ISSN 1407-8015 TRANSPORT AND ENGINEERING MAŠĪNZINĀTNE UN TRANSPORTS 2008-8015

MECHANICS
MEHĀNIKA

PLANE MOTION WITH COLLISIONS OF SOLID WITH ADDITIONAL UNILATERAL CONSTRAINT

CIETA ĶERMEŅA AR NENOTUROŠO PAPILDSAITI KOMPLĀNA KUSTĪBA AR TRIECIENIEM

Svetlana Polukoshko, leading researcher, Dr.sc.ing.
Ventspils University College, Engineering Research Centre
Address: 101 Inzenieru Street, Ventspils, LV-3601, Latvia
E-mail: pol.svet@inbox.lv

Svetlana Sokolova, lecturer, Mg. sc.
Riga Technical University, Institute of Mechanics
Address: 6 Ezermalas Street, Riga, LV-1006, Latvia
E-mail: svetavla@inbox.lv

Key words: rigid body, collision, friction, unilateral constraint, separation, sticking, sliding

1. Introduction

Two-dimensional motion of solids with inelastic impacts against a rough surface is discussed in this work. Before the collision a body has an additional immovable point of contact with a surface, therefore the impact occurs in two points. It is considered that contact points with zero velocity can transfer normal impact impulse; collisions in every point start and come to the end simultaneously. The state of unilateral constraints is changing during time interval of impact and transition from non-separation to separation, from stick to sliding, from sliding to sticking must be predicted. Motion of solid with impacts in two or more points is difficult to analyze because of necessity to define the state of contact and after-impact behaviour of body. In this study the problem is solved in the framework of rigid body, lows of impact is modeled after Poison, deformation does not taking into account, friction is taking into account after Coulomb with dry friction factor, impact is assumed instantaneous and simultaneous in both points of contact. Three types of bodies are investigated: homogenous rigid rod, plate and cylinder. It is shown that the different types of resulted motion of body are possible: stop of body; sliding on two points of support; rotation round the immovable new point of support; sliding on the new point of support and rotation round it. We define conditions at which one or another type of motion takes place after impact. The domains of existence of each mode of motion are determined for some special cases for different kinds of body. Calculated results were examined on experimental setup for the rigid rod.
This problem is of special interest for motion of walking machines study, motion of unfixed objects on ship deck and etc.

2. Plane motion of rigid rod with impact in two points

The purpose of this chapter is to define the scopes of applicability of theory of solids bodies impact to the multi-point impact at two-dimentional motion.

2.1. Model of impact

Non-sliding rotation of uniform rod with mass ***m***, length ***l*** accompanied by by collision with fixed supports is investigated (Fig. 1). Both supports are unilateral and placed on one horizontal level, ***a*** and ***b*** are the distance between mass center of rod and supports location. In-plane moving solid rests upon a supporting surface in a point ***A***, which has a zero velocity. Resulting impact with a supporting surface occurs in the new point ***B***, impact is assumed perfectly inelastic and after impact the new point of contact remains on a supporting surface. Rotation motion of rod without sliding around fixed point ***A***, is described by differential equation:

$$\left(J_c + ma^2\right)\ddot{\varphi} = mga\cos\varphi . \tag{2.1}$$

Angular rate of the rod when it reaches the support ***B***, i.e. pre-impact velocity***:***

$$\omega_0 = \sqrt{\frac{2mga}{J_c + ma^2}\sin\alpha} = \sqrt{\frac{24ga}{l^2 + 12a^2}\sin\alpha} \ ,$$

where α – the angle of initial ascent, $Jc = ml^2/12$ – moment of inertia of rod.

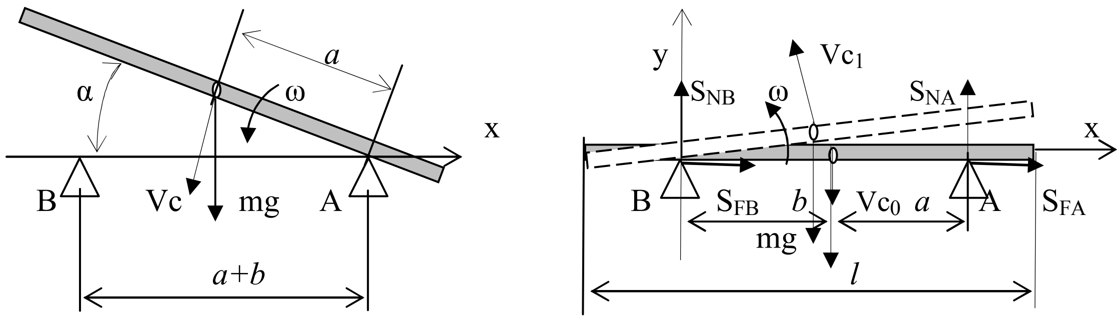


Fig. 1. Rotation motion of rod with impacts against fixed supports.

