ISSN 1407-8015 TRANSPORT AND ENGINEERING
MAŠĪNZINĀTNE UN TRANSPORTS 2008-8015

MECHANICS
MEHĀNIKA

CONTACT FORCES IN VIBRO-IMPACT SYSTEMS

KONTAKTA SPĒKI VIBRO-TRIECIENU SISTĒMĀS

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Key words: impact analysis, impact forces, contact models, impact deformations

1. Introduction

The impact is very important phenomenon in many mechanical systems. The selection of the most adequate contact force model plays a key role in analysis and design of the mechanical systems with intermittent motion and clearance. In general, there are two different approaches for impact analysis: the discontinuous and continuous approach. The first approach is to assume that impact occur instantaneously. The analysis is divided into two phases – before and after impact, interconnection between phases is made by impulse-momentum equation $\overline{S} = m \cdot \Delta\overline{v}$, connecting pre-impact and post-impact velocity, and coefficient of restitution, representing the energy dissipation in the impact. At that forces and duration of impact cannot be determined. The duration of the contact period governs the choice of method used to analyze the impact. If duration of contact is large enough that significant changes occur in the configuration of multibody system, then assumption of instantaneousness of impact is no valid. The continuous approach assumes that the forces and deformations vary continuously and motion of all body during impact is described by the differential equations of motion of a rigid body (the equation force – acceleration $\overline{F} = m \cdot \overline{a}$).
The forces acting during impact time

take into account viscous-elastic properties of real bodies and are modeled by a set of elastic spring, viscous damper, stiff-plastic elements (Fig.1). In this paper the low-speed direct impact of homogeneous isotropic bodies is considered. The examined bodies have such forms and size that only local deformations in the small area are significant and wave propagating in the bodies may be neglected.

The forces models are based on Hertz contact force. The consideration is based on equations of contacts mechanics. The purpose of this research is to develop the method of preliminary estimation of impact character, taking into account the forms of colliding bodies and properties of its material, in order to correctly create the impact force models for using it in vibro-impact systems.

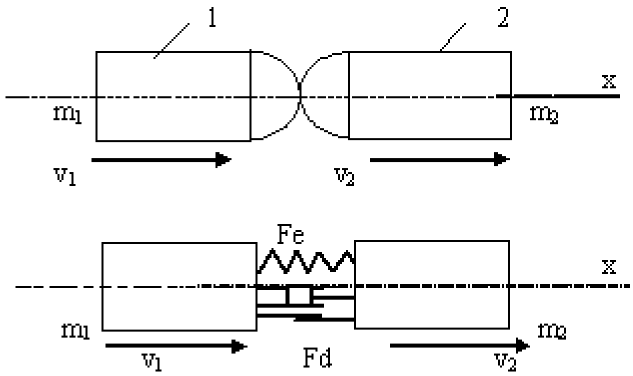


Fig. 1. Scheme of rigid bodies impact

2. Quasistatic Hertz's Model of Spherical Bodies Impact

A theory of contact interaction of the deformed bodies was developed by Hertz on the base of the theory of elasticity for the case of bodies, confined in the point of contact by curved surfaces. Three hypotheses lie in basis of model of Hertz: 1) at intersection of colliding bodies local deformations in the area of contact are substantial, the characteristic sizes of contact area are small as compared to sizes each of bodies and with the radiuses of curvature of their surfaces; 2) dependence of contact force on contact deformation remains the same as at the static compression of bodies; 3) deformations is elastic.

Contact impact force, expressed through approach of mass centers:

$$F_e = kx^n , \tag{1}$$

where: n- index of power, for the spherical bodies $n = \frac{3}{2}$;

$x = x_1 - x_2$ - relative approach or identation between the surfaces of the two spheres;

$k-$ proportionality factor, depending on curvature of surfaces in the point of contact and properties of material:

$$k = \frac{4}{3\eta}\sqrt{\frac{R_1 R_2}{R_1 + R_2}} , \tag{2}$$

where R_1, R_2 - radii of curvature of contact surfaces of colading bodies;

η - elastic factor is equal:

$$\eta = \eta_1 + \eta_2 = \frac{1-\mu_1^{\ 2}}{E_1} + \frac{1-\mu_2^{\ 2}}{E_2} , \tag{3}$$

μ_1, μ_2 – Poisson's ratio; E_1, E_2 – Young's modulus associated with each solid.

