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SYMBOLICAL COMBINATORY MODEL OF PARALLEL ALGORITHM OF IDENTIFICATION THAT USES METHOD OF LEAST SQUARES

SIMBOLISKS KOMBINATORISKAIS MODELIS PARALĒLAM IDENTIFIKĀCIJAS ALGORITMAM, KAS IZMANTO MAZĀKO KVADRĀTU METODI

Genadijs Burov, Dr.sc.ing., researcher
Riga Technical University, Faculty of Computer Science and Information Technology
Environment Modelling Centre
Address: 1/4 Meza Street, LV-1048, Riga, Latvia
Phone: +371 67089511
E-mail: emc@cs.rtu.lv

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Introduction

When using of algorithms of identification for the management of technical objects and their diagnosis, it is necessary to process large amounts of information. It requires using faster computers; however, their hardware possibilities are limited. Therefore, this problem better solved using software methods by applying parallel processing of information. When doing so, it is difficult to coordinate the architecture of the algorithms with the architecture of the computers working in the modes of parallel calculation. In particular, it is true for the algorithms realizing the method of the least squares (MLS), used for smoothing the noise. The transformation of the initial system of conditional equations into the system of normal equations leads to the reduction of numerical stability of the algorithm. The application of the traditional methods of regularization is inefficient.

To develop a parallel MLS algorithm, it is necessary to present it in the form of decomposition of fragments that can be processed independently from each other. Thus, there is a problem of development of a special formalized model for the transformation of the traditional algorithm into an algorithm, possessing such decomposition.

In the article, the specified problems are solved on the basis of a new approach. In it, the new mathematical apparatus of symbolical analytical calculations, based on the application of symbolical combinatory models [5, 6, 7, 8, 9, 10, 11], is applied. To reduce the size of the proof, the necessary references to the earlier published results are made.

Using such approach, the numerical algorithm can be mapped into a symbolical combinatory model (SC model) that is equivalent to the used computing algorithm.

Using method of least squares in algorithms for the test mode of identification

The numerical stability of MLS algorithm depends on the character of the matrix of initial system of

conditional equations. This dependence shows especially strongly if the matrix of the system of conditional equations is a Toeplitz matrix. Such matrices arise in the problems of the test control of analog dynamic objects and in the problems of auto-regression.
We shall consider an analog object, parameters of which are identified from the results of the measurements of dynamic process during discrete moments of time

$$y(kT) = \sum_{i=1}^{n} C_i q_i^{\ k} ; \ q_i = \exp(-a_i T). \tag{1}$$

The system of the identification equations is formed as a system of difference equations on the basis of the operator

$$D(z) = \frac{A(z)}{B(z)} = \frac{\sum_{i=1}^{m} \alpha_i \cdot z^{-(k+i)}}{\sum_{i=1}^{n} \beta_i \cdot z^{-n}}; D(z) = \varphi Z * \{F(p,z) \cdot W(p)\}. \tag{2}$$

In generally, it is found from the analog transfer function of the object *W(p)*, taking into account the operator of interpolation filter *F(p, z)*. This operator designates the mathematical operation of smoothing of mistakes of discrete approximation of the input signal [4, 6, 8, 9].
In the mode of the test control, the estimates of the factors of polynomial-denominator (2) can be found by solving the system of conditional equations

$$Y^{(N\times n)} \overline{\beta}^{(n)} = \overline{y}^{(N)} ; \quad [Y]_{r\,L} = \sum_{i=1}^{n} C_i q_i^{(L+r-1)}. \tag{3}$$

For finding the solution, the system of normal equations is defined

$$B^{(n\times n)} \overline{\beta}^{(n)} = \overline{u}^{(n)} ; B^{(n\times n)} = Y^{(N\times n)\ T} \cdot Y^{(N\times n)} ; \overline{u}^{(n)} = Y^{(N\times n)\ T} \cdot \overline{y}^{(n)}. \tag{4}$$