The investigation of collision shows that there are five possible variants of the rod behavior:
1 - the rod doesn't separate from supports and the motion ends; 2- non-separation in the both support and sliding of rod in the horizontal direction to the left, 3 - separation of the rod from support ***A*** and further rotation round point B, 4- separation of the body from the support ***A***, sliding on a surface in the new point of contact ***B*** and rotation round it.
In order to determine the condition of the sticking of rod to the supports or separation from it rigid body momentum theorem in projection on Cartesian axes and principle of moment of momentum in regard to mass center is drown up. It is assumed that rod separates from support when calculated from the system of the general dynamics equations normal reactive impulse force is negative or equal zero, if the rod sticks to both support it angular speed equal zero. Taking into account, that the velocity of mass center before and after impact are: $v_{c0y} = \omega_0 a$, $v_{c0x} = \omega_0 0.5h$ (h is the thickness of rod) and $\omega_1 = 0$, hence $v_{c1y} = 0$, $vx = 0$, we have a system:

$$\begin{cases} -(-m\omega_0 a) = S_{NB} + S_{NA} \\ -(-m\omega_0 0.5h) = S_{FB} + S_{FA} \\ -J_c\omega_0 = -S_{NB}b + S_{NA}a + (S_{XB} + S_{XA})0.5h \end{cases} \tag{2.2}$$

Solving the system (2.2) gives the normal impulses in point of contact:

$$S_{NA} = \omega_0 \frac{mba - J_c}{b+a} = \omega_0 m \frac{12ba - l^2 - 3h^2}{12(b+a)}, \qquad S_{NB} = \omega_0 \frac{ma^2 + J_c}{b+a} = \omega_0 m \frac{12a^2 + l^2 + 3h^2}{12(b+a)}.$$

It is evidently that at the beginning of impact the rod doesn't rebound from support ***B***, and doesn't separate from ***A*** support only subject to: $12ab > l^2 + 3h^2$, where $3h^2$ may be neglected because it is too small, hence the condition of non-separation:

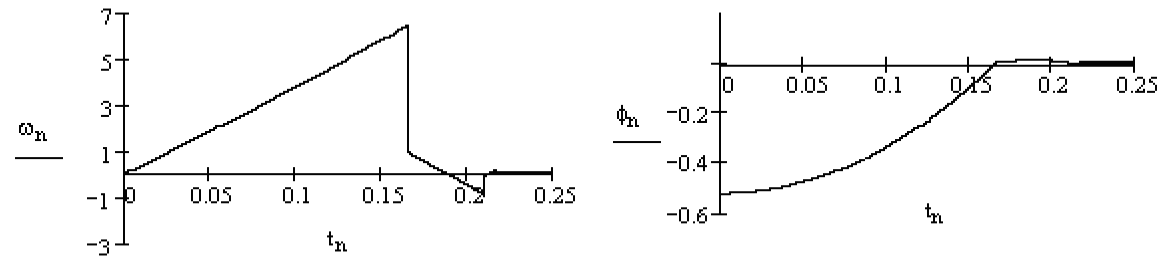
$$12ab > l^2 \;. \tag{2.3}$$

In case of $a=b$, and this condition will be: $0.5l > a > 0.289l$.

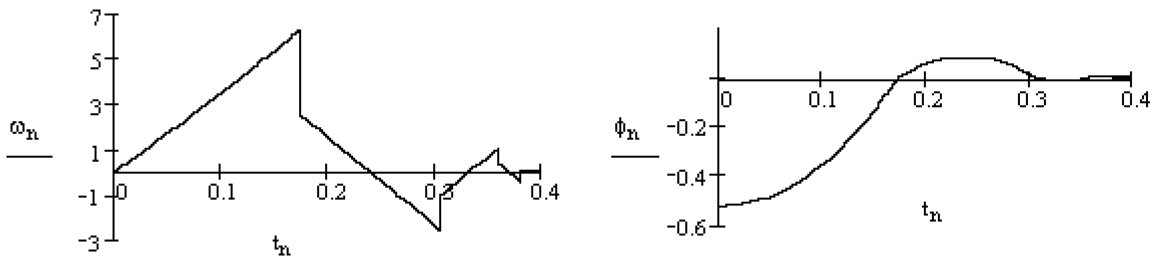
The condition of non-sliding is: $\left|S_{FB} + S_{FA}\right| \le (S_{NB} + S_{NA})f$, i.e. $f \ge h/2a$. (2.4)

