ENERĢĒTIKA
UN ELEKTROTEHNIKA
ISSN 1407-7345 2008-23
POWER AND ELECTRICAL
ENGINEERING

ANALYSIS OF ASYMMETRIC SUPPLY MODE OF THE INDUCTION MOTOR IN SYSTEM OF d,q COORDINATES

ASINHRONĀ DZINĒJA DARBA ANALĪZE d,q KOORDINĀTU SISTĒMĀ PIE BAROŠANAS ASIMETRIJAS

Marina Koņuhova, Ph. D. Student, Mg.Sc.Ing.
Riga Technical University
Faculty of Power and Electrical Engineering
Address: Kronvalda boulevard 1, LV 1010, Riga, Latvia
Phone: +371 7089926
e-mail: konuhova@eef.rtu.lv

Karlis Ketners, Prof., Dr.sc.ing.
Riga Technical University
Faculty of Power and Electrical Engineering
Address: Kronvalda boulevard 1, LV 1010, Riga, Latvia
Phone: +371 7089926, Fax: 7089418
e-mail: ketners@eef.rtu.lv

Elena Ketnere, Asoc. Prof., Dr. sc. Ing.
Riga Technical University
Faculty of Power and Electrical Engineering
Address: Kronvalda boulevard 1, LV 1010, Riga, Latvia
Phone: +371 7089929
e-mail: ketnere@eef.rtu.lv

Svetlana Kļujevska, Doc., Dr.sc.ing.
Riga Technical University
Faculty of Power and Electrical Engineering
Address: Kronvalda boulevard 1, LV 1010, Riga, Latvia
Phone: +371 7089929
e-mail: svzimina@eef.rtu.lv

Keywords: analysis, asymmetric supply mode, induction motor, mathematical model, modeling, Park-Gorev's equations, research of the dynamic mode

Wide usage of induction motors (IM) in the different fields of economy is motivating necessity for research of physical phenomenon, which occurs in its working process
Big quantity of researches is devoted to investigation and analysis of the asymmetric modes of the IM [3,4,5]. However there remains a problem of simultaneously assurance of accuracy and simplicity of solution until to now.
The assessment of three-phase IM operation at asymmetry of voltage is rather actual problem under the conditions of development of new technologies.
The mathematical model consists of 3-phase feeder and connects the induction motor with an indefinitely infinite -power network was developed for analysis of asymmetric supply mode of the IM. This model gives opportunity to simulate asymmetric damage on stator terminals of the IM (Fig.1.).

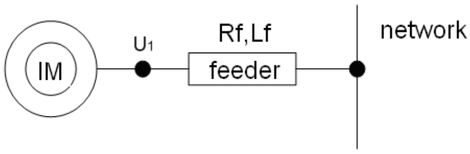

Figure 1. The scheme of the model

The mathematical model of transients modes of alternating current electric machines presently often the bases on Park-Gorev's equations traditionally operated in d,q,0 coordinate system [2]. At absence of a zero component the complete system of the equation of mathematical model of the induction motor in means of relative units is:

$$\left.\begin{aligned} U_{1d} &= R_1 i_{1d} - \omega_{0el}\psi_{1q} + \frac{d\psi_{1d}}{d\tau} \\ U_{1q} &= R_1 i_{1q} + \omega_{0el}\psi_{1d} + \frac{d\psi_{1q}}{d\tau} \\ U_{2d} &= R_2 i_{2d} - (\omega_{0el} - \omega)\psi_{2q} + \frac{d\psi_{2d}}{d\tau} \\ U_{2q} &= R_2 i_{2q} + (\omega_{0el} - \omega)\psi_{2d} + \frac{d\psi_{2q}}{d\tau} \end{aligned}\right\} \tag{1}$$

$$T_M \frac{d\omega}{d\tau} = \left[M_{em} - M_l\right] \tag{2}$$

$$\left.\begin{aligned} \psi_{1d} &= X_1 i_{1d} + X_{ad} i_{2d}; \\ \psi_{1q} &= X_1 i_{1q} + X_{ad} i_{2q}; \\ \psi_{2d} &= X_2 i_{2d} + X_{ad} i_{1d}; \\ \psi_{2q} &= X_2 i_{2q} + X_{ad} i_{1q}; \end{aligned}\right\} \tag{3}$$

where T_M - time constant of the motor in electrical radians;

$M_{em} = X_{ad}(i_{2d}i_{1q} - i_{2q}i_{1d})$ - electromagnetic torque.