The solution is found as

$$\overline{\beta}^{(n)} = H \cdot \overline{u}^{(n)} ; \ H = B^{-1}. \tag{5}$$

Using the numerical methods for inverting the matrix *B*, there can be big errors. The application of the traditional methods of regularization, connected to the introduction of auxiliary functional in the solution of the problem to reduce the errors, appeared to be inefficient. The functional itself brings methodical errors into the solution of the problem. A new method of regularization can be developed, if the calculation of *H* will be made on the basis of its analytical expressions. However, solving this problem using the traditional methods is impossible. It can be done by mapping the MLS algorithm into a symbolical combinatory model (SC model). It can be developed on the basis of ordered numerical sequences *R(m, n)* [7, 8, 9, 10], formed in the space of integers

$$R(m,n) \Rightarrow \sum_{\nu=1}^{m} \left[\varphi KC(\nu) * \overline{1.n}\right] * \varphi Arng(Z_\nu); Z_\nu \Rightarrow \varphi Perm * \left[\varphi Part(\nu) * m\right]. \tag{6}$$

The operator $\varphi Arng(Z_\nu)$ is intended for the accommodation of the elements of set Z_ν over the elements of other set Z_ν itself can be formed with the help of the combinatory operators from the components of other set. In this case, Z_ν is formed as a set of permutations of the elements [8, 10, 11]. In [6, 7, 8] it is proved, that the numerical sequence (1) is a generating sequence for the other numerical sequence

$$G(m,n) \Rightarrow \overline{(0.m-1)} \oplus R(m,n) \Rightarrow \varphi KC(m) * \left(\overline{1.n}\right). \tag{7}$$

This expression can be generated using the positional principle [7]. It represents the index form for mapping the Kronecker product of *m* sets. The operator of its formation in the lexicographic form we shall designate as

$$\varphi \Pi r(m) * \left(\overline{1.n}\right) \Rightarrow R(m,n) \ . \tag{8}$$

Taking into account (7), this expression can be represented as

$$\varphi \Pi r(m) * \left(\overline{1.n}\right) \Rightarrow \varphi KC(m) * \left(\overline{1.n}\right) - \overline{(0.m-1)} \, . \tag{9}$$

Using (6), we find the expression for an element of the matrix of system of normal equations

$$B_{rL} = \overline{y}_r^{\,T} \cdot \overline{y}_L \Rightarrow \varphi Sum(j \in \overline{1.N}) * \left\{ \left(\sum_{i=1}^{n} C_i q_i^{\ r+j} \right) \cdot \left(\sum_{i=1}^{n} C_i q_i^{\ L+j} \right) \right\}. \tag{10}$$

The specification with the use of the positional principle can be used as [1]

$$A^{(n)} \Rightarrow \bigcup_{i=1}^{n} \theta_i^{[Z_i]} \Rightarrow \bigcup_{i=1}^{n} \left(\theta_i \otimes \circ z_i \right) \Rightarrow \left(\bigcup_{i=1}^{n} \theta_i \right) \otimes \circ \left(\bigcup_{i=1}^{n} z_i \right). \tag{11}$$

The symbolical expression for the element of the matrix of system of normal equations according to MLS should satisfy the formula of Kronecker product of sets

$$S(r,L) \Rightarrow \left\{ \left(\bigcup_{k=1}^{n} \theta_k \right) * \varphi Arang \left(\bigcup_{k=1}^{n} (r+k) \right) \right\} \times \circ \left\{ \left(\bigcup_{k=1}^{n} \theta_k \right) * \varphi Arang \left(\bigcup_{k=1}^{n} (L+k) \right) \right\}. \tag{12}$$

SC model of matrix of system of normal equations

The elements of the matrix (4) are formed as scalar products of column vectors of the matrix (3). They depend on the coordinates of the cell of matrix

$$[Y]_{rL} \Rightarrow \left[\overline{q} * Arang(r) \right]^T \cdot Diag(\overline{C}^{(n)}) \cdot \left[\overline{q} * Arang(L) \right]; (r,L) \in \left[\operatorname{Im} s = (\overline{r} \times \overline{L}) \right]. \tag{13}$$

The expression for the elements of the matrix of normal equations is

$$B_{rL} = \overline{y}_r^{\,T} \cdot \overline{y}_L \Rightarrow \varphi Sum(j \in \overline{1.N}) * \left\{ \left(\sum_{i=1}^{n} C_i q_i^{\ r+j} \right) \otimes \left(\sum_{i=1}^{n} C_i q_i^{\ L+j} \right) \right\}. \tag{14}$$

