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RANKING OF BRIDGE DESIGN ALTERNATIVES: A TOPSIS-FADR METHOD

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Abstract. Bridges are considered as essential structures of the transport infrastructures, which play an essential role in any road network. Therefore, the process of planning and designing bridges needs to be made efficiently. The design of bridges usually consists of two stages: conceptual design and detailed design. Designers make decisions on the overall form of the structure in the conceptual design process. This process is defined as Multi-Criteria Decision-Making problems. In this study, a modified fuzzy Technique for Order of Preference by Similarity to Ideal Solution method to deal with the conceptual design process under uncertainty is proposed. The proposed method uses an area-based deviation ratio to determine the degree of difference between alternatives and reference solutions of the Technique for Order of Preference by Similarity to Ideal Solution method. Using this ratio incorporates the effects of the membership functions into the evaluation process. To illustrate the procedure of the proposed method, an example of multi-criteria assessment of bridge design including three Multi-Criteria Decision-Making problems with quantitative and qualitative criteria is used. For validation of the results of the

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modified fuzzy Technique for Order of Preference by Similarity to Ideal Solution method, a comparative analysis is also made. The analysis shows that the results of the proposed method are consistent with the other method.

Keywords: area-based deviation, bridge, fuzzy MCDM, MCDM, ranking of design alternatives, TOPSIS, TOPSIS-FADR.

Introduction

Bridges are considered as cruical structures of the transport infrastructure systems and fundamental elements in any road network. A combination of art and compromise forms the basis of the process of planning and designing bridges. The design of bridges is the most significant aspect of structural engineering and a visible sign of creativity of designers (Duan & Chen, 1999). Generally, the service life of a bridge is divided into different phases, and the design and construction is the first phase and the most important one (Rashidi & Gibson, 2012). The conceptual design and detailed design are the two main steps in designing any structures. In the conceptual design, decisions about the overall form of the structure are made.

On the other hand, more detailed analysis and calculations are carried out in the detailed design for verification of the conceptual design choice (Miles & Moore, 1991). An excellent detailed analytical design barely compensates a poor conceptual design, therefore the importance of making correct decisions in the early stages of the design process is well-understood now (Machwe & Parmee, 2007). The focus of this research is on the conceptual design process. This process incorporates many sub-processes such as assessment of structural systems, construction methods, and materials into the design of bridges. Each of these sub-processes is usually affected by several dimensions.

Because of the multidimensional nature of the conceptual design of bridges, this process is classified as a Multi-Criteria Decision-Making (MCDM) process (Ohkubo, Dissanayake, & Taniwaki, 1998). There are several MCDM methods and techniques which have been used in different fields of science and engineering (Keshavarz Ghorabaee, Amiri, Zavadskas, & Antucheviciene, 2017; Mardani, Jusoh, Nor, Khalifah, Zakwan, & Valipour, 2015; Mardani, Jusoh, Zavadskas, Kazemilari, Ungku, & Khalifah, 2016; Mardani, Zavadskas, Khalifah, Jusoh, & Nor, 2016). SAW (Simple Additive Weighting), AHP (Analytic Hierarchy Process), WASPAS (Weighted Aggregated Sum Product ASsessment), COPRAS (COmplex PRoportional ASsessment), PROMETHEE (Preference Ranking Organization METHod for

Enrichment Evaluations), VIKOR (translation from Serbian – VIseKriterijumska Optimizacija I Kompromisno Resenje), TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution), EDAS (Evaluation based on Distance from Average Solution) and SWARA (Step-wise Weight Assessment Ratio Analysis) are some of the efficient MCDM methods (Keshavarz Ghorabaee, Zavadskas, Turskis, & Antucheviciene, 2016). Many studies have been made on the applications of MCDM approaches in different stages of bridge construction such as design, maintenance, risk assessment (de Lurdes Antunes, Marecos, Neves, & Morgado, 2016; Mojtahedi, Mousavi, & Makoui, 2008). In real-world problems, the information used in the process of evaluation is rarely precise (Makui, Mojtahedi, & Mousavi, 2010; Mousavi, Vahdani, & Behzadi, 2016). Because of the uncertainties, ambiguity, and imprecision of information, researchers use different tools to deal with the MCDM problems. The fuzzy sets theory is one of the most efficient tools to deal with the problems under uncertainty. Table 1 presents some of the studies, which utilize MCDM approaches to handle different problems in the bridge construction process.

The TOPSIS method is one the most popular MCDM methods, and fuzzy extensions of this method have also been applied to many real-world MCDM problems (Zavadskas, Mardani, Turskis, Jusoh, & Nor, 2016). As seen in Table 1, the TOPSIS method is suitable to use in the field of bridge construction. The applications of the fuzzy variants of this method are enormous (Mojtahedi, Mousavi, & Aminian, 2008; Roshanaei, Vahdani, Mousavi, Mousakhani, & Zhang, 2013). Table 2 briefly presents some recent applications of fuzzy TOPSIS methods in different fields and categorizes them into two types: single approaches and hybrid approaches.

Table 1. Applications of the Multi-Criteria Decision-Making approaches in bridge construction

No.	Author(s) and year	Multi-Criteria Decision-Making approach	Description
1	Youssef, Anumba, & Thorpe (2005)	The AHP method	An intelligent Decision Support System based on the AHP method that helps construction professionals and designers during the early stages of a construction project. An application to evaluate construction methods alternatives for different concrete bridges in Egypt.

No.	Author(s) and year	Multi-Criteria Decision-Making approach	Description
2	Wang & Elhag (2006)	Fuzzy TOPSIS method	A fuzzy TOPSIS method based on alpha level sets and nonlinear programming, and its application in bridge risk assessment.
3	Liu & Frangopol (2006)	Multiple Attribute Utility Theory (MAUT)	A Decision Support System for maintenance planning of bridge networks.
4	Wang & Elhag (2007)	Fuzzy weighted average	A fuzzy group decision-making approach for bridge risk assessment.
5	Lu, Lin, & Ko (2007)	Analytic Network Process (ANP)	Application of the ANP method to handle the risk of principal activities of an urban bridge project.
6	Wang, Liu, & Elhag (2008)	AHP and Data Envelopment Analysis (DEA)	An integrated AHP–DEA methodology for evaluation of bridge risks in different bridge structures based on the maintenance priorities.
7	Pan (2008)	Fuzzy AHP	A fuzzy AHP that employs fuzzy numbers and α-cut to handle the uncertainties of subjective judgments and a case study of bridge construction methods evaluation.
8	Malekly, Mousavi, & Hashemi (2010)	TOPSIS and Quality Function Deployment (QFD)	A fuzzy integrated approach based on TOPSIS and QFD methods and an application to evaluation of conceptual bridge design.
9	Wang, Fan, & Hastak (2011)	The AHP method	A methodology based on the AHP method to determine the weights of criteria for bridge performance in different bridge structures.
10	Gervásio & Da Silva (2012)	PROMETHEE and AHP	A hybrid decision-making approach based on the PROMETHEE and AHP method, and an application to bridge assessment concerning environmental, economic and social criteria.
11	Aghdaie, Hashemkhani Zolfani, & Zavadskas (2012)	COPRAS and AHP	Application of the AHP and COPRAS methods for evaluation and selection of locations for constructing footbridges in Iran.
12	Salem, Miller, Deshpande, & Arurkar (2013)	The AHP method	A decision-making system based on the AHP method that is utilized to extract weights of both quantitative and qualitative criteria to select a bridge construction plan.