Provided that condition (2.3) doesn't satisfy and since impact is perfectly inelastic, after separating from support ***A*** and energy losing during impact, the rod goes on rotation round support ***B*** in accordance with equation:

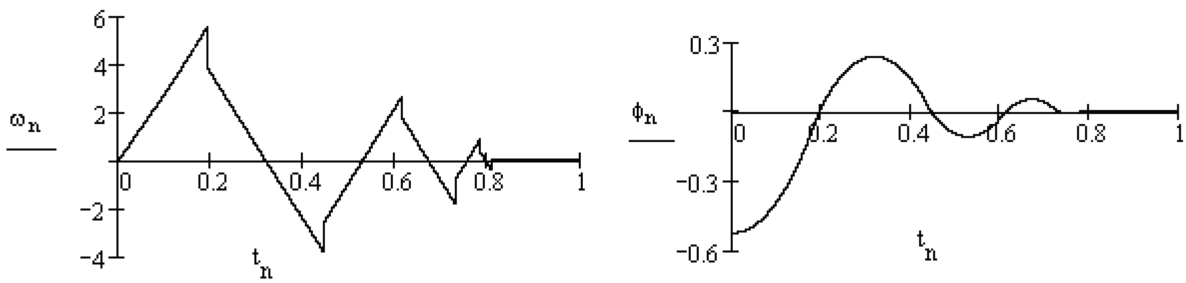
$$\left(J_c + mb^2\right)\ddot{\varphi} = -mgb\cos\varphi \;. \tag{2.5}$$



a) l = 0.40 m, a=b=0.10 m, k=0.143



b) l = 0.40 m, a=b=0.075 m, k=0.407



c) l = 0.40 m, a=b=0.05 m, k=0.684

Fig. 2. Angular speed ω= ω(t) and rotation angle φ=φ (t) plots depending on time

The initial angular speed of this motion is determined from the system of the general dynamics equations for the case of point ***A*** separation, noting that $v_{c1y} = \omega_1 b$, $v_{c1x} = \omega_0 0.5h$:

$$\begin{cases} m\omega_1 b - (-m\omega_0 a) = S_{NB} \\ m\omega_1 0.5h - (-m\omega_0 0.5h) = S_{FB} \\ J_c\omega_1 - J_c\omega_0 = -S_{NB}b + S_{FB}0.5h \end{cases} \qquad (2.6)$$

Solving this system gives: $\omega_1 = \omega_0 \dfrac{l^2 - 12ba + 3h^2}{l^2 + 12ba + 3h^2}$; value of $3h^2$ is too small for rod and may be neglected, then coefficient of decreasing of angular velocity will be: $k = \dfrac{l^2 - 12ab}{l^2 + 12ab}$.

Necessary condition of this mode existing is satisfying an equation (2.4), otherwise it will be sliding with rotation on the support B.

In Fig. 2 angular speed ω and rotation angle φ plots depending on time are shown for the rod with *l= 0.40m* in cases of *a=b=0.10 m*, *a=b=0.075* m and *a=b=0.05* m; the initial angle of ascent is 30º.

2.2. Experimental setup

For verification of theoretical results obtained above correctness and conditions of applicability of classical theory of solid body's impact, the simplest experimental setting, shown in Fig. 3, was created.