As the coordinate systems for the algebraic complements of matrix elements we use vectors consisting from the components of the numerical sequences $G(m,n) \Rightarrow \overline{(0.m-1)} \oplus R(m,n)$. The algebraic complement is formed from a sub-matrix determined by the coordinate system

$$\operatorname{Im} s = G(m,n) \times \circ G(m,n) \, . \tag{15}$$

The components (16) are used as the arguments for the operators of addressing and sampling

$$h_{i} \Rightarrow \overline{C} * \varphi Adres(G(n,m)_{i}) \quad h_{j} \Rightarrow \overline{C} * \varphi Adres(G(n,m)_{j}) . \tag{16}$$

Using (12) we shall write down

$$B_{rL} \Rightarrow \varphi\, Sum(i \in \overline{1.N}) * [S(r+i, L) \otimes S(r, L+i)], \tag{17}$$

or in an expanded form

$$B(r,L) \Rightarrow \varphi\, Sum(i \in \overline{1.N}) * \left\{ \left[\left(\bigcup_{k=1}^{n} \theta_{k} \right) * Arang\left(\bigcup_{i=1}^{n} (r+i) \right) \right] \times \circ \right.$$
$$\left. \times \circ \left[\left(\bigcup_{k=1}^{n} \theta_{k} \right) * Arang\left(\bigcup_{k=1}^{n} (L+i) \right) \right] \right\}. \tag{18}$$

We introduce in (18) the indices of rows *r* and columns *L* of the matrix

$$B(r,L) \Rightarrow \varphi\, Sum(i \in \overline{1.N}) * \left\{ \left[\left(\bigcup_{k=1}^{n} \theta_{k} \right) * Arang(r) \right] \otimes \circ \left[\left(\bigcup_{k=1}^{n} \theta_{k} \right) * Arang(i) \right] \times \circ \right.$$
$$\times \circ \left[\left(\bigcup_{k=1}^{n} \theta_{k} \right) * Arang(L) \right] \otimes \circ \left[\left(\bigcup_{k=1}^{n} \theta_{k} \right) * Arang(i) \right]. \tag{19}$$

We use the symbol of direct lexicographic product

$$B(r,L) \Rightarrow \varphi\, Sum(i \in \overline{1.N}) * (A \otimes \circ U) \Rightarrow A \otimes \circ \varphi\, Sum(i \in \overline{1.N}) * U . \tag{20}$$

Here the second factor not dependent on the variable *i*, is taken out of the summation:

$$A \Rightarrow \left\{ \left[\left(\bigcup_{k=1}^{n} \theta_{k} \right) * \varphi Arang(r) \right] \times \circ \left[\left(\bigcup_{k=1}^{n} \theta_{k} \right) * \varphi Arang(L) \right] \right\}; \tag{21}$$

$$U \Rightarrow \left[\left(\bigcup_{k=1}^{n} \theta_{k} \right) * \varphi Arang(i) \right] \times \circ \left[\left(\bigcup_{k=1}^{n} \theta_{k} \right) * \varphi Arang(i) \right]; \tag{22}$$

$$B(r,L) \Rightarrow A \otimes \circ V; \quad V \Rightarrow \varphi\, Sum(i \in \overline{1.N}) * U . \tag{23}$$

The expression (22) we shall write down in the form of a structure

$$G(2,n) \Rightarrow (0.1) \oplus \sum_{\nu=1}^{2} \left[\varphi KC(\nu) * \overline{1.n} \right] * \varphi Arng(Z_{\nu}); \; Z_{1} = 2; \; Z_{2} = 11 , \tag{24}$$

and then we have

$$[U]_{ij} \Rightarrow (C_{i} \cdot C_{j}) \cdot (q_{i} \cdot q_{j}). \tag{25}$$

Using (25), we can write down

$$V \Rightarrow \varphi\, Sum(i \in \overline{1.N}) * \left\{ \begin{array}{l} \bigcup_{i=1}^{N} \left\{ \left[\left(\bigcup_{k=1}^{n} q_k \right) \otimes \left(\bigcup_{k=1}^{n} C_k \right) \right] * Arang(i,i) \right\} \times \\ \times \left\{ \left[\left(\bigcup_{k=1}^{n} q_k \right) \otimes \left(\bigcup_{k=1}^{n} C_k \right) \right] * Arang(i,i) \right\} \end{array} \right\}. \tag{26}$$