No.	Author(s) and year	Multi-Criteria Decision-Making approach	Description
13	Chaphalkar & Shirke (2013)	Fuzzy AHP and fuzzy TOPSIS	An integrated MCDM approach and its application to selection of bridge type in Pune city.
14	Ardeshir, Mohseni, Behzadian, & Errington (2014)	Geographic information system (GIS) and fuzzy AHP	A hybrid decision-making method based on the GIS and AHP methods and its application to bridge construction site selection.
15	Bitarafan, Zolfani, Arefi, Zavadskas, & Mahmoudzadeh, (2014)	SWARA and WASPAS	An integrated MCDM approach based on the SWARA and WASPAS methods to evaluate the real-time intelligent sensors for monitoring the structural health of bridges.
16	Yadollahi, Ansari, Abd Majid, & Yih (2015)	The AHP method	Study on bridge sustainability issues concerning environmental, economic and social aspects and application of MCDM to assess the sustainability of a bridge in Malaysia.
17	Jakiel & Fabianowski (2015)	Fuzzy AHP	Evaluation of structural and technological aspects of the highway reinforced concrete bridges using a fuzzy AHP method.
18	Sultana & Rasel (2016)	Evidential Reasoning	Selection of an appropriate location for bridge construction based on both qualitative and quantitative criteria using Evidential Reasoning.
19	Rashidi, Samali, & Sharafi (2016)	The AHP method	A requirements-driven MCDM methodology based on AHP for remediation of concrete bridges for maintaining bridges in an acceptable limit of serviceability, safety, and sustainability.
20	Bansal, Singh, & Singh (2017)	Fuzzy VIKOR	Application of a fuzzy VIKOR method for sustainability evaluation of iconic bridge corridors.
20	Bansal, Singh, & Singh (2017)	Fuzzy VIKOR	Application of a fuzzy VIKOR method for sustainability evaluation of iconic bridge corridors.

Table 2. Applications of fuzzy Technique for Order of Preference by Similarity to Ideal Solution approach in different fields

No.	Author(s) and year	Type of the approach	Description
1	Taylan, Bafail, Abdulaal, & Kabli (2014)	Hybrid	Integration of the fuzzy AHP and fuzzy TOPSIS methodologies for construction projects selection and risk assessment.
2	Kannan, de Sousa Jabbour, & Jabbour (2014)	Single	A framework based on the green supply chain management practices and application of the fuzzy TOPSIS method for supplier selection.
3	Guo & Zhao (2015)	Single	A Multi-Criteria Decision-Making approach to consider sustainability criteria for electric vehicle charging station site selection.
4	Roszkowska & Wachowicz (2015)	Single	Application of the fuzzy TOPSIS method in ill-structured negotiations to support the process of making the scoring system for negotiation offer.
5	Beikkhakhian, Javanmardi, Karbasian, & Khayambashi (2015)	Hybrid	Integration of fuzzy AHP and fuzzy TOPSIS methods and using interpretive structural model for evaluating agile suppliers.
6	Şengül, Eren, Shiraz, Gezder, & Şengül (2015)	Single	Application of the fuzzy TOPSIS method for ranking renewable energy supply systems in Turkey.
7	Sang, Liu, & Qin (2015)	Single	An analytical solution to the fuzzy TOPSIS method and its application to personnel selection for knowledge-intensive enterprises.
8	Beskese, Demir, Ozcan, & Okten (2015)	Hybrid	Application of the fuzzy AHP and fuzzy TOPSIS methods to multi-criteria evaluation and selection of landfill sites.
9	Onat, Gumus, Kucukvar, & Tatari (2016)	Single	Application of an intuitionistic fuzzy TOPSIS method to the evaluation of the life cycle sustainability performance of vehicle technologies.
10	Zare, Nouri, Abdoli, & Atabi (2016)	Hybrid	Using the fuzzy TOPSIS method and Life Cycle Assessment for industrial waste management in the aluminium industry.

No.	Author(s) and year	Type of the approach	Description
11	Mittal, Chandra Tewari, Khanduja, & Kaushik (2016)	Single	Application of the fuzzy TOPSIS approach for evaluation processes in the problems of plywood industry.
12	Selim, Yunusoglu, & Yılmaz Balaman (2016)	Hybrid	A framework based on the fuzzy TOPSIS and Failure Mode and Effects Analysis for maintenance planning in a company.
13	Suder & Kahraman (2016)	Single	Evaluation of innovation investments based on conflicting tangible and intangible criteria using the fuzzy TOPSIS method.
14	Cavallaro, Zavadskas, & Raslanas (2016)	Hybrid	Application of the fuzzy Shannon Entropy and the fuzzy TOPSIS methods for evaluation of combined heat and power systems.
15	Ravasan, Hanafizadeh, Olfat, & Taghavifard (2017)	Single	Application of the fuzzy TOPSIS method for evaluation and selection of the appropriate E-banking outsourcing strategy.
16	Yang, Chen, & Zhang (2017)	Single	Application of an intuitionistic fuzzy TOPSIS method in supplier selection problem and the evaluation of murals in a metro line.
17	Hatami-Marbini & Kangi (2017)	Single	A modified fuzzy TOPSIS method and its application to selection of stocks in the Tehran stock exchange.
18	Polat, Eray, & Bingol (2017)	Hybrid	Application of fuzzy AHP and fuzzy TOPSIS for the supplier selection problem and selecting the best rail supplier.
19	Estay-Ossandon, Mena-Nieto, & Harsch (2018)	Single	Using a scenario analysis based on the fuzzy TOPSIS method to improve municipal solid waste forecasting and planning.
20	Rostamzadeh, Ghorabaee, Govindan, Esmaeili, & Nobar (2018)	Hybrid	Application of the fuzzy TOPSIS and CRITIC (CRiteria Importance Through Inter-criteria Correlation) methods for multi-criteria evaluation of sustainable supply chain risk management.
21	Shen, Ma, Li, Xu, & Cai (2018)	Hybrid	An extended intuitionistic fuzzy TOPSIS method and its application to credit risk evaluation of potential strategic partners.