$u_{1d}, u_{1q}, u_{2d}, u_{2q}$ - components of stator and rotor voltage;

$u_{2d} = u_{2q} = 0$ for induction motor with a squirrel-cage rotor;

$\psi_{1d}, \psi_{1q}, \psi_{2d}, \psi_{2q}$ - components of flux linkages;
$i_{1d}, i_{1q}, i_{2d}, i_{2q}$ - components of stator and rotor currents;
ω - angular rotation speed of rotor;
ω_{0el} - angular rotation speed of coordinate system;
$x_1, x_2, x_{ad}, r_1, r_2$ - parameters of the induction motor;
M_{em}, M_{sl} - electromagnetic moments and load.

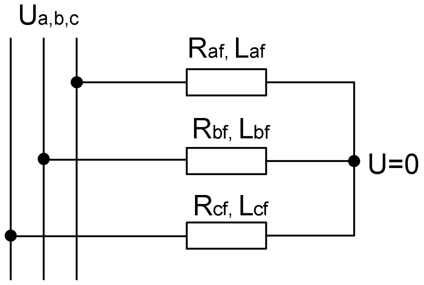

Figure 2. The scheme of bonding process of active-reactive devices

The initial equations of active-reactive devices (fig.2.) in phase coordinates might be presented in the following kind:

$$\begin{vmatrix} U_a \\ U_b \\ U_c \end{vmatrix} = \begin{vmatrix} R_a & 0 & 0 \\ 0 & R_b & 0 \\ 0 & 0 & R_c \end{vmatrix} \times \begin{vmatrix} i_a \\ i_b \\ i_c \end{vmatrix} + \begin{vmatrix} L_a & 0 & 0 \\ 0 & L_b & 0 \\ 0 & 0 & L_c \end{vmatrix} \times \frac{d}{dt} \begin{vmatrix} i_a \\ i_b \\ i_c \end{vmatrix}, \qquad (4)$$

where $U_{a,b,c}$ – phase voltages;

$i_{a,b,c}$ – phase currents;

$R_{a,b,c}$ – active resistances;

$L_{a,b,c}$ – inductances.

Or in contact notation:

$$\left[U_{ph}\right] = \left[R_{ph}\right] \cdot \left[i_{ph}\right] + \left[L_{ph}\right] \cdot \frac{d}{dt}\left[i_{ph}\right] \qquad (5)$$

Indexes a,b,c at sizes designate corresponding phases. Using transformations of Park it is possible to get the differential equations of active-reactive resistances. The matrix of direct linear transformation of Park looks like:

$$\left|M_{\Pi}\right| = \frac{2}{3}\begin{vmatrix} \cos\theta & \cos(\theta-120°) & \cos(\theta+120°) \\ -\sin\theta & -\sin(\theta-120°) & -\sin(\theta+120°) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{vmatrix} \qquad (6)$$

$$\frac{d\theta}{dt} = \omega \qquad (7)$$

The matrix of return linear transformation of Park is represented as follows:

$$\left|M_{\Pi}\right| = \frac{2}{3}\begin{vmatrix} \cos\theta & -\sin\theta & 1 \\ \cos(\theta-120°) & -\sin(\theta-120°) & 1 \\ \cos(\theta+120°) & -\sin(\theta+120°) & 1 \end{vmatrix} \qquad (8)$$