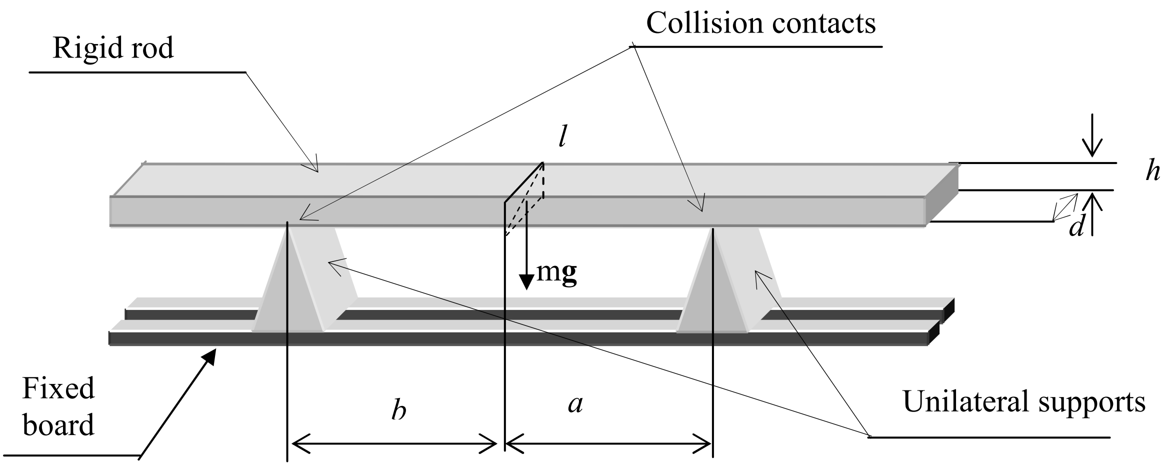


Fig. 3. Experimental setup

Test setup consists of the fixed lower supporting board with two unilateral supports on it and set of wooden, metal and plastic rods with length ***l*** = 40 cm, width ***d*** =3см and thickness ***h***=2.0, 1.0, 0.5 and 0.2 cm. The removable fixation of the supports on a board allows easy changing its positions, distance ***a*** and ***b*** changes from 0 till 20cm. Rotational motion of rod started with the initial angles of ascent from α=15º till 30º. Plane motion of all rods with thickness of 2, 1 and 0,5 cm with one fixed point (rotational motion) prove the classic theory of impact, sticking of rod took place at $12ab > l^2$, i.e. if *a=b* then *a>11.55 cm.* The rod with thickness 0.2 cm rebounded from both points of contact regardless of the location of unilateral supports. This fact confirms the scopes of applicability of classical theory of impact for solids: if the cross section of body is large in comparison with the local deformations area, the theory of solid bodies' impact is applicable. In the case of thin beam (with h=2 mm in experiment) cross section and local deformations area are comparable and it is necessary to apply wave theory of impact. Sliding of the rod on wooden surface didn't observe.

At the asymmetrical location of supports and at a very small ***a*** value (for this beam there is about 1 cm), in spite of satisfaction $12ab > l^2 + 3h^2$, non-separation of rod in a point *A* and separation in a point ***B*** with a further rotation round a point ***A*** in opposite direction are observed. In this case because of small distance from the axis of rotation to the beam mass center and small ascent angles very small initial velocity of impact is received, at which impact forces are comparable with gravity force.

3. Plane motion of rigid plate with impact in two points

Massive homogenous plate placed on rough surface rotates round point ***A*** till collision with surface (Fig. 4), impact with surface is considered perfectly inelastic. If the body doesn't separate from surface in point of contact ***A*** and impact is non-sliding, then post-impact velocity of mass center and angular velocity are equal to zero. Momentum theorem in projection on x and y axes and principle of moment of momentum in regard to center of mass, taken into account, that $\omega_1 = 0$, v_1=0 is:

$$\begin{cases} m\omega_0 h = S_{FB} + S_{FB} \\ m\omega_0 a = S_{NA} + S_{NB} \\ -J_c\omega_0 = +S_{FA}h + S_{FB}h + S_{NA}a - S_{NB}b \end{cases} \quad (3.1)$$

