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ON INTERSPECIFIC COMPETITION OF LOGISTIC POPULATIONS WITH THE ASSUMPTION OF RAPID RANDOM CONTACTS

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Abstract: We analyze a logistic type Markov impulsive differential model for interspecific competition of two populations under assumption that they come into contact at random time moments only but between the contacts they grow independently. Assuming the intervals be sufficiently small we apply the stochastic averaging procedure and construct an ordinary 2-dimensional differential equation for population dynamics in the mean and a linear 2-dimensional stochastic differential equation for deviations on the mean trajectories.

Keywords: interspecific competition; random contacts; stochastic approximation

Mathematics Subject Classification: Primary 92B05,60H30; Secondary 60G57.

1 Introduction

The famous mathematical model for competition of two populations for the first time appears in the papers [1,2]. Using the notations $x(t)$ for the mass density of the first species and $y(t)$ for the mass density of the second species at time moment t the mentioned authors assume that the dynamics of populations may be studied as the system of ordinary differential equations:

$$\frac{dx}{dt} = r_1 x(K_1 - x - \alpha y)K_1^{-1}, \frac{dy}{dt} = r_2 y(K_2 - y - \beta x)K_2^{-1} \tag{1}$$

where $r_1 > 0$ and $r_2 > 0$ are the birth rates in absence of intraspecific and interspecific competition, $K_1 > 0$ and $K_2 > 0$ are the asymptotic masses of the first and of the second species when they grow separately, and the coefficients $\alpha > 0, \beta > 0$ show the influences of one species to another, that defines decreasing opportunity of growth. Later this model was analyzed in detail by many authors (see, for example, [3,4,5,6,7] and references there). The phase portrait of the dynamical system (1) has a very simple structure. There are only four stationary states

$$A_1:\bar{x}_1=0,\bar{y}_1=0;\ A_2:\bar{x}_2=0,\bar{y}_2=K_2;\ A_3:\bar{x}_3=K_1,\bar{y}_3=0;A_4:x_4=\frac{K_1-\alpha K_2}{1-\alpha\beta},y_4=\frac{K_2-\beta K_1}{1-\beta\alpha} \quad (2)$$

These points define the following possibilities for the longtime behavior of the populations: A_1 - both populations become extinct; A_2 - the *x*-population converges to a certain value, bet *y*-population disappears; A_3 - the *y*-population converges to a certain value, bet *x*-population disappears; A_4 - both population co-exist. As it has been shown in [8] the defined by the equations (1) dynamics of populations for any initial values $x(0)>0,y(0)>0$ may be described in the following way:

1. the point A_1 is unstable, that is, for a long time exists at least one of the above populations;
2. if $r_1K_1>\alpha K_2,r_2K_2<\beta K_1$ then the point A_2 is asymptotically stable, that is, $\lim_{t\to\infty}x(t)=K_1,\lim_{t\to\infty}y(t)=0$;
3. if $r_1K_1<\alpha K_2,r_2K_2>\beta K_1$ then the point A_3 is asymptotically stable, that is, $\lim_{t\to\infty}x(t)=0,\lim_{t\to\infty}y(t)=K_2$;
4. if $r_1K_1>\alpha K_2,r_2K_2>\beta K_1$ then the point A_4 is asymptotically stable, that is, $\lim_{t\to\infty}x(t)=\frac{K_1-\alpha K_2}{1-\alpha\beta},\lim_{t\to\infty}y(t)=\frac{K_2-\beta K_1}{1-\beta\alpha}$;
5. if $r_1K_1<\alpha K_2,r_2K_2<\beta K_1$ then subject to initial conditions $\lim_{t\to\infty}x(t)=K_1,\lim_{t\to\infty}y(t)=0$ or $\lim_{t\to\infty}x(t)=0,\lim_{t\to\infty}y(t)=K_2$.

