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ROBUST EVOLUTIONARY ALGORITHMS FOR MULTI-ECHELON SUPPLY CHAIN CYCLIC PLANNING AND OPTIMISATION TASK

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1. Introduction

Multi-echelon supply chain cyclic planning and optimisation problem is complicated by the presence of uncertainty, which could be provided, for example, by deviations of customer demand and lead times of the processes. When the number of supply chain echelons is increased, it could result in magnified performance variance in a supply chain. As a result, the solution considered as optimal could perform different when used in practice. Therefore, it is important to create the so-called robust solutions, which are tolerated to certain deviations in environmental variables without a total loss of quality.

There are various classifications of uncertainties described in the literature [1-3]. The most common classification is provided by Jin and Branke [4], where uncertainties are divided into four categories based on the evolutionary computation theory:

1. **Noise in fitness evaluations.** The fitness evaluation often includes a noise, which could come from various sources. For example, the simulation model used to estimate an objective function could produce a noise while running different simulation replications.
2. **Deviations in the environmental variables.** Deviations of environmental variables could occur after the optimal solution is determined. For example, the end-customer demand variation could increase after a cyclic planning decision is made.
3. **Fitness approximation errors.** When the fitness function is expensive to evaluate, its approximation, called a meta-model, is often applied. This meta-model usually contains the approximation error.
4. **Errors in time – varying optimum evaluations.** The fitness function is deterministic, but it is time-dependent. As a result, optimum also changes over time.

Different methods and algorithms are developed to solve supply chain planning and optimisation problems. However, only few of them take into account the presence of uncertainties. This paper investigates the category of uncertainties labelled as 'deviations in environmental variables' in the above-described classification.

2. Optimisation problem statement

In this section the optimisation model and assumptions are presented, as well as the properties of the optimal solution are discussed.

The multi-objective optimisation problem with the presence of uncertainty in environmental variables can be symbolically represented in compact form as:

$$\text{Min } \mathbf{E}[F(\mathbf{x},\delta_e)] = \mathbf{E}[f_1(\mathbf{x},\delta_e),...,f_M(\mathbf{x},\delta_e)], \tag{1}$$
$$\text{subject to: } \mathbf{g}(\mathrm{x}, \delta_e) = E[\mathbf{r}(\mathbf{x}, \delta_e)] \le 0 \text{ and } \mathbf{h}(\mathbf{x}, \delta_e) \le 0,$$

where $E[\cdot]$ is a mathematical expectation; $\mathbf{x} = (x_1,...x_K) \in X$, $\mathbf{f} = (f_1,...,f_M) \in F$; $\mathbf{x}$ is called a vector of decision variables; $\mathbf{f}$ is called a vector of objective functions; $\mathbf{g}$ is a vector of stochastic constraints; $\mathbf{h}$ is a vector of deterministic constraints on the decision variables; $\mathbf{r}$ is a random vector that represents several responses of the simulation model for a given $\mathbf{x}$; δ_e represents the uncertainty associated with environmental variables; $x_1,...x_K$ denote K decision variables; $f_1,...,f_M$ denote M objective functions; X is called the decision space; F is the objective space.

As regards the problem of cyclic planning within multi-echelon supply chain network, we deal with two objective functions [5]. The first one is to minimise the average total cost represented by the sum of inventory holding, production and ordering costs. The second is aimed to maximise the average order fill rate *FR* that is defined as the percentage of end-customers' orders filled from the available inventory.

Proceeding from (1), the solution of multi-objective stochastic optimisation problem is a vector of decision variables $\mathbf{x}$ that satisfies all feasible constraints and provides the best trade-off between multiple objectives. This problem is characterised by two conflicting objectives, i.e. average total cost and average order fill rate, to be optimised simultaneously. Therefore, instead of a single optimal solution, there is a whole set of optimal trade-offs solutions of equivalent quality. An optimal trade-off solution, also called a Pareto – optimal, is the solution that is not dominated by any other solution in the search space. It means that there doesn't exist any other solution that is better in all objectives. The entire set of these solutions is called a Pareto – optimal set [6].

For example, Figure 1 illustrates the problem with two conflicting objectives. Solutions *A* and *B* dominate the others by *objective1* and *objective2*, respectively. Solution *C* dominates the other solutions at least by one objective.

