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NEW PRACTICAL METHODS OF ANALYSIS OF SECOND ORDER CURVES ON A PLANE

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Abstract. In this work second order curves have been studied in all their diversity as they appear in applications. These curves have been examined without using the concept of rotation, a concept which is introduced artificially in textbooks in order to simplify the basic form of the canonical equations of these curves: a concept which is only used for inclined second order curves, and which is both intimidating and difficult for newly accepted university students. A new methodological approach based only on completing the perfect square has been proposed, and generalized formulas of equations for an ellipse, a hyperbola and a parabola have been obtained. The use of these formulas has been shown in particular examples of the study of several inclined second order curves, analyzing their basic characteristics and graphing. The new generalized formulas of ellipse and hyperbola equations seem absent in literature.

Keywords: mathematical education, teaching methods, second order curves

Mathematics Subject Classification: 97G70, 97D40, 97C70, 97A80, 97U20

1 Introduction

Employers often say that an educator should not scare students with mathematics. Consequently, complicated mathematics need to be taught to students by using the simplest techniques, combining straightforward and well-known methods. However, the formulas derived by these methods may be complex, since we do not require students to memorize them. Moreover, these complex formulas must be derived, if they have useful applications [4], and it must be shown to students how these formulas can be applied to solve specific problems!

Therefore, the method of completing the perfect square [2] is the most appropriate technique to be used regarding the study of the second order curves, as it is a very useful and well-known method for transforming algebraic expressions. For newly accepted university students, the method of rotation of coordinate axes and coordinate transformations [1] is a completely new and unknown method, a method which should be taught later, not as a starting point in the analysis and graphing of the second order curves on the plane. This is the main idea of the considered methodological approach.

Thus, the aim of the present work is to consider arbitrary second order curves on the coordinate plane and corresponding second degree equations in Cartesian coordinates in their original form. It means a refusal of the concept of rotation on consideration of arbitrary second order curves on the plane. The analogical approach is used, for example, for the simplest equation of a straight line on the plane $y = mx + b$, where m is a slope of the line and b is the y-intercept, when nobody mentions that this straight line is obtained by rotating the Ox axis by the angle α, where $\mathrm{tg}\,\alpha = m$. In the current work, the same approach is applied for the study of second order curves on the plane.

The authors are suggesting the implementation of this approach which avoids both the time-consuming transformations and the complicated trigonometry, both of which intimidate the 1st year students so much. This is the step-by-step method based on the simplest and most basic technique - the method of completing the perfect square without using the concept of rotation. There will be no trigonometry at all in the study of an inclined parabola, and just the use of the definition of sine and cosine functions in the study of an inclined ellipse and hyperbola. We should have already looked at the world of the second degree equations with "open eyes" some time ago, and acknowledge that there are opportunities to teach in a more effective way, rather than artificially introduce the theory of rotation.

It is well-known that the second order curve is described by the general second degree equation

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0, \tag{1}$$

where A, B, C, D, E, F are the constant coefficients such as $A^2 + B^2 + C^2 \neq 0$. Depending on these coefficients, equation (1) can define one of the 3 classes of real non-degenerate second order curves, i.e., a parabola, an ellipse or a hyperbola. In the non-degenerate case, if $B^2 - 4AC = 0$, then equation (1) describes a parabola; if $B^2 - 4AC > 0$, then equation (1) describes a hyperbola; if $B^2 - 4AC < 0$, then equation (1) describes an ellipse or a circle.

The class of the second order curves, their main characteristics and graphical representations can be easily defined from the canonical forms of the general equation (1). As every student knows, in the case of absence of the term Bxy in equation (1), the simplest canonical form equation of a parabola, ellipse or hyperbola can be found by only completing the perfect square ([1], [2]). However, if $B \neq 0$, then a rotation of the coordinate axes is usually applied, which involves the use of trigonometric functions and work with their bulky expressions.