When the contact is caused by a collision the second-ordinary differential equation is directly relation of the contact force and the amount of indentation of two solid is:

$$m\ddot{x} = -F_e, \quad \text{or} \quad m\dot{x}\frac{d\dot{x}}{dx} = -kx^{\frac{3}{2}}, \tag{4}$$

where $m = \dfrac{m_1 m_2}{m_1 + m_2}$ - reduced mass of two bodies, for the collision a sphere with massive plane we can accept, that $R_2 = \infty$, $m_2 = \infty$, $m = m_1$.

Equation (4) can be integrated analytically taking into account initial conditions: $t = 0$, $x = 0$, $\dot{x} = v_0$:

$$\frac{1}{2}m\left(\dot{x}^2 - v_0^{\ 2}\right) = -\frac{2}{5}\cdot kx^{\frac{5}{2}}.$$

At the instant of maximum compression the indentation velocity is zero and maximum approach of masses center is:

$$x_{\max} = \left(\frac{5}{4}\frac{mv_0^{\ 2}}{k}\right)^{\frac{2}{5}}. \tag{5}$$

The maximum contact force

$$F_{\max} = k^{\frac{2}{5}}\left(\frac{5}{4}mv_0^{\ 2}\right)^{\frac{3}{5}}. \tag{6}$$

Approximate formula for the time of contact:

$$t_c = 2.9432\cdot\left(\frac{5}{4}\frac{m}{k}\right)^{\frac{2}{5}}\cdot v_0^{-\frac{1}{5}}. \tag{7}$$

In contact mechanics for the case of Hertz's contact force the following parameters were analytically developed and experimentally confirmed [1], [2]:

sizes of contact area:

$$a = b = \sqrt[3]{\frac{3}{4}\eta F\frac{R_1 R_2}{R_1 + R_2}}, \tag{8}$$

value of maximal pressure between contacting bodies:

$$p_0 = 0.5784\cdot\sqrt[3]{\frac{F}{\eta^2}\left(\frac{R_1 + R_2}{R_1 R_2}\right)^2}, \tag{9}$$

approach of bodies - from it formula (1) is developed:

$$x = \sqrt[3]{\left(\frac{3}{4}\right)^2(\eta F)^2\frac{R_1 + R_2}{R_1 R_2}}. \tag{10}$$

For the case of contact of sphere and plane with identical properties of material

$$a = b = \sqrt[3]{\frac{3}{\eta}\cdot FR}, \quad p_0 = 0.5784\cdot\sqrt[3]{\frac{F}{\eta^2 R^2}}.$$

To illustrate process of the parameters estimation a numerical examples is considered for direct impact of steel spheres against massive steel plate and of the same size aluminium alloy sphere against aluminium plate ($R_2 = \infty$).The parameters of colliding bodies are given in the

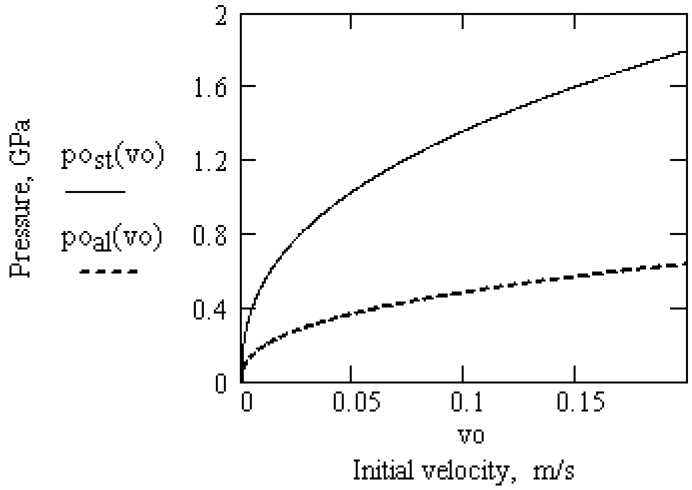


Fig. 2. Plot of dependence of the maximal pressure on the pre-impact velocity: po_{st}- for steel sphere impact against plate, po_{al}- aluminium alloy sphere impact

table 1, where the next characteristics are defined as a function of pre-impact velocity v_0: maximal approach as a function maximal force, time of impact, radius of circular contact area, maximal pressure. For two spheres the reduced mass and relative velocity are indicated. In Fig. 2 the plot of dependence of the maximal pressure on the pre-impact velocity are presented.