The TOPSIS method uses distances of alternatives from positive-ideal and negative-ideal solutions for evaluating alternatives. In fuzzy variants of this method, the distances determined based on fuzzy distance methods which are unlikely to include the membership function in the calculation of distances. In this paper, a modified TOPSIS method is proposed based on an area-based deviation ratio (TOPSIS-FADR). The deviation ratio is used to determine the difference between alternatives and the positive-ideal and negative-ideal solutions based on the area under the membership function of fuzzy subtractions. Using the proposed approach, the effect of membership functions is incorporated into the evaluation process of alternatives. In this study, Triangular Fuzzy Numbers (TFNs) are used to deal with the uncertainty. An illustrative example is presented to show the application of the proposed modified TOPSIS method in the multi-criteria assessment of bridge design. In addition, the results of the proposed method are compared to the results of the SAW, WASPAS, COPRAS, TOPSIS, VIKOR and EDAS methods. The validity of the results and efficiency of the proposed method are show

The rest of this paper is organized as follows. In Section 1, the methodology is detailed. Some essential concepts of the fuzzy sets theory are presented. Definitions of the TFNs and the definition related to the calculation of area-based deviations are listed in the first sub-section of Section 1. Then the modified fuzzy TOPSIS method is proposed in this section. Section 2 presents an illustrative example in which the proposed method is applied to multi-criteria bridge design assessment problems. Also, the results of the comparative analysis are shown in this section. Finally, conclusions are discussed.

1. Methodology

Some concepts and definitions about the fuzzy set theory and operations on fuzzy numbers are first presented in this section. Then a modified TOPSIS method is proposed based on the presented definitions.

1.1. Concepts and definitions

If the complexity of a system increases, making a precise and meaningful model for describing its behaviours will be difficult. Uncertainty in such complex systems is modelled by different theories. The quantity of available information and the type of it, the requirements of the observer, and the causes of uncertainty are some of the critical factors. They affect the choice of the way for modelling of systems. The fuzzy sets theory is one of the theories that is used use in specific circumstances. This theory was introduced by Zadeh (1965)

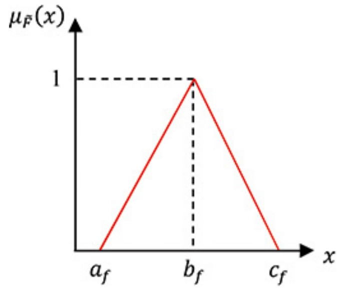

Figure 1. A Triangular Fuzzy Number

to reach approximate solutions for real-world problems efficiently. The fuzzy set theory helps to reduce the complexity of information using linguistic variables or fuzzy data analysis. In the following, some definitions of this theory are presented.

Definition 1. Let U denotes the universe which is a classical set of objects, and x denotes the generic elements of U. A fuzzy set $\tilde{F}$ is defined using a membership function $\mu_{\tilde{F}}(x) \in [0,1]$ associated with each element of the universe. A fuzzy set is mathematically defined by a set of pairs shown as follows:

$$\tilde{F} = \{(x, \mu_{\tilde{F}}(x)), x \in U\}. \tag{1}$$

Definition 2. The fuzzy numbers are individual cases of fuzzy sets which have some properties like being convex and normal (Wang & Lee, 2007).

Definition 3. The following membership function is used to describe a TFN $\tilde{F}$. This fuzzy number can also be defined as a triplet $\tilde{F} = (a_f, b_f, c_f)$.

$$\mu_{\tilde{F}}(x) = \begin{cases} (x - a_f)/(b_f - a_f), & a_f \le x \le b_f \\ (c_f - x)/(c_f - b_f), & b_f \le x \le c_f \\ 0, & otherwise. \end{cases} \tag{2}$$

This fuzzy number is depicted in Figure 1.

Definition 4. Suppose that $\tilde{F} = (a_f, b_f, c_f)$ and $\tilde{G} = (a_g, b_g, c_g)$ are two TFNs where $a_f \ge 0$ and $a_g \ge 0$, and is a crisp number. The arithmetic operations of these fuzzy numbers are as follows (Chen & Hwang, 1992):

– addition

$$\tilde{F} \oplus \tilde{G} = (a_f + a_g, b_f + b_g, c_f + c_g), \tag{3}$$

$$\tilde{F} + d = (a_f + d, b_f + d, c_f + d); \tag{4}$$

– subtraction

$$\tilde{F} \ominus \tilde{G} = (a_f - c_g, b_f - b_g, c_f - a_g), \tag{5}$$

$$\tilde{F} - d = (a_f - d, b_f - d, c_f - d); \tag{6}$$

– multiplication

$$\tilde{F} \otimes \tilde{G} = (a_f \cdot a_g, b_f \cdot b_g, c_f \cdot c_g), \quad (7)$$

$$\tilde{F} \cdot d = \begin{cases} (a_f \cdot d, b_f \cdot d, c_f \cdot d) & \text{if} \quad d \geq 0 \\ (c_f \cdot d, b_f \cdot d, a_f \cdot d) & \text{if} \quad d < 0 \end{cases}; \quad (8)$$

– division

$$\tilde{F} \oslash \tilde{G} = \left(\frac{a_f}{c_g}, \frac{b_f}{b_g}, \frac{c_f}{a_g}\right), \quad (9)$$

$$\frac{\tilde{F}}{d} = \begin{cases} \left(\frac{a_f}{d}, \frac{b_f}{d}, \frac{c_f}{d}\right) & if \quad d > 0 \\ \left(\frac{c_f}{d}, \frac{b_f}{d}, \frac{a_f}{d}\right) & if \quad d < 0. \end{cases} \quad (10)$$

Definition 5. According to the centroid of the fuzzy numbers defined by Wang, Yang, Xu, & Chin (2006), the defuzzified value of a TFN $\tilde{F} = (a_f, b_f, c_f)$ is be calculated as follows:

$$\mathfrak{D}(\tilde{F}) = \frac{1}{3}(a_f + b_f + c_f). \quad (11)$$

Definition 6. Let us define $\tilde{F} = (a_f, b_f, c_f)$ and $\tilde{G} = (a_g, b_g, c_g)$ as two TFNs, and $\tilde{D} = \tilde{F} \ominus \tilde{G}$ is the fuzzy subtraction of $\tilde{F}$ from $\tilde{G}$ where $\tilde{D} = (a_d, b_d, c_d)$. A deviation measure based on the area under the membership function of $\tilde{D}$ is defined. The areas under the membership function of $\tilde{D}$ for $x \geq 0$ and $x \leq 0$ are called Positive Area-Based Deviation (PAD) and Negative Area-Based Deviation (NAD), respectively. The values of PAD and NAD are denoted by S_p and S_n and depicted in Figure 2. In this Figure, two different circumstances for the membership function of $\tilde{D}$ are shown.