In the simplest case of non-inclined curves, the corresponding canonical forms of equations of a parabola, an ellipse and a hyperbola ([1], [2]), respectively, are

$$(y - y_0)^2 = \pm 2p \cdot (x - x_0) \quad \text{or} \quad (x - x_0)^2 = \pm 2p \cdot (y - y_0), \tag{2}$$

$$\frac{(x - x_0)^2}{a^2} + \frac{(y - y_0)^2}{b^2} = 1, \tag{3}$$

$$\frac{(x - x_0)^2}{a^2} - \frac{(y - y_0)^2}{b^2} = \pm 1, \tag{4}$$

where p is parabola's focal parameter, a is the semi-major axis, b is the semi-minor axis, $C(x_0, y_0)$ is the centre or the vertex, respectively.

At the present time of intellectual progress and internet freedom, it makes sense to abandon the simple, well-known canonical form equations of the ellipse, hyperbola and parabola, and in their place to derive more complex, generalized form equations, which would describe the second order curves on the plane in all their diversity and magnificence. This is seen most often in the case of the inclined second order curves.

2 New Formula of a Parabola Equation

The present study of a parabola is based on considering its general second degree equation without using the concept of rotation, and consequently without using any trigonometry. First of all, the following alternative formula of equation of an arbitrary parabola is suggested.

$$(x - x_F)^2 + (y - y_F)^2 = \frac{(y - mx - b)^2}{m^2 + 1}, \tag{5}$$

where $F(x_F, y_F)$ is the focus of a parabola, $y = mx + b$ is the directrix of a parabola.

In the authors' opinion, formula (5) should be introduced into all textbooks of higher education in mathematics, alongside the already widely used "simplest" formulas of a parabola, $y^2 = \pm 2px$ and $x^2 = \pm 2py$. Formula (5) of a parabola equation immediately follows from the definition of a parabola as a geometric locus of points of the plane, which are equidistant both to the focus and directrix of the parabola (see Fig. 1).

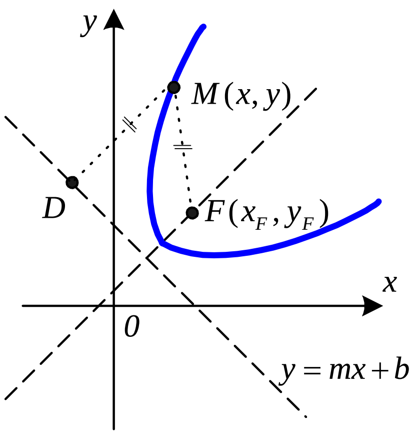

Fig. 1. Definition of a parabola.

For example, if there is a task [3] to analyse and sketch a graph of the parabola

$$16x^2 - 24xy + 9y^2 + 25x - 50y + 50 = 0, \tag{6}$$

then the focus, vertex, directrix and axis of symmetry of the parabola should be found beforehand. The solving must start from the verification that the given second degree equation (6) actually represents a parabola on the plane. It can be easily checked by using the condition $B^2-4AC=0$ on the coefficients of the second degree equation (1), when $A=16$, $B=-24$, $C=9$ in equation (6). However, this also can be verified without using the classification of the second order lines, since in the course of analytical geometry at university, the classification is the final topic of the study of the second order lines.

The study of the parabola starts from the completing the perfect square and reducing equation (6) into the simplest form equation of a parabola, $y^2=\pm 2px$:

$$16x^2-24xy+9y^2+25x-50y+50=0$$
$$(4x-3y)^2=50y-25x-50.$$

Using the substitution $4x-3y=u$, $\Rightarrow$ $y=\frac{4}{3}x-\frac{1}{3}u$, one gets

$$u^2=\frac{200}{3}x-\frac{50}{3}u-25x-50$$
$$u^2+\frac{50}{3}u=\frac{125}{3}x-50$$
$$\left(u+\frac{25}{3}\right)^2=\frac{125}{3}\left(x+\frac{7}{15}\right). \qquad (7)$$

Equation (7) describes the equation of the parabola in the (x,u) - coordinate system.