Simple elastic-plastic theory suggest that at the point of yield average indentation pressure is given by $p_a \approx 1.1\sigma_y$, where σ_y is uniaxial yield stress; full plasticity limit of $p_p \approx 2.8\sigma_y$. This allows estimating at what velocity full plasticity will be achieved for given material. It is evidently that this velocity is a function of material properties and does not depend on the radius of spheres in the case of sphere - plate collision. Such preliminary estimation allows choosing the model of contact force more correctly.

Table 1. The parameters of the colliding bodies

Parameter	Units	Steel sphere Steel plate	Steel sphere Steel plate	Aluminium alloy sphere Aluminium alloy plate
Sphere radius R	m	0.005	0.01	0.01
Mass m	kg	0.004	0.033	0.011
Elasticity modulus E	Pa	2.10^{11}	2.10^{11}	7.10^{10}
Poisson's ratio μ		0.3	0.3	0.33
Factor η	m^2/N	$9.1\cdot10^{-12}$	$9.1\cdot10^{-12}$	$2.546\cdot10^{-11}$
Factor k	$N/m^{3/2}$	$1.036\cdot10^{10}$	$1.465\cdot10^{10}$	$5.237\cdot10^{9}$
maximal approach $x_{\max}(v_0)$	m	$1.194\cdot10^{-5}v_0^{\frac{4}{5}}$	$2.388\cdot10^{-5}v_0^{\frac{4}{5}}$	$2.358\cdot10^{-5}v_0^{\frac{4}{5}}$

radius of circular contact area $a(v_0)$	m	$2.443 \cdot 10^{-4} \cdot v_0^{\frac{2}{5}}$	$4.886 \cdot 10^{-4} \cdot v_0^{\frac{2}{5}}$	$4.889 \cdot 10^{-4} \cdot v_0^{\frac{2}{5}}$
maximal force $F_{\max}(v_0)$	N	$427 \cdot v_0^{\frac{6}{5}}$	$1710 \cdot v_0^{\frac{6}{5}}$	$600 \cdot v_0^{\frac{6}{5}}$
time of impact $t(v_0)$	s	$3.515 \cdot 10^{-5} v_0^{\frac{-1}{5}}$	$7.029 \cdot 10^{-5} v_0^{\frac{-1}{5}}$	$6.940 \cdot 10^{-5} v_0^{\frac{-1}{5}}$
maximal pressure $p_0(v_0)$	Pa	$3419 \cdot 10^6 v_0^{\frac{2}{5}}$	$3419 \cdot 10^6 v_0^{\frac{2}{5}}$	$1213 \cdot 10^6 v_0^{\frac{2}{5}}$
σ_y, Pa	Pa	$780 \cdot 10^6$	$780 \cdot 10^6$	$340 \cdot 10^6$
$v_{0y}(1.1\,\sigma_y)$	m/s	0.032	0.032	0.053
$v_{0p}(2.8\,\sigma_y)$	m/s	0.326	0.326	0.545

3. Quasistatic Impact Model of Cylindrical Bodies

For the contact between two parallel cylinders (Fig.3) there are few and approximate force-displacement relationships. ESDU-78035 Tribology Series [2] presented for cylindrical contact formulas for contact area, pressure and approach the same as Briger-Panovko [1].
For two cylinders with radii R_1 and R_2 with parallel axis and with length L is derived:

size of contact area – half of width:

$$b = \sqrt{\frac{4}{\pi}\eta \frac{F}{L}\frac{R_1 R_2}{R_1 + R_2}} \quad , \tag{11}$$

value of maximal pressure between contacting bodies:

$$p_0 = \frac{2F}{\pi b L} = \sqrt{\frac{1}{\pi}\frac{F}{L}\frac{1}{\eta}\left(\frac{R_1 + R_2}{R_1 R_2}\right)} \quad , \tag{12}$$

approach of bodies:

$$x = \frac{2}{\pi}\frac{F}{L}\left[\eta_1\left(\ln\frac{2R_1}{b} + 0.407\right) + \eta_2\left(\ln\frac{2R_2}{b} + 0.407\right)\right], \tag{13}$$

where elasic factor η is the same as in equation (3). Since equation (13) is nonlinear implicit function for F with known indentation depth x, F can be evaluated using an iterative scheme.