For a steady object, the elements of the matrix (26) are

$$[V]_{ks} \Rightarrow \varphi\, Sum(i \in \overline{1.N}) * \{[1 \circ (q_k\, q_s)] * (Arang[\varphi Part(2) * N]\}. \tag{27}$$

Using the obtained results, we get

$$B_{rL} \Rightarrow \varphi Sum * \left\{ \left[\left(\overline{q} \times \overline{q}^{T} \right) * Arang(r, L) \right] \otimes V \right\}. \tag{28}$$

SC model of algebraic complements of matrix of normal equations

Using the properties of the operator $\varphi Dpv(\arg)$ [1, 10, 11], we shall find the result of its application to the structure $R(m,n)$ (6). As it is shown in [5], this result can be represented by a graph structure

$$\varphi Gr(\overline{r}, \overline{L}): \ \tilde{q}_1^{(n)} \Rightarrow \varphi Dpv * [\varphi Kc(m_1) * \tilde{q}_1] \times \varphi Dpv * [\varphi Kc(m_2) * \tilde{q}_2] \times \cdots$$
$$\cdots \times [\varphi Dpv * [\varphi Kc(m_k) * \tilde{q}_k]]; \quad \sum_{i=1}^{k} m_i = n \tag{29}$$

The graph has the form of a branching tree, and it is adapted to the formation of computing algorithms with parallel architecture. The properties of the operator $\varphi Dpv(\arg)$ are found from the analysis of such graphs:

$$\varphi Dvp(\arg) * (Q \otimes \circ P) \Rightarrow \varphi Sum * [Q * \varphi Arang(\varphi\, Gr(P))]; \tag{30}$$

$$\varphi Dvp(\overline{r}) * Q^{(n)} \Rightarrow Q^{(n)} * \varphi Arang\left(\varphi\, Gr(\arg) * \overline{r}\right) \quad \arg = \bigcup_{i=\overline{1.k}} \nu_i \quad \sum_{i=\overline{1.k}} \nu_i = n; \tag{31}$$

$$\varphi Dvp(\overline{r}) * Q^{(n)} \Rightarrow \varphi Sum * \left[Q^{(n)} * \left\{Arang\left[\varphi\, Gr(\arg) * \overline{r}\right]\right\}\right]$$
$$* [\varphi Perm * Q] \Rightarrow Q * \varphi Arang\left(\varphi\, Gr(\arg) * \overline{r}\right) \tag{32}$$

Using these properties, we have

$$\varphi Dpv(\arg) * G(m,n) \Rightarrow \varphi Dpv(\arg) * \left\| \varphi Perm * \left(\varphi KC(m) * \overline{1.n}\right) \right\| \Rightarrow$$
$$\Rightarrow \varphi Perm * \left[\varphi Dpv(\arg) * \left(\varphi KC(m) * \overline{1.n}\right)\right] \tag{33}$$

$$\varphi Dvp\left\{(\varphi Perm * \overline{r}^{(n)}) \times \overline{L}^{(n)}\right\} * \tilde{q}^{(n)} \Rightarrow \tilde{q}^{(n)} * Arang\left[\left(\varphi Dvp * \overline{r}^{(n)}\right) \oplus \overline{L}^{(n)}\right]. \tag{34}$$

We use the relation based on the property of operators conjugation

$$(\varphi Perm * \tilde{q}) * Arang(\overline{L}) \Rightarrow \tilde{q} * Arang(\varphi Perm * \overline{L}). \tag{35}$$

On the systems of accommodation of degrees we shall generate the components

$$\alpha_i \Rightarrow \left[\varphi Dpv(\overline{r_i}) * \overline{q}^{(n)}\right] \Rightarrow \overline{q}^{(m)} * \varphi Arang(\varphi Perm * \overline{r_i});$$ (36)

$$\beta_j \Rightarrow \left[\varphi Dpv(\overline{L_j}) * \overline{q}^{(n)}\right] \Rightarrow \overline{q}^{(m)} * \varphi Arang(\varphi Perm * \overline{L_j}).$$ (37)

