the correlation receiver elements;
$b_i(t)$ – NRZ information bits of the *i*-th transmitter;
$c_i(t)$ – NRZ spreading chaotic sequences of the *i*-th transmitter;
τ_i – time delay of the *i*-th transmitter signal;

i – index of the transmitter;
K – number of the transmitters;
T_b – duration of one bit;
$\eta(t)$ – white Gausian noise AWGN;
$c_n(t)$ – NRZ spreading chaotic sequence of the n-th receiver.

The threshold detector compares the obtained value U_n to zero, thus, the received signal $r(t)$ becomes converted back to the information signal $b_n'(t)$. At the detector input, the corresponding maximum value of the autocorrelation function and the values of the cross-correlation functions are present. Thereby, the high values of cross-correlation functions have a strong negative influence on the detection correctness.

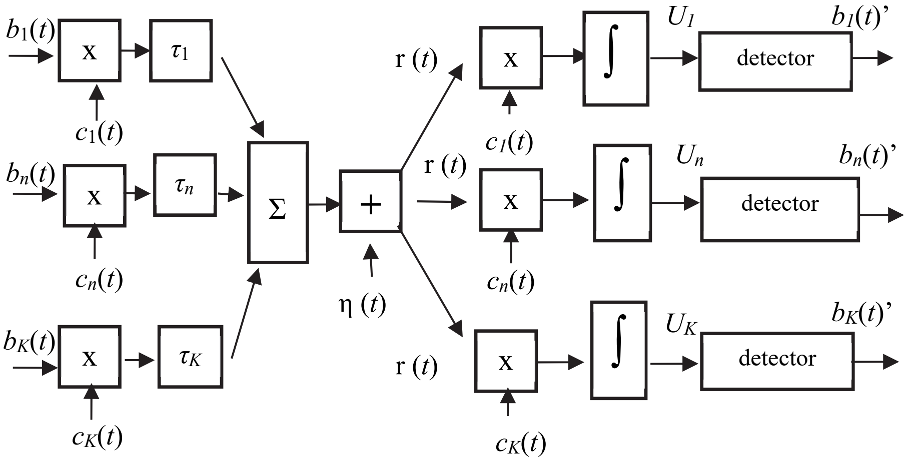

Fig. 3.4. Block diagram of an asynchronous DS-CDMA baseband communication system model.

Table 3.2. Parameters of the simulated chaotic DS-CDMA system

Parameters	
Information bits b	Random NRZ sequences
Chaotic spreading sequence c	NRZ sequences generated by 1-D maps and comparison rule, selected according to Algorithm 3.1., or Algorithms 3.1. and 3.2., or without selection (all generated sequences)
Spreading sequence length	L_c = 15, 31, 63, 127
Initial parameters for the generation of chaotic sequences x_0	Randomly generated from x definition interval corresponding to the map
Number of transmitter-receiver pairs K	K = [4, 8] or K = [8, 20]
Transfer mode	Synchronous or asynchronous
Starting moment τ	Synchronous mode: $\tau = 0$ Asynchronous mode: $\tau \in [0, L_c-1]$
Message duration	128 bits
Number of generated sequences N_{all}	Variable depending on the selection algorithm

The parameters of simulation are shown in Table 3.2. The transmitted information is sent by fixed duration messages. During the simulation, all transmitters are continuously sending signals and all bit errors are reported.

3.3.2.1. BER evaluation in the case of using minimization of the maximum absolute values of periodic cross-correlation

This section presents an effectiveness analysis of the chaotic sequence selection algorithm based on minimization of the maximum absolute values of periodic cross-correlation function. Seven 1-D maps (2.4.–2.10.) are used for generation of chaotic spreading sequences. For both asynchronous and synchronous communication scenarios, system performance is assessed using the average bit error ratio (BER). Furthermore, starting moment optimization is required in order to achieve the minimum MAI level in the synchronous transmission mode. This optimization procedure is based on minimization of the sum of inner products between all pairs of selected sequences by adjusting the mutual (cyclic) shift among the sequences. It is worth to mention that such sequence "synchronization" is computationally very demanding, since it requires to compare L_c^K sums of $L_c!/2(L_c-2)!$ inner products, where each inner product involves L_c multiplications and L_c additions.

The impact of the following parameters on BER performance in a DS-CDMA system has been explored:

A. selection ratio,
B. number of users,
C. sequence length.

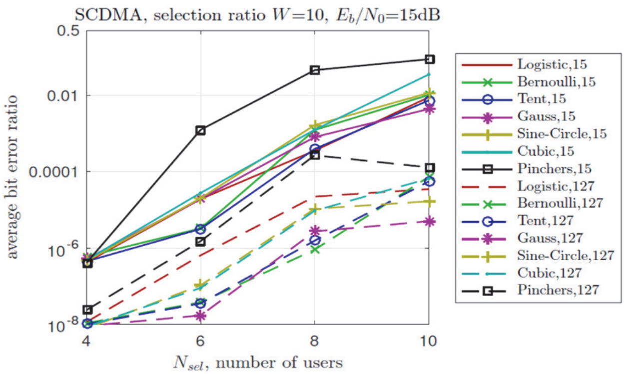

Fig. 3.5. Synchronous DS-CDMA: Impact of the number of users on the average BER, if E_b/N_0 = 15dB, selection ratio W = 4.

The proposed sequence generation and selection method has demonstrated the following features:

- selecting sequences with low periodic cross-correlation peaks and using starting moment optimization provide a BER reduction by more than 100 times for sequences of all studied lengths and for almost all studied chaotic maps;

- performance gain is more variable in the case of asynchronous DS-CDMA transmission. However, a positive impact of sequence selection on the reduction of BER is evident in the case of shorter sequences.

According to the simulation results (see Fig. 3.5 and Fig. 3.6), if the length of chaotic spreading sequences is 15 chips, a DS-CDMA system employing the sequence selection can handle communication of up to 10 simultaneous users with a 10^{-2} BER level, whereas, if the sequence length is 127 chips, then for the same number of simultaneous users the BER level is 10^{-4}.