Using transformation of Park it is possible to get the differential equations of active-reactive resistances connected to the network in d,q,0 coordinate system.
In equations (6), (8) θ - a corner of turn of d,q,0 coordinate system, relating to a,b,c phases coordinates.
Having applied forward transformation of Park and multiply both sides of equation term by term, from the left on $\left[M_{\Pi}\right]$ matrix, we get:

$$\left[M_{\Pi}\right]*\left[U_{ph}\right]=\left[M_{\Pi}\right]*\left[R_{ph}\right]*\left[i_{ph}\right]+\left[M_{\Pi}\right]*\left[L_{ph}\right]*\frac{d}{dt}\left[i_{ph}\right] \qquad (9)$$

where $\left[M_{\Pi}\right]=\left[U_{ph}\right]=\begin{bmatrix} U_d \\ U_q \\ U_0 \end{bmatrix}=\left[U_{dq0}\right]$, using $\left[M_{\Pi}{}^{-1}\right]=\left[i_{ph}\right]=\left[i_{dq0}\right]$, we might get:

$$\left[U_{d,q0}\right]=\left[M_{\Pi}\right]*\left[R_{ph}\right]*\left[M_{\Pi}{}^{-1}\right]\cdot\left[i_{dq0}\right]+\left[M_{\Pi}\right]*\left[L_{ph}\right]\cdot\frac{d}{dt}\left\{\left[M_{\Pi}\right]\cdot\left[i_{dq0}\right]\right\} \qquad (10)$$

Differentiating product of matrix, $\left\{\left[M_{\Pi}\right]\cdot\left[i_{dq0}\right]\right\}$ we might get:

$$\left[U_{d,q0}\right]=\left[M_{\Pi}\right]*\left[R_{ph}\right]*\left[M_{\Pi}{}^{-1}\right]\cdot\left[i_{dq0}\right]+\left[M_{\Pi}\right]*\left[L_{ph}\right]*\left[M_{\Pi}{}^{-1}\right]*\frac{d}{dt}\left[i_{dq0}\right]+\left[M_{\Pi}\right]*\left[L_{ph}\right]*\frac{d}{dt}\left[M_{\Pi}{}^{-1}\right]*\left[i_{dq0}\right] \qquad (11)$$

If to designate:

$$|R| = |M_{\Pi}| \times \begin{vmatrix} R_a & 0 & 0 \\ 0 & R_b & 0 \\ 0 & 0 & R_c \end{vmatrix} \times |M_{\Pi}{}^{-1}| \tag{12}$$

$$|L| = |M_{\Pi}| \times \begin{vmatrix} L_a & 0 & 0 \\ 0 & L_b & 0 \\ 0 & 0 & L_c \end{vmatrix} \times |M_{\Pi}{}^{-1}| \tag{13}$$

$$|X| = |M_{\Pi}| \times \begin{vmatrix} L_a & 0 & 0 \\ 0 & L_b & 0 \\ 0 & 0 & L_c \end{vmatrix} \frac{d}{dt} |M_{\Pi}{}^{-1}| \tag{14}$$

Taking into account (12)-(14) expression (9) becomes:

$$\left[U_{dq0}\right] = \left[R\right] * \left[i_{dq0}\right] + \left[L\right] \cdot \frac{d}{dt}\left[i_{dq0}\right] + \left[X\right] * \left[i_{dq0}\right] \tag{15}$$

If open the system of active-reactive resistances and plug in the stator windings of the induction motor the voltage on the stator terminal we can quantify on the further equation:

$$[U_1] = [U_S] - \left\{ [R] * \begin{bmatrix} i_d \\ i_q \end{bmatrix} + [L] \cdot \frac{d}{dt} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + [X] * \begin{bmatrix} i_d \\ i_q \end{bmatrix} \right\} \tag{16}$$

Equation (16) is free of zero component because of switching the IM stator the zero wire is absent.