The vectors consisting of these components are

$$\overline{\alpha} \Rightarrow \overline{q}^{(m)} * \varphi Arang(\varphi Perm * \bigcup_{i \in 1.n-1} \overline{r_i});$$ (38)

$$\overline{\beta} \Rightarrow \overline{q}^{(m)} * \varphi Arang(\varphi Perm * \bigcup_{j \in 1.n-1} \overline{L_j}).$$ (39)

Here the character of permutations is taken into the account. For $S(r, L)$ (12) we shall get

$$\varphi Dpv(\arg) * S(\overline{r} \times \circ \overline{L}) \Rightarrow \left[\varphi Dpv(\overline{r}) * \overline{q}^{(m)\,T}\right] \cdot \left[\varphi Dpv(\overline{r} \times \circ \overline{L}) * Diag\left(\overline{C}^{(m)}\right)\right] \cdot \left[\varphi Dpv(\overline{L}) * \overline{q}^{(m)}\right]$$ (40)

We get the result of application of the operator $\varphi Dpv(\arg)$ to the sub-matrix in the symbolical form as

$$\varphi Dpv(ims_1 \times \circ ims_2) * S(\overline{r} \times \circ \overline{L}) \Rightarrow \left[\varphi Dpv(\overline{r}) * \overline{q}^{(m)\,T}\right] \cdot \left[\varphi Dpv(\overline{r} \times \circ \overline{L}) * Diag\left(\overline{C}^{(m)}\right)\right] \cdot \left[\varphi Dpv(\overline{L}) * \overline{q}^{(m)}\right]$$ (41)

The result of application of the operator $\varphi Dpv(\overline{r}, \overline{L})$ has a positional character relative to a fixed component of the indices of columns. It allows to determine the algebraic complements for the elements of the matrix $B = (Y^T \cdot Y)$. The vector of values of algebraic complements for the elements of the matrix $B = (Y^T \cdot Y)$ is positionally attached to a component of the discrete poles

$$\varphi Dpv(\overline{r}, \overline{L}) * \widetilde{q}_j \Rightarrow \left\{\varphi Sum * \left[\widetilde{q} * \varphi Arang\left(\varphi \Pr m * \overline{r}\right)\right]\right\} \cdot \left[\widetilde{q} * \varphi Arang(\overline{L})\right].$$ (42)

Minimization of algorithm complexity using method of decomposition

We use the method of decomposition of coordinate components into regular fragments of [2]:

$$\varphi Part(2) * n \Rightarrow \bigcup_{i=0}^{n} \left(i.n-1\right) \quad \varphi Fg\left\{\bigcup_{i=0}^{n} \left(i.n-1\right)\right\} * \left[\overline{G(i,n)} \otimes \overline{G(n-i,n)}\right]$$ (43)

$$\overline{r} \Rightarrow \varphi Part(1.s) * \widetilde{r}^{(n)} \Rightarrow \bigcup_{i \in \overline{1\ S}} z_j \oplus (1.k_i).$$ (44)

To each of the regular fragments the following operator is applied:

$$\varphi Fg(ims) * \widetilde{q} \Rightarrow \varphi \Pi r_{j \in \overline{1.S}} * \left\{\varphi \Pi r * \left(\widetilde{q}_j * Arang(z_j)\right) \cdot FG(\widetilde{q}_j)\right\}.$$ (45)

Therefore we have

$$\varphi Dpv * \left[\bar{q} * \ Arang\,(\varphi \Pr m * \bar{r})\right] \Rightarrow \varphi Sum * \{[\varphi\, Fg * G(\nu_1, n)] \otimes$$
$$\otimes \left[\varphi\, Fg * G(n - \nu_1, n)\right] \otimes \bar{g}_r\}. \qquad (46)$$

Thus, the following condition is observed:

$$G(\nu_1, n)_i \circ G(n - \nu_1, n)_i = n\,. \qquad (47)$$

Here,

$$\varphi Fg * G(k, n) \Rightarrow \coprod_{i, j \in 1.k} (q_i - q_j)\,. \qquad (48)$$