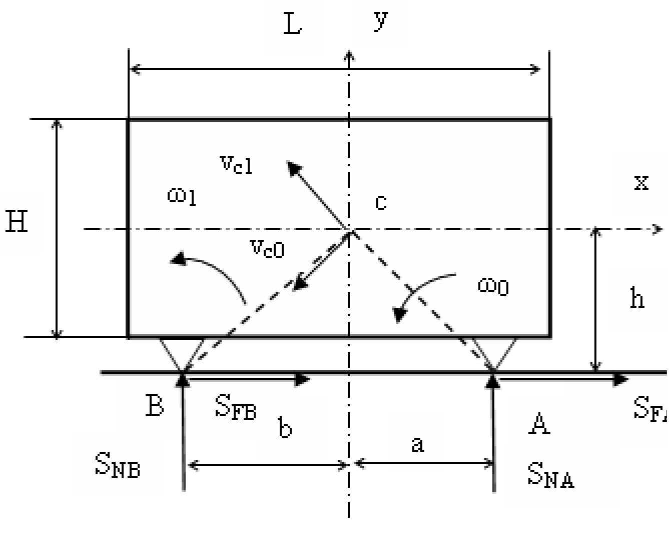


Fig. 4. Plane motion of the plate with two-point impacts

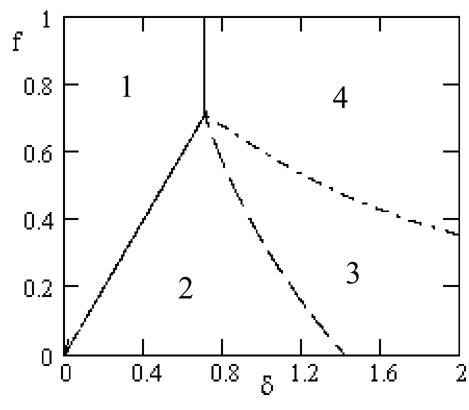


Fig. 5. Plot of the dependence of factor *f* on δ

Taking into account that $J_C = m\rho^2$, we can define, that S_{NB} is always positive and S_{NA} may be negative: $S_{NA} = m\omega_0 \dfrac{ab - \rho^2 - h^2}{a+b}$, $\qquad S_{NB} = m\omega_0 \dfrac{h^2 + a^2 + \rho^2}{a+b}$.

The condition of non-separation of point ***A*** is $S_{NA} \geq 0$, i.e. $ab \geq \rho^2 + h^2$.

In accordance to the Routh theory during impact the friction force is assumed as a dry friction with factor f, then, if the sliding take place, impact friction forces are:

$$S_{FA} = S_{NA} f\,, \quad S_{FB} = S_{NB} f\,.$$

The summarized horizontal impact force: $S_{FA} + S_{FB} = m\omega_0 h$ (the first equation in (3.1)) must be less than $(S_{NA} + S_{NB})f$, i.e. the condition of sticking is: $f \geq h/b$.

For the case of non-separation of ***A*** and full sliding on both supports with velocity v_s:

$$\begin{cases} -mv_s + m\omega_0 h = S_{NA} f + S_{NB} f \\ m\omega_0 a = S_{NA} + S_{NB} \\ -J_C\omega_0 = S_{NA} fh + S_{NB} fh + S_{NA} a - S_{NB} b \end{cases} \tag{3.2}$$

Solving (3.2) with respect to S_{NA} S_{NB}, v_s we receive:

$$S_{NA} = m\omega_0 \frac{ab - \rho^2 - fha}{a+b}, \quad S_{NB} = m\omega_0 \frac{\rho^2 + fha + a^2}{a+b}, \ v_s = \omega_0 (h - fa)\,.$$

The conditions of existing of this mode are: $S_{NA} \geq 0$, $v_s > 0$, i.e.

$$ab \geq \rho^2 + h^2 \ \text{and} \ f < h/a\,, \text{or} \ ab < \rho^2 + h^2 \ \text{and} \ f \leq (ab - \rho^2)/ha\,.$$

For the case of point A separation and rotation around point ***B*** and non-sliding:

$$\begin{cases} -m\omega_1 h + m\omega_0 h = S_{FB} \\ m\omega_1 b + m\omega_0 a = S_{NB} \\ J_C\omega_1 - J_C\omega_0 = S_{FB} h - S_{NB} b \end{cases} \tag{3.3}$$

Solving (3.3) with respect to S_{NB}, S_{FB} and ω_1 we receive:

$$S_{NB} = m\omega_0 \frac{(\rho^2 + h^2)(a+b)}{\rho^2 + h^2 + b^2}, \quad S_{FB} = m\omega_0 \frac{hb(a+b)}{\rho^2 + h^2 + b^2}, \quad \omega_1 = \omega_0 \frac{\rho^2 + h^2 - ab}{\rho^2 + h^2 + b^2}\,.$$

The conditions of existing of this mode are: $S_{NB} f \geq S_{FN}$, $\omega_1 > 0$, i.e.:

$$ab < \rho^2 + h^2, \ f \geq hb/(\rho^2 + h^2)\,.$$

For the case of separation of point ***A***, rotation around point ***B*** and sliding:

$$\begin{cases} -m\omega_1 h - mv_s + m\omega_0 h = S_{NB} f \\ m\omega_1 b + m\omega_0 a = S_{NB} \\ J_C\omega_1 - J_C\omega_0 = S_{NB} fh - S_{NB} b \end{cases} \tag{3.4}$$

Solving (3.4) with respect to S_{NB}, ω_1, v_s we receive:

$$S_{NB} = m\omega_0 \rho^2 \frac{(a+b)}{\rho^2 + b^2 - fhb}, \quad \omega_1 = \omega_0 \frac{\rho^2 + fha - ab}{\rho^2 + b^2 - fhb}, \quad v_S = \omega_0 \frac{hab - f(a+b)(\rho^2 + h^2)}{\rho^2 + b^2 - fhb}\,.$$

