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MATHEMATICAL MODELLING OF BRUSHLESS MOTOR TRANSIENT

VENTIĻDZINĒJU PĀREJAS PROCESA MATĒMATISKĀ MODELĒŠANA

Irina Latve, *ph.D. student. Mag. sc. ing.*
Riga Technical University
Faculty of Power and Electrical Engineering
Address: Kronvalda boulevard 1, LV 1010, Riga, Latvia
Phone: +371 7089926, Fax: 7089418
e-mail: latveirina@inbox.lv

Janis Dirba, *Prof., Dr. habil. sc. ing.*
Riga Technical University
Faculty of Power and Electrical Engineering
Address: Kronvalda boulevard 1, LV 1010, Riga, Latvia
Phone: +371 7089926, Fax: 7089418
e-mail: dirba@eef.rtu.lv

Karlis Ketners, *Prof.,Dr. sc. ing.*
Riga Technical University
Faculty of Power and Electrical Engineering
Address: Kronvalda boulevard 1, LV 1010, Riga, Latvia
Phone: +371 7089926, Fax: 7089418
e-mail: ketners@eef.rtu.lv

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Introduction

Alternating current electric machines aspire to use as motors in such operation cycles where rotation frequency regulation in a wide range is not required. For electric drives with rotation frequency regulation in a wide range often use electric motors of a direct current. However opportunities of rotation frequency regulation of alternating current motor, caused by advanced achievements in power electronics, essentially expand sphere of their application.

Function fulfilled by modern industrial machines and mechanisms, and requirements to the driver of these mechanisms are very different. The implementation of these requirements can be achieved not only by using direct current commutator motors possessing a variety of characteristics, but even more efficiently, by using synchronous motors (SM) as a driver to mechanisms. Synchronous motors operating in special modes together with an electronic commutator and a rotor position sensor are enclosed in electronically commutated motors (ECM). [2]

Brushless motor represents the electromechanical system that includes the electronic commutator (EC), the synchronous motor (SM) and the rotor position sensor (RPS) (fig. 1). The general attribute for all circuits is presence of a rotor position feedback. The signal of a rotor position feedback concerning stator windings position is formed by rotor position sensor (RPS). This signal goes on the electronic commutator (EC) and is used for taking of opening impulses of the semi-conductor keys switching stator windings in demanded sequence [4]. In one of possible operating modes of brushless motor is provided constant value load angle (θ=const). The load angle θ is angle between voltage U_1 and idle of electromotive force of the first harmonic.

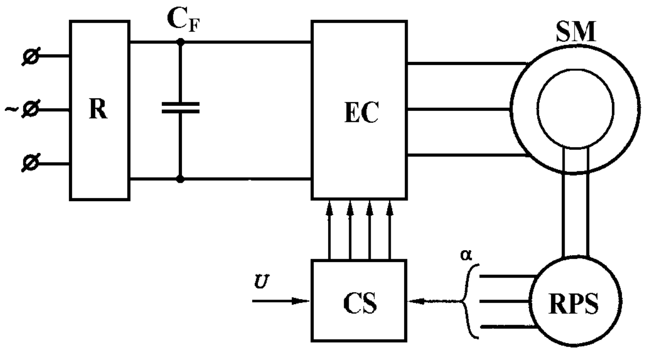


Figure 1. Function circuit of brushless motor

Present research is devoted to mathematical modelling of brushless motor transient on the base of mathematical modelling of synchronous machine.

In other words it is possible to tell, that brushless motor is alternating current electric motor, which stator winding feeding from the frequency converter that realize current switching in function of rotor angle turn with frequency, corresponding angular speed of rotor.

The electronic commutator (EC) can be independent inverter receiving a feed from direct current source or from the rectifier (R), connected with alternating current network (fig.1). The electronic commutator with pulse-width modulation (EC) control position of stator magnetomotive forces resultant vector concerning to rotor magnetomotive forces vector and adjusts value of the first harmonic of voltage that is brought to stator winding phases. Spatial position of a stator MMF vector is defined by a corresponding combination of simultaneously open keys of the voltage inverter, and voltage quantity is defined by open condition duration of these keys. [3]

Proceeding from that brushless motor can be considered as the synchronous machine it is possible to draw a conclusion, that in the basis of mathematical model of brushless motor it is possible to use mathematical model of the synchronous machine.