The simplest method is to establish set of value of F, calculate x from (11) and (13), create two vectors: F and x and to find the unknowns *a, n, c* of power equation

$F(x) = ax^n + c$,

for example, with help of **"pwrfit"** MathCAD function. If we want to estimate the collision of cylinder and plane surface ($R_2 \to \infty$) the equation (13) cannot be used. Then based on Hertz theory the expression of Dubovsky - Freudenstein (for the indentation as a function of force on the internal pin inside the cylinder) may be used:

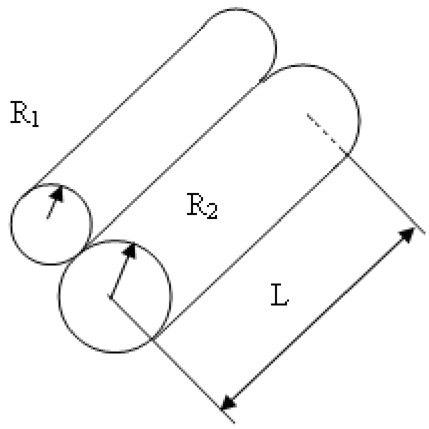


Fig. 3. Contact between two external cylinders

$$x = \frac{F}{L}(\eta_1 + \eta_2)\left[\ln\left(\frac{L^3(R_1 - R_2)}{FR_1R_2(\eta_1 + \eta_2)}\right) + 1\right], \tag{14}$$

if we assume, that $R_1 \to \infty$.

In the expression $F = kx^n$ power index n is changing in the range: 1.5> n >1.0.

For numerical example we calculated the parameters of collision of two steel spheres and sphere with plate. The parameters of impact as function of pre-impact velocity v_0 are resulted in the table 2.
For two cylinders the reduced mass and relative velocity are indicated.

Table 2. Parameters of the colliding circular cylindrical bodies

Parameter	Units	Steel cylinder Steel cylinder R_1=R_2=R	Steel cylinder Steel plate
Cylinder radius R	m	0.02	0.02
Cylinder length L	m	0.10	0.10
Mass m	kg	0.50	1.00
Elasticity modulus E	Pa	$2{\cdot}10^{11}$	$2{\cdot}10^{11}$
Poisson's ratio μ		0.3	0.3
Factor η	m^2/N	$9.1\cdot10^{-12}$	$9.1\cdot10^{-12}$
Factor k	N/m^n	$8.435{\cdot}10^9$	$1.91{\cdot}10^9$
Power index n		1.090	1.081
maximal approach $x_{max}(v_0)$	m	$1.306\cdot10^{-5}v_0^{0.957}$	$3.535\cdot10^{-5}v_0^{0.961}$
width of contact area $b(v_0)$	m	$2.154\cdot10^{-4}v_0^{0.522}$	$2.353\cdot10^{-4}v_0^{0.519}$
maximal force $F_{max}(v_0)$	N	$4.004\cdot10^4v_0^{1.043}$	$4.779\cdot10^4v_0^{1.039}$
maximal pressure $p_0(v_0)$	Pa	$1183\cdot10^6v_0^{0.522}$	$914\cdot10^6v_0^{0.519}$

Yield point σ_y	Pa	$780 \cdot 10^6$	$780 \cdot 10^6$
$v_{0y}(1.1\sigma_y)$	m/s	0.539	0.885
$v_{0p}(2.8\sigma_y)$	m/s	3.259	5.340

4. Models of Normal Impact Forces Based on Hertz's Contact Force

Quasistatic impact model does not represent the energy dissipation process. The dissipation of energy may be taking into account in the form of internal damping of colliding solid. Some studies have been performed to extend the theory to include dissipation [4, 7, 8]. The models of Hunt–Grossley and Lankarani–Nikravesh are given below; Yang-Sun model is similar to Hunt–Grossley model. We propose two models of impact force on the base of Hertz's Contact Force: for very-low-speed impact without permanent deformation and for impact with the velocity causing plastic deformations. As a numerical example the steel sphere impact against plate are taken with $R=0.01$m, impact parameters for which are given in the middle column of the table 1 in accordance with quasistatic impact model.

4.1. The model of force for impact without permanent deformation

We take the dissipative part of the impact force in the form of dry friction, multiplied by approach x in order to eliminate the tensile and plastic deformation appearance, the equation of motion during impact:

$$m \cdot \ddot{x} = -k \cdot x^{\frac{3}{2}} - P \cdot x \cdot sign(\dot{x}) \tag{15}$$

Energy loses can be estimated by integrating the force

$$\Delta T = \frac{1}{2} m v_0^2 (1 - e^2) = \oint F dx = 2 \int_0^{x_{\max}} P x dx = P x_{\max}^2 \tag{16}$$

Taking into account equation (5) the force P for spherical bodies may be expressed from (16):

$$P = \frac{1}{2} \frac{m v_0^2 (1 - e^2)}{x_{\max}^2} = 0.418 \cdot (1 - e^2) \cdot \sqrt[5]{m k^4 v_0^2} \tag{17}$$

Numerical example for $v_0=0.03m/s$, coefficient of restitution $e=0.9$ is presented in Fig.4.