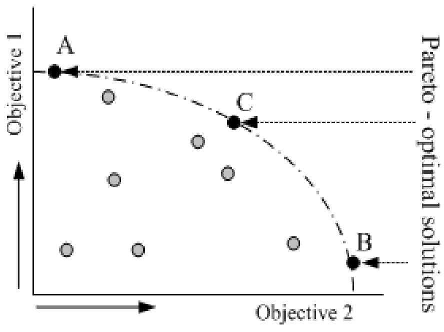


Figure 1. Concept of Pareto optimality

3. Multi – objective evolutionary algorithms

Evolutionary algorithms are well suited to solving multi-objective optimisation problems, because they could evolve a set of non-dominated solutions instead of a single solution. Historically, multi-objective evolutionary algorithms can be divided into three groups: Algorithms based on aggregating functions, Population-based algorithms and Pareto - based algorithms.

The idea behind aggregating functions is to combine all objective functions into a single composite objective function using arithmetic operations. Population – based algorithms apply a separate sub-population for each objective function. Pareto – based algorithms use the selection schemes based on the concept of Pareto optimality. The last group of multi-objective evolutionary algorithms can be historically studied as covering two generations. Algorithms that belong to the first generation use fitness sharing and niching combined with Pareto ranking. The second generation of algorithms is characterised by the concept of elitism [7]. In the proposed work the last group of evolutionary algorithms are described and analysed.

4. Multi – objective robust evolutionary algorithms

Multi-objective evolutionary algorithms described in Section 3 are not intended to search for robust solutions. Let's remind that robust are such solutions, which are not sensitive to slight changes in the environment.

There exist a range of enhanced evolutionary algorithms improved to find robust solutions mostly by enlarging the search space, i.e. by analysing neighbours of candidate solutions. Most popular robust evolutionary algorithms are:

- Single/Multi – objective Inverse Robust Evolutionary (SMIRE). The algorithm does not make assumptions about the uncertainty structure in the formulation of the optimization search process. For searching robust solutions it uses different IRE schemes, for example Single, Bi – objective, Tri – objective IRE schemes [8].
- Genetic Algorithm with Robust Searching Scheme (GA/ RS3). In this algorithm the probable noise vector is added to genotype before fitness evaluation. In order to generate phenotype, genotype is coded with noise and then it interacts with the environment [9].
- Enhanced Genetic Algorithm. This algorithm combines a simulation model with stochastic non-dominating multi-objective optimization method and genetic algorithms. The concept of robustness is implemented in selection process of non-dominated solutions. In this case solution *A* dominates solution *B* with confidence level (1 - α)%, and probability $\prod_{i=1}^{n} P(f_i(A) < f_i(B)) \geq 1-\alpha$, where α is a confidence level ($0 \leq \alpha \leq 1$) [10].
- Robust Multi – Objective Evolutionary Algorithm (RMOEA). The RMOEA considers robustness as independent optimization criteria and implements the features of micro – genetic algorithm, Tabu restriction, and archival re-evaluation [11].
- Evolutionary Approach for Assessing the Degree of Robustness of Solutions to Multi – Objective Models. The concept of degree of robustness is incorporated into evolutionary algorithm and is used in the fitness evaluation process. Non – dominated solutions are classified by their degree of robustness [12].
- Evolutionary Multi – Objective Approach. In order to find robust solution the fitness of solution is evaluated by averaging different points from a set of its neighbours [13].

Genetic algorithms, such as Non – dominated Sorting Genetic Algorithm – II (NSGA II) [14], do not search for robust solutions in a direct way. Instead, they evaluate the robustness of trade-offs solutions included in the Pareto – optimal set. This approach allows decreasing the computational time. However, it could lead to losing some potential solutions.

5. Development of robust multi-objective genetic algorithm

A robust multi-objective genetic algorithm is developed on the basis of MOSGA [5] and is called rMOSGA. It is aimed to provide robust solutions to multi-echelon supply chain cyclic planning and optimisation problem.