Is considered mathematical models and characteristics of brushless motor with constant value of load angle (*θ =const*). Thus, for a generality, we shall believe that stator winding is biphase sine curve, and the rotor winding has electromagnetic excitation. It will allow us to consider brushless motor as a special case of the generalized electric machine. Stator three-phase winding of real motor can be transformed to an equivalent biphase winding according to rules of phase transformations.

Transient research for electric machine has large practical importance that make possible correctly understand this conductions, to foresee the possible brake actions, take precautionary measures, hold down the brake actions and liquidate problems.

Necessity for the analysis of transients arises in connection with that productivity of some responsible mechanisms, quality of performance of many technological operations, mechanical and electric overloads of the equipment is defined by speed of transients.

The primary goal of transients studying is definition of currents in machine windings in time when is change operating mode that is why it is necessary to make and solve electromotive forces equations system balance in electric circuits of the machine.

In order to study the transient processes of the rotating electrical machines, including the synchronous, it is necessary to obtain the equations of stator and rotor electric circuits and the equation of rotor movements in the differential form. The form of these equations depends on the choice of coordinates axes as well as the positive direction of current.

The differential equations of machine, which describe the dynamic processes, are written in the system of relative units (system X_{ad}).

Mathematical model of synchronous machine in *d, q* axes:

$$\left.\begin{aligned} u_d &= \frac{d\psi_d}{d\tau} - \psi_q \omega + i_d r; \\ u_q &= \frac{d\psi_q}{d\tau} + \psi_d \omega + i_q r; \\ u_f &= \frac{d\psi_f}{dt} + i_f r; \\ 0 &= \frac{d\psi_D}{d\tau} + i_D r_{D;} \\ 0 &= \frac{d\psi_Q}{d\tau} + i_Q r_Q \end{aligned}\right\} \qquad (1)$$

Flux linkage of all machine circuits, written in *d, q* axes, contain only constants independent on an induction time:

$$\left.\begin{aligned} \psi_d &= x_d i_0 + x_{ad} i_f + x_{ad} i_D; \\ \psi_q &= x_q i_q + x_{aq} i_Q; \\ \psi_f &= x_{ad} i_d + x_f i_f + x_{ad} i_D; \\ \psi_D &= x_{ad} i_d + x_{ad} i_f + x_D i_{D;} \\ \psi_Q &= x_{aq} i_q + x_Q i_Q \end{aligned}\right\} \qquad (2)$$

– Equation of rotor movement:

$$\frac{d\omega}{d\tau} = \frac{1}{T_M}\left[\left(\psi_q i_d - \psi_d i_q\right) - M_L\right] \qquad (3)$$

where T_M - inertial constant of the machine in electrical radians.

To obtain the final algorithm of SM simulation in alternating currents, we shall substitute (1) for (2) in the set of equations and solve it relatively to the derivatives of currents. Then we receive in matrix form:

$$\frac{d}{d\tau}\begin{bmatrix} i_d \\ i_q \\ i_f \\ i_D \\ i_Q \end{bmatrix} = \begin{bmatrix} Q_1 & 0 & 0 & 0 & 0 \\ 0 & Q_2 & 0 & 0 & 0 \\ 0 & 0 & Q_3 & 0 & 0 \\ 0 & 0 & 0 & Q_4 & 0 \\ 0 & 0 & 0 & 0 & Q_5 \end{bmatrix} * \begin{bmatrix} u_d \\ u_q \\ u_f \\ u_d \\ u_q \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{bmatrix} * \begin{bmatrix} i_d \\ i_q \\ i_f \\ i_D \\ i_Q \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \end{bmatrix} =$$

$$= \begin{bmatrix} Q_1 & 0 & 0 & 0 & 0 \\ 0 & Q_2 & 0 & 0 & 0 \\ 0 & 0 & Q_3 & 0 & 0 \\ 0 & 0 & 0 & Q_4 & 0 \\ 0 & 0 & 0 & 0 & Q_5 \end{bmatrix} * \begin{bmatrix} u_d \\ u_q \\ u_f \\ u_d \\ u_q \end{bmatrix} + \begin{bmatrix} H_1 \\ H_2 \\ H_3 \\ H_4 \\ H_5 \end{bmatrix} \quad \text{or} \quad \frac{d}{dt}\left[I_S\right] = \left[Q_S\right]\left[U_S\right] + \left[H_S\right] \qquad (4)$$