The decomposition of the coordinate components is mapped into decomposition of the graph structure on the basis of which the operator $\varphi Dvp(\arg)$ is realized:

$$\varphi Gr((k_1, k_2) * \tilde{q}^{(n)} \Rightarrow \left[\varphi Dvp(\bar{r} \times \circ \bar{L}) * \overline{G(k_1, n)}\right] \otimes$$
$$\otimes \left[\varphi Dvp(\bar{r} \times \circ \bar{L}) * \overline{G(k_2, n)}\right]. \qquad k_1 + k_2 = n \qquad (49)$$

From here we get

$$\varphi Gr((k_1, k_2) * \tilde{q}^{(n)} \Rightarrow \left[\varphi Fg(k_1) * \overline{G(k_1, n)}\right] \otimes \left[\varphi Fg(k_2) * \overline{G(k_2, n)}\right] \otimes \bar{\psi}(k_2)\,. \qquad (50)$$

Here $\bar{\psi}(k_2) \Rightarrow \overline{G(k_2, n)} * Arang(k_2)$ is a multiplier, the degree of which is determined by the parameters of decomposition. The results of application of the operator $\varphi Dpv(\arg)$ to every component of $\varphi Perm * \tilde{q}_i \in \overline{G(n-1, m)}$ can be expressed as Kronecker lexicographic product

$$\varphi Gr((k_1, k_2) * \tilde{q}^{(n)} \Rightarrow \left\{\left[\varphi Fg(k_{11}) * \overline{G(k_{11}, n)}\right] \otimes \left[\varphi Fg(k_{12}) * \overline{G(k_{12}, n)}\right] \otimes \bar{\psi}(k_{12})\right\} \times \circ$$
$$\times \circ \left\{\left[\varphi Fg(k_{21}) * \overline{G(k_{21}, n)}\right] \otimes \left[\varphi Fg(k_{22}) * \overline{G(k_{22}, n)}\right] \otimes \bar{\psi}(k_{22})\right\}. \qquad (51)$$

Using the obtained results, the SC the model for the inverse matrix of dynamic process can be written down in the following form:

$$\left(Y^T \cdot Y\right)^{-1} \Rightarrow \varphi Dpv\left\{\left(\overline{G(n, m)} \times \circ \overline{G(n, m)}\right\} * Y \Rightarrow \left[\varphi Dpv * Q(\overline{G(n, m)})\right] \cdot$$
$$\cdot \left[\varphi Dpv\left\{\left(\overline{G(n, m)} \times \circ \overline{G(n, m)}\right\} * Z^{(k \times k)}\right] \cdot \left[\varphi Dpv * Q(\overline{G(n, m)})\right] \qquad (52)$$

Conclusions

In the derived analytical expressions for the inverse matrix of the system of normal equations, the factors influencing the numerical stability of the MLS algorithm are isolated. In particular, the weight matrix that corresponding to the MLS in which the influence of the amount of processed information is displayed, is isolated. These expressions are derived as a decomposition consisting of separate fragments that can be processed independently from each other. The forms of such decomposition can be varied by changing the parameters of the symbolical combinatory model. It allows to flexibly reconstruct the parallel architecture of the MLS algorithm with software methods, coordinating it with the parallel architecture of the computer working in the parallel mode of calculation.

The system of conditional equations to which Y. Merkuryev has given a new name: "Metamodel" in his doctoral thesis, should be transformed into a system of normal equations on the basis of strict mathematical relations. An unaware reader can have a deceptive impression, that the term "Metamodel" designates some new direction in information technologies. Actually, it is only a designation of the initial description which in itself is not valid. Therefore, on its basis only, any methods of imitation modeling, about which Y. Merkuryev speaks, cannot be constructed [14].

In the new form of the algorithm, the features of the dynamic character of identified object are reflected. The numerical stability of the algorithm depends on the values of distances of poles of the object's discrete operator. They should be distinct enough from the background of working noise and it imposes certain restrictions on the choice of the sampling rate of signals *T*. Therefore, the method of "the nearest neighbours," which is submitted as a new achievement in the field of information technologies by its authors Y. Merkuryev, L. Rastrigin, and G. Vulfs, is mathematically incorrect [12, 13, 15]. Actually it is offered to solve the problem of identification and imitation modelling in the area of degenerate systems of equations where reliable results cannot be received.