The concept of worst scenario robustness measure, which reflects the degree of variation resulting from the worst objective function value, is taken from RMOEA [11] and incorporated in rMOSGA. This robustness measure could be defined for the *i*-th objective as follows [12]:

$$f_i^{'}(\mathbf{x},\delta_e) = \frac{\max_{\mathbf{x}'\in X^{n_\mathbf{x}}}(f_i(\mathbf{x}',\delta_e)) - f_i(\mathbf{x},\delta_e)}{f_i(\mathbf{x},\delta_e)}, \text{ where } \mathbf{x}'\in[\mathbf{x}'-lb, \mathbf{x}'+up], \quad (2)$$

where $X^{n_\mathbf{x}}$ is a set of neighbours of the candidate solution **x**, [*lb, up*] denotes the lower and upper bounds of replenishment cycles in the neighbourhood of the candidate solution.
The multi – objective problem to be solved can be re-formulated as follows:

$$\min f(\mathbf{x},\delta_e) = \{f_1(\mathbf{x},\delta_e),..., f_M(\mathbf{x},\delta_e), f_{M+1}(\mathbf{x},\delta_e)\}, \quad (3)$$

where $f_{M+1}(\mathbf{x},\delta_e) = \max_{i=1,M}(f_i^{'}(\mathbf{x},\delta_e))$.

The following blocks of the original RMOEA have been implemented in rMOSGA:

1. fitness evaluation based on robustness measure (3),
2. selection strategy modified to search for compromise solutions for non-robust and robust Pareto-optimal sets.

Table 1 represents the main blocks of MOSGA and rMOSGA.

Table 1

Developed blocks of the MOSGA and rMOSGA algorithms

Algorithms / Blocks	MOSGA	rMOSGA
Fitness assignment	Pareto-based ranking	Pareto-based ranking and robustness measure
Crossover and mutation	Uniform crossover One point mutation	
Reproduction strategy	To the next generation go *N* individuals from the union of parent and offspring populations	
Selection strategy	Tournament selection based on crowded comparison operator	Tournament selection based on the crowded comparison operator and robustness measure

6. Computational study

In this section, an illustrative example of multi-echelon cyclic planning and optimisation problem is provided to compare the performance of non-robust (MOSGA) and robust (rMOSGA) genetic algorithms with respect to the robustness of solutions.

6.1. Input data description

The chemical manufacturing supply chain network is used as a test bed to compare MOSGA and rMOSGA. Both algorithms are applied to find an optimal cyclic plan of a chemical product, i.e. liquid based raisin, in order to *minimise average total cost* and *maximise average order fill rate*. Total cost consists of inventory holding, ordering and production costs. Order fill rate is defined as the percentage of end-customers' orders filled from the available inventory. Decision variables are *replenishment cycles*, which determine the reorder period for each mature product in the network. Order-up-to levels which define the quantity to be ordered or produced each cycle are calculated using analytical calculi. The following assumptions are introduced in the problem: the end-customer demand is normally distributed and lead times of the processes are constant. Replenishment cycles are defined according to the power-of-two policy, and are presented in weeks as follows: 7, 14, 28 and 56, where 56 days is the maximal cycle that corresponds to one full turn of a "planning wheel". Initial stocks are equal to order-up-to levels plus average demand multiplied by cycle delays. Backorders are delivered in full.

6.2. Experimental setup

Both algorithms, i.e. MOSGA and rMOSGA, try to find solutions with a minimised average total cost and maximised average order fill rate. Additionally, rMOSGA takes into account the robustness of candidate solutions. To estimate the objective functions values, the algorithms use a supply chain simulation model. This procedure could be explained as follows: the genetic algorithm chooses values of decision variables and uses the responses generated by the simulation model to make decisions regarding the selection of the next potential solution. To solve the problem, the algorithms are executed with the parameters summarised in Table 2.