where

$$
\begin{aligned}
&a_{11} = -r_a Q_1, \quad a_{12} = x_q \omega Q_1, \quad a_{13} = -r_f Q_3, \quad a_{14} = -r_D Q_4, \quad a_{15} = x_{aq}\omega Q_1;\\
&a_{21} = -\omega x_d Q_2, \quad a_{22} = -r_a Q_2, \quad a_{23} = -x_{ad}\omega Q_2, \quad a_{24} = -x_{ad}\omega Q_2, \quad a_{25} = -r_Q Q_5;\\
&a_{31} = -r_a Q_3, \quad a_{32} = x_q \omega Q_3, \quad a_{33} = -\frac{x_d x_D - x_{ad}^2}{\Delta d} r_f, \quad a_{34} = \frac{x_s x_{ad}}{\Delta d} x_D, \quad a_{35} = x_{aq}\omega Q_3;\\
&a_{41} = -r_a Q_4, \quad a_{42} = x_q \omega Q_4' \quad a_{43} = \frac{x_s x_{ad}}{\Delta d} r_f, \quad a_{44} = -\frac{x_d x_B - x_{ad}^2}{\Delta d} r_D, \quad a_{45} = x_{aq}\omega Q_4;\\
&a_{51} = -x_d \omega Q_5, \quad a_{52} = -r_a Q_5, \quad a_{53} = -x_{ad}\omega Q_5, \quad a_{54} = -x_{ad}\omega Q_5, \quad a_{55} = -\frac{x_q}{\Delta q} r_Q.
\end{aligned}
$$

$$Q_1 = \frac{x_D x_f - x_{ad}^2}{\Delta d}; \quad Q_2 = \frac{x_Q}{\Delta q}; \quad Q_3 = -\frac{x_{sD} x_{ad}}{\Delta d}; \quad Q_4 = -\frac{x_{sf} x_{ad}}{\Delta d}; \quad Q_5 = -\frac{x_{aq}}{\Delta q},$$

$$\Delta d = x_d x_f x_D - x_{ad}^2 \left(x_d + x_D + x_B - 2x_{ad}\right),$$

$$\Delta q = x_q x_Q - x_{aq}^2$$

$$B_1 = Q_3 u_B; \quad B_2 = 0; \quad B_3 = \frac{x_d x_D - X_{ad}^2}{\Delta d} u_B; \quad B_4 = -\frac{x_s x_{ad}}{\Delta d} u_B; \quad B_5 = 0.$$

This system of the equations is supplemented with the equation of movement

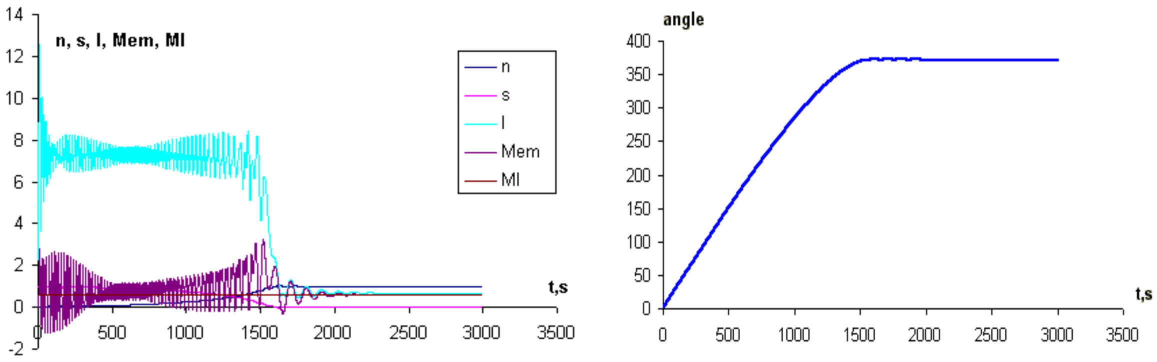
$T_M \dfrac{d\omega}{d\tau} = \left(\psi_q i_d - \psi_d i_q\right) - M_{LM}$ and the set moment of loading

Using basic model of the synchronous machine made on the full Park-Gorev's equations, calculating concerning derivative currents it is possible to consider influence of loading and regulators of excitation on processes of start-up. Voltage submitted on stator windings U_{ds}, U_{qs} controlled by control system is necessary to transform to axes d, q rotating together with a synchronous machine rotor.

$$
\begin{aligned}
U_d &= U_{ds} \cos\theta - U_{qs} \sin\theta \\
U_q &= U_{ds} \sin\theta - U_{qs} \cos\theta
\end{aligned}
\tag{5}
$$

Also it is necessary to consider change of corner θ as function of a difference of rotation frequencies of electronically commutator and the rotor of the synchronous machine.