The results, found for the MLS algorithms using SC models, allow to make the computing process observable and to correct its stability.

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G. Burov. Symbolical Combinatory Model of Parallel Algorithm of Identification That Uses Method of Least Squares

The problem of development of parallel algorithms for the method of the least squares (MLS), used in the identification of analog dynamic objects, is considered. With the help of symbolical combinatory models, the analytical expression of the inverse matrix of the system of normal equations is found. It has allowed to apply non-conventional methods of regularization and to reveal the factors that influence the numerical stability of the MLS algorithm. The reduction of the distances between the discrete poles of the object can lead to degeneration of the algorithm. The increase in the linear dependence between the vectors of the matrix of initial system of equations leads to the same results. The developed theoretical model of the method of least squares (MLS) shows, that the improvement in solving the problem of approximation is achieved due to the reduction of the stability of computing algorithm. In the inverse matrix of the system of normal equations, as a matrix factor, there arises a matrix constructed from fragments of the products of mutual distances between discrete poles with absolute value less than one. The determinant of this matrix depends on the number of equations in the system of conditional equations. It decreases in a nonlinear way with the increase in the number of the determined parameters and the amount of processed information. The use of symbolical combinatory models allows, to some extent, to overcome the computation difficulties.

G. Burovs. Simbolisks kombinatoriskais modelis paralēlam identifikācijas algoritmam, kas izmanto mazāko kvadrātu metodi

Rakstā apskatīta paralēlu algoritmu izstrāde mazāko kvadrātu metodei (MKM), ko izmanto analogo dinamisko objektu identifikācijai. Izmantojot simboliskos kombinatoriskos modeļus iegūta analītiska izteiksme inversajai normālo vienādojumu matricai. Tas ļauj izmantot netradicionālas regularizācijas metodes un noteikt faktorus, kas ietekmē MKM algoritma skaitlisko stabilitāti. Attālumu starp objekta diskrētajiem poliem samazināšanās var novest pie algoritma deģenerācijas. Pie tāda pat rezultāta noved lineārās atkarības starp sākotnējās matricas vektoriem palielināšanās. Iegūtais MKM teorētiskais modelis parāda, ka aproksimācijas uzdevuma risinājuma uzlabošana notiek samazinot skaitļošanas algoritma skaitlisko stabilitāti. Normālo vienādojumu sistēmas inversās matricas sastāvā ietilpst matricas reizinātājs, kas veidots no diskrēto polu savstarpējo attālumu reizinājumu fragmentiem, kas pēc absolūtās vērtības ir mazāki par vienu. Šīs matricas determinants ir atkarīgs no vienādojumu skaita nosacīto vienādojumu sistēmā. Palielinoties nosakāmo parametru skaitam un apstrādājamās informācijas daudzumam, tas samazinās nelineārā veidā. Simbolisko kombinatorisko modeļu izmantošana ļauj zināmā mērā pārvarēt radušās izskaitļošanas problēmas.

Г. Буров. Символьная комбинаторная модель параллельного алгоритма идентификации с использованием метода наименьших квадратов

Рассмотрена задача получения параллельных алгоритмов для метода наименьших квадратов (МНК), применяемых для идентификации аналоговых динамических объектов. С помощью символьных комбинаторных моделей получено формульное выражение обратной матрицы системы нормальных уравнений. Это позволило применить нетрадиционные методы регуляризации и выявить факторы, влияющие на вычислительную устойчивость алгоритма МНК. Уменьшение расстояний между дискретными полюсами объекта может привести к вырождению алгоритма. К этому же приводит увеличение линейной зависимости между векторами матрицы исходной системы уравнений. Полученная теоретическая модель МНК показывает, что улучшение решения задачи аппроксимации достигается за счет уменьшения устойчивости вычислительного алгоритма. В обратную матрицу системы нормальных уравнений входит матричный множитель, построенный из фрагментов произведений взаимных расстояний между дискретными полюсами, которые по абсолютной величине меньше единицы. Детерминант этой матрицы зависит от числа уравнений, входящих в систему условных уравнений. Он уменьшается по нелинейному закону с увеличением числа определяемых параметров и объема обрабатываемой информации. Использование символьных комбинаторных моделей позволяет в определенной степени преодолеть возникающие вычислительные трудности.