In order to obtain the basic characteristics of a parabola (6), its given equation and the new formula of a parabola equation (5) must be re-written in the following form.

$$(y-mx-b)^2-(m^2+1)\left((x-x_F)^2+(y-y_F)^2\right)=0, \qquad (8)$$
$$-x^2+\frac{3}{2}xy-\frac{9}{16}y^2-\frac{25}{16}x+\frac{25}{8}y-\frac{25}{8}=0. \qquad (9)$$

The next step is to compare the coefficients of the like monomials in (8) - (9):

$$\begin{array}{rl}
x^2\,| & m^2-(m^2+1)=-1 \\
xy\,| & \frac{3}{2}=-2m, \quad \Rightarrow \quad m=-\frac{3}{4}. \\
x\,| & -\frac{25}{16}=2mb+(m^2+1)\cdot 2x_F \\
y\,| & \frac{25}{8}=-2b+(m^2+1)\cdot 2y_F \\
1\,| & -\frac{25}{8}=b^2-(m^2+1)(x_F^2+y_F^2).
\end{array} \qquad (10)$$

As a result, the following system of equations for the unknown parameters b, x_F, y_F is obtained.

$$\begin{cases} -\frac{3}{2}b+\frac{25}{8}x_F=-\frac{25}{16} \\ -b+\frac{25}{16}y_F=\frac{25}{16} \\ b^2-\frac{25}{16}(x_F^2+y_F^2)=-\frac{25}{8} \end{cases}. \tag{11}$$

The solution of system (11) is $b=\frac{15}{16}$, $x_F=-\frac{1}{20}$, $y_F=\frac{8}{5}$. It means that the focus of the parabola is the point $F\left(-\frac{1}{20};\frac{8}{5}\right)$, and the directrix equation is $y=-\frac{3}{4}x+\frac{15}{16}$. The symmetry axis of the parabola is a straight line which passes through the focus and is perpendicular to the directrix. Thus, using the condition of perpendicularity of two lines, the symmetry axis of the parabola is $y=\frac{4}{3}x+\frac{5}{3}$. The parabola's vertex is $C\left(-\frac{1}{5};\frac{7}{5}\right)$, which is found as a midpoint of the segment of the symmetry axis, which is located between the focus and directrix of the parabola.

Then the equation of the considered parabola takes the form

$$\left(x+\frac{1}{20}\right)^2+\left(y-\frac{8}{5}\right)^2=\frac{\left(\frac{3}{4}x+y-\frac{15}{16}\right)^2}{1\frac{9}{16}}. \tag{12}$$

Using the obtained characteristics, the parabola's graph can be sketched on the plane rapidly and properly, and that means that the problem is solved (see Fig. 2).

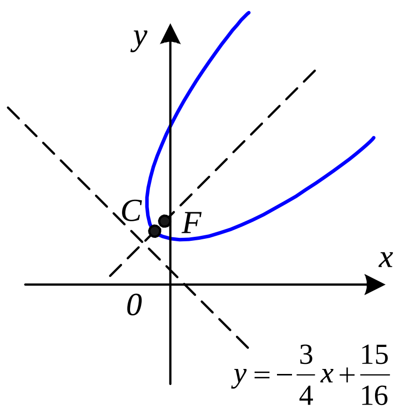

Fig. 2. A sketch of the graph of the given inclined parabola.

3 New Formula of an Ellipse Equation

This chapter is dedicated to the study of an ellipse, which is based on considering only the general second degree equation of ellipse, without using the concept of rotation. The study starts from obtaining the most general form equation of an inclined ellipse, the equation which takes into account the angle of inclination of the axis of the ellipse.

Thus, the task is to derive an equation of the ellipse, whose major axis makes the angle α with the coordinate axis *0x*, the centre is located in the point $C(x_0, y_0)$, and the distance between the focus points is equal to $2c$ (see Fig. 3).

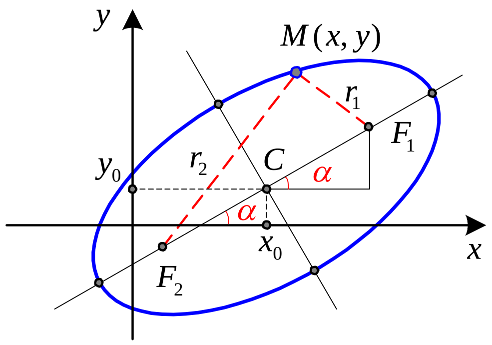

Fig. 3. Definition of an inclined ellipse.