In research results of modeling of process of start-up of brushless motor are presented.



a) b)

Figure 2. a) the characteristic of change of the rotation frequency, sliding, current, electromagnetic moment and load moment from the time; b) the characteristic of change of the angle θ from the time

From the characteristic we can see that motor start – up was successful, angle reached stable condition at the preset value of the load moment M_L = 0,6. It is possible to say, that the developed mathematical model of brushless motor successfully works. The developed mathematical model of brushless motor successfully works.

Conclusion

Present research is devoted to mathematical modelling of brushless motor transient on the base of mathematical modelling of synchronous machine. Using similarity of the synchronous machine and brushless motor the equations of electric contours voltage balance on stator and rotor and the equation of movement of a rotor in the differential form are written. The differential equations of machine, which describe the dynamic processes, are written in the system of relative units. In research results of modeling of process of start-up of brushless motor are presented.

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Latve I., Dirba J., Ketners K. Ventiļdzinēju pārejas procesa matemātiskā modelēšana

Darbā tiek apskatīts ventiļdzinēja pārejas procesa matemātiskais modelis, kura pamatā ir sinhronās mašīnas matemātiskais modelis. Ventiļdzinēju pārejas procesu analīzes aktualitāti nosaka tas apstāklis, ka daudzu tehnoloģisko procesu kvalitāte un darba ražīgums ir atkarīgs no šo procesu norises ātruma. Ir piedāvātas ventiļdzinēja struktūras un funkcionālās shēmas. Aplūkots ventiļdzinējs, kas sastāv no sinhronās mašīnas, elektroniskā komutatora un rotora stāvokļa devēja. Apskatīts arī ventiļdzinēja matemātiskais modelis ar nemainīgo slodzes leņķi (θ=const). Izmantojot sinhronās mašīnas un ventiļdzinēja līdzības, ir uzrakstīti elektrisko kontūru sprieguma līdzsvara uz statora un rotora vienādojumi, kā arī rotora kustības vienādojumi diferenciālā formā. Vienādojumi pierakstīti relatīvajās vienībās. Darbā piedāvāti palaišanas režīma modelēšanas rezultāti.

Latve I., Dirba J., Ketners K. Mathematical modeling of brushless motor transient

Present research is devoted to mathematical modelling of brushless motor transient on the base of mathematical modelling of synchronous machine. Necessity for the analysis of transients arises in connection with that productivity of some responsible mechanisms, quality of performance of many technological operations, mechanical and electric overloads of the equipment is defined by speed of transients. It is presented structural and function circuits of brushless motor. Brushless motor includes the electronic commutator, the synchronous motor and the rotor position sensor. Is considered mathematical models of brushless motor with constant value of load angle (θ =const). Using similarity of the synchronous machine and brushless motor the equations of electric contours voltage balance on stator and rotor and the equation of movement of a rotor in the differential form are written. The differential equations of machine, which describe the dynamic processes, are written in the system of relative units. In research results of modeling of process of start-up are presented.

Латве И., Дирба Я., Кетнер К. Математическое моделирование переходных процессов вентильного двигателя

В работе рассмотрена математическая модель переходных процессов вентильного двигателя на основе модели синхронной машины. Необходимость в анализе переходных процессов возникает в связи с тем, что производительность ряда ответственных механизмов, качество выполнения многих технологических операций, механические и электрические перегрузки оборудования определяется быстротой протекания переходных процессов. Представлена структурная и функциональные схемы вентильного двигателя. Вентильный двигатель включает в себя электронный коммутатор, синхронный двигатель и датчик положения ротора. Рассмотрены математические модели вентильного двигателя с неизменной величиной угла нагрузки (θ=const). Используя подобия синхронной машины и вентильного двигателя написаны уравнения равновесия напряжений электрических контуров на статоре и роторе и уравнение движения ротора в дифференциальной форме. Все уравнения записаны в системе относительных единиц. В работе представлены результаты моделирования процесса пуска.