To use the numerical methods of integration (such as Runge-Kuta) it is proposed to resolve the system (1), (3) concerning derivative currents:

$$\frac{d}{dt}\begin{bmatrix} i_{1d} \\ i_{1q} \\ i_{2d} \\ i_{2q} \end{bmatrix} = \begin{bmatrix} Q_1 & 0 & 0 & 0 \\ 0 & Q_2 & 0 & 0 \\ 0 & 0 & Q_3 & 0 \\ 0 & 0 & 0 & Q_4 \end{bmatrix} \times \begin{bmatrix} U_{1d} \\ U_{1q} \\ U_{1d} \\ U_{1q} \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \times \begin{bmatrix} i_{1d} \\ i_{1q} \\ i_{2d} \\ i_{2q} \end{bmatrix}$$

where $a_{11} = -\frac{r_1}{x'_d}$; $a_{12} = \left(\omega_k - \frac{x_{ad}^2 \omega}{x_2 x'_d}\right)$; $a_{13} = \frac{x_{ad} r_2}{x_2 x'_d}$; $a_{14} = \frac{x_{ad}\omega}{x'_d}$;

$a_{21} = -\left(\omega_k + \frac{x_{ad}^2 \omega}{x_2 x'_d}\right)$; $a_{22} = -\frac{r_1}{x'_d}$; $a_{23} = -\frac{x_{ad}\omega}{x'_d}$; $a_{24} = \frac{r_2 x_{ad}}{x'_d x_2}$;

$a_{31} = \frac{r_2 x_{ad}}{x_2 x'_d}$; $a_{32} = -\frac{x_1 x_{ad}}{x_2 x'_d}\omega$; $a_{33} = -\frac{r_2 x_1}{x_2 x'_d}$; $a_{34} = \left(\omega_k - \frac{x_1 \omega}{x'_d}\right)$;

$$a_{41}=\frac{x_{ad}x_1}{x_2x'_d};\qquad a_{42}=\frac{r_2x_{ad}}{x'_d\,x_2};\qquad a_{43}=\left(-\omega_k+\frac{x_1\omega}{x'_d}\right);\qquad a_{44}=-\frac{x_1r_2}{x_2x'_d};$$

$$x'_d=x_1-\frac{x^2{}_{ad}}{x_p};\qquad Q_1=Q_2=\frac{1}{x'_d};\qquad Q_3=Q_4=-\frac{x_{ad}}{x_px'_d};$$

Considering that the currents and derivative of currents of the induction motor and active-reactive resistances are identical in (16) it is possible to substitute matrix expressions of stator currents of the induction motor. Than equation of stator currents can be written as:

$$[U_1]=[U_s]-\left\{[C_1][R]\left[C_2^{-1}\right][i]+[C_1][L]\left[C_2^{-1}\right]*\left\{\begin{bmatrix}Q_1 & 0\\ 0 & Q_2\end{bmatrix}*[U_1]+\begin{bmatrix}H_1\\ H_2\end{bmatrix}\right\}+[C_1][L]\frac{d}{dt}\left[C_2^{-1}\right]*[i]\right\}\tag{17}$$

If exchange in (17) $\frac{d}{dt}\left[C_2^{-1}\right]$ to $\left[C_3\right]$ we might get:

$$\left\{[U_1]+[C_1][L]\left[C_2^{-1}\right]*[Q]*[U_1]\right\}=[U_s]-[C_1][R]\left[C_2^{-1}\right]*[i]-[C_1][L]\left[C_2^{-1}\right]*[H]-[C_1][L][C_3]*[i];$$

$$\left\{[1]+[C_1][L]\left[C_2^{-1}\right]*[Q]\right\}[U_1]=[U_s]-[C_1][R]\left[C_2^{-1}\right]*[i]-[C_1][L]\left[C_2^{-1}\right]*[H]-[C_1][L][C_3]*[i]$$

The voltage on the induction motor terminals in final is:

$$[U_1]=\left\{[1]+[C_1][L]\left[C_2^{-1}\right]*[Q]\right\}^{-1}*\left\{[U_s]-[C_1][R]\left[C_2^{-1}\right]*[i]-[C_1][L]\left[C_2^{-1}\right]*[H]-[C_1][L][C_3]*[i]\right\}\tag{18}$$

Thus setting values of active-reactive elements included in equation (18) there is opportunity to simulate an asymmetrical feed of the induction motor, with use of usual symmetric model of the induction motor in d,q coordinate system.