Using Fig. 3 and the definition of an ellipse as a geometric locus of the points $M(x, y)$ on the plane, such that the sum of the distances from each point $M(x, y)$ to the two points (the foci F_1 and F_2) is a constant value $2a$ (provided that $2a > 2c$), one gets the following.

$$r_1 + r_2 = 2a\,, \tag{13}$$

$$CF_1 = CF_2 = c\,,$$

$$F_1(x_0 + c\cos\alpha,\ y_0 + c\sin\alpha),\quad F_2(x_0 - c\cos\alpha,\ y_0 - c\sin\alpha),$$

$$r_1^2 = |MF_1|^2 = (x - x_0 - c\cos\alpha)^2 + (y - y_0 - c\sin\alpha)^2\,, \tag{14}$$

$$r_2^2 = |MF_2|^2 = (x - x_0 + c\cos\alpha)^2 + (y - y_0 + c\sin\alpha)^2\,. \tag{15}$$

It follows from equations (13)-(15) that

$$\sqrt{(x - x_0 + c\cos\alpha)^2 + (y - y_0 + c\sin\alpha)^2} = 2a - \sqrt{(x - x_0 - c\cos\alpha)^2 + (y - y_0 - c\sin\alpha)^2}\,. \tag{16}$$

After eliminating the square roots in equation (16) and re-grouping the terms, the new formula of an ellipse equation is obtained in the form

$$a^2((x - x_0)^2 + (y - y_0)^2 + c^2) = a^4 + c^2((x - x_0)\cos\alpha + (y - y_0)\sin\alpha)^2\,. \tag{17}$$

Let us show how this method works if there is a task [3] to analyse and sketch a graph of the ellipse

$$25x^2 - 14xy + 25y^2 + 64x - 64y - 224 = 0, \tag{18}$$

then beforehand, it must be proven that the curve given by (18) describes an ellipse on the plane. It can be easily done by verifying the condition $B^2 - 4AC < 0$ in equation (1). Then it follows from (8) by completing the perfect square that

$$\left(5x - \frac{7}{5}y\right)^2 - \frac{49}{25}y^2 + 25y^2 + 64x - 64y = 224. \tag{19}$$

Using the substitution $\left[5x - \frac{7}{5}y = u, \ \Rightarrow \ x = \frac{1}{5}u + \frac{7}{25}y\right]$ in (19), one gets

$$u^2 + \frac{576}{25}y^2 + \frac{64}{5}u + \frac{448}{25}y - 64y = 224,$$

$$\left(u + \frac{32}{5}\right)^2 + \left(\frac{24}{5}y - \frac{24}{5}\right)^2 = 224 + \frac{32^2}{25} + \frac{24^2}{25},$$

$$\left(u + \frac{32}{5}\right)^2 + \frac{24^2}{25}(y-1)^2 = 288. \tag{20}$$

Equation (20) describes the equation of an ellipse in the (y,u)- coordinate system with the centre coordinates $y_0 = 1$ and $u_0 = -\frac{32}{5}$. The centre coordinates in the (x,y)- coordinate system are $y_0 = 1$ and $x_0 = \frac{1}{5}u_0 + \frac{7}{25}y_0 = -1$.

The centre coordinates can be also found from the following parametric equations of the given ellipse.

$$\begin{cases} u + \frac{32}{5} = 12\sqrt{2}\cos t \\ \frac{24}{5}(y-1) = 12\sqrt{2}\sin t \end{cases}, \quad \text{or} \quad \begin{cases} x = \sqrt{2}\left(\frac{12}{5}\cos t + \frac{7}{10}\sin t\right) - 1 \\ y = \frac{5}{\sqrt{2}}\sin t + 1 \end{cases}. \tag{21}$$

Equation (21) describes the parametric equations of the ellipse with the centre $C(-1;1)$, where $x_0 = -1$, $y_0 = 1$.