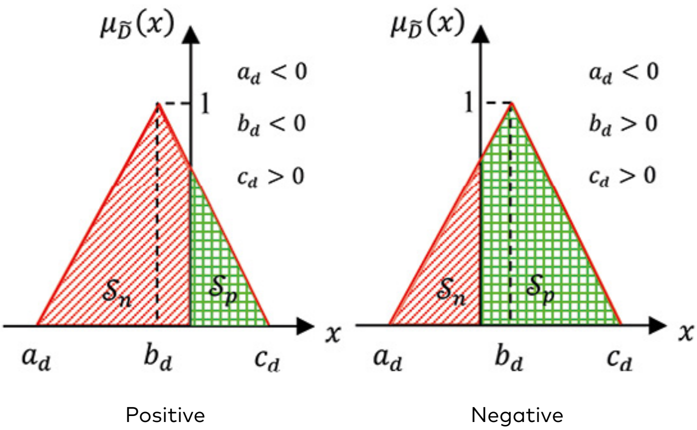

Figure 2. The graphical representation of Area-Based Deviation in two circumstances

According to Figure 2, it is clear that if $c_d \leq 0$ then $S_p = 0$, and if $a_d \geq 0$ then $S_n = 0$.

– Example:

Suppose that there is $\tilde{F} = (2, 3, 5)$ and $\tilde{G} = (3, 5, 8)$. Then the subtraction $\tilde{F} \ominus \tilde{G}$ results $\tilde{D} = (-6, -2, 2)$. According to Definition 6, the values of PAD and NAD are $S_p = 0.5$ and $S_n = 3.5$.

1.2. A Modified fuzzy TOPSIS method

As already mentioned, the TOPSIS method is one of the most popular MCDM methods which have been used in many studies for different purposes. This method has been extended to deal with the uncertainty in real-world problems. Using the fuzzy set theory to extend the TOPSIS method has been shared among researchers. The TOPSIS method uses the distances of alternatives from the positive-ideal and negative-ideal solution for evaluation of them. Different types of distances are used in this process. Fuzzy TOPSIS methods usually use simple fuzzy distances like Vertex distance, Hamming distance, Euclidean distance and Chebyshev distance (Ertuğrul, 2011). The effect of the membership function of the fuzzy elements in the decision-matrix is neglected in these types of fuzzy distances. This section proposes a modified fuzzy TOPSIS method which uses an area-based deviation measure described in Definition 6 (TOPSIS-FADR). Although the described measure is used in the fuzzy TOPSIS method, it is possible to integrate it with the other distance-based MCDM methods. Using this deviation measure helps to incorporate the effect of membership functions in the process of evaluation in the TOPSIS method.

Suppose that there are alternatives that need to be evaluated on criteria. The following steps describe the procedure of the proposed fuzzy TOPSIS method to deal with MCDM problems:

Step 1. Defining the MCDM problem including the evaluation criteria and alternatives, and constructing the decision-matrix according to the problem as follows:

$$\tilde{X} = [\tilde{x}_{ij}]_{n \times m}, \tag{12}$$

$$\tilde{W} = [\tilde{w}_j]_{1 \times m}, \tag{13}$$

where $\tilde{x}_{ij}$ and $\tilde{w}_j$ are the performance values of alternatives and the weights of criteria, respectively.

Step 2. Normalization of decision-matrix using the vector normalization technique and defuzzified values of the decision-matrix as follows:

$$\tilde{X}^N = [\tilde{x}_{ij}^n]_{n \times m}, \tag{14}$$

$$\tilde{x}_{ij}^n = \frac{\tilde{x}_{ij}}{\sqrt{\sum_{i=1}^{n} \left(\mathfrak{D}(\tilde{x}_{ij}^n)\right)^2}}. \tag{15}$$

Step 3. Calculation of weighted normalized decision-matrix by use of the following equations:

$$\tilde{X}^{NW} = \left[\tilde{x}_{ij}^{nw}\right]_{n\times m}, \tag{16}$$

$$\tilde{x}_{ij}^{nw} = \tilde{w}_j \otimes \tilde{x}_{ij}^{n}. \tag{17}$$

Step 4. Determination of the fuzzy positive-ideal solution ($\widetilde{PIS}$) and negative-ideal solution ($\widetilde{NIS}$) for each criterion as follows:

$$\widetilde{PIS}_j = \begin{cases} \widetilde{\max_i}\, \tilde{x}_{ij}^{nw} & if \quad j \in BC \\ \widetilde{\min_i}\, \tilde{x}_{ij}^{nw} & if \quad j \in NC, \end{cases} \tag{18}$$

$$\widetilde{NIS}_j = \begin{cases} \widetilde{\min_i}\, \tilde{x}_{ij}^{nw} & if \quad j \in BC \\ \widetilde{\max_i}\, \tilde{x}_{ij}^{nw} & if \quad j \in NC, \end{cases} \tag{19}$$

where

$$\widetilde{\max_i}\, \tilde{x}_{ij}^{nw} = \left\{\tilde{x}_{kj}^{nw} \mid \mathfrak{D}(\tilde{x}_{kj}^{nw}) = \max_i \mathfrak{D}(\tilde{x}_{ij}^{nw})\right\}, \tag{20}$$

$$\widetilde{\min_i}\, \tilde{x}_{ij}^{nw} = \left\{\tilde{x}_{kj}^{nw} \mid \mathfrak{D}(\tilde{x}_{kj}^{nw}) = \min_i \mathfrak{D}(\tilde{x}_{ij}^{nw})\right\}, \tag{21}$$

and denotes the set of beneficial criteria, and NC is the symbol of non-beneficial criteria.

Step 5. Computation of the fuzzy deviations from positive-ideal ($\tilde{\tilde{D}}_{ij}^{P}$) and negative-ideal ($\tilde{\tilde{D}}_{ij}^{N}$) solutions using the following equations:

$$\tilde{\tilde{D}}_{ij}^{P} = \begin{cases} \widetilde{PIS}_j \ominus \tilde{\tilde{x}}_{ij}^{nw} & if \quad j \in BC \\ \tilde{\tilde{x}}_{ij}^{nw} \ominus \widetilde{PIS}_j & if \quad j \in NC, \end{cases} \tag{22}$$

$$\tilde{\tilde{D}}_{ij}^{N} = \begin{cases} \tilde{\tilde{x}}_{ij}^{nw} \ominus \widetilde{NIS}_j & if \quad j \in BC \\ \widetilde{NIS}_j \ominus \tilde{\tilde{x}}_{ij}^{nw} & if \quad j \in NC. \end{cases} \tag{23}$$

Step 6. Computation of the PAD ratio based on Definition 6 as follows:

$$PR_i = \sum_{j=1}^{m} \left(\frac{1+PS_{ij}^{P}}{1+PS_{ij}^{N}}\right), \tag{24}$$

where $PS_{ij}^{P} = \mathcal{S}_p(\tilde{D}_{ij}^{P})$ and $PS_{ij}^{N} = \mathcal{S}_n(\tilde{D}_{ij}^{P})$.