It should be noted that, before practical implementation, selection efficiency in the case of a fading channel should be additionally studied.

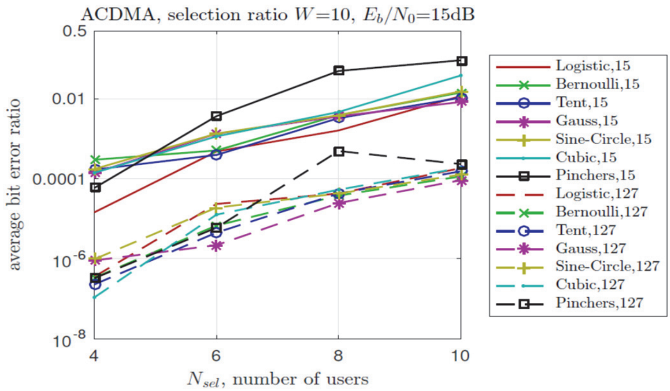

Fig. 3.6. Asynchronous DS-CDMA: Impact of the number of users on the average BER, if E_b/N_0 = 15dB, selection ratio W = 4.

3.3.2.2. BER evaluation in the case of using adaptive limitation of the maximum absolute values of periodic cross-correlation

To validate the chaotic spreading sequence selection algorithm based on adaptive limitation of the maximum absolute values of periodic cross-correlation, BER performance analysis of an asynchronous DS-CDMA system has been done. To limit computational complexity, 8–20 sequences are selected from 100 chaotic spreading sequences of the same length and generated using the same algorithm.

The impact of the following parameters on BER performance in an asynchronous DS-CDMA system has been explored:

A. number of users,
B. sequence length,
C. selection effectiveness.

According to the obtained simulation results, the following conclusions have been made:

- even if the threshold for chaotic sequence selection has the same value for different generation algorithms (1-D maps), the performance of an asynchronous DS-CDMA system can differ;
- the proposed algorithm for chaotic spreading sequence selection reduces the average BER by more than 10 times and is more efficient when the number of users is equal

to or less than 10. To increase selection effectiveness when there are more than 10 users, the set of generated sequences must be extended;

- in an asynchronous DS-CDMA system based on chaotic spreading sequences, the BER level of 10^{-2} for $E_b/N_0 = 20$ dB (Fig. 3.7) can be achieved in the case of:
  a) 8 simultaneous users if the length of sequences is 15 chips;
  b) 10 simultaneous users if the length of sequences is 31 chips;
  c) 14 simultaneous users if the length of sequences is 63 chips;
  d) 20 simultaneous users if the length of sequences is 127 chips.

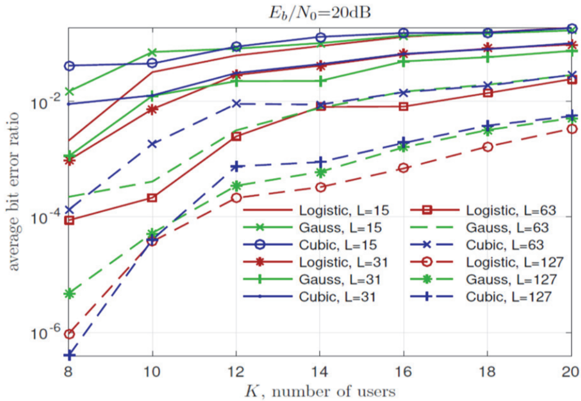

Fig. 3.7. Asynchronous DS-CDMA: Impact of the number of users on the average BER, if $E_b/N_0 = 20$dB.

3.4. Summary of the application of chaotic sequences for DS-CDMA systems

The proposed research presents the analysis of multi-access interference (MAI) distribution in a synchronous and an asynchronous DS-CDMA communication system based on relatively short chaotic spreading sequences (up to 127 chips). For both (synchronous and asynchronous) transfer modes, the approach for multi-user system performance evaluation based on a Gaussian approximation of the MAI cannot be applied if the length of sequences is up to 127 chips. For these cases, a Monte Carlo simulation can be applied.

Two chaotic spreading sequence selection algorithms have been proposed to enhance the performance of a chaotic DS-CDMA system.

The first selection algorithm realizes the minimization of the maximum absolute values of periodic cross-correlation function. For this reason, a set of chaotic sequences producing the minimum sum of the maximum absolute values of periodic cross-correlation are chosen from generated sequences.

The second selection algorithm realizes an iterative adaptive limitation of the maximum absolute values of periodic cross-correlation function, significantly reducing the amount and time of calculation by up to 10^{19} times. For sequence selection, a set of 100 chaotic sequences have been generated by each generation algorithm for different sequence lengths and number of users.

The efficiency evaluation of the proposed binary chaotic sequence selection methods, using averaged BER estimation of a chaotic DS-CDMA system under perfect chip synchronization assumption and with an AWGN channel, has been done.

The proposed chaotic spreading sequence generation and selection methods have demonstrated the following features:

1) positive impact on the performance is more evident in the case of shorter sequences;
2) a chaotic DS-CDMA system can provide a BER level of 10^{-2}:
    a) if the length of sequences is 15 chips, then up to 10 simultaneous users are possible using the first selection method and up to 8 users are possible using the second method,
    b) if the length of sequences is 127 chips, then up to 20 simultaneous users are possible using the second selection method;
3) a chaotic DS-CDMA system can provide a BER level of 10^{-4} for up to 10 simultaneous users if the length of sequences is 127 chips and using any of the two selection algorithms.

4. APPLICATION OF CHAOTIC SEQUENCES FOR OFDM SYSTEMS

The application of chaotic sequences for multicarrier communication systems is explored in this chapter. The principles, strengths and weaknesses of OFDM systems are analyzed. A comprehensive analysis of PAPR reduction methods for an OFDM signal is done. The PAPR reduction and OFDM system performance enhancement methods based on the application of chaotic sequences for multipath propagation environments are proposed.