In order to find the other basic characteristics of the ellipse, the new formula of an ellipse equation (17), with the substituted $x_0 = -1$ and $y_0 = 1$, must be compared with the given line equation (18) by the coefficients of the like monomials:

$$a^2((x+1)^2 + (y-1)^2 + c^2) - a^4 - c^2((x+1)\cos\alpha + (y-1)\sin\alpha)^2 = 0, \tag{22}$$

$$xy|\quad 2c^2\sin\alpha\cos\alpha = 14k$$

$$x^2|\quad a^2 - c^2\cos^2\alpha = 25k \tag{23}$$

$$y^2|\quad a^2 - c^2\sin^2\alpha = 25k\,. \tag{24}$$

Subtracting equation (24) from equation (23), it yields that $c^2\cos 2\alpha = 0$ and, consequently, $\alpha = 45^\circ$, on assuming that $k>0$. Then equation (22) takes the form

$$a^2((x+1)^2+(y-1)^2+c^2)-a^4-c^2\left((x+1)\frac{\sqrt{2}}{2}+(y-1)\frac{\sqrt{2}}{2}\right)^2=0\,,$$

$$a^2(x^2+2x+y^2-2y+c^2+2)-a^4-\frac{1}{2}c^2(x+y)^2=0,\quad \Big|\div a^2$$

$$x^2+2x+y^2-2y+c^2+2-a^2-\frac{c^2}{2a^2}(x+y)^2=0,\quad \Big|\cdot 32$$

$$32x^2+64x+32y^2-64y+32\left(c^2+2\right)-32a^2-\frac{16c^2}{a^2}(x+y)^2=0\,. \tag{25}$$

The comparison of the like monomials in (25) and (18) gives

$$xy|\quad 2\cdot\frac{16c^2}{a^2}=14 \tag{26}$$

$$x^2|\quad 32-\frac{16c^2}{a^2}=25 \tag{27}$$

$$1|\quad 32\left(c^2+2\right)-32a^2=-224\,. \tag{28}$$

It follows from equations (26) and (27) independently, that $\frac{c}{a}=\frac{\sqrt{7}}{4}$, and from (28) that

$$32\left(c^2-a^2\right)=-288\,,\quad \Rightarrow\quad a^2-c^2=9\,,\quad \Rightarrow\quad a^2-\frac{7}{16}a^2=9\,.$$

Thus, $a=4$, $c=\sqrt{7}$, $b=\sqrt{a^2-c^2}=3$ so that the considered ellipse has the semi-major axis $a=4$ and the semi-minor axis $b=3$. The coordinates of the foci F_1 and F_2 are

$$F_1(x_0+c\cos\alpha,\ y_0+c\sin\alpha)=F_1\left(\frac{\sqrt{14}}{2}-1,\ \frac{\sqrt{14}}{2}+1\right),$$

$$F_2(x_0-c\cos\alpha,\ y_0-c\sin\alpha)=F_2\left(-\frac{\sqrt{14}}{2}-1,\ 1-\frac{\sqrt{14}}{2}\right).$$

Equation (18) of the given ellipse takes the form

$$16((x+1)^2+(y-1)^2+7)=256+7((x+1)\cos\tfrac{\pi}{4}+(y-1)\sin\tfrac{\pi}{4})^2\,. \tag{29}$$

Using the obtained characteristics, a graph of the ellipse can be sketched on the plane, and that means that the problem is solved (see Fig. 4).

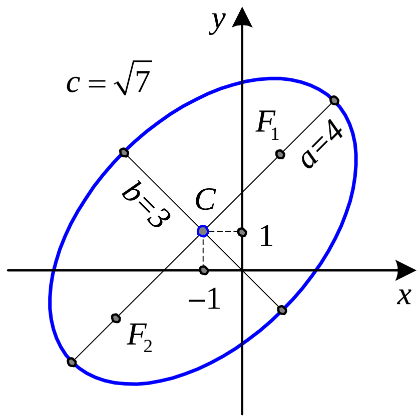

Fig. 4. A sketch of the graph of the given inclined ellipse.