Step 7. Computation of the NAD ratio based on Definition 6 as follows:

$$NR_i = \sum_{j=1}^{m} \left(\frac{1+NS_{ij}^{P}}{1+NS_{ij}^{N}}\right), \tag{25}$$

where $NS_{ij}^{P} = \mathcal{S}_p(\tilde{D}_{ij}^{N})$ and $NS_{ij}^{N} = \mathcal{S}_n(\tilde{D}_{ij}^{N})$.

Step 8. Determination of the closeness coefficient of each alternative using the following formula:

$$CC_i = \frac{NR_i}{PR_i + NR_i}. \tag{26}$$

Step 9. Ranking the alternatives according to the descending order of the closeness coefficient values, the alternative with the highest value of CC_i is the best alternative.

In Steps 6 and 7, when the PAD values increase, the values of PR_i and NR_i also, increase (there is a direct relationship between the PAD values and the ratios). On the other hand, when the NAD values increase, the values of PR_i and NR_i decrease (there is an inverse relationship between the NAD values and the ratios).

2. Illustrative example

In this section, to illustrate the procedure of the proposed modified fuzzy TOPSIS method, it is applied for ranking bridge design alternative solutions. The problem which is used in this section is an adapted version of a case study presented by Balali, Mottaghi, Shoghli, & Golabchi (2014). The case study includes three MCDM problems for selecting an appropriate structural system, construction method and material to design a bridge. The bridge has a length of 320 meters, and a width of a deck is 10 meters. In normal conditions, the width of the river at the location of bridge construction is 97.5 meters, and in a 100-year flood, it expands to about 170 meters. The maximum needed and the minimum possible spans are 105 meters and 70 meters, respectively. The maximum distance between water level and the deck of the bridge varies from 42 meters to 53 meters (in standard and 100-year flood conditions).

2.1. Using the proposed method

Figure 3 represents the hierarchical structure of the MCDM problems of the case study.

As seen in this Figure, there are 4 alternatives and 11 criteria in the Structural System problem. The Construction Methods problem consists of 4 alternatives and 7 criteria. The Materials problem consists of 4 alternatives and 4 criteria in.

The steps of the proposed approach are used to deal with these problems as follows:

Step 1. The decision-matrices related to these problems are presented in Tables 3 to 5. In these Tables, the type of each criterion (beneficial or non-beneficial) is indicated. Also, the criteria for each problem are categorized into quantitative (QT) and qualitative (QL). A spectrum from “Very low” (VL) to “Very high” (VH) is used to show the performance of the alternatives in qualitative criteria. These linguistic variables are transformed into fuzzy numbers according to Table 6. Weights of each criterion in each problem are also presented in Tables 3 to 5.