The three-phase coordinate system is symmetrical if the voltages in all phases are identical in magnitudes and in phases. In reality there are some deviations from symmetry: in some cases they are larger, in others - smaller. The reason of asymmetry usually is non-uniformly distribution of loading on the phases, created by single-phase load [3]. Heating of the engine happens larger at asymmetry of a voltage, than at the same loading and a feed from a network with a symmetric voltage. The engine work deterioration is explicable by current in phases become unequal on value, increasing in one and decreasing in other phases in comparison with a symmetric mode at the same load. The value of the safe load on a shaft depends on

value of a current in the most loaded phase. At rated load one of phases may appear overloaded [5].

Calculation shows, that long admissible capacity of engines at asymmetry of voltage decreases in comparison with nominal.

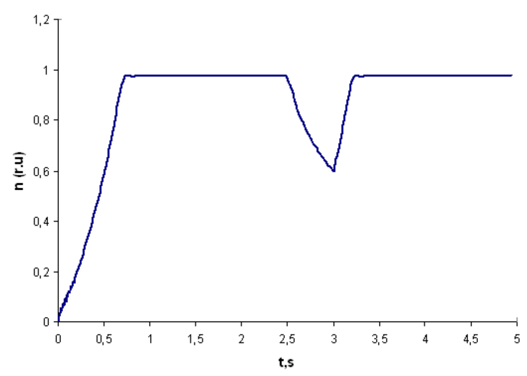

Figure 3. The curve of change of speed of rotation of the IM

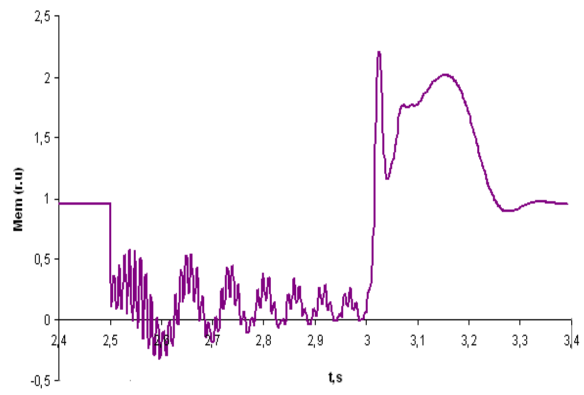

Figure 4. The curve of change of the electromagnetic torque of the IM

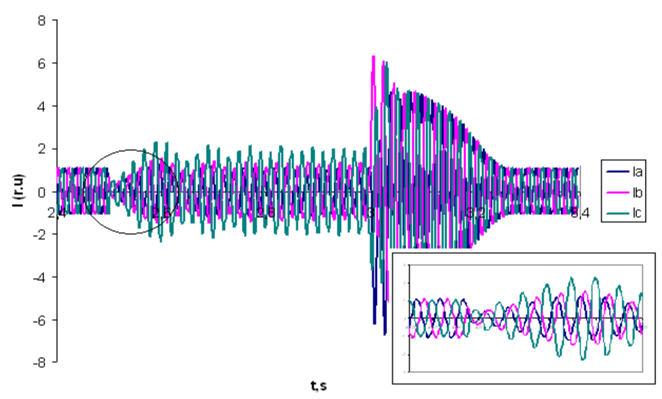

Figure 5. The curve of change of currents of the IM

From the figures 3-5 displaying curves of change of speed of rotation, the electromagnetic torque and currents from time, got as a result of modeling of two-phase asymmetry of a feed of the induction motor, it is possible to make the following conclusions: the rotating electromagnetic moment of the motor is proportional to a current and a magnetic flux. At a voltage reducing the magnetic flux of the machine decreases therefore decreases the running torque of the motor. As for the moment of the working machine resistance becomes larger than the moment of the motor, there comes braking, rotor spinning speed decreases. Thus the current in a rotor and stator increases up to such value at which the broken balance is restored. Thus, despite of reducing of a voltage, a current of the motor increases.

References

1. И. И. Трещев «Электромеханические процессы в машинах переменного тока». – Л. Энергия, 1980. – 344.

2. Ю. А. Мощинский, А. П. Петров «Математические модели трехфазных асинхронных двигателей, включенных в однофазную сеть», журнал «Электричество» Nr. 2/2000.