4.1. Principles, strengths and weaknesses of OFDM communication systems

4.1.1. Principles of OFDM system

The scheme of orthogonal frequency division multiplexing (OFDM) implements information data stream transmission using several subcarriers. The transfer rate of subcarriers is lower than the rate of information signal. All subcarriers are mutually orthogonal.

An OFDM signal is the sum of phase-shift keyed (PSK) or quadrature amplitude modulated (QAM) subcarriers. If all subcarriers in are BPSK (Binary Phase-Shift Keying) manipulated, then by transmitting one OFDM symbol N_{FFT} bits are sent. If T_{FFT} is the duration of an OFDM symbol, then the transmission rate is N_{FFT} / T_{FFT} bit/s.

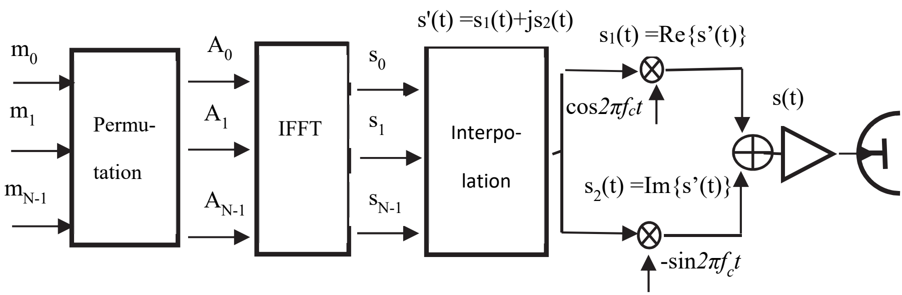

Fig. 4.1. Block diagram of the transmitter in an OFDM communication system.

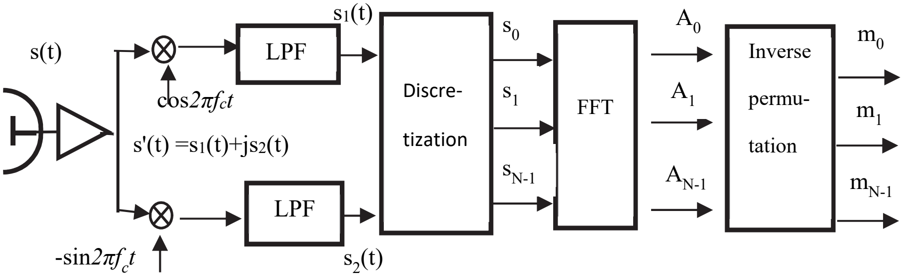

Fig. 4.2. Block diagram of the receiver in an OFDM communication system.

An OFDM signal can be expressed as [65]:

$$s(t) = \sum_{k=-N_{FFT}/2}^{N_{FFT}/2-1} m_{k+N_{FFT}/2} \cos(2\pi(f_c + k\Delta f)t), \text{ if } 0 \le t < T_{FFT}, \qquad (4.1.)$$

where m_i – information symbol;
N_{FFT} – size of FFT and IFFT;
k – OFDM signal subcarrier index, $k \in [-N_{FFT}/2, N_{FFT}/2 - 1]$;
T_{FFT} – duration of an OFDM symbol;
f_c – carrier frequency;
Δf – frequency band of a subcarrier or frequency band between the side subcarriers of an OFDM signal.

To generate the signal of the expression (4.1), the IFFT (Inverse Fast Fourier Transform) operation and the FFT (Fast Fourier Transform) operation to restore transmitted information are performed in the transmitter and receiver, respectively. The creation and detection of an OFDM signal are shown in the block diagrams in Fig. 4.1 and Fig. 4.2, respectively.

An OFDM symbol in the channel is a linear combination of subcarriers. If the orthogonality distortion of an OFDM signal (such as Doppler shift) is not caused in the channel, the receiver can easily separate and demodulate the signal of each subcarrier. Thus, OFDM data are transmitted in parallel. This approach provides the same data rate and a larger symbol duration compared to the single-carrier modulation method.

4.1.2. Strengths and weaknesses of OFDM systems

Two features of an OFDM system should be noted:

1) insertion of a cyclic prefix CP in front of each OFDM symbol increases ISI (Inter-Symbol Interference) resistance;
2) distribution of subcarriers in the frequency band of an OFDM symbol provides an efficient usage of frequency bandwidth and ICI resistance.

The mentioned OFDM features are closely related. If the signal is transmitted in multipath environment, then the received signal consists of several delayed copies of the transmitted signal. If the duration of the cyclic prefix is greater than the multipath propagation delay, then the previously transmitted symbol does not overlap the useful part of the current OFDM symbol. The interference of cyclically shifted copies of the same symbol can be eliminated if the channel frequency response is known.

Pilot signals are used for the estimation and equalization of channel frequency response in the frequency domain after the FFT operation. By passing a block of pilots (block-type estimation) or a comb of pilots (comb-type estimation) through a channel, linear frequency/phase distortions of useful subcarriers can be detected and reduced using “subcarrier-by-subcarrier” amplitude and phase correction [66].

OFDM systems have some weaknesses:

1) sensitivity to carrier frequency shifts, caused by frequency instability of the local generator or by Doppler shift;
2) increased transmission latency. Creating and decoding of an OFDM signal requires computing resources and time. A full OFDM symbol must be received to decode the signal;
3) OFDM signals have high PAPR levels. High levels of PAPR increase the requirements on the dynamic range of analog-to-digital and digital-to-analog converters and reduce the efficiency of the transmitter RFPA (Radio Frequency Power Amplifier). If RFPA works in saturated mode, then the shape of the OFDM signal is distorted, the orthogonality of subcarriers is lost, and the signal spectrum is spread. As a result, ICI appears and demodulation of each subcarrier signal independently of the other subcarriers becomes impossible.