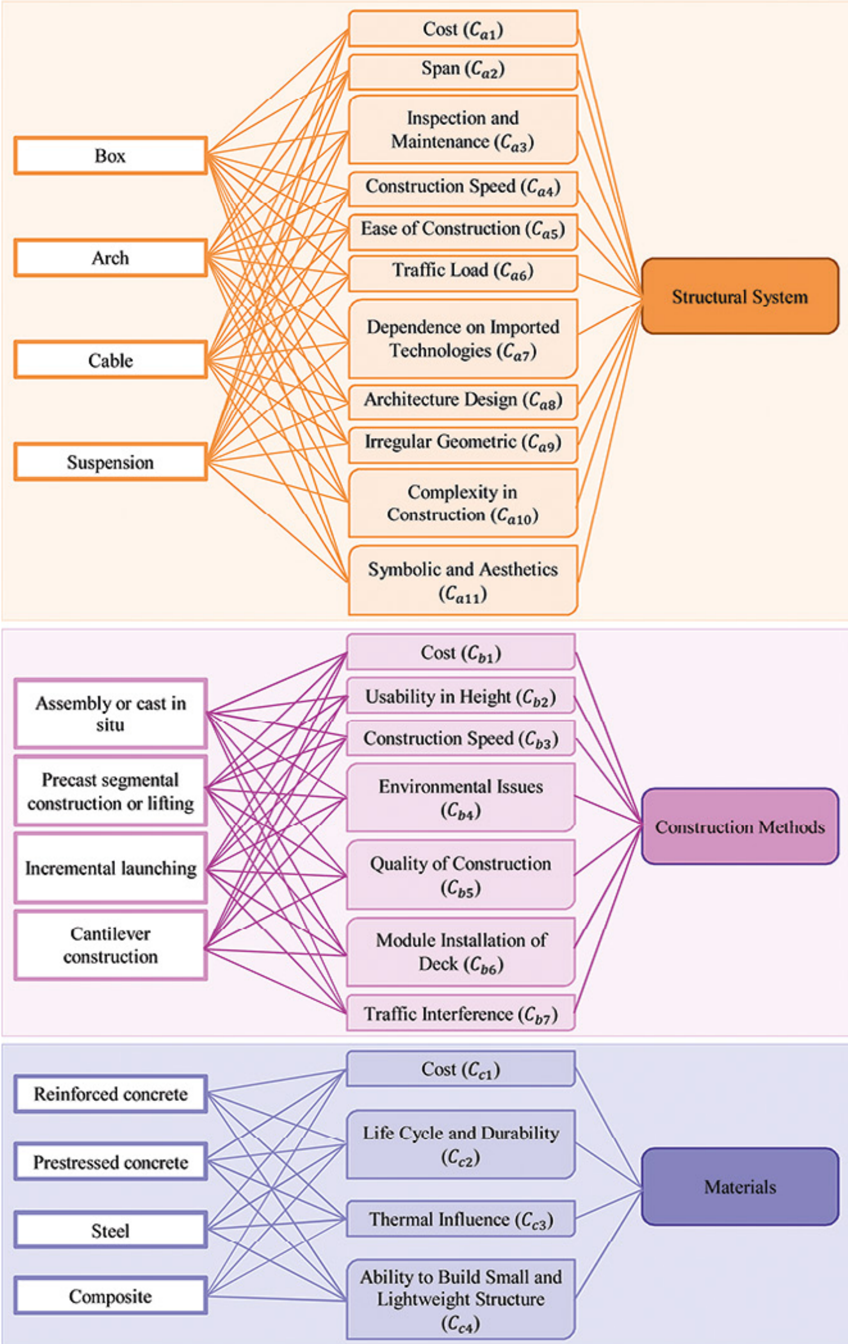

Figure 3. The hierarchical structure of the problems

Table 3. The decision-matrix of the Structural Systems problem

Category	Criteria	Type	Weights	Box	Arch	Cable	Suspension
QT	C_{a1}	*NC*	(0.054, 0.060, 0.066)	(1.35, 1.50, 1.65)	(2.25, 2.50, 2.75)	(2.7, 3.0, 3.3)	(4.5, 5.0, 5.5)
	C_{a2}	*BC*	(0.135, 0.150, 0.165)	(135, 150, 165)	(225, 250, 275)	(360, 400, 440)	(1080, 1200, 1320)
	C_{a3}	*NC*	(0.117, 0.130, 0.143)	(0.27, 0.30, 0.33)	(0.27, 0.30, 0.33)	(0.72, 0.80, 0.88)	(0.9, 1.0, 1.1)
	C_{a4}	*BC*	(0.117, 0.130, 0.143)	(22.5, 25.0, 27.5)	(22.5, 25.0, 27.5)	(13.5, 15.0, 16.5)	(6.3, 7.0, 7.7)
QL	C_{a5}	*BC*	(0.171, 0.190, 0.209)	H	L	L	VL
	C_{a6}	*BC*	(0.081, 0.090, 0.099)	VH	VH	H	H
	C_{a7}	*NC*	(0.117, 0.130, 0.143)	L	L	H	H
	C_{a8}	*BC*	(0.018, 0.020, 0.022)	M	VH	VH	VH
	C_{a9}	*BC*	(0.018, 0.020, 0.022)	M	VL	VL	VL
	C_{a10}	*NC*	(0.054, 0.060, 0.066)	L	L	H	VH
	C_{a11}	*BC*	(0.018, 0.020, 0.022)	L	H	VH	VH

Table 4. The decision-matrix of the Construction Methods problem

Category	Criteria	Type	Weights	Assembly or cast in situ	Precast segmental construction or lifting	Incremental launching	Cantilever construction
QT	C_{b1}	*NC*	(0.117, 0.130, 0.143)	(0.09, 0.10, 0.11)	(0.09, 0.10, 0.11)	(0.225, 0.250, 0.275)	(0.135, 0.150, 0.165)
	C_{b2}	*BC*	(0.207, 0.230, 0.253)	(9, 10, 11)	(9, 10, 11)	(900, 1000, 1100)	(900, 1000, 1100)
	C_{b3}	*BC*	(0.099, 0.110, 0.121)	(45, 50, 55)	(405, 450, 495)	(360, 400, 440)	(13.50, 15.00, 16.50)
QL	C_{b4}	*NC*	(0.171, 0.190, 0.209)	H	M	VL	L
	C_{b5}	*BC*	(0.117, 0.130, 0.143)	M	VH	VH	M
	C_{b6}	*BC*	(0.180, 0.200, 0.220)	VL	M	VH	H
	C_{b7}	*NC*	(0.018, 0.020, 0.022)	VH	H	L	L

Table 5. The decision-matrix of the Materials problem

Category	Criteria	Type	Weights	Reinforced concrete	Prestressed concrete	Steel	Composite
QT	C_{c1}	*NC*	(0.180, 0.200, 0.220)	(0.72, 0.80, 0.88)	(1.08, 1.20, 1.32)	(2.16, 2.40, 2.64)	(1.62, 1.80, 1.98)
	C_{c2}	*BC*	(0.243, 0.270, 0.297)	(90, 100, 110)	(108, 120, 132)	(63, 70, 77)	(72, 80, 88)

Category	Criteria	Type	Weights	Reinforced concrete	Prestressed concrete	Steel	Composite
QL	C_{c3}	*NC*	(0.117, 0.130, 0.143)	VL	VL	VH	H
	C_{c4}	*BC*	(0.360, 0.400, 0.440)	VL	M	VH	VH

Table 6. The Triangular Fuzzy Numbers related to linguistic variables.

Linguistic variables	Triangular Fuzzy Number
Very low (VL)	(0, 1, 3)
Low (L)	(1, 3, 5)
Medium (M)	(3, 5, 7)
High (H)	(5, 7, 9)
Very high (VH)	(7, 9, 10)

Steps 2 to 4. According to the decision-matrices, Table 6 and Eqs. (14) and (15), the normalized decision-matrices ($\tilde{X}^N$) are be obtained. Based on the normalized decision-matrices and Eqs. (16) and (17), the weighted normalized matrices ($\tilde{X}^{NW}$) are calculated.

Table 7 represents the weighted normalized matrix of the Structural Systems problem. Here, this matrix is only shown for the first problem, and the weighted normalized matrices of the Construction Methods and Materials problems are calculated in the same way. In Table 7, the values of the fuzzy positive-ideal and negative-ideal solutions are also presented.

Step 5. The fuzzy deviations from positive-ideal and negative-ideal solutions are calculated for the problems addressed based on the results of Steps 2 to 4 and Eqs. (22) and (23). To make the procedure of the proposed approach clear, the results of this step for the Structural Systems problem are presented in Table 8. The results of this step for the Construction Methods and Materials problems are determined in the same manner.

Steps 6 to 9. The positive and negative area-based deviation ratios are calculated in this step according to the results of Step 5, Definition 6 and Eqs. (24) and (25). The results of this step for the problems are presented in Table 9. Based on these ratios and Eq. (26) the closeness coefficients are calculated. The values of closeness coefficients are

Table 7. The matrices $\tilde{X}^{NW}$, $\widetilde{PIS}$ and $\widetilde{NIS}$ for the Structural Systems problem

Criteria	Type	$\tilde{X}^{NW}$				$\widetilde{PIS}_j$	$\widetilde{NIS}_j$
		Box	Arch	Cable	Suspension		
C_{c1}	NC	(0.011, 0.014, 0.017)	(0.019, 0.023, 0.028)	(0.022, 0.028, 0.033)	(0.037, 0.046, 0.056)	(0.011, 0.014, 0.017)	(0.037, 0.046, 0.056)
C_{c2}	BC	(0.014, 0.017, 0.021)	(0.023, 0.029, 0.035)	(0.037, 0.046, 0.056)	(0.112, 0.139, 0.168)	(0.112, 0.139, 0.168)	(0.014, 0.017, 0.021)
C_{c3}	NC	(0.023, 0.029, 0.035)	(0.023, 0.029, 0.035)	(0.062, 0.077, 0.093)	(0.078, 0.096, 0.117)	(0.023, 0.029, 0.035)	(0.078, 0.096, 0.117)
C_{c4}	BC	(0.067, 0.083, 0.101)	(0.067, 0.083, 0.101)	(0.040, 0.050, 0.060)	(0.019, 0.023, 0.028)	(0.067, 0.083, 0.101)	(0.019, 0.023, 0.028)
C_{c5}	BC	(0.103, 0.160, 0.227)	(0.021, 0.069, 0.126)	(0.021, 0.069, 0.126)	(0.000, 0.023, 0.076)	(0.103, 0.160, 0.227)	(0.000, 0.023, 0.076)
C_{c6}	BC	(0.036, 0.051, 0.063)	(0.036, 0.051, 0.063)	(0.026, 0.040, 0.057)	(0.026, 0.040, 0.057)	(0.036, 0.051, 0.063)	(0.026, 0.040, 0.057)
C_{c7}	NC	(0.011, 0.036, 0.066)	(0.011, 0.036, 0.066)	(0.054, 0.084, 0.119)	(0.054, 0.084, 0.119)	(0.011, 0.036, 0.066)	(0.054, 0.084, 0.119)
C_{c8}	BC	(0.003, 0.006, 0.010)	(0.008, 0.011, 0.014)	(0.008, 0.011, 0.014)	(0.008, 0.011, 0.014)	(0.008, 0.011, 0.014)	(0.003, 0.006, 0.010)
C_{c9}	BC	(0.010, 0.018, 0.028)	(0.000, 0.004, 0.012)	(0.000, 0.004, 0.012)	(0.000, 0.004, 0.012)	(0.010, 0.018, 0.028)	(0.000, 0.004, 0.012)
C_{c10}	NC	(0.005, 0.015 ,0.028)	(0.005, 0.015, 0.028)	(0.023, 0.035, 0.050)	(0.032, 0.045, 0.055)	(0.005, 0.015, 0.028)	(0.032, 0.045, 0.055)
C_{c11}	BC	(0.001, 0.004, 0.008)	(0.006, 0.010, 0.014)	(0.009, 0.012, 0.015)	(0.009, 0.012, 0.015)	(0.009, 0.012, 0.015)	(0.001, 0.004, 0.008)