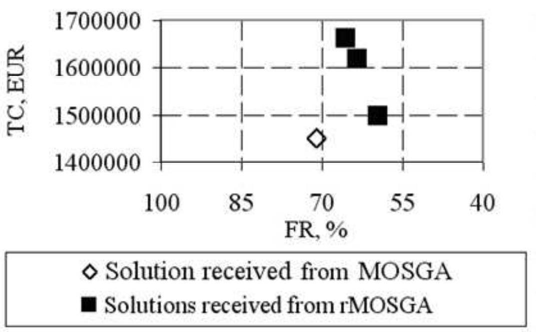
Table 2

Parameters of the algorithms

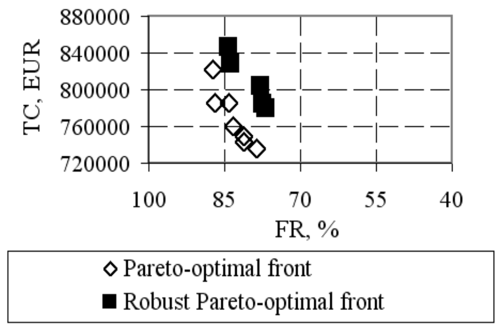
Parameters \ **Algorithms**	**MOSGA**	**rMOSGA**
Population size	10	10
Iterations	16	16
Neighbours of each individual in population	-	2
Number of decision variables	33	33
Number of objective functions	2	2 + robustness measure
The increase in customer demand standard deviation	1%, 2%, 5%	1%, 2%, 5%

7. Results and discussion

The results provided by MOSGA are better than those provided by rMOSGA with respect to objective functions values (see Figure 2). The reason is that MOSGA selects optimal solutions based on dominance relation. However, rMOSGA takes into account robustness of candidate solutions as an additional objective function.



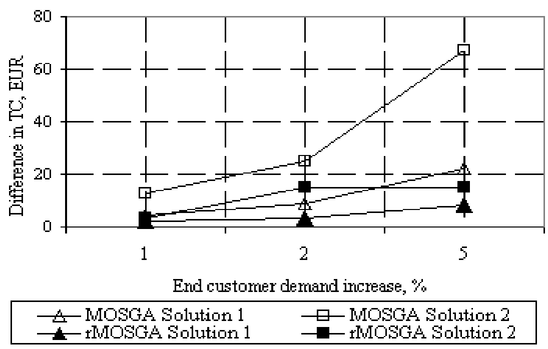
a) MOSGA and rMOSGA (1st generation)



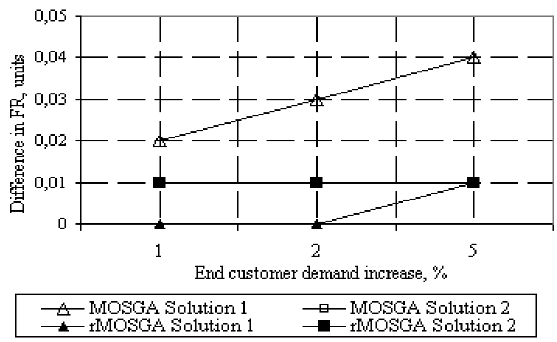
b) MOSGA and rMOSGA (16th generation)

Figure 2. Pareto-optimal fronts generated by MOSGA and rMOSGA

Therefore, to determine an algorithm able to provide robust solutions, it is necessary to check the stability of solutions to uncertainties in environmental variables. For that purpose, two solutions are randomly selected from non-robust and robust Pareto-optimal fronts. Subsequently, standard deviations of end-customer demand are increased by 1%, 2% and 5%. From Figure 3 it can be seen that rMOSGA provides solutions that are less sensitive to demand variations. For example, the total cost of the first solution is increased by 0.001% only when standard deviations of end-customer demand are increased by 5%. Solutions found by MOSGA are less tolerated to deviations of the demand. For example, the total cost of the second solution is increased after changing standard deviation by 1%.



a) Comparison of the algorithms (on TC)



b) Comparison of the algorithms (on FR)

Figure 3. The offset of solution points in the presence of uncertainty

8. Conclusions

In this paper the importance of searching for robust solutions is emphasized. The robust evolutionary algorithm rMOSGA is developed on the basis of the MOSGA algorithm. The robustness measure is added to fitness assignment block of rMOSGA in order to evaluate the robustness of candidate solutions.

Both algorithms are applied to the multi-echelon supply chain cyclic planning and optimization problem. Two types of optimization experiments were performed with these algorithms. The experimental results indicate that MOSGA performs better on objective functions, but in the case of solutions sensitivity to uncertainties, it is less tolerated to deviations of the demand than rMOSGA whose solutions proved to be more stable under stochastic demand conditions.