4.2. Application of chaotic sequences for PAPR reduction of OFDM signals

4.2.1. PAPR reduction methods for OFDM signals

A variety of PAPR reduction methods for OFDM signals have been proposed, and these methods can be divided into two types: distortion approaches and scrambling approaches [67]. Both approaches have been intensively studied and described [67]–[79].

Distortion approaches reduce the high peaks of an OFDM signal by changing the signal waveform. However, this type of method can lead to an in-band and an out-of-band distortion of the OFDM signal spectrum. Distortion approaches are: clipping [68], partial shift mapping [69], companding [70].

Scrambling approaches include all methods where signals are encoded before IFFT. Some methods require the transmission of additional information, which reduces throughput and increases redundancy. Scrambling approaches are: Hadamard transform [71], SLM (Selected Level Mapping) method [72], [73], block coding method [74], [75], dynamic constellation method [76], and a scrambling-based method described in [77], which is popular due to its efficiency and simplicity.

Combined approaches have also been proposed. Combined approaches are: Hadamard transform combined with companding [78], coding combined with clipping [79], as well as a method proposed by the author based on signal scrambling and clipping. A variety of different factors should be taken into account before choosing a specific PAPR reduction method: efficiency of PAPR reduction, signal power increase, reduction of system immunity, reduction of data transmission rate, computational complexity, etc.

If the average power is increased, then, after normalization, the system noise immunity decreases and the BER performance degrades. Some methods lead also to noise immunity degradation, such as clipping and filtering methods. This applies also to methods where successful signal decoding requires additional information transfer, such as the SLM method. If the additional information is not correctly received, then errorless OFDM signal decoding is impossible.

Some methods require data rate reduction, for example, in the case of block coding method, PAPR control data are sent periodically instead of information signal transfer.

Before practical implementation of any of PAPR reduction approach, the costs and influence on system performance should be carefully analyzed under conditions as close as possible to a real propagation environment. Table 4.1 shows the comparison of potential negative effects of different PAPR reduction methods. The last method in Table 4.1 is proposed by the author and described in the next subsection.

Table 4.1. Comparison of PAPR reduction methods

Method	Spectrum distortion	Loss in data rate	Power increase	Computa-tional complexity	BER degra-dation	ISI resis-tance
Companding	Yes	No	No	Low	Yes	No [1]
Clipping	Yes / decreased [2]	No	No	Low	Yes	N.F.[3]
Partial shift mapping	No	Yes	No	Medium	No	N.F.
Dynamic constellation shaping	No	No	Yes	Medium	No	N.F.
Block coding	No	Yes	No	Medium	No	N.F.
SLM	No	Yes	No	High	No[4]	N.F.
Pre-scrambling	No	No	No	Low	No	Yes
Hadamard transform	No	No	No	Low	No	No
Hadamard transform and companding	Yes	No	No	Medium	No	No
Coding and clipping	Yes / decreased [5]	No	No	Medium	No	Yes [6]
Pre-scrambling and clipping	**Yes**	**No**	**No**	**Low**	**Yes** / decreased[7]	**Yes**

4.2.2. PAPR reduction by OFDM signal scrambling and clipping

A new PAPR reduction method based on OFDM signal pre-scrambling using modified m-sequences or chaotic sequences before the IFFT operation and clipping after the IFFT is proposed and explored in this section.

[1] If equalization in the receiver is done after the signal dynamic range expansion. If equalization is carried out before, then the system complexity increases.
[2] If filtering with distortion limitation is applied, then distortions are reduced.
[3] Information not found in the literature.
[4] If side information is correctly received.
[5] If filtering with distortion limitation is applied, then distortions are reduced.
[6] Depends on coding.
[7] The BER level depends on the clipping level. If it is reasonably chosen, then the BER degradation is reduced (is negligible).

4.2.2.1. Selection of pre-scrambling sequences and OFDM system simulation parameters

The simulation parameters for pre-scrambling sequence selection are presented in Table 4.2. The length of scrambling sequence is equal to the number of non-zeroed subcarriers, thus the length is 52 chips.

Table 4.2. Simulation parameters for sequence selection with a minimum PAPR level

Baseband OFDM signal	
FFT size	64
Right zeroed subcarriers	5
Left zeroed subcarriers	6
Zeroed carrier	1
Data subcarriers	52
Permutation	Yes
Radio OFDM signal	
Oversample	32
Duration of a radio OFDM symbol	64*32=2048 samples
Radio OFDM symbol approximation method	Zero padding
Carrier period	2048/256= 8 samples
Carrier phase	0

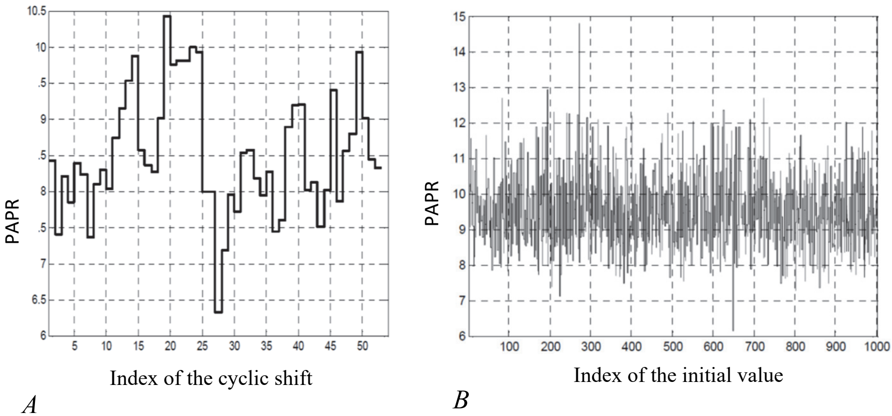

Fig. 4.3. A – PAPR level [dB] dependence on the cyclic shift for a modified m-sequence after IFFT; B – PAPR level [dB] dependence on the initial value index for binary chaotic sequences after IFFT.