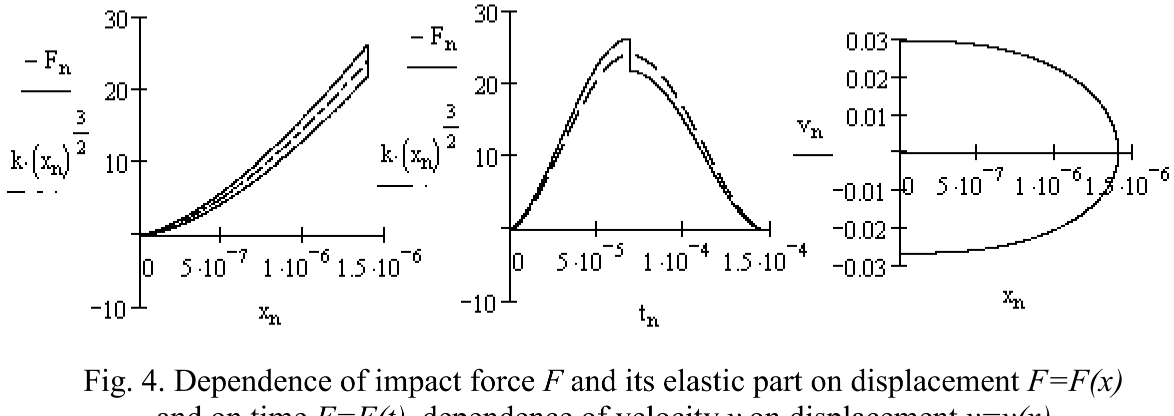


Fig. 4. Dependence of impact force F and its elastic part on displacement $F=F(x)$ and on time $F=F(t)$, dependence of velocity v on displacement $v=v(x)$ $(m=0.033kg, k=1.465 \cdot 10^{10} N/m^{3/2})$

4.2. The model of force for impact with permanent deformation

The dissipative part of the impact force is taken in the form of viscous friction. We assume that impact force depends on both contact Hertz force, and on the rate of its growing, i.e. damping part of force emerges derivative of the elastic force, multiplied by damping factor. Damping factor *b* represents the incorporated value, equal to product of n (for sphere n=3/2) by factor *k* and on the corresponding value of damping:

$$F = k \cdot x^n + b \cdot x^{n-1} \cdot \dot{x}\,. \tag{18}$$

When $F=0,\ v = -v_0 e$, $x = x_{pl}$ and factor *b* may be determined from this condition: $b = \frac{kx_{pl}}{v_0 e}$.

Plastic deformation x_{pl} is the part of full deformation x_{max} (5), depending on coefficient *e.* The energy loss during impact may be defined by numerical integration of the equation:

$$\Delta T = \oint bx^{n-1}\dot{x}dx = b\int_0^{x_{max}} x^{n-1}\dot{x}dx - b\int_{x_{max}}^{x_{pl}} x^{n-1}\dot{x}dx\,.$$

On the other hand $\Delta T = \frac{1}{2} m v_0^2 (1 - e^2)$, hence coefficient of restitution *e* may be estimated.

4.3. Model of Hunt – Grossley impact force

Hunt and Grossley [4] derived the model of impact force with nonlinear spring-damper, satisfying the boundary condition. Nonlinear model of force is described as:

$$F = k \cdot x^n + \frac{3}{2} a \cdot k \cdot x^n \cdot \dot{x} = kx^n \left[1 + \frac{3}{2} a \cdot \dot{x} \right]. \tag{19}$$

The restoring part of force is represented by Hertz's contact force and its damping part is the product of Hertz's force, velocity of deformation and parameter *a.* Parameter *a* is found from the straight line approximation of the coefficient of restitution *e* as a function of initial impact velocity v_0: $e = 1 - av_0$, power index $n=3/2$ for the point contact.