4 New Formula of a Hyperbola Equation

This chapter is dedicated to the study of a hyperbola, which is based on considering only the general second degree equation of hyperbola, without using the concept of rotation. The study starts from obtaining the most general form equation of an inclined hyperbola, the equation which takes into account the angle of inclination of the axis of the hyperbola.

Thus, the task is to derive an equation of the hyperbola, whose major axis makes the angle α with the coordinate axis $0x$, the centre is located in the coordinate origin, and the distance between the focus points is equal to $2c$ (see Fig. 5).

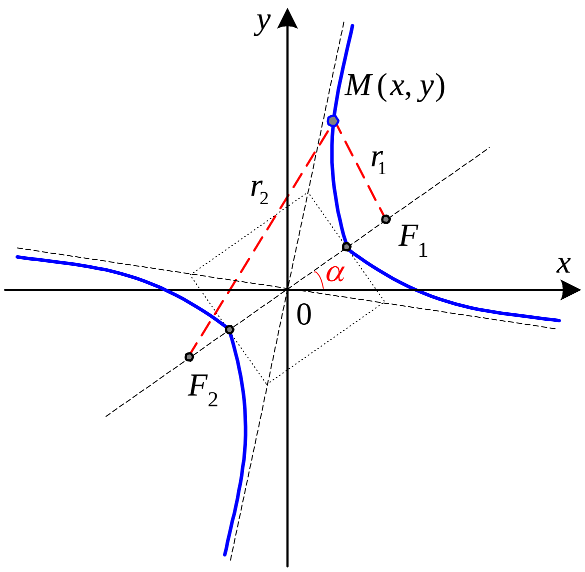

Fig. 5. Definition of an inclined hyperbola.

Using Fig. 5 and the definition of a hyperbola as a geometric locus of the points $M(x, y)$ on the plane, such that the absolute difference between distances from each point $M(x, y)$ to the two points (the foci F_1 and F_2) is a constant value $2a$ (provided that $2a < 2c$), one gets the following.

$$|r_1 - r_2| = 2a, \tag{30}$$

$$OF_1 = OF_2 = c,$$

$$F_1(c\cos\alpha, c\sin\alpha), \quad F_2(-c\cos\alpha, -c\sin\alpha),$$

$$r_1^2 = |MF_1|^2 = (x - c\cos\alpha)^2 + (y - c\sin\alpha)^2, \tag{31}$$

$$r_2^2 = |MF_2|^2 = (x + c\cos\alpha)^2 + (y + c\sin\alpha)^2. \tag{32}$$

It follows from equations (30)-(32) that

$$\left| \sqrt{(x - c\cos\alpha)^2 + (y - c\sin\alpha)^2} - \sqrt{(x + c\cos\alpha)^2 + (y + c\sin\alpha)^2} \right| = 2a. \tag{33}$$

After eliminating the square roots in equation (33) and re-grouping the terms, the new formula of a hyperbola equation is obtained in the form

$$a^2(x^2 + y^2 + c^2) = a^4 + c^2(x\cos\alpha + y\sin\alpha)^2. \tag{34}$$

It is to be noted that the new formula of the hyperbola equation (34) coincides with the new formula of the ellipse equation (17). This is good news for students since they have to memorize one formula less. Why does it happen that the different second kind lines are defined by the same formula? We will raise this question with our curious readers, those who are interested in this.

Let us show how this method works, if there is a task [3] to analyse and sketch a graph of the hyperbola

$$3x^2 + 10xy + 3y^2 - 2x - 14y - 13 = 0, \tag{35}$$

then beforehand, it must be proven that the curve given by (30) describes a hyperbola on the plane. It can be easily done by verifying the condition $B^2 - 4AC > 0$ in equation. Then it follows from equation (30) by completing the perfect square that

$$x^2 + \frac{10}{3}xy + y^2 - \frac{2}{3}x - \frac{14}{3}y = \frac{13}{3},$$

$$\left(x + \frac{5}{3}y\right)^2 - \frac{25}{9}y^2 + y^2 - \frac{2}{3}x - \frac{14}{3}y = \frac{13}{3}. \tag{36}$$