The conditions of existing of this mode are: $S_{NB} \geq 0$, $v_S > 0$, $\omega_1 > 0$, i.e.:

$$ab < \rho^2 + h^2, \ (ab - \rho^2)/ha < f < hb/(\rho^2 + h^2)\,.$$

As example the rectangular plate with length *L*, height *H* and inertia radius $\rho^2 = (L^2 + H^2)/12$ is considered if $a = b = L/2$. The height of supported legs is negligible and *h* may be assumed as $h = H/2$. The conditions of existence of each mode of impacts are derived as function of non-dimensional parameter $\delta = H/L$: 1- non-separation of ***A***, non-sliding: $\delta \leq 1/\sqrt{2}$, $f \geq \delta$; 2-non-separation, two-points sliding: $\delta \leq 1/\sqrt{2}$, $f < \delta$; or $1/\sqrt{2} < \delta$ and $f < (2 - \delta^2)/3\delta$;

3-separation, rotation round ***B***, sliding: $1/\sqrt{2} < \delta$ and $(2-\delta^2)/3\delta < f < 3\delta/(4\delta^2+1)$;

4-separation, sticking, rotation round ***B***: $1/\sqrt{2} < \delta$ and $f \geq 3\delta/(4\delta^2+1)$.

In Fig. 5 the domains of existence of each mode are presented for the rectangular plate with length *L* and height *H*, where 1- non-separation of point A, non-sliding impact, sticking; 2- non-separation of point A, sliding on two points; 3- separation of point ***A***, rotation round point ***B***, sliding. 4- separation of point ***A***, rotation round point ***B***, non-sliding.

4. Plane motion of rigid cylinder with impact in two points

Homogeneous cylinder of radius ***a*** rolls on rough surface, and before impact it rotates without sliding **round** point ***A***, γ_1, γ_2 – central angles of contact points (Fig. 6). Impact with surface in the point ***B*** is considered perfectly inelastic. If the point ***A*** non-separate from fixed point of contact and impact is sliding, then post-impact velocity of mass centre is equal to zero: v_1=0, but angular velocity nonzero: $\omega_1 \neq 0$ Momentum theorem and principle of moment of momentum regard to center of mass for this case:

$$\begin{cases} m\omega_0 a\cos\gamma_1 = -S_{NA}\sin\gamma_1 + S_{NB}\sin\gamma_2 - S_{NA}f\cos\gamma_1 - S_{NB}f\cos\gamma_2 \\ m\omega_0 a\sin\gamma_1 = S_{NA}\cos\gamma_1 + S_{NB}\cos\gamma_2 - S_{NA}f\sin\gamma_1 + S_{NB}f\sin\gamma_2 \\ J_c\omega_1 - J_c\omega_0 = -S_{NA}fa - S_{NB}fa \end{cases} \quad (4.1)$$

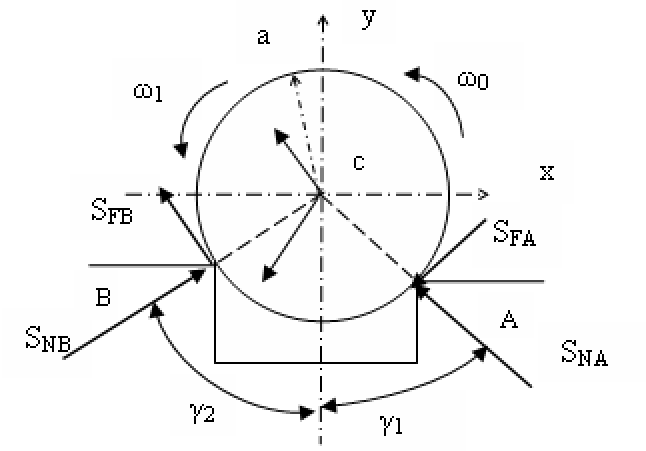


Fig. 6.Rolling motion of the cylinder with impacts

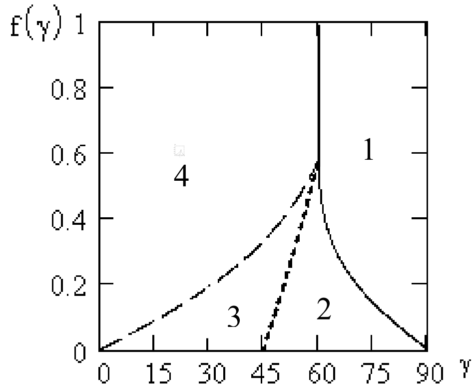


Fig. 7. Plot of the dependence of factor f on angle γ, provided $\gamma_1 = \gamma_2$