4.4. Model of Lankarani - Nikravesh Impact Force

Lankarani – Nikravesh [7] developed a contact force model, in which hysteresis damping function was incorporated in the model that represent the energy dissipated during the impact This model can be expressed as $F = kx^n + D\dot{x}$, where $D = \mu x^n$, factor μ is determined from the energy loss equation by integration of the contact force around the hysteresis loop:

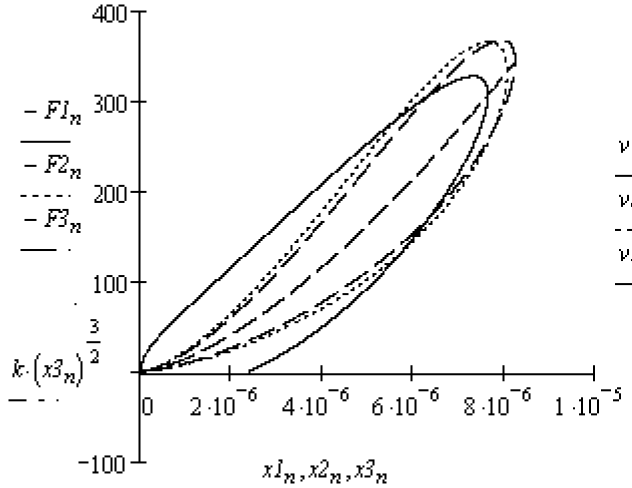
$$F = kx^n + \mu x^n \dot{x} = kx^n \left[1 + \frac{3\left(1 - e^2\right)}{4} \frac{\dot{x}}{v_0} \right] \tag{20}$$

4.5. Numerical example

Equation of motion during impact: $m\ddot{x} = -F$ is solved numerically for the proposed force for impact with permanent indentation (18), Hunt-Grossley force (19) and Lankarani - Nikravesh force (20). Plots of dependence of force *F* and velocity *v* on displacement are given in Fig. 5. ($n=1.5$, $m=0.033$ kg, $k=1.465\cdot 10^{10}$ N/m$^{3/2}$, $v_0=0.3$ m/s, $x_{pl}=2.3\cdot 10^{-6}$ m, $e=0.6$).

In accordance with equations (5), (6), (7) maximal approach $x_{max}= 9.11\cdot 10^{-6}$ m, maximal contact force $F_{max}=403$ N, time of impact $t = 8.9\cdot 10^{-5}$s. Hunt – Grossley and Lankarani –

Nikravesh forces show the close results: post-impact velocities are larger than $v_0 e = 0.18$ *m/s* (*0.21m/s* for Hunt – Grossley and *0.23m/s* for Lankarani – Nikravesh); maximum penetration x_{max} is $8.04 \cdot 10^{-6}$ and $8.22 \cdot 10^{-6}$ *m,* maximal contact force *Fmax is 367 N,* and *369 N* respectively, time of impact $t = 9.2 \cdot 10^{-5}$*s*. Proposed force model shows maximal approach $x_{max} = 7.62 \cdot 10^{-6}$ *m,* maximal contact force *Fmax=329N,* impact time $t = 8.3 \cdot 10^{-5}$*s,* post-impact velocity is $v_0 e = 0.18$ *m/s.*



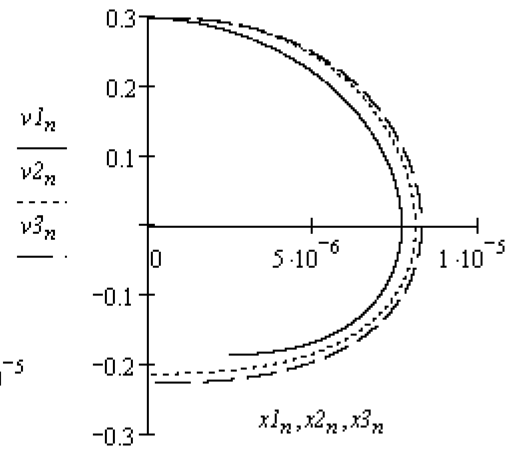


Fig. 5. Dependence of impact force F and velocity v on displacement: (——) in case of proposed impact force F1=*F1(x1), v1=v1(x1)*; (·······) in case of Hunt – Grossley force *F2= F2(x2), v2=v2(x2)*; (- - -) *in* case of Lankarani – Nikravesh force *F3=F3(x3), v3=v3(x3)*

5. Conclusion

The models of continuously acting impact forces represent the impact cycle from the beginning of contact to its end, these models are the most convenient for systems with frictionless impacts analysis, as they allow using the differential equations of dynamics with additional contact forces for solution of the problems of motion. If the contacts between solid are of hertzian type, it is suitable to use Hertz contact theory for the creating impact force model. For the modeling the elastic components of the impact force Hertz contact forces $F = kx^n$ are used for spherical surfaces with power exponent *n=1.5* and for cylindrical surfaces $1.0 