Table 8. The values of $\tilde{\tilde{D}}_{ij}^{P}$ and $\tilde{\tilde{D}}_{ij}^{N}$ for the Structural Systems problem

Deviations	Criteria	Box	Arch	Cable	Suspension
$\tilde{\tilde{D}}_{ij}^{P}$	C_{a1}	(-0.006, 0.000, 0.006)	(0.002, 0.009, 0.017)	(0.006, 0.014, 0.022)	(0.021, 0.032, 0.044)
	C_{a2}	(0.091, 0.121, 0.154)	(0.077, 0.110, 0.144)	(0.056, 0.092, 0.130)	(-0.055, 0.000, 0.055)
	C_{a3}	(-0.012, 0.000, 0.012)	(-0.012, 0.000, 0.012)	(0.027, 0.048, 0.070)	(0.043, 0.067, 0.093)
	C_{a4}	(-0.033, 0.000, 0.033)	(-0.033, 0.000, 0.033)	(0.007, 0.033, 0.060)	(0.039, 0.060, 0.082)
	C_{a5}	(-0.124, 0.000, 0.124)	(-0.023, 0.092, 0.206)	(-0.023, 0.092, 0.206)	(0.027, 0.137, 0.227)
	C_{a6}	(-0.027, 0.000, 0.027)	(-0.027, 0.000, 0.027)	(-0.021, 0.011, 0.037)	(-0.021, 0.011, 0.037)
	C_{a7}	(-0.056, 0.000, 0.056)	(-0.056, 0.000, 0.056)	(-0.012, 0.048, 0.109)	(-0.012, 0.048, 0.109)
	C_{a8}	(-0.002, 0.005, 0.010)	(-0.006, 0.000, 0.006)	(-0.006, 0.000, 0.006)	(-0.006, 0.000, 0.006)
	C_{a9}	(-0.018, 0.000, 0.018)	(-0.002, 0.015, 0.028)	(-0.002, 0.015, 0.028)	(-0.002, 0.015, 0.028)
	C_{a10}	(-0.023, 0.000, 0.023)	(-0.023, 0.000, 0.023)	(-0.005, 0.020, 0.045)	(0.004, 0.030, 0.051)
	C_{a11}	(0.001, 0.008, 0.014)	(-0.005, 0.003, 0.009)	(-0.007, 0.000, 0.007)	(-0.007, 0.000, 0.007)

Deviations	Criteria	Box	Arch	Cable	Suspension
$\tilde{\tilde{D}}_{ij}^{N}$	$C_{\alpha 1}$	(0.021, 0.032, 0.044)	(0.009, 0.023, 0.037)	(0.004, 0.018, 0.033)	(-0.018, 0.000, 0.018)
	$C_{\alpha 2}$	(-0.007, 0.000, 0.007)	(0.002, 0.012, 0.021)	(0.016, 0.029, 0.042)	(0.091, 0.121, 0.154)
	$C_{\alpha 3}$	(0.043, 0.067, 0.093)	(0.043, 0.067, 0.093)	(-0.015, 0.019, 0.054)	(-0.039, 0.000, 0.039)
	$C_{\alpha 4}$	(0.039, 0.060, 0.082)	(0.039, 0.060, 0.082)	(0.012, 0.027, 0.042)	(-0.009, 0.000, 0.009)
	$C_{\alpha 5}$	(0.027, 0.137, 0.227)	(-0.055, 0.046, 0.126)	(-0.055, 0.046, 0.126)	(-0.076, 0.000, 0.076)
	$C_{\alpha 6}$	(-0.021, 0.011, 0.037)	(-0.021, 0.011, 0.037)	(-0.031, 0.000, 0.031)	(-0.031, 0.000, 0.031)
	$C_{\alpha 7}$	(-0.012, 0.048, 0.109)	(-0.012, 0.048, 0.109)	(-0.065, 0.000, 0.065)	(-0.065, 0.000, 0.065)
	$C_{\alpha 8}$	(-0.006, 0.000, 0.006)	(-0.002, 0.005, 0.010)	(-0.002, 0.005, 0.010)	(-0.002, 0.005, 0.010)
	$C_{\alpha 9}$	(-0.002, 0.015, 0.028)	(-0.012, 0.000, 0.012)	(-0.012, 0.000, 0.012)	(-0.012, 0.000, 0.012)
	$C_{\alpha 10}$	(0.004, 0.030, 0.051)	(0.004, 0.030, 0.051)	(-0.018, 0.010, 0.033)	(-0.024, 0.000, 0.024)
	$C_{\alpha 11}$	(-0.006, 0.000, 0.006)	(-0.001, 0.006, 0.012)	(0.001, 0.008, 0.014)	(0.001, 0.008, 0.014)

also presented in Table 9. According to these values, the final rank of alternatives is determined. As seen in Table 9, in the Structural System problem "Box" is the best alternative and "Suspension" is the

Table 9. The deviation ratios, the closeness coefficients, and rank of alternatives

Problems	Alternatives	PR_i	NR_i	CC_i	Rank
Structural System	Box	11.043	11.270	0.5051	1
	Arch	11.169	11.238	0.5015	2
	Cable	11.315	11.155	0.4964	3
	Suspension	11.270	11.043	0.4949	4
Construction Methods	Assembly or cast in situ	7.255	7.030	0.4921	4
	Precast segmental construction or lifting	7.198	7.222	0.5009	3
	Incremental launching	7.044	7.252	0.5073	1
	Cantilever construction	7.176	7.242	0.5023	2
Materials	Reinforced concrete	4.164	4.132	0.4981	4
	Prestressed concrete	4.158	4.266	0.5064	1
	Steel	4.139	4.120	0.4989	3
	Composite	4.138	4.221	0.5050	2

worst alternative. In the Construction Methods problem, "Incremental launching" is a better alternative than the others, and finally "Prestressed concrete" is chosen as the best alternative in the Materials problem.