Naturally, in reality the above analyzed model (1) gives just a simplified view on the population dynamics. The deterministic approach to analysis of ecosystems for the first time was questioned by Feller [4]. Analyzing the logistic model for the real logistic type population dynamics he found that population trajectories could equally well be fitted by scaled normal distribution curves with time-dependent argument. Therefore, later the deterministic model (1) was converted by many authors (see, for example, [4] and references there) into the Ito stochastic differential equations. In our paper we also analyze the stochastic modification of the two-species competition model, using as basis the following assumptions. Note that the model (1) supposes a time-continuous contact of populations. This means that within a very small time interval $[t,t+\Delta)$ the impact of population *y* on population *x* may be given by term $\alpha r_1K_1^{-1}x(t)y(t)\Delta$ and the impact of population *x* on population *y* by term $\beta r_2K_2^{-1}x(t)y(t)\Delta$. Our model supposes that the populations come in contact at the random time moments $\tau_1<\tau_2<\cdots<\tau_n<\cdots$ and time-intervals $\tau_n-\tau_{n-1}$ are independent exponentially distributed random variables, $\mathbf{E}\{\tau_n-\tau_{n-1}\}=\varepsilon$, where ε is small positive parameter. The impacts of the populations against each other at the time moment τ_n also are sufficiently small and random: the impact of the population *y* on population *x* is given by term $\varepsilon a(\xi_n)r_1K_1^{-1}x(\tau_n-)y(\tau_n-)$ and the impact of the population *x* on population *y* by term $\varepsilon b(\xi_n)r_2K_2^{-1}x(\tau_n-)y(\tau_n-)\varepsilon$, where $\{\xi_n,n\in\mathbf{N}\}$ is the independent on $\{\tau_n,n\in\mathbf{N}\}$ sequence of identically Uniform(0,1)-distributed random variables. Specify that here and later we use a notation $f(t-)$ for the left limit, that is, $f(t-) := \lim_{\tau \uparrow t} f(\tau)$. The formal definition of the above-mentioned model will be proposed in the next section.

2 The model

As it has been mentioned in the previous section our paper deals with interspecific competition model for populations $\{x_\varepsilon(t), t \geq 0\} \subset \mathbf{R}$ and $\{y_\varepsilon(t), t \geq 0\} \subset \mathbf{R}$ under assumption that the populations come in contacts only at random time moments given by the increasing sequence $\mathrm{T} = \{\tau_k, k \in \mathbf{N}\}$. Index $\varepsilon \in (0, \varepsilon_0)$ is a small positive parameter, which we will use for asymptotic analysis of population dynamics. The sequence T consists of the switching time moments of the Markov process $\{\xi(t), t \geq 0\}$, which is defined on the probability space $(\Omega, \mathcal{F}, \mathbf{P})$ by the infinitesimal operator [8]

$$(Qv)(\xi) = \varepsilon^{-1} \int_0^1 (v(z) - v)(\xi)) dz \tag{3}$$

where $\{v(z), z \in [0,1]\}$ is an arbitrary real continuous function. We assume that in absence of contacts the populations evolve independently in compliance with the logistic law:

$$\frac{dx_\varepsilon(t)}{dt} = r_1 x_\varepsilon(t)(K_1 - x_\varepsilon(t))K_1^{-1}, \frac{dy_\varepsilon(t)}{dt} = r_2 y_\varepsilon(t)(K_2 - y_\varepsilon(t))K_2^{-1} \tag{4}$$

and at moments of contacts they have the jumps given by the equations:

$$x_\varepsilon(t) - x_\varepsilon(t-) = -\varepsilon K_1^{-1} a(\xi(t-)) x_\varepsilon(t-) y_\varepsilon(t-), y_\varepsilon(t) - y_\varepsilon(t-) = -\varepsilon K_2^{-1} b(\xi(t-)) x_\varepsilon(t-) y_\varepsilon(t-) \tag{5}$$

where $a(z), b(z)$ are real continuous functions on segment [0,1]. Let $\{\mathcal{F}^t, t \geq 0\}$ be the minimal filtration [8] for the Markov process $\{\xi(t), t \geq 0\}$. By definition the two-dimensional random process $\{x_\varepsilon(t), y_\varepsilon(t), t \geq 0\}$ is $\mathcal{F}^t$-adopted and possesses the Markov property [8]. Besides, the dynamical characteristics given by equations (4)-(5) are homogeneous in time. Therefore, the process $\{x_\varepsilon(t), y_\varepsilon(t), t \geq 0\}$ may be uniquely defined by the infinitesimal generator [8], which one can find calculated the limit

