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Stochastic Stability of a Pipeline Affected by Pulsate Fluid Flow

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Abstract

This paper deals with stability analysis of elastic pipeline containing water flow, the velocity of which is perturbed harmonically under an action of pulsate fluid flow. The stability conditions of the pipeline section are analyzed under assumption of the mathematical model of fluid caused by longitudinal force with Poisson characteristics and application of the stochastic modification of the second Lyapunov method.

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Keywords: Pipeline dynamics; Perturbation theory; Second Lyapunov method; Random harmonic oscillator

1. Introduction

Pipelines are among the most critical elements of aircraft design. The requirement for weight reduction is associated with decreasing thickness of pipes' walls. An important question for pipes with thin walls is how the pipe cross-section impact to the stability of the pipelines' dynamics with liquid flowing through it. In particular, the actual purpose of the research is pipelines' parametric oscillations. There is plenty of literature devoted to parametric resonance excited fluid flow with low ripple flow rate in pipeline[1,2]. The parametric oscillations of the pipeline were studied, perturbed simultaneously by pulsed fluid flow and a variable longitudinal force also[3]. The purpose of this study is to evaluate the effect of transverse strains cross section of the pipe under pipelines' internal pressure pulsations pipeline as another destabilizing factor causing parametric oscillations.

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There are pipelines' sections temporarily overlain by simple or other shut-off valves in the hydraulic circuits of different technical systems, such as propulsion systems of aircraft, communications ground launch facilities, energy equipments. Moreover, the intense pressure pulsations before these valves are often observed during working processes of hydraulic units. Therefore, it is necessary to calculate the pipeline stability conditions, which are located upstream of the valve.

Let us consider the straight section of the pipeline length L, filled with an incompressible in viscid liquid. Since the liquid is non-viscous, friction between the walls of the pipeline and the medium is absent. The tube has a constant circular cross-section (the average diameter D_0 and wall thickness h). The overpressure at the entrance of the pipeline is created. The conditions of pipe fixing are hinging ends of the pipe segment. The overpressure has both permanent components and components, which are changing by the law $P(t) = P_0 + P_1(y(t))$, where $y(t)$ we decompose later.

Therefore, the equation of small transverse vibrations of the pipeline taking into account the effect of the cross-sectional view of the rotational inertia has the form[3]:

$$EJ\frac{\partial^4 u}{\partial x^4} + P(t)\frac{\partial^2 u}{\partial x^2} + D\frac{\partial u}{\partial t} + m\frac{\partial^2 u}{\partial t^2} = 0 \tag{1}$$

This equation was developed[4] for analysis of the transverse oscillations of a pipeline section of length L under the action of pulse fluid flow. Here we use the following notations:

- EJ - flexural rigidity of pipeline
- $P(t)$ - disturbance longitudinal force, involved by fluid flow
- m - mass of unit of pipeline length
- D - dissipation factor

This model has been delineated[5] for pin-ended pipeline section assuming boundary condition in a form of equalities:

$$u(t,0) = u(t,L) = 0; \tag{2}$$

$$\left(\frac{\partial^2 u}{\partial x^2}\right)(t,0) = \left(\frac{\partial^2 u}{\partial x^2}\right)(t,L) = 0. \tag{3}$$

and a longitudinal force in the form, which is already mentioned before:

$$P(t) = P_0 + P_1(y(t)), \tag{4}$$

where $y(t)$ is Poisson process with exponential distribution with parameter λ between switches with values ξ_n - on $\boldsymbol{R}(0,1)$. The infinitesimal operator of the Poisson process is:

$$Qv(y) = \lambda\int_0^1 [v(z) - v(y)]dz. \tag{5}$$

As the author of paper[5], we decompose solution of equation (l-3) in Fourier series:

$$u(t,x) = \sum_{n=1}^{\infty} T_n(t)\sin\left(\frac{\pi n x}{L}\right) \tag{6}$$

and discuss behaviour of the first shape amplitude $T_n(t)$. Under assumption of sufficiently great mass for dissipation factor, they apply well known Bogolyubov-Mitropolsky method[6] to the second order ordinary differential equation Mathieu for function $T_n(t)$, and discuss pipeline stability.

After applying[7] the stochastic stability analysis method derived in our paper, we also study the dependence on the intensity λ and the frequency mismatch of a necessary dissipation factor D to continue our investigation of stability conditions that have been started[8].

2. Stochastic oscillator

As in the paper[8], we apply decomposition (6) to the solution of equation (1)-(3) with above (4) longitudinal force $P(t)$ and derive the second order stochastic differential equation having the following form:

$$\ddot{x} + \omega^2 x = -2\delta\varepsilon^2 \dot{x} - \varepsilon x\, h(y(t)), \tag{7}$$

where the random process $y(t)$ – is the Poisson process with infinitesimal operator $Qv(y) = \lambda \int_0^1 [v(z) - v(y)]dz$ (that means, that due to condition of normal solvability: $\int_0^1 g(y)dy = 0$, and therefore $Qg(y) = -\lambda g(y)$).

Using the substitution, the equation for the radius and phase can be derived:

$$x_1 = r\cos(\omega\varphi),\ x_2 = -r\omega\sin(\omega\varphi),\ \psi = 2\omega\varphi. \tag{8}$$

$$\begin{cases} \dot{\psi} = 2\omega + \varepsilon \dfrac{1}{\omega}[1 + \cos\psi]h(y(t)) - \varepsilon^2 2\delta \sin\psi, \\ \dot{r} = -\varepsilon^2 r\delta[1 - \cos\psi] + \varepsilon \dfrac{1}{2\omega} r h(y(t))\sin\psi. \end{cases} \tag{9}$$

To analyze α-exponential stability of the solution for equation (7) we will apply the second Lyapunov method and derive mean square stability conditions. First of all let's denote the Lyapunov function $F(r,\psi,y) = r^\alpha V^\varepsilon(\psi,y)$. As we know[9] the Lyapunov function should be satisfied the Lyapunov inequalities:

$$\exists\varepsilon_0, \exists c > 0, \forall\{\psi, y, |\varepsilon| < \varepsilon_0\}:$$
$$\mathbf{L}F \le -cF,\ c_1 r^\alpha \le F(r,\psi,y) \le c_2 r^\alpha,\ \textit{for any } 0 < c_1 < c_2. \tag{10}$$

The Lyapunov derivative can be written by the form:

$$LF(r,\psi,y) = \left(\dot{\psi}\frac{\partial V}{\partial \psi} + \dot{r}\frac{\partial V}{\partial r} + QV\right) r^\alpha \tag{11}$$

or

$$\begin{aligned}(\mathbf{L}F)(r,\psi,y) &= r^\alpha\left[-\alpha\varepsilon^2\delta[1-\cos\psi] + \alpha\varepsilon\frac{1}{2\omega}\sin\psi\, h(y)\right]V^\varepsilon(\psi,y) + \\ &+ r^\alpha\left(2\omega + \varepsilon\frac{1}{\omega}[1+\cos\psi]h(y) - \varepsilon^2 2\delta\sin\psi\right)\frac{\partial}{\partial\psi}V^\varepsilon(\psi,y) + r^\alpha QV^\varepsilon(\psi,y),\end{aligned} \tag{12}$$

Next, let's rewrite the equation (11) with small positive parameter ε by the following way: $(\mathbf{L}F)(r,\psi,y) = r^\alpha\left(L(\varepsilon)V^\varepsilon\right)(\psi,y)$, where $L(\varepsilon)$ is infinitesimal operator of the process (r,ψ,y) defined on the space $\boldsymbol{L}_2(S^2)$ for sufficiently smooth functions by the equality:

$$\begin{aligned}L(\varepsilon) &= \left\{2\omega\frac{\partial}{\partial\psi} + Q\right\} + \varepsilon\left\{\alpha\frac{1}{2\omega}h(y)\sin\psi + \frac{1}{\omega}[1+\cos\psi]h(y)\frac{\partial}{\partial\psi}\right\} + \\ &+ \varepsilon^2\left\{-2\alpha\delta[1-\cos\psi] - 2\delta\sin\psi\frac{\partial}{\partial\psi}\right\}.\end{aligned} \tag{13}$$

Therefore function $V^\varepsilon(\psi,y)$ can be found from the equation:

$$L(\varepsilon)V^\varepsilon(\psi,y) = -1 \tag{14}$$

and α-exponential stability conditions means:

$$-r^\alpha \le -\frac{1}{c_2} r^\alpha V^\varepsilon(\psi,y) = -\frac{1}{c_2} r^\alpha F.$$

Then we apply the method of the small parameter and find the functions $L(\varepsilon)$ and $V^\varepsilon(\psi,y)$ through the series by small parameter ε. First, we rewrite equation (13) for $L(\varepsilon)$ by the following form $L(\varepsilon) = Q_0 + \varepsilon Q_1 + \varepsilon^2 Q_2$. Therefore, from (13) we can defined operators Q_0, Q_1, Q_2 :

$$Q_0 V(\psi, y) = 2\omega \frac{\partial}{\partial \psi} V(\psi, y) + QV(\psi, y),$$

where

$$QV(\psi, y) = \lambda \int_0^1 [V(\psi, z) - V(\psi, y)] dz. \tag{15}$$

$$Q_1 V(\psi, y) = \alpha \frac{1}{2\omega} \sin\psi\, h(y) + \frac{1}{\omega}[1 + \cos\psi] h(y) \frac{\partial}{\partial \psi} V(\psi, y),$$

$$Q_2 V(\psi, y) = -\alpha\delta\left[1 - \cos\psi\right] - 2\delta \sin\psi \frac{\partial}{\partial \psi} V(\psi, y).$$

Let's decide to have the following solution for the Lyapunov function $V^{\varepsilon}(\psi, y)$:

$$V^{\varepsilon}(\psi, y) = \varepsilon^{-2} V_0(\psi, y) + \varepsilon^{-1} V_1(\psi, y) + V_2(\psi, y). \tag{16}$$

Now we apply the stochastic modification of the second Lyapunov method[9] and we deal with the equation (14). The necessary and sufficient condition for exponential decreasing of the second moment $\mathbf{E}\{|z(t)|^2\} = \mathbf{E}\{|r(t)^{2\alpha}\}$ is positivity of function $V^{\varepsilon}(\psi, y)$ for all $\{\psi, y\} \in S^2$ and sufficiently small ε > 0. The family of operators $L(\varepsilon)$ defined in (15) is a holomorphic family and we can look[10] for function $V^{\varepsilon}(\psi, y)$ in a form of the Laurent series $V^{\varepsilon}(\psi, y) = \sum_{k=-2}^{\infty} \varepsilon^k q_k(\psi, y)$. Substituting this series in (14) and equating the terms near the same powers of ε we can find the first function q_{-2} from equation $Q_0 q_{-2} = 0$ as an arbitrary constant $V_0(\psi, y) \equiv q$. The equation $Q_0^{*} p(\psi, y) = 0$ also has only constant solution and we can set $p(\psi, y) = 1$.

Next, taking into account equality $Q_1 q_{-2} = 0$ we should analyze the equation:

$$L(\varepsilon) V^{\varepsilon}(\psi, y) = Q_0 q \varepsilon^{-2} + \varepsilon^{-1}\left[Q_1 q + Q_0 V_1(\psi, y)\right] + Q_2 q + Q_1 V_1(\psi, y) + Q_0 V_2(\psi, y) + O(\varepsilon), \tag{17}$$

The first step is to find function $V_1(\psi, y)$ from the equation (16):

$$Q_1 q + Q_0 V_1(\psi, y) = 0 \Rightarrow$$

$$\left(Q + 2\omega \frac{\partial}{\partial \psi}\right) V_1(\psi, y) = -Q_1 q \Rightarrow$$

$$\left(Q + 2\omega \frac{\partial}{\partial \psi}\right) V_1(\psi, y) = -\alpha q \frac{1}{2\omega} h(y) \sin\psi \tag{18}$$

Let's the solution of the equation (18) will be the following:

$$V_1(\psi, y) = -\frac{\alpha h(y)}{2\omega} q\left[\left(C_1 \sin\psi + C_2 \cos\psi\right)\right],$$

where C_1 and C_2 are arbitrary constants we just find:

$$\left(Q+2\omega\frac{\partial}{\partial\psi}\right)\left[-\frac{\alpha h(y)}{2\omega}q\left[\left(C_1\sin\psi+C_2\cos\psi\right)\right]\right]=-\alpha q\frac{1}{2\omega}h(y)\sin\psi \Rightarrow$$

$$\left(Q+2\omega\frac{\partial}{\partial\psi}\right)\left[\left(C_1\sin\psi+C_2\cos\psi\right)h(y)\right]=-\left(C_1\sin\psi+C_2\cos\psi\right)\lambda h(y)+$$

$$+2\omega\left(C_1\cos\psi-C_2\sin\psi\right)h(y)=h(y)\sin\psi .$$

$$\begin{cases}-\lambda C_1-2\omega C_2=1,\\ -\lambda C_2+2\omega C_1=0\end{cases} \Rightarrow C_2=\frac{2\omega}{\lambda}C_1,\ -\left(\lambda+\frac{4\omega^2}{\lambda}\right)C_1=1 \Rightarrow$$

$$C_1=-\frac{\lambda}{\lambda^2+4\omega^2};\quad C_2=-\frac{2\omega}{\lambda^2+4\omega^2}.$$

Therefore,

$$V_1(\psi,y)=\frac{\alpha}{2\omega(\lambda^2+4\omega^2)}qh(y)\left(\lambda\sin\psi+2\omega\cos\psi\right)\cdot \tag{19}$$

The second step is to find the constant q from the equation:

$$Q_2q+Q_1V_1(\psi,y)+Q_0V_2(\psi,y)=-1 \tag{20}$$

or

$$Q_0V_2(\psi,y)=-1-Q_2q-Q_1V_1(\psi,y) \tag{21}$$

To fulfil the condition of normal solvability of equation (20) we should consider the equation (21), where $V_1(\psi,y)$ is already known:

$$V_1(\psi,y)=\alpha q\frac{h(y)}{2\omega(\lambda^2+4\omega^2)}\left(\lambda\sin\psi+2\omega\cos\psi\right).$$

Therefore,

$$\underbrace{\overline{\left[-1+\alpha q\delta\left[1-\cos\psi\right]\right]}}_{-1+\alpha\delta q}-\overline{\left\{\left[\alpha\frac{1}{2\omega}\sin\psi h(y)+\frac{1}{\omega}[1+\cos\psi]h(y)\frac{\partial}{\partial\psi}\right]V_1(\psi,y)\right\}}=0$$

$$\Rightarrow -1+\alpha q\delta-\alpha q\frac{1}{2\omega(\lambda^2+4\omega^2)}\overline{\left\{\left[\alpha\frac{1}{2\omega}\sin\psi h(y)+\frac{1}{\omega}[1+\cos\psi]h(y)\frac{\partial}{\partial\psi}\right]h(y)\left(\lambda\sin\psi+2\omega\cos\psi\right)\right\}}=0;$$

$$\overline{\sin\psi h(y)}=0;$$

so,

$$\overline{\frac{1}{\omega}h^2(y)[1+\cos\psi]\frac{\partial}{\partial\psi}\left(\lambda\sin\psi+2\omega\cos\psi\right)}=$$

$$=\underbrace{\overline{h^2(y)\frac{\partial}{\partial\psi}\left(\lambda\sin\psi+2\omega\cos\psi\right)}}_{0}+\overline{h^2(y)\cos\psi\frac{\partial}{\partial\psi}\left(\lambda\sin\psi+2\omega\cos\psi\right)}=\frac{\lambda}{2\omega}\int_0^1 h^2(y)dy;$$

$$\overline{\frac{\alpha}{2\omega}h^2(y)\sin\psi\left(\lambda\sin\psi+2\omega\cos\psi\right)}=\frac{\alpha\lambda}{4\omega}\int_0^1 h^2(y)dy;$$

we use, that $\gamma^2:=\int_0^1 h^2(y)dy.$

Then,

$$\alpha q\delta-\alpha q\frac{\lambda\gamma^2(\alpha+2)}{8\omega^2(\lambda^2+4\omega^2)}=1 \Rightarrow \alpha q=\left[\delta-\frac{\lambda\gamma^2(\alpha+2)}{8\omega^2(\lambda^2+4\omega^2)}\right]^{-1} \tag{22}$$

And the last step is to write stability condition for solution of equation (1) :

$$q > 0 \Leftrightarrow \delta > \frac{\lambda \gamma^2 (\alpha + 2)}{8\omega^2(\lambda^2 + 4\omega^2)}. \tag{23}$$

3. Linearized equation for the pipeline under stochastic perturbation

In our previous notations the linearized equation for the pipeline shape $T_n(t)$ is the following:

$$EJ\left(\frac{\pi n}{L}\right)^4 T_n(t) + \left(\frac{\pi n}{L}\right)^2 (P_0 + P(y(t))T_n(t) + DT_n'(t) + mT_n''(t) = 0 \cdot \tag{24}$$

Assuming the mass m and the flexural rigidity factor EJ are sufficiently large and dissipation factor D is sufficiently small we can introduce a small parameter ε > 0 and rewrite equation (23) in a following form:

$$T_n''(t) + 2\varepsilon^2 \delta T_n'(t) + \omega_n^2 T_n(t) + \varepsilon h_n(y(t))T_n(t) = 0, \tag{25}$$

where

$$2\varepsilon^2\delta = \frac{D}{m},\ \omega_n^2 = n^2\frac{EJ\,\pi^4}{mL^4}(n^2+p) = \theta n^2(n^2+p),\ p = \frac{P_0L^2}{EJ\,\pi^2},\ h_n = \left(\frac{\pi n}{L}\right)^2 P_1 \cdot \tag{26}$$

Substituting the parameters (26) in the formula (23) we can derive the stochastic stability borders δ_n for shapes $T_n(t)$**,** $n \in$ N in a following form:

$$\delta_n = \frac{1}{8\theta n^2(n^2+p)}\frac{\lambda\gamma^2(\alpha+2)}{(\lambda^2+\theta n^2(n^2+p))}, \tag{27}$$

where $\theta = \dfrac{EJ\,\pi^4}{mL^4}$.

Note that for a qualitative analysis of the stochastic stability shape borders δ_n without loss of generality we can make substitution: $\theta = 1$, and analyze the formulae

$$\hat{\delta}_n = \frac{1}{8n^2(n^2+p)}\frac{\lambda\gamma^2(\alpha+2)}{(\lambda^2+n^2(n^2+p))}, \tag{28}$$

4. Conclusion

Now we can discuss the impact of the parameters p and λ on giving by the above formula stability border for the shapes $T_n(t)$; the parameter p in this case can be considered as the frequency mismatch (see Fig. 1).

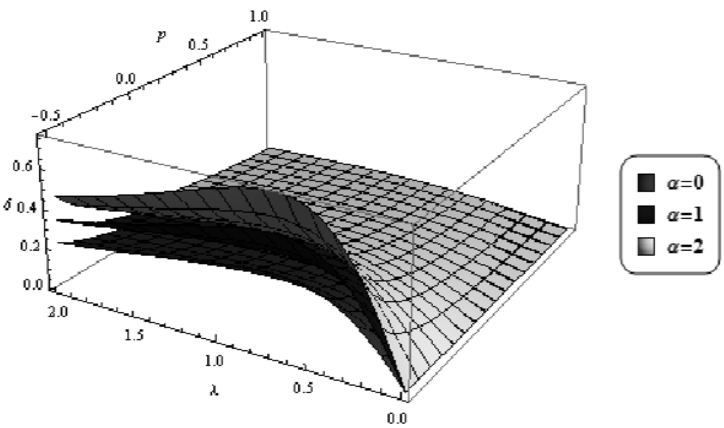

Fig. 1. Dependence of stochastic stability border on mismatch p and intensity λ resonance region, n = 1.

First of all the defined by formula (28) surface for any $n \in N$ has a very similar form to shown in Fig.1 surface for $n = 1$. Both for negative $p > -1$ and positive p and sufficiently big intensity the pipeline oscillations decay even for a relatively small dissipation (see Fig. 2).

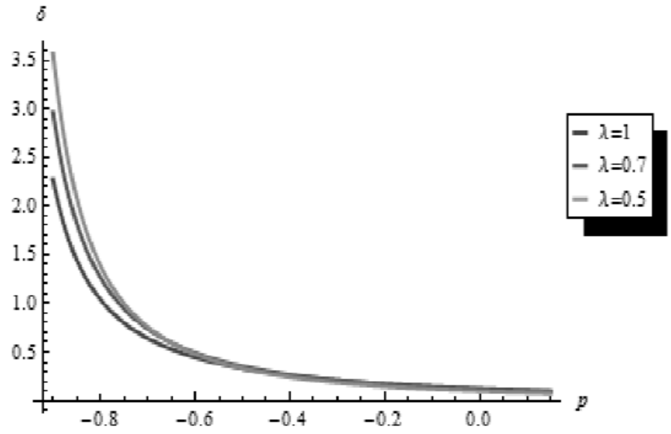

Fig. 2. $n = 1$. Dependence of p with several values of intensity λ.

But it should be mentioned that the stochastic stability border non-monotonic depends on the intensity and mismatch for all shapes as function on intensity λ have maximum and decrease with increasing of intensity (see Fig. 3).

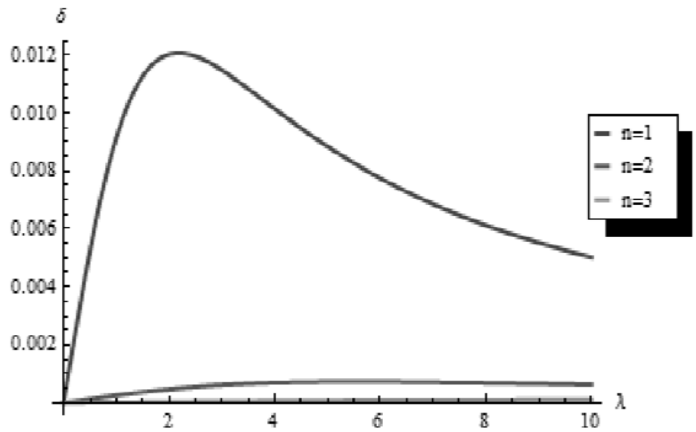

Fig. 3. $p = 1$. Dependence on λ.

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