3. И. А. Сыромятников «Режимы работы асинхронных и синхронных двигателей»/Под ред. Л. Г. Мамиконянца.-4-е изд., перераб. И доп. – М.: Энергоатомиздат, 1984. – 217-223 стр.

4. И. П. Копылов «Математическое моделирование электрических машин. Учебник для вузов.-3-е изд.-М.:Высшая шк., 2001.

5. Leonhard, Werner. 2001. Control of Electrical Drives. Springer-Verlag Berlin Heidelberg New York.

Koņuhova M., Ketners K., Ketnere E., Kļujevska S. Asinhronā dzinēja darba analīze d,q koordinātu sistēmā barošanas asimetrijas apstākļos
Asinhronā dzinēja darba nesimetriskā sprieguma ietekmē novērtējums moderno tehnoloģiju attīstības apstākļos ir ļoti aktuāls uzdevums. Piedāvātajā darbā pārejas procesu izpētei asinhronajā dzinējā tiek piedāvāts veidot asimetrijas apstākļus ar aktīvi induktīviem elementiem, kas tiek attēloti a, b, c fāžu koordinātu sistēmā, kura saista asinhrono dzinēju ar bezgalīgi lielas jaudas elektrisko tīklu. Izmantojot tiešo un atgriezenisko Parka pārveidojumu, tika noteikta sprieguma asimetrijas vērtība asinhronā dzinēja spailēs. Tas ļauj imitēt asimetrisko bojājumu uz asinhronā dzinēja spailes. Asinhronā dzinēja matemātiskais modelis tiek īstenots uz Parka-Goreva vienādojumu pamata d, q koordinātu sistēmā, vienādojumu, kas atrisināti saistībā ar strāvu atvasinājumiem, pārveidojot Koši formā. Darbā tiek parādīti matemātiskās modelēšanas rezultāti, kā arī rotēšanas frekvences, elektromagnētiskā momenta un strāvas laika grafiki asinhronā dzinēja barojošā sprieguma divfāžu asimetrijas apstākļos.

Kohunova M., Ketner K., Ketnere E., Klujevska S. Analysis of asymmetric supply mode of the induction motor in system of d,q coordinates
The assessment of three-phase induction motor operation at asymmetry of voltage is rather actual problem under the conditions of development of new technologies. In proposed work for asymmetric supply modes investigation is offered conditions of asymmetry to set with the help of active-reactive elements, presented in a, b, c phases coordinates system that connecting the induction motor with indefinitely high power network. Using direct and reverse Park conversion it is defined voltage asymmetry value on the induction motor clamps. That allows simulating asymmetric damage on induction motor stator terminal. Induction motor mathematical model is implemented on the base of the Park-Gorev's equations in d,q coordinate system, reducing equations to Kochi form, solved relative to current derivative. In proposed work are given mathematic modeling results and curves of spinning speed, electromagnetic torque and currents in time scale at induction motor two-phase feed asymmetry.

Конюхова М., Кетнер К., Кетнере Е., Клюевская С. Анализ работы асинхронного двигателя при несимметрии питания в системе координат d,q
Оценка работы асинхронного двигателя при несимметрии напряжения в условиях развития новейших технологий является весьма актуальной задачей. В предлагаемой работе для исследования переходных режимов асинхронного двигателя предлагается условия несимметрии задавать с помощью активно-индуктивных элементов представленных в фазной системе координат a,b,c, связывающих асинхронный двигатель с сетью бесконечно большой мощности. Используя прямое и обратное преобразования Парка, определено значение несимметрии напряжения на зажимах асинхронного двигателя. Это позволяет имитировать несимметричное повреждение на зажимах асинхронного двигателя. Математическая модель асинхронного двигателя реализуется на основе уравнениях Парка-Горева в системе координат d,q, с приведением уравнений в форме Коши, разрешенных относительно производных токов. В работе приводятся результаты математического моделирования и графики зависимости частоты вращения, электромагнитного момента, токов от времени при двухфазной несимметрии питающего напряжения асинхронного двигателя.