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Merkurjeva Gaļina, Lagzdiņa Tatjana. Robustie evolūcijas algoritmi daudz ešelonu piegādes ķēžu ciklisko plānu optimizācijas uzdevumam

Daudz ešelonu piegādes ķēdes ciklisko plānu optimizācijas uzdevumā ir nepieciešams, lai risinājumi tiktu meklēti nenoteiktības apstākļos. Nenoteiktības var nākt no pasūtījuma apstrādes aizkavēšanas laikiem un svārstībām gala patērētāju pieprasījumā. Galvenās grūtības šajā problēmā ir saistītas ar to, ka atrastais risinājums var nozīmīgi izmainīties pie nenozīmīgām izmaiņām ārējos faktoros. Tāpēc ir svarīgi definēt tā saucamos robustos risinājumus, kuri ir mazāk jūtīgi pret šīm izmaiņām. Rakstā ir piedāvāta robusto evolūcijas algoritmu analīze. Šo algoritmu mērķis ir meklēt robustus risinājumus, kuri pieļauj noteiktas nobīdes ārējos faktoros tajā pašā laikā nezaudējot risinājuma kvalitāti. Rakstā ir izpētītas divas evolūcijas algoritmu paaudzes un izanalizēta virkne robusto evolūcijas algoritmu daudzmērķu problēmu risināšanai. Visbeidzot, rakstā tiek piedāvāti daudzmērķu ģenētiskā un robustā ģenētiskā algoritma pielietošanas piemēri piegādes ķēžu ciklisko plānu optimizācijas uzdevumā. Tika veikts arī abu algoritmu salīdzinājums nenoteiktību iespaidā, pievienojot svārstības gala patērētāju pieprasījumam, tā iemesla dēļ, ka nav lietderīgi salīdzināt ģenētiskā un robustā ģenētiskā algoritma darbības rezultātus pēc mērķa funkcijām, jo pirmais meklē optimālus, bet otrais - robustus risinājumus.

Merkuryeva Galina, Lagzdina Tatyana. Robust evolutionary algorithms for multi-echelon supply chain cyclic planning and optimisation task

Multi-echelon supply chain cyclic planning and optimisation problem requires searching for solutions in the presence of uncertainty. Uncertainty could be provided by deviations in lead times of the processes and customer demand. A key difficulty in this problem is that solutions found could falter completely, when a slight change of the environment occurs. Therefore, it is important to define the so-called robust solutions, which are less sensitive to such changes. This paper focuses on the analysis of robust multi-objective evolutionary algorithms. These algorithms are aimed to search robust solutions, which are tolerated to certain deviations of environmental variables without a total loss of quality. In this paper, two generations of non-robust evolutionary algorithms and a range of robust evolutionary algorithms for multi-objective problems are investigated. Finally, the paper provides the application examples of multi-objective robust and non-robust genetic algorithms for solving multi-echelon supply chain cyclic planning and optimisation problem. Also a comparison of results of both algorithms under the influence of uncertainties is made, when fluctuations are added to end customer demand, since a comparison of Genetic Algorithm with robust Genetic Algorithm on objective functions will not allow to draw an objective conclusion because the first searches for optimal, but the second – for robust solutions.

Меркурьева Галина, Лагздыня Татьяна. Робастные эволюционные алгоритмы для задачи оптимизации циклического планирования в цепях поставок

Проблема циклического планирования и оптимизации многоэшелонной системы поставок требует, чтобы поиск решений происходил в условиях неопределенности. Неопределенность может быть вызвана отклонениями во временах поставок и спросе конечного потребителя. Ключевая сложность этой проблемы состоит в том, что найденные решения могут сильно колебаться при небольших изменениях в окружающих факторах. Поэтому важно определить так называемые робастные решения, которые менее чувствительны к изменениям такого рода. В данной статье предложен анализ робастных многоцелевых эволюционных алгоритмов. Эти алгоритмы нацелены на поиск робастных решений, которые допускают определенные отклонения окружающих переменных без потери качества решения. В статье исследованы два поколения неробастных эволюционных алгоритмов и ряд робастных эволюционных алгоритмов для многоцелевых проблем, приведены примеры применения многоцелевых робастного и неробастного генетических алгоритмов для решения проблемы циклического планирования и оптимизации многоэшелонной системы поставок. Также проведено сравнение результатов обоих алгоритмов под влиянием неопределенностей, с добавлением колебания в спрос конечного потребителя, так как сравнение генетического алгоритма с робастным генетическим алгоритмом по целевым функциям не позволит сделать объективный вывод по той причине, что первый ищет оптимальные, а второй - робастные решения.