2.2. Comparative analysis

For the validation of the results obtained in the previous section, a comparison is made in this section. The ranking results determined by the proposed TOPSIS-FADR method are compared to the original results the PROMETHEE method by Balali, Mottaghi, Shoghli, & Golabchi (2014). Also, the decision-matrices presented in Tables 3 to 5 are defuzzified. Then the SAW, WASPAS, COPRAS, TOPSIS, VIKOR and EDAS methods are utilized for ranking the alternatives with crisp performance values. Table 10 represents the results obtained.

As seen in Table 10, the ranking results of the proposed method are entirely consistent with the original results presented in the research of Balali, Mottaghi, Shoghli, & Golabchi (2014). Moreover, it is visible that the results determined by using the proposed fuzzy approach are relatively consistent with the results of the other methods considered

Table 10. The results of the comparative analysis

Problems	Alternatives	Proposed method	Balali, Mottaghi, Shoghli, & Golabchi (2014)	SAW	WASPAS	COPRAS	TOPSIS	VIKOR	EDAS
Structural System	Box (S1)	1	1	1	1	1	1	2	1
	Arch (S2)	2	2	2	2	2	2	1	2
	Cable (S3)	3	3	3	3	4	4	3	3
	Suspension (S4)	4	4	4	4	3	3	4	4
Construction Methods	Assembly or cast in situ (C1)	4	4	4	4	4	4	4	4
	Precast segmental construction or lifting (C2)	3	3	3	3	3	3	3	3
	Incremental launching (C3)	1	1	1	1	1	1	1	1
	Cantilever construction (C4)	2	2	2	2	2	2	2	2
Materials	Reinforced concrete (M1)	4	4	4	4	3	4	4	3
	Prestressed concrete (M2)	1	1	1	1	1	2	1	1
	Steel (M3)	3	3	3	3	4	3	3	4
	Composite (M4)	2	2	2	2	2	1	2	2

for the comparison. Figure 4 shows the congruity of the ranking results in a more clear way. Therefore, it is concluded that the proposed fuzzy TOPSIS method is efficient, and results in valid ranks for alternatives in MCDM problems under uncertainty.

Conclusions

Because of the significant role of bridges in the field of transportation and road networks, designing these structures is a critical process that has to be made efficiently. Conceptual design, as well as detailed design, is a fundamental stage of the process of designing bridges. The conceptual design of a bridge usually presents decisions on the general form of it, and the process of making such decisions is commonly subjective and includes multiple factors. Therefore, the Multi-Criteria Decision-Making methods and techniques are helpful to reach an efficient conceptual design. Based on an efficient conceptual design, designers are able to make the analytical design process more efficient. To handle a real-world multi-criteria bridge design problem, designers are usually confronted with the uncertainties of data. In this study, the authors have proposed a modified fuzzy TOPSIS method to deal with the

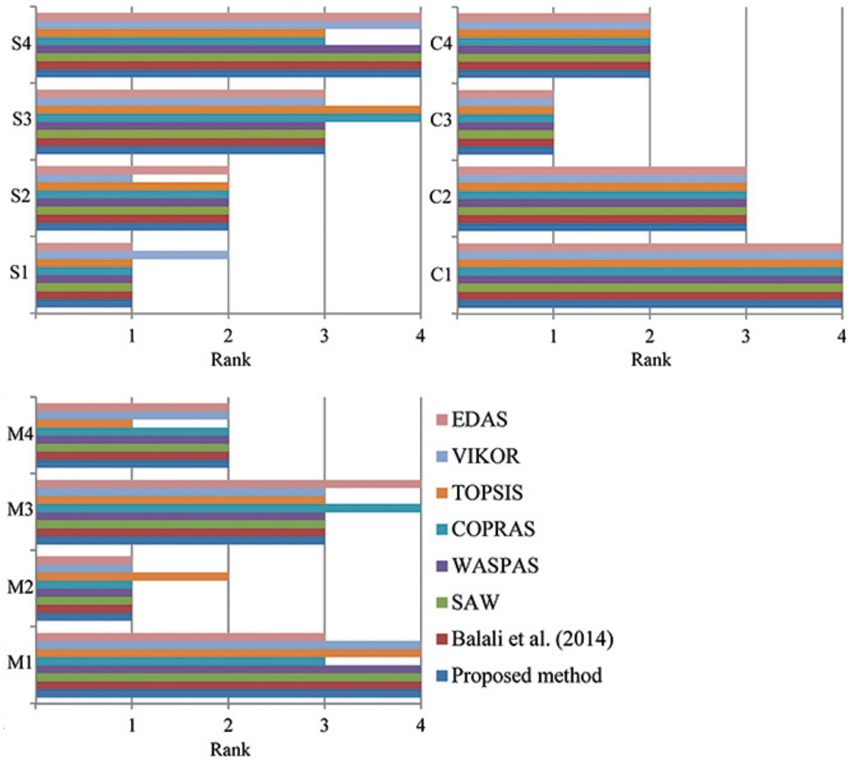

Figure 4. The graphical representation of the comparative analysis

multi-criteria assessment of alternative solutions in the bridge design process under uncertainty.

In the procedure of the proposed method, the degree of difference between the alternatives and the positive-ideal and negative-ideal solutions has been determined based on an area-based deviation ratio. This ratio is defined based on the area under the membership function of the fuzzy numbers determined by subtracting ratings of alternatives from PIS and NIS. Unlike many of the fuzzy extensions of the Technique for Order of Preference by Similarity to Ideal Solution method, the proposed method incorporates the membership degrees in the evaluation process. To present the application of the proposed method in designing bridges, it has been utilized it in a multi-criteria bridge design assessment problem. In this regard, three Multi-Criteria Decision-Making problems have been solved using the proposed method.

Moreover, the results have been compared to the results of some other methods. The comparative analysis shows that the results of the proposed modified method are valid and congruent with those of the other Multi-Criteria Decision-Making methods. Although the proposed method has been applied to the bridge design process, it is possible to use the method in many other real-world Multi-Criteria Decision-Making problems in future research. The area-based deviation approach is also suitable to be extended for the other types of fuzzy sets like interval type-2 fuzzy sets. In addition, in future research it is possible to extend this study by using the area-based deviation ratio in the other distance-based methods like the VIKOR method.

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