Using the substitution $\left[x + \frac{5}{3}y = u, \ \Rightarrow \ x = u - \frac{5}{3}y\right]$ in (36), one gets

$$u^2-\frac{16}{9}y^2-\frac{2}{3}u+\frac{10}{9}y-\frac{14}{3}y=\frac{13}{3},$$

$$\left(u-\frac{1}{3}\right)^2-\frac{16}{9}y^2-\frac{32}{9}y=\frac{13}{3}+\frac{1}{9},$$

$$\left(u-\frac{1}{3}\right)^2-\frac{16}{9}(y+1)^2=\frac{8}{3}. \qquad (37)$$

Equation (37) describes the equation of a hyperbola in the (y,u)- coordinate system with the centre coordinates $y_0=-1$ and $u_0=\frac{1}{3}$. The centre coordinates in the (x,y)- coordinate system are $y_0=1$ and $x_0=u_0-\frac{5}{3}y_0=2$.

The centre coordinates can be also found from the following parametric equations of the given hyperbola.

$$\begin{cases} u-\frac{1}{3}=\frac{2\sqrt{6}}{3}\cosh t \\ \frac{4}{3}\left(y+1\right)=\frac{2\sqrt{6}}{3}\sinh t \end{cases}, \quad \text{or} \quad \begin{cases} x=\sqrt{6}\left(\frac{2}{3}\cosh t-\frac{5}{6}\sinh t\right)+2 \\ y=\frac{\sqrt{6}}{2}\sinh t-1 \end{cases}. \qquad (38)$$

Equation (38) describes the parametric equations of the hyperbola with the centre $C\left(2;-1\right)$, where $x_0=2$, $y_0=-1$.

In order to find the other basic characteristics of the hyperbola, the coefficients of the like monomials in the new formula of the hyperbola equation (34), with the substituted $x_0=2$ and $y_0=-1$, and in the given line equation (35) must be compared:

$$a^2((x-2)^2+(y+1)^2+c^2)=a^4+c^2((x-2)\cos\alpha+(y+1)\sin\alpha)^2, \qquad (39)$$

$$3x^2+10xy+3y^2-2x-14y-13=0, \qquad (35)$$

$$xy\,|\quad 2c^2\sin\alpha\cos\alpha=10k$$

$$x^2\,|\quad c^2\cos^2\alpha-a^2=3k \qquad (40)$$

$$y^2\,|\quad c^2\sin^2\alpha-a^2=3k. \qquad (41)$$

Subtracting equation (41) from equation (40), it yields that $c^2\cos 2\alpha=0$ and, consequently, $\alpha=45^\circ$, on assuming that $k>0$. Then equation (39) takes the form

$$x^2-4x+y^2+2y+c^2+5=a^2+\frac{c^2}{2a^2}\left(x+y-1\right)^2. \qquad (42)$$

The comparison of the like monomials in equations (42) and (34) gives

$$
\begin{array}{ll} xy\,| & \dfrac{c^2}{2a^2}\cdot 2=10k \\ x^2\,| & \dfrac{c^2}{2a^2}-1=3k \end{array}, \qquad \Rightarrow \quad \begin{cases} \dfrac{c^2}{a^2}=10k \\ 5k-1=3k \end{cases}, \qquad \Rightarrow \quad k=\frac{1}{2}. \tag{43}
$$

Thus, $k=1/2$, $c^2=5a^2$ and equation (42) becomes

$$
x^2-4x+y^2+2y+5a^2+5=a^2+\frac{5}{2}\left(x^2+y^2+1+2xy-2x-2y\right), \qquad |\cdot 2
$$

$$
2x^2-8x+2y^2+4y+10a^2+10=2a^2+5\left(x^2+y^2+1+2xy-2x-2y\right).
$$

$$
1|\quad 5-8a^2-10=-13.
$$

Thus, $a=1$, $c=\sqrt{5}$, $b=\sqrt{c^2-a^2}=2$ so that the considered hyperbola has the semi-major axis $a=1$ and the semi-minor axis $b=2$. The coordinates of the foci F_1 and F_2 are

$$
F_1(x_0+c\cos\alpha,\ y_0+c\sin\alpha)=F_1\left(2+\frac{\sqrt{10}}{2},\ \frac{\sqrt{10}}{2}-1\right),
$$

$$
F_2(x_0-c\cos\alpha,\ y_0-c\sin\alpha)=F_2\left(2-\frac{\sqrt{10}}{2},\ -\frac{\sqrt{10}}{2}-1\right).
$$