Parts of a 63-chip long m-sequence are selected for OFDM signal pre-scrambling. Because of this length, six m-sequences can be matched with the corresponding 6th degree primitive polynomials. Part of an m-sequence consisting of the first 52 chips can be used as a pre-scrambling sequence due to the possibility to use its 62 cyclic shifts. Each 52-chip long pre-scrambling sequence can also be cyclically shifted. A search for "the best" pre-scrambling sequence has been made in the first 52-chip blocks of six cyclically shifted m-sequences using the peak-factor criterion for a modified m-sequence after IFFT depending on the cyclic shift. The initial condition for generation of all m-sequences is "1 1 1 1 1 1". The resulting pre-scrambling sequence is generated by the polynomial $p(x) =$

$x^6 \oplus x^5 \oplus x^4 \oplus x \oplus 1$ and with 46 cyclic shifts. The PAPR of IFFT of this zero-padded sequence depending on its cyclic shift indexes is shown in Fig. 4.3-A.

A binary chaotic sequence generated by logistic map is also used as a pre-scrambling sequence. In the case of chaotic sequences, the length and the number of sequences with the same length are more flexible compared with m-sequences, therefore, it is possible to generate a larger number of sequences and to select a sequence with the least peak factor. Initial values $x^i_0 = y_i$ for generation of a binary chaotic sequence $c_i(t)$ are selected using another logistic map $y(n+1) = r\, y(n)\, (1 - y(n))$, where $r \neq 4$, but close to it. Besides, the initial value y_0 is randomly selected from interval (0, 1), $y_0 = 0.1066527702$. Thus, 1000 chaotic binary sequences have been generated. The peak factor of an IFFT zero-padded sequence depending on the index of its initial value is shown in Fig 4.3-B. The index of the resulting selected sequence is 649, and its initial value is $x^{649}_0 = y_{649} = 0.189008601531864$. The peak factor of this sequence is 6.1439 dB, and it is less than the peak factor of the "best" modified m-sequence.

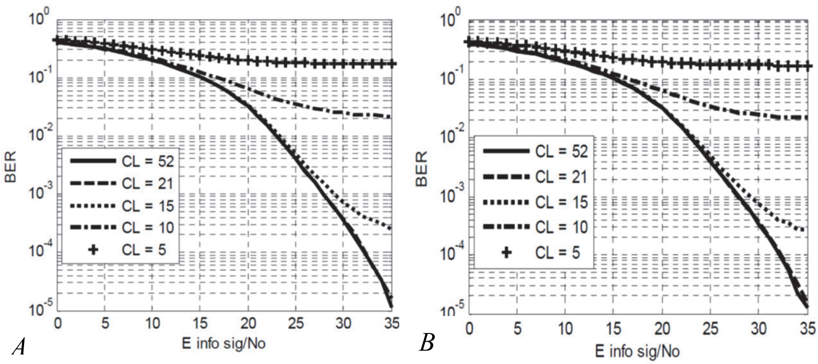

Fig. 4.4. BER dependence on the clipping level for an OFDM 64 QAM signal, prescrambled with: A – modified m-sequence; B – binary chaotic sequence.

4.2.2.2. Evaluation of ISI resistance and BER performance in the case of OFDM signal pre-scrambling and clipping

For stability evaluation of pre-scrambled OFDM signals in the case of "hard" clipping, a simulation of OFDM radio signal transferring has been done using MATLAB software. All simulation parameters are shown in Table. 4.3.

Fig. 4.4 shows the BER performance of the simulated OFDM system operating over a multipath channel with AWGN, when a 64 QAM signal is prescrambled before IFFT and clipped after IFFT. The clipping level is varied from CL = 52 (the maximum possible OFDM signal value) to CL = 5 (1/10 of the maximum possible OFDM signal value).

In Fig. 4.4, the BER performance shows that the efficiency of pre-scrambling with a modified m-sequence or a binary chaotic sequence is practically the same in either case. The clipping level can be reduced by more than 8 dB relative to the maximum envelope peak of OFDM signals without a significant change of the noise immunity. Signals are considerably distorted at the clipping level CL = 10 and below.

Table 4.3. Simulation parameters for BER performance evaluation

Baseband OFDM signal	
Modulation	64 QAM
FFT size	64
Right zeroed subcarriers	5
Left zeroed subcarriers	6
Zeroed carrier	1
Data subcarriers	48
Pilot subcarriers	4
Permutation	Yes
Cyclic prefix CP	CP = 1/4 OFDM symbol
Radio OFDM signal	
Oversample	32
Duration of a radio OFDM symbol	(64+16)*32=2560 samples
Radio OFDM symbol approximation method	Zero padding
Carrier period	8 samples
Carrier phase	0
Radio channel	
Direct path	Gain G = 1, Delay D = 0
Side path	Gain AL= 0.2, Delay TD= CP/16
Noise	AWGN
Clipping	
Clipping level of a radio OFDM signal	CL= 52, 31, 15, 10, 5. (100%, 50%, 29%, 19%, 9.6% of the maximum possible value of OFDM signal)
Equalization	
Comb-type channel estimation and frequency domain equalization	Zero Forcing [82]

4.3. Application of chaotic sequences for OFDM system performance enhancement in multipath environment

The application of chaotic sequences for precoding has several benefits. Broadband signals created by chaos generators have the required properties for scrambling and precoding. Precoding increases the transmission security and enhances the performance of an OFDM system in multipath propagation environment [80].

In this section, the application of chaotic sequences for OFDM signal precoding and the application of sequences generated by Chua circuit for both precoding and synchronization are proposed.