$$(L(\varepsilon)v)(x, y) := \lim_{t \downarrow 0} \frac{1}{t} \left[\mathbf{E}\{v(x_\varepsilon(t), y_\varepsilon(t)) / x_\varepsilon(0) = x, y_\varepsilon(0) = y\} - v(x, y) \right] \tag{6}$$

for an arbitrary sufficiently smooth bounded function $v(x, y)$. Remind that by definition [8] of the Poisson process with infinitesimal generator (3) the time-intervals $\{\Delta_k = \tau_k - \tau_{k-1}, k \in \mathbf{N}, \tau_0 = 0\}$ are independent, do not dependent on the sequence $\{\xi(\tau_k), k \in \mathbf{N}\}$ and have an exponential distribution with parameter ε^{-1}. Not so difficult to calculate the limit (6):

$$(L(\varepsilon)v)(x,y)=v'_x(x,y)\left[rx(K_1-x)\right]K_1^{-1}+v'_y(x,y)\left[ry(K_2-y)\right]K_2^{-1}y+$$
$$+\varepsilon^{-1}\left[\int_0^1\mathbf{E}\{v(x-\varepsilon a(\xi)xy,y-\varepsilon b(\xi)xy)\}d\xi-v(x,y)\right] \tag{7}$$

3 Stochastic asymptotic analysis of population dynamics

The assumption that the populations have rapid contacts implies the smallness of time intervals Δ_k that have the exponential distribution with parameter ε. This parameter we will use for approximation of trajectories $\{x_\varepsilon(t),y_\varepsilon(t),t\geq 0\}$ applying the stochastic approximation procedure proposed in [5,6,7]. The first step of this procedure is a construction of the ordinary differential equations for the mean values $\overline{x}(t):=\mathbf{E}\{x_\varepsilon(t)\}$ and $\overline{y}(t):=\mathbf{E}\{y_\varepsilon(t)\}$ of populations species. For that we should pass to limit in (7) rushing ε to zero: $\lim\limits_{\varepsilon\to 0}(L(\varepsilon)v)(x,y)=v'_x(x,y)x\left[r(1-K_1^{-1}x)+\alpha y\right]+v'_y(x,y)y\left[ry(1-K_2^{-1}y)+\beta x\right]$

where $\alpha=\mathbf{E}\{a(\xi)\}=\int_0^1 a(\xi)d\xi,\beta=\mathbf{E}\{b(\xi)\}=\int_0^1 b(\xi)d\xi$. This means that the mean values of the populations species satisfy the same equations as (1):

$$\frac{d\overline{x}}{dt}=r_1\overline{x}(K_1-\overline{x}-\alpha\overline{y})K_1^{-1},\frac{d\overline{y}}{dt}=r_2\overline{y}(K_2-\overline{y}-\beta\overline{x})K_2^{-1} \tag{8}$$

Not so difficult to ensure that A_1, A_2 and A_3 are equilibrium points for the dynamical system (4)-(5) and the system (8). As it has been proved in [6] for sufficiently small $\varepsilon>0$ the dynamical systems (4)-(5) and (8) have the same local stability properties of the above equilibriums. The sample trajectories of the solutions of (4)-(5) jointly with solutions of equations (8) (magnitudes of parameters $r_1=1,\ r_2=1,\ K_1=40,\ \varepsilon=0.05, K_2=50$) are shown below on the Fig. 1. As we can see the solutions of the system (4)-(5) preserve behavior pattern of the corresponding solutions of the equation (8). The equilibrium point A_5 of the system (8) is not an equilibrium point for the dynamical system (4)-(5). If $r_1K_1>\alpha K_2, r_2K_2>\beta K_1$, then as we can see at the Fig.1, the sample trajectories of the dynamical system (4)-(5) have relatively large random deviations $\left\{x_\varepsilon(t)-\overline{x}(t),y_\varepsilon(t)-\overline{y}(t)\right\}$ from the corresponding trajectories of the average equations (8). To estimate the above mentioned deviations we need to do the second step of the diffusion approximation procedure.