Equations of asymptotes of the given hyperbola are found as equations of straight lines passing through the hyperbola's centre and making the angles $\alpha_1=\arctan 2+\dfrac{\pi}{4}$, $\alpha_2=\arctan\dfrac{1}{2}+\dfrac{3\pi}{4}$ with the Ox axis, respectively. Thus, the equation of the 1st and 2nd asymptotes, respectively, are

$$
y+1=\tan\left(\frac{\pi}{4}+\arctan 2\right)(x-2), \quad \text{or} \quad y=-3x+5,
$$

$$
y+1=\tan\left(\frac{3\pi}{4}+\arctan\frac{1}{2}\right)(x-2), \quad \text{or} \quad y=-\frac{1}{3}(x+1).
$$

Equations of symmetry axes are found as equations of two perpendicular straight lines passing through the centre, if one of them also passes through the focus. Thus, the equations of two symmetry axes are

$$
y=x-3 \quad \text{and} \quad y=-x+1.
$$

Equation (35) of the given hyperbola takes the form

$$
(x-2)^2+(y+1)^2+4=5((x-2)\cos\tfrac{\pi}{4}+(y+1)\sin\tfrac{\pi}{4})^2. \tag{44}
$$

Using the obtained characteristics, the given hyperbola can be sketched on the plane, and this means that the problem is solved (see Fig. 6).

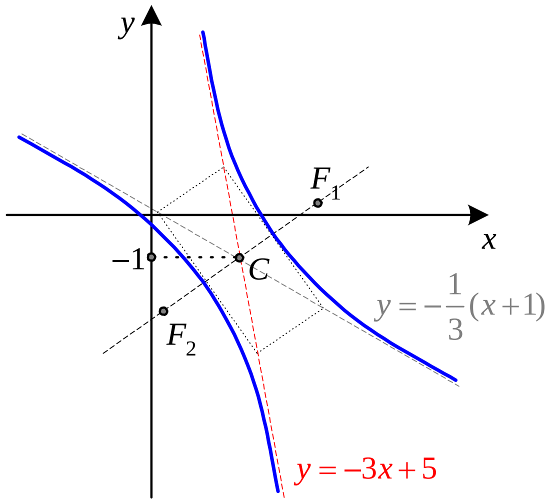

Fig. 6. A sketch of the graph of the given inclined hyperbola.

5 Conclusions

The necessity of considering arbitrary second order curves in applied problems has lead the authors to consider the study of these curves in the most general case. This also includes inclined curves, and deriving the generalized formulas of these curves in contrast to well-known, but simple canonical equations.

Such generalized formulas extend the possibilities of applying second order curves in the course of higher mathematics for first year students, and allow students to use the generalized equations in various applied problems more widely, due to the allowance for the slope of these curves.

Using the proposed methodological approach, the analysis of the inclined second order curves is based on the simple method of completing the perfect square, avoiding the rotation of coordinate axes, which is time-consuming and complicated for newly accepted university students. Simple methods are always greatly appreciated by students.

In the authors' opinion, it would be useful to complement mathematical textbooks with a supplement containing the considered methodology. The authors suggest introducing the "generalized" equations of arbitrary second order curves, calling them new canonical equations of arbitrary second order curves. As to the equations currently called "canonical" equations, they should be re-named as the simplest equations of an ellipse, hyperbola and parabola.

The new formulas of an ellipse and hyperbola equations seem absent in literature, although the formula of a parabola equation of a more or less similar form can be found in the literature.

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