4.3.1. Model of a linear precoded OFDM system

A linear precoded OFDM (LP-OFDM) system is constructed by adding an inverse orthogonal transformation in the transmitter before the IDFT (Inverse Discrete Fourier Transform) block and an orthogonal transformation in the receiver after the DFT (Discrete Fourier Transform) block and the frequency domain equalizer (FDE). This leads to spreading the information signal over the whole frequency band of the system before data are transferred onto subcarriers using IDFT. Thus, each subcarrier contains information about all bits (or QAM symbols) of the transmitted OFDM symbol, and even if one of the subcarriers is distorted, the information can be reconstructed from the signals on the rest of the subcarriers. This operation improves the frequency diversity of the transmitted symbol, which leads to better performance over frequency selective channels and probably decreases the PAPR level of the transmitted OFDM signal. In this case, the Walsh-Hadamard transform (WHT) and the orthogonalized chaotic transform (OCT) are used as orthogonal transformation.

4.3.1.1. Walsh-Hadamard transform WHT

The WHT is an orthogonal transform with a real or complex transformation matrix. It performs an orthogonal, symmetric, involutional, linear operation on 2^n numbers. The transformation matrix of the real WHT of the order n can be defined as follows:

$$U_n = U_1 \otimes U_{n-1} , \quad (4.2.)$$

where $U_1 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$.

Since the WHT matrix is Hermitian, the inverse WHT transform is equal to the direct transform.

4.3.1.2. Orthogonalized chaotic transform OCT

The orthogonalized chaotic transform (OCT) is carried out in two steps. The first step is generation of a chaotic sequence, the second step is orthonormalization. Logistic map (2.4) and Chua circuit (2.1–2.3) are used to generate chaotic sequences.

When Logistic map is used, then a non-binary chaotic sequence x with length N^2 is generated. If Chua circuit is used for generation, then a chaotic sequence x is a discrete or a discrete and decimated chaotic signal χ (2.3). If the number of the samples of the discrete chaotic signal is the same as sequence length, then $x = \chi$. If the number of samples is MN^2 , then the chaotic signal is decimated M times.

The chaotic sequence x of length N^2 is reshaped into a matrix V with dimensions N x N. The number of columns and rows is equal to the number of OFDM subcarriers.

Unfortunately, these chaotic sequences are not fully orthogonal and have some level of cross-correlation. Therefore, the chaotic matrix V must be orthonormalized. The Gram-Schmidt orthonormalization process [81] has been chosen for this purpose.

The result of Gram-Schmidt process is stored in the matrix $U = [u_1\ u_2\ \dots\ u_N]$. The parameters of Chua circuit used for chaotic sequence generation are presented in Table 2.1.

4.3.2. Baseband OFDM model

An LP-OFDM system model is shown in Fig. 4.5. The parallel pilot or the QAM modulated data signal is transformed from respective transformation (WHT or OCT) domain into frequency domain by means of the inverse orthogonal transform U^{-1}:

$$p = U^{-1} m , \quad (4.3.)$$

where U^{-1} – inverse orthogonal transformation (WHT or OCT);
p – precoded informational signal;
m – QAM symbol or pilot symbol.

The precoded information signal p is multiplexed by the IDFT, and the corresponding time domain signal s is generated:

$$s = T^{-1} p = T^{-1} U^{-1} m , \quad (4.4.)$$

where T^{-1} – inverse discrete Fourier transform (IDFT);
U^{-1} – inverse orthogonal transformation (WHT or OCT);
s – baseband OFDM symbol;
p – precoded informational signal;
m – QAM symbol or pilot symbol.

The parallel signal is converted into serial, and the CP is added before each OFDM symbol at the transmitter side:

$$s_{CP} = [\, s(N-L+1)\; s(N-L+2) \dots s(N)\; s(1)\; s(2) \dots\; s(N)] \,, \qquad (4.5.)$$

where s_{CP} – baseband OFDM symbol with CP;
s – baseband OFDM symbol;
L – duration of CP (number of samples);
N – number of OFDM signal subcarriers.

After that, the signal is up-converted to the carrier frequency and sent to the communication channel. The impact of the equivalent baseband channel can be modeled by the finite impulse response filter with complex taps h and additive white Gaussian noise w (AWGN). Therefore, the received baseband signal is described as follows:

$$r_{CP} = h * s_{CP} + w \,, \qquad (4.6.)$$

where *denotes circular convolution;
h – impulse response of the channel;
w – additive white Gaussian noise (AWGN);
s_{CP} – baseband OFDM symbol with CP;
r_{CP} – received baseband signal.

At the receiver, the CP is removed from the received signal:

$$r = [r_{CP}(L+1)\; r_{CP}(L+2) \dots r_{CP}(L+N)\,] \,, \qquad (4.7.)$$

where r_{CP} – received baseband signal;
r – received baseband signal without CP;
L – duration of CP (number of samples);
N – number of OFDM signal subcarriers.

The reconstruction of the transmitted data signal from the precoded OFDM signal is carried out by the DFT:

$$q = Tr \,, \qquad (4.8.)$$

where T – DFT matrix;
r – received baseband signal without CP;
q – received informational signal, which is spread over subcarriers.

Before de-precoding, the received signal is equalized in the frequency domain using the direct orthogonal transformation:

$$p' = \frac{q}{H} \,, \qquad (4.9.)$$

where p' – received and equalized informational signal before orthogonal transformation;
q – the received informational signal, which is spread over subcarrier;
H – channel estimation carried out by pilot symbol processing.

It should be noted that if the pilot signal at the transmitter side has been precoded, then, to perform the channel estimation, the pilot signal at the receiver side should also be orthogonally transformed (WHT or OCT). However, an LP-OFDM system can be implemented without pilot signal precoding.

The reconstruction of the QAM data samples from the spread equalized ones is performed by the orthogonal transformation:

$$m' = U\, p' \,, \qquad (4.10.)$$

where U – the orthogonal transformation (WHT or OCT);
p' – received and equalized informational signal before orthogonal transformation;
m' – reconstructed QAM symbol.