For the case of point A separation and non-sliding impact the system of equation is:

$$\begin{cases} -m\omega_1 a\cos\gamma_2 + m\omega_0 a\cos\gamma_1 = S_{NB}\sin\gamma_2 - S_{FB}\cos\gamma_2 \\ m\omega_1 a\sin\gamma_2 + m\omega_0 a\sin\gamma_1 = S_{NB}\cos\gamma_2 + S_{FB}\sin\gamma_2 \\ J_c\omega_1 - J_c\omega_0 = -S_{FB}a \end{cases} \quad (4.2)$$

For the case of separation of point A and sliding impact the system is:

$$\begin{cases} -m\omega_1 a\cos\gamma_2 + mv_s\cos\gamma_2 + m\omega_0 a\cos\gamma_1 = S_{NB}\sin\gamma_2 - S_{NB}f\cos\gamma_2 \\ m\omega_1 a\sin\gamma_2 - mv_s\sin\gamma_2 + m\omega_0 a\sin\gamma_1 = S_{NB}\cos\gamma_2 + S_{NB}f\sin\gamma_2 \\ J_c\omega_1 - J_c\omega_0 = -S_{NB}fa \end{cases} \quad (4.3)$$

Thereafter these systems are solved for the homogeneous cylinder with $J_C = 0.5ma^2$, in case of symmetrical contact points, i.e. $\gamma_1 = \gamma_2$. Solving (4.1) with respect to S_{NA}, S_{NB}, ω_1 gives:

$$S_{NA} = m\omega_0 a \frac{\tan^2(\gamma) - 2f\tan(\gamma) - 1}{2\tan(\gamma)(1+f^2)}, \qquad S_{NB} = m\omega_0 a \frac{\tan^2(\gamma)+1}{2\tan(\gamma)(1+f^2)},$$

$$\omega_1 = \omega_0 \frac{1+3f^2 - 2f\tan(\gamma)}{2\tan(\gamma)(1+f^2)}.$$

From this we can conclude that point B never separate from contact point and the conditions $S_{NA} > 0$ of existing of this mode - point A non-separate and impact is sliding:

$$\frac{1+3f^2}{2f} > \tan(\gamma) \geq f + \sqrt{f^2+1}.$$

The condition non-sliding impact in the case of non-separation of point ***A***: $\frac{1+3f^2}{2f} \leq \tan(\gamma)$.

Soling system (4.2) for the case of point A separation and non-sliding impact gives:

$$S_{NB} = m\omega_0 a \frac{2\tan\gamma}{\tan^2\gamma+1}, \quad S_{FB} = m\omega_0 a \frac{2\tan\gamma}{3(\tan^2\gamma+1)}, \quad \omega_1 = \omega_0 \frac{3-\tan^2(\gamma)}{3(\tan^2(\gamma)+1)}.$$

The conditions of existing of this mode: $\omega_1 > 0$, $S_{NB} f > S_{FB}$, i.e. $\gamma < \pi/3$, $(\tan\gamma) > 3f$.

Soling system (4.3) for the case of point A separation and sliding impact gives:

$$S_{NB} = m\omega_0 a \frac{2\tan\gamma}{\tan^2\gamma+1}, \quad \omega_1 = \omega_0 \frac{\tan^2(\gamma) - 4f\tan\lambda + 1}{(\tan^2(\gamma)+1)}, \quad v_s = 2\omega_0 a \frac{3f - \tan(\gamma)}{(\tan^2(\gamma)+1)}.$$

The conditions of existing of this mode: $\omega_1 > 0$, $v_s > 0$, i.e., $(2f + \sqrt{4f^2 - 1}) > (\tan\gamma) \geq 3f$.

In Fig. 7 the domains of existence of each mode are presented for the cylinder with radius ***a*** and $J_c = 0.5ma^2$ for the case of $\gamma_1 = \gamma_2$; here: 1- non-separation of point A, sticking, impact is non-sliding, 2-non-separation of point A, impact is sliding; 2-separation of point A, sliding; 4-separation of point A, non-sliding.

5. Conclusion

It is proved that approximate treatment of rigid body collision with Poisson impact law, which does not take into account deformation during collision, may be successfully used for determination of the uniform rigid body's behaviour in some cases, and such as analyzed above motion with simultaneous impacts in two or more points. It is established that the post-impact motion of the solid bodies with additional point of contact depends on the state of contact – separation, non-separation, sticking, sliding or reversed sliding. Friction factor and linear dimensions of the bodies – distance between supports and moment of inertia - influence on the state of contact and on the angular speed decreasing coefficient.