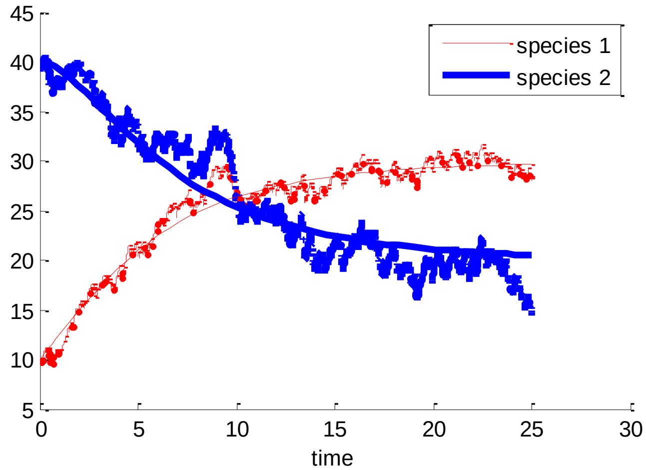

Fig. 1. $r_1K_1 < \alpha K_2, r_2K_2 < \beta K_1, x(0) = 10, y(0) = 40, \alpha = 0.5, \beta = 1$

As it has been shown in [6] for this aim we have to introduce the two-dimensional random processes of *the normalized deviations:*

$$z_\varepsilon(t) := \frac{x_\varepsilon(t) - \bar{x}(t)}{\sqrt{\varepsilon}}, u_\varepsilon(t) := \frac{y_\varepsilon(t) - \bar{y}(t)}{\sqrt{\varepsilon}} \tag{9}$$

which by definition satisfies differential equations

$$\begin{cases} \dfrac{dz_\varepsilon(t)}{dt} = \dfrac{r_1K_1^{-1}}{\sqrt{\varepsilon}}\left[\alpha\bar{x}(t)\bar{y}(t) + \sqrt{\varepsilon}z_\varepsilon(t)(K_1 - 2\bar{x}(t)) - \varepsilon\left(z_\varepsilon(t)\right)^2\right], \\ \dfrac{dy_\varepsilon(t)}{dt} = \dfrac{r_2K_2^{-1}}{\sqrt{\varepsilon}}\left[\beta\bar{x}(t)\bar{y}(t) + \sqrt{\varepsilon}u_\varepsilon(t)(K_2 - 2\bar{y}(t)) - \varepsilon\left(u_\varepsilon(t)\right)^2\right] \end{cases} \tag{10}$$

for $t \notin \mathrm{T}$ and have the jumps

$$\begin{cases} z_\varepsilon(t) - z_\varepsilon(t-) = -\sqrt{\varepsilon}K_1^{-1}a(\xi(t-))\left[\bar{x}(t)\bar{y}(t) + \sqrt{\varepsilon}\left(z_\varepsilon(t-))\bar{y}(t) + \bar{x}(t)u_\varepsilon(t-)\right) + \varepsilon z_\varepsilon(t-)u_\varepsilon(t-)\right], \\ u_\varepsilon(t) - u_\varepsilon(t-) = -\sqrt{\varepsilon}K_2^{-1}b(\xi(t-))\left[\bar{x}(t)\bar{y}(t) + \sqrt{\varepsilon}(u_\varepsilon(t-))\bar{x}(t) + z_\varepsilon(t-)\bar{y}(t)) + \varepsilon z_\varepsilon(t-)u_\varepsilon(t-)\right] \end{cases} \tag{11}$$

at time moments $t \in \mathrm{T}$. $t \in \mathrm{T}$. Like in previous section we can prove that defined by equations (8)-(10)-(11) four-dimensional process $\{\bar{x}(t), \bar{y}(t), z_\varepsilon(t), u_\varepsilon(t), t \geq 0\}$ on the probability space $(\Omega, \mathfrak{F}, \mathfrak{F}^t, \mathbf{P}, t \geq 0)$ is homogeneous and possess the Markov property. To avoid cumbersome formulas, we will use the following notation $\mathbf{E}_{xy}^{zu}\{\bullet\} = \mathbf{E}\{\bullet / \bar{x}(0) = x, \bar{y}(0) = y, z_\varepsilon(0) = z, u_\varepsilon(0) = u\}$ for conditional expectation. The weak infinitesimal operator of this Markov process $\{\bar{x}(t), \bar{y}(t), z_\varepsilon(t), u_\varepsilon(t), t \geq 0\}$ should be calculated by formula:

$$
\begin{aligned}
&(\mathcal{L}(\varepsilon)v)(x,y,z,u)=\lim_{t\downarrow 0}\frac{1}{t}\Big[\mathbf{E}_{xy}^{zu}\{v(\overline{x}(t),\overline{y}(t),z_{\varepsilon}(t),u_{\varepsilon}(t))\}-v(x,y,z,u)\Big]=\\
&=v'_x(x,y,z,u)r_1x(K_1-x-\alpha y)K_1^{-1}+v'_y(x,y,z,u)r_2y(K_2-y-\beta x)K_2^{-1}+\\
&+\frac{1}{\sqrt{\varepsilon}}[v'_z(x,y,z,u)r_1K_1^{-1}\alpha xy+\beta xyv'_u(x,y,z,u)r_2K_2^{-1}\beta xy]+\\
&+v'_z(x,y,z,u)r_1K_1^{-1}z(K_1-2x)+v'_u(x,y,z,u)r_2K_2^{-1}u(K_2-2y)-\\
&-\sqrt{\varepsilon}[v'_z(x,y,z,u)r_1K_1^{-1}z^2+v'_u(x,y,z,u)r_2K_2^{-1}u^2]+\\
&+\varepsilon^{-1}\mathbf{E}\Big\{v\Big(x,y,z-\sqrt{\varepsilon}K_1^{-1}a(\xi)\Big[xy+\sqrt{\varepsilon}\left(zy+xu\right)+\varepsilon zu\Big],u-\sqrt{\varepsilon}K_2^{-1}b(\xi)\Big[xy+\sqrt{\varepsilon}(ux+zy)+\varepsilon zu\Big]\Big)\Big\}-\\
&-\varepsilon^{-1}v(x,y,z,u)
\end{aligned}
$$

According to stochastic approximation procedure [5] we should pass to limit in the previous formula for $\varepsilon\to 0$, assumed that $v(x,y,z,u)$ is sufficiently smooth function:

$$
\begin{aligned}
&(\hat{\mathcal{L}}v)(x,y,z,u):=\lim_{\varepsilon\to 0}(\mathcal{L}(\varepsilon)v)(x,y,z,u)=\\
&=v'_x(x,y,z,u)r_1x(K_1-x-\alpha y)K_1^{-1}+v'_y(x,y,z,u)r_2y(K_2-y-\beta x)K_2^{-1}+\\
&+v'_z(x,y,z,u)r_1K_1^{-1}[z(K_1-2x)-\alpha\left(zy+xu\right)]+v'_u(x,y,z,u)r_2K_2^{-1}[u(K_2-2y)-\beta\left(zy+xu\right)]+\\
&+\frac{1}{2}(xy)^2\Big[K_1^{-2}\sigma_1^2v''_{zz}(x,y,z,u)+K_2^{-2}\sigma_2^2v''_{uu}(x,y,z,u)+2K_1^{-1}K_2^{-1}\sigma_{12}v''_{uz}(x,y,z,u)\Big]
\end{aligned}
\tag{12}
$$

where $\sigma_1^2=\mathbf{E}\{a^2(\xi)\},\sigma_2^2=\mathbf{E}\{b^2(\xi)\},\sigma_{12}=\mathbf{E}\{a(\xi)b(\xi)\}$. The defined by this formula operator $\hat{\mathcal{L}}$ may be interpreted [5,7] as the weak infinitesimal operator of the Markov process $\{\overline{x}(t),\overline{y}(t),\hat{z}(t),\hat{u}(t),t\ge 0\}$. This infinitesimal operator may be used to define on the probability space $(\Omega,\mathcal{F},\mathbf{P})$ the systems of Ito stochastic linear inhomogeneous differential equations with dependent on $\{\overline{x}(t),\overline{y}(t)\}$ drift and diffusion:

$$
\begin{cases}
d\hat{z}=r_1K_1^{-1}[\hat{z}(K_1-2\overline{x}(t))-\alpha\left(\hat{z}\overline{y}(t)+\overline{x}(t)\hat{u}\right)]dt+\dfrac{1}{\sqrt{2}}\overline{x}(t)\overline{y}(t)\Big[K_1^{-1}\sigma_1dw_1(t)+\sqrt{2K_1^{-1}K_2^{-1}\sigma_{12}}dw_2(t)\Big],\\
d\hat{u}=r_2K_2^{-1}[\hat{u}(K_2-2\overline{y}(t))-\beta\left(\hat{z}\overline{y}(t)+\overline{x}(t)\hat{u}\right)]dt+\dfrac{1}{\sqrt{2}}\overline{x}(t)\overline{y}(t)\Big[K_2^{-1}\sigma_2dw_1(t)+\sqrt{2K_1^{-1}K_2^{-1}\sigma_{12}}dw_2(t)\Big]
\end{cases}
\tag{13}
$$

where $w_1(t)$ and $w_2(t)$ are standard independent Wiener processes and $\{\overline{x}(t),\overline{y}(t),t\ge 0\}$ is the solution of the equation system (8) with initial condition $\overline{x}(0)=x_{\varepsilon}(0)$, $\overline{y}(0)=y_{\varepsilon}(0)$ and $\{\hat{z}(t),\hat{u}(t),t\ge 0\}$ be the solution of the system (13). The two-dimensional process $\{X_{\varepsilon}(t):=\overline{x}(t)+\hat{z}(t)\sqrt{\varepsilon},\mathrm{Y}_{\varepsilon}(t)=:\overline{y}(t)+\hat{u}(t)\sqrt{\varepsilon},t\ge 0\}$ where $\{\hat{z}(t),\hat{u}(t)\}$ is the solution of the equations (13) with the trivial initial conditions $\hat{z}(0)=\hat{u}(0)=0$ is called *the Gaussian approximation* of the process $\{x_{\varepsilon}(t),y_{\varepsilon}(t),t\ge 0\}$.

Therefore $\mathrm{E}\{\hat{z}(t)\}\equiv 0$ and $\mathrm{E}\{\hat{u}(t)\}\equiv 0$. Not so difficult to ensure that if $\lim_{t\to\infty}\overline{x}(t)\overline{y}(t)=0$ (the cases 1,2,3 and 5, section 1, the second page of this article) then $\lim_{t\to\infty}\mathrm{E}\{\hat{z}^2(t)\}=0$ and $\lim_{t\to\infty}\mathrm{E}\{\hat{u}(t)\}=0$. This means if $\lim_{t\to\infty}\overline{x}(t)\overline{y}(t)=0$ then the Gaussian approximation process has the same behavior as the corresponding solutions of the initial impulsive differential equations (4)-(5). But if $r_1K_1 > \alpha K_2, r_2K_2 > \beta K_1$ then $\lim_{t\to\infty} \overline{x}(t) = \frac{K_1 - \alpha K_2}{1-\alpha\beta} := \overline{\overline{x}}$ and $\lim_{t\to\infty} \overline{y}(t) = \frac{K_2 - \beta K_1}{1-\beta\alpha} := \overline{\overline{y}}$, that is, the drift and diffusion coefficients of the equation (12) stabilize. The corresponding to this limit stochastic differential equation for the vector $\vec{z}(t) = \begin{pmatrix} \hat{z}(t) \\ \hat{u}(t) \end{pmatrix}$ may be given as follows:

$$d\vec{z}(t) = A\vec{z}(t)dt + \Sigma d\vec{w}(t) \tag{14}$$

where $\vec{z}(t) = \begin{pmatrix} \hat{z}(t) \\ \hat{u}(t) \end{pmatrix}, \vec{w}(t) = \begin{pmatrix} w_1(t) \\ w_2(t) \end{pmatrix}$, $\Sigma = \frac{1}{\sqrt{2}} \overline{\overline{xy}} \begin{pmatrix} K_1^{-1}\sigma_1 & \sqrt{2K_1^{-1}K_2^{-1}\sigma_{12}} \\ K_2^{-1}\sigma_2 & \sqrt{2K_1^{-1}K_2^{-1}\sigma_{12}} \end{pmatrix}$,

$$A = \begin{pmatrix} r_1K_1^{-1}(K_1 - 2\overline{\overline{x}} - \alpha\overline{\overline{y}}) & -r_1K_1^{-1}\alpha\overline{\overline{x}} \\ -\beta r_2K_2^{-1}\overline{\overline{y}} & r_2K_2^{-1}(K_2 - 2\overline{\overline{y}} - \beta\overline{\overline{x}}) \end{pmatrix} \tag{15}$$

Applying the Cauchy formula [5] we can find a vector solution of the equation (14) with initial condition $\vec{Z}(t_0) = \vec{Z}_0$ as follows:

$$\vec{Z}(t) = \exp\{(t-t_0)A\}\vec{Z}_0 + \int_{t_0}^{t} \exp\{(t-s)A\}\Sigma d\vec{w}(s) \tag{16}$$

Recall that we should apply the formula (16) with $t_0 = 0$ and $\vec{Z}(0) = \vec{0}$. The sample trajectories for the coordinates $\{X_\varepsilon(t), Y_\varepsilon(t)\}$ of corresponding to magnitudes of parameters $r_1 = 1,\ r_2 = 1,\ K_1 = 40,\ \varepsilon = 0.05,\ K_2 = 50, \alpha = 0.5,\ \beta = 1,\ x(0) = 10, y(0) = 40$ Gaussian approximation are shown below on the Fig. 2.

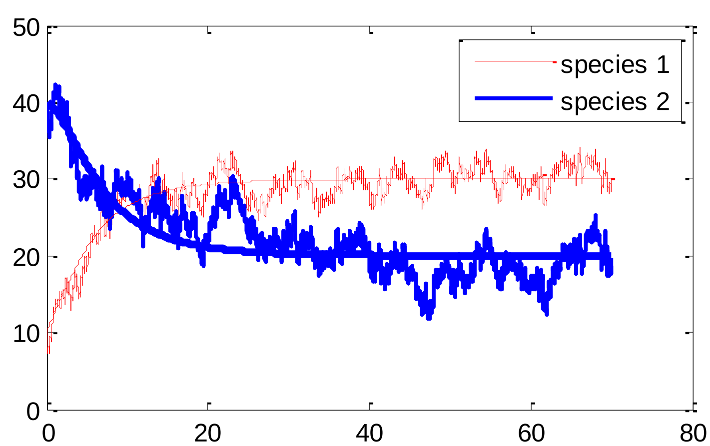

Fig.2. The sample trajectories of the Gaussian approximation.

Under assumptions $r_1K_1 > \alpha K_2, r_2K_2 > \beta K_1$ the eigenvalues of the matrix A are negative. Therefore, with probability one there exists a limit [5]:

$$\lim_{t_0 \to -\infty} \vec{Z}(t) = \tilde{Z}(t) := \begin{pmatrix} \tilde{z}(t) \\ \tilde{u}(t) \end{pmatrix} = \int_{-\infty}^{t} \exp\{(t-s)A\} \Sigma d\vec{w}(s) \tag{17}$$

Not so difficult to ensure that this integral defines the two-dimensional Gaussian process with zero mean and constant covariance matrix $\hat{Q} = \int_0^\infty \exp\{tA\} \Sigma^2 dt$. This means that for sufficiently large *t >0* the probability distribution function for the coordinates of process (17) may be approximated as a two-dimensional Gaussian distribution
$\mathrm{P}\left(X_\varepsilon(t) \le X, Y_\varepsilon(t) \le Y \right) \approx \mathrm{P}\left(\bar{x}(t) + \sqrt{\varepsilon}\xi \le X, \bar{y}(t) + \sqrt{\varepsilon}\eta \le Y \right)$ where random vector $\{\xi,\eta\}$ has the zero mean and covariance matrix $\hat{Q}$ defined by equation $A\hat{Q} + \hat{Q}A^T = -\Sigma^2$.

4 Conclusion

Applying the asymptotic methods of the contemporary stochastic analysis [5,6,7] we have constructed the average differential equations for the described in Section 2 Markov impulsive differential model for interspecific competition of two populations in a form (1) and the stochastic differential equations for the Gaussian deviations of the random trajectories on the corresponding solutions of the average equations.

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