Finally, the binary data are detected by means of the QAM demodulator.

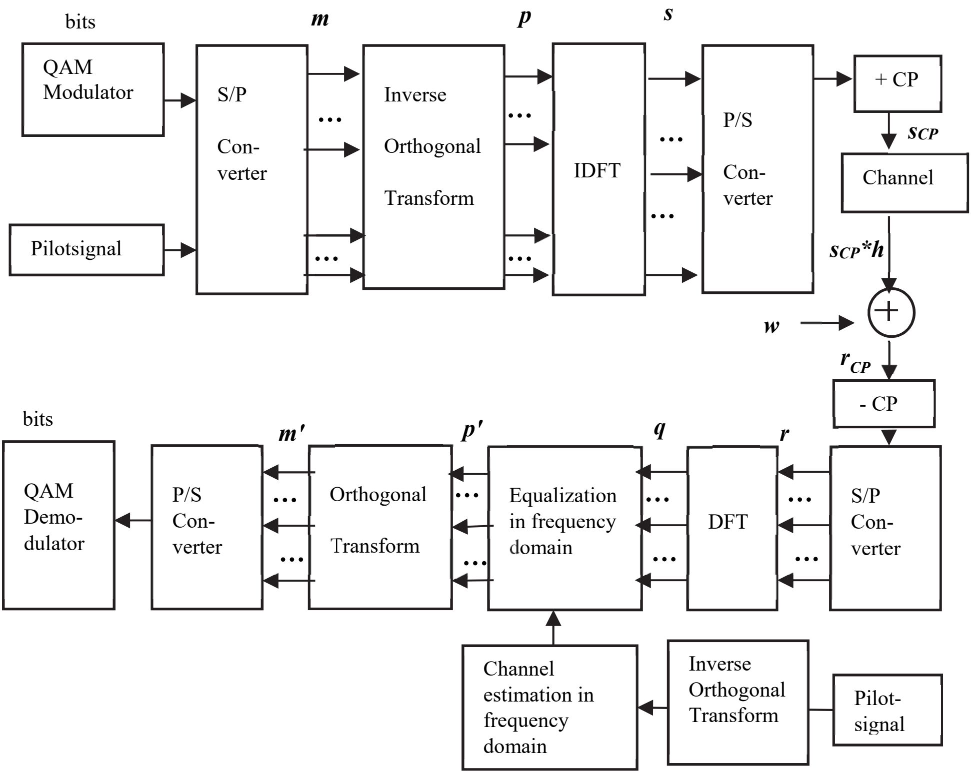

Fig. 4.5. Block diagram of a baseband LP-OFDM system without pilot signal precoding.

4.3.3. Simulation parameters of LP-OFDM system

Table 4.4. Simulation parameters of OFDM communication system

Parameter	Value
Modulation type	4QAM
Total number of subcarriers	64
Number of data subcarriers	64
Length of CP	16
Number of data symbols per frame	20
Number of training symbols per frame	4
Total number of symbols per frame	24
Pilot signal	Zadoff-Chu sequence [83]
Number of transmitted sequences	10^8

In order to investigate the validity of the chaotic precoding, systems using OFDM, WHT-OFDM with and without pilot signal precoding, and OCT-OFDM with and without pilot signal precoding have been simulated over the same relay channel. To compensate the channel effects, zero forcing (ZF) equalizer [82] and block type channel estimation are used in all models. Processing of the received signal has been performed with the assumption of perfect block timing offset and carrier frequency offset synchronization. MathWorks Simulink environment has been used for creation of the baseband communication system models. The parameters of the communication system models are listed in Table 4.4.

4.3.4. BER performance

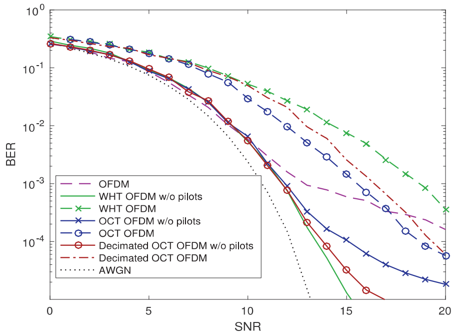

Fig. 4.6. BER performance for different modulation schemes: non-precoded OFDM, OFDM preceded with WHT or OCT (chaotic sequences generated by Chua circuit).

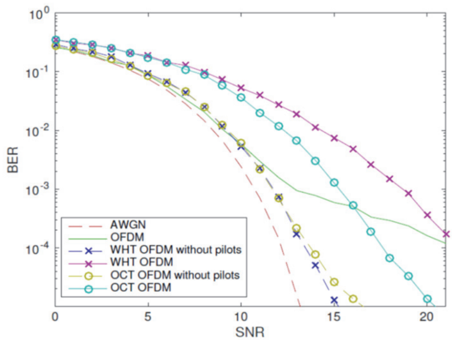

Fig. 4.7. BER performance for different modulation schemes: non-precoded OFDM, OFDM precoded with WHT or OCT (chaotic sequences generated by Logistic map).

Fig. 4.6 shows BER simulation results of the multicarrier communication system without a precoder, i.e., the OFDM, and the communication systems with a precoder based on the chaotic sequences generated by Chua circuit or a precoder based on the WHT. Fig. 4.7 presents BER simulation results of the non-precoded OFDM system and the systems with a precoder based on the chaotic sequences generated by Logistic map or a precoder based on the WHT. It can be observed that the LP of data subcarriers provides gains in terms of communication system throughput for high (>10dB) signal-to-noise ratio (SNR) scenario.

The simulated communication systems (WHT- and OCT-based) with precoded pilot symbols perform significantly worse than the OFDM. The main advantage of Zadoff-Chu sequences [83] – uniform amplitude of the training samples is damaged by the precoding. Consequently, the frequency domain channel estimation on some frequencies becomes poor, which leads to an increased error rate. The use of another pilot signal can significantly change BER simulation results for WHT and OCT linear precoded OFDM systems with pilot signal precoding.