References

1. Formalsky A., Ghevallereau C., Perrin B. About Impact Interaction of Body with Support // In: Bulletin of Moscow University: Mathematics, Mechanics, 2000, № 1 - p. 27-32.
2. Lapshin V. V. 2-D impact of body against rough surface. Preprint - Institute of Applied Mathematics, the Russian Academy of Science - Moscow, 2001, №10.

3. Lapshin V. V. A body collision with a surface in the presence of an additional point of contact. Preprint - Institute of Applied Mathematics, the Russian Academy of Science - Moscow, 2002.
4. Nagaev R.F. Mechanical processes with repeated damped collisions - Moscow, Nauka, 1985.
5. Pljavnieks V. J. Calculation of Oblique Impact against an Obstacle. // In: Questions of dynamics and strength, № 18 - Riga, Zinatne, 1969 - p. 87-109.
6. Pfeiffer F., Glocker Chr. Dynamical system with unilateral contacts. International University Course in framework of COMETT II // In: Engineering Applications of the Dynamics of Chaos - Wien, 1994.

Polukoško S., Sokolova S. Cieta ķermeņa ar nenoturošo papildsaiti komplāna kustība ar triecieniem
Darbā tika izpētīta cietu ķermeņu komplāna kustība ar sadursmēm pret nekustīgu grubuļainu virsmu: homogēna stieņa, plāksnes un cilindra. Pirms trieciena ķermenim ir papildus kontakta punkts ar virsmu. Pierādīts, ka vispārīgā neelastīga trieciena gadījumā pēc trieciena var būt sekojoši ķermeņa kustības veidi: 1- ķermeņa neatvienošanās no abiem kontakta punktiem un kustības pārtraukšana (pielipšana), 2- neatvienošanās divos punktos un ķermeņa slīdēšana pa virsmu, 3 - ķermeņa atvienošanās no pirmā punkta un rotācija bez slīdes ap jaunu kontakta punktu, 4- ķermeņa atvienošanās no pirmā punkta un slīdēšana pa virsmu jaunā kontakta punktā un griešanās ap to. Visiem ķermeņiem ir noteikti katra kustības režīma eksistences apgabali. Uzdevuma atrisinājumam ir izmantota klasiskā cietu ķermeņu trieciena teorija savienojumā ar Rausa hipotēzi, kas ņem vērā berzi pie trieciena pēc Kulona likuma. Viendabīgam stienim klasiskās teorijas secinājumi ir pārbaudīti eksperimentāli.

Polukoshko S., Sokolova S. Plane motion with collisions of solid with additional unilateral constraint
In this article in-plane motion of solids with collisions against the immovable rough surface is considered; examined bodies - homogeneous rod, plate and cylinder - have an additional point of contact with a surface before the impact. It is established that in general case of perfectly inelastic impact the followings modes of post-impact motion of body may be realized: 1- non-separation in the both points of contact, non-sliding and stopping of motion (sticking), 2- non-separation in the both points of contact and sliding of body on a surface two points, 3– separation of bodies from the first point and rotation without sliding round a new point of contact, 4- separation of the body from the first point, sliding on a surface in the new point of contact and rotation round it. The domains of existence of each mode of motion are determined for all bodies. For solving of this problem the classical theory of solid bodies' impact is used in combination with the Routh's hypothesis, taking into account a friction by law of Coulomb. The conclusions of classic theory are tested experimentally for uniform rod.

Полукошко С., Соколова С. Плоское движение с ударами твёрдого тела с дополнительной неудерживающей связью
В работе рассматривается плоское движение с ударами о неподвижную шероховатую поверхность твёрдых тел, имеющих до удара дополнительную точку контакта с поверхностью: однородного стержня, пластины и цилиндра. Установлено, что в общем случае абсолютно неупругого удара могут быть реализованы следующие виды послеударного движения тела: 1- неотрыв в обеих точках контакта и прекращение движения (прилипание), 2- неотрыв в двух точках и скольжение тела по поверхности, 3- отрыв тела от первой точки и вращение без скольжения относительно новой точки контакта, 4- отрыв тела от первой точки, скольжение по поверхности в новой точке контакта и вращение вокруг неё. Для каждого тела определены области существования каждого режима движения. Для решения задачи использована классическая теория удара твёрдых тел в сочетании с гипотезой Рауса, учитывающей трение при ударе по закону Кулона. Выводы классической теории проверены экспериментально для случая однородного стержня.