4.3.5. Chaotic synchronization in an LP-OFDM system

To check the applicability of the proposed linear precoding sequences for chaotic synchronization, a discrete-time model of chaotic drive–response system pair has been studied. During this research, the impact of sampling and decimation on the chaotic synchronization between two continuous-state discrete-time models of modified Chua's circuits has been tested. In this research, the so-called observer-based chaotic synchronization [84] shown in Fig. 4.8 has been used.

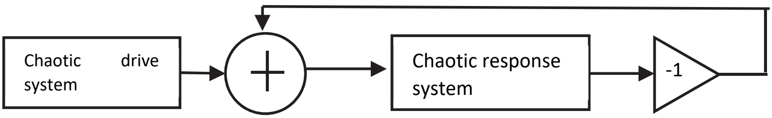

Fig. 4.8. Observer-based chaotic synchronization.

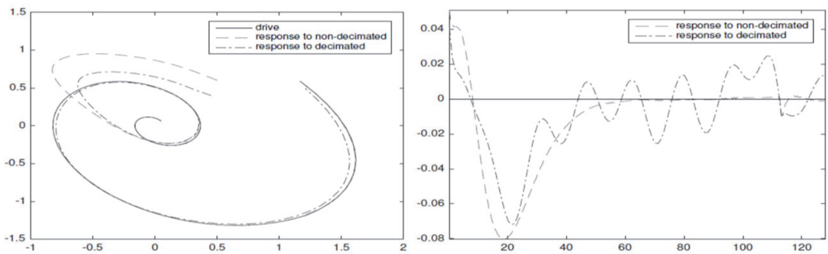

Fig. 4.9. Phase trajectories of the chaotic systems (left), and synchronization error versus applied chip number (right).

In the first experiment, the sampled and recorded output of the chaotic drive system consisting of 128 samples (the first two rows of V matrix) is sent to the chaotic response system model. In the second experiment, 10 times decimated (i. e., low-pass filtered and downsampled) output of the drive system is used, but, before sending to the response system, the sequence is 10 times interpolated. The aim of the experiment is to observe the impact of decimation/interpolation on the chaotic synchronization. The initial conditions of the drive and response systems are different in both experiments, and the phase trajectories of these chaotic systems are shown in Fig. 4.9. The system without decimation has stable synchronization within approximately 50 samples as illustrated in the error plot in Fig. 4.9. The system with decimation is still able to be synchronized, although with reduced accuracy.

4.4. Summary of the application of chaotic sequences for OFDM systems

A PAPR reduction method based on OFDM signal pre-scrambling before the IFFT and descrambling after the FFT with a modified m-sequence or a binary chaotic sequence has been proposed. In addition, the impact of the clipping effect on the immunity of pre-scrambled OFDM signals has been considered. A MATLAB simulation of an OFDM system with comb-type estimation and frequency domain equalization in a two-path channel with AWGN has been performed. The proposed method decreases the PAPR value from 20.17 dB to 10.5–12.3 dB without: additional data transfer, complicated processing on the transmitter and receiver sides, average power change of the OFDM signal, and significant degradation of BER performance in multipath propagation environment.

A novel linear precoding method based on chaotic sequences has been proposed for performance enhancement of OFDM communication systems in multipath propagation environments. A linear precoded OFDM system is developed by adding an inverse orthogonal transformation before the IDFT block at the transmitter side and an orthogonal transformation after the DFT block at the receiver side. Orthogonalized chaotic sequences are used for precoding. Logistic map and Chua circuit with and without 1/10 signal decimation are used to generate chaotic sequences. Orthonormalization is performed by Gram-Schmidt process.

The validation of the proposed precoding method is performed using Simulink simulation of a baseband LP-OFDM system in a frequency selective channel. This has produced the following results:

1) if SNR at the input of the receiver exceeds 10 dB, the proposed OCT-based precoder allows achieving higher communication system throughput compared to the classic OFDM and approximately the same throughput compared to the WHT-precoded OFDM;
2) discrete chaotic signals with and without decimation, generated by Chua circuit, can be used for precoding and synchronization tasks. Moreover, stable synchronization can be achieved with 50 samples, providing the possibility of synchronization using only the first row of OCT;
3) the proposed OCT-based precoder increases the security aspects of communications systems and, therefore, has potential for encryption applications.

CONCLUSIONS

This work clearly demonstrates the possibility to develop a DS-CDMA system based on relatively short (up to 127 chips) binary chaotic sequences and presents the following results:

- generation and selection of chaotic sequences with lower cross-correlation levels allow developing a DS-CDMA system with higher capacity than in the case of m-sequences;
- analytical BER performance estimation methods based on MAI approximation by Gaussian distribution cannot be used for DS-CDMA systems based on binary chaotic sequences;
- proposed chaotic DS-CDMA system can provide a reliable link for 20 users at the BER level of 10^{-2} and for 10 users at the BER level of 10^{-4}.

The following applications of chaotic sequences for multicarrier communication systems are proposed:

- PAPR reduction method for OFDM signals, which is based on scrambling with a binary chaotic sequence and clipping;
- performance improvement of OFDM systems in multipath propagation environment using linear precoding which is based on the orthogonalized chaotic transform.

Binary chaotic sequences generated according to simple algorithms can be used in secure DS-CDMA systems with a relatively low number of users. The proposed methods for OFDM system performance enhancement provide also increased confidentiality and could be used, for example, in the development of secure wireless modems.

The developed adaptive algorithm for chaotic sequences selection provides an efficient search of spreading sequences for a DS-CDMA system. It leads to reduction of the cross-correlation between the sequences and increases the performance of the system. This algorithm significantly decreases the computation time making possible examination of a huge number of combinations.

The proposed systems can be used for highly secure sensor networks with simple signal processing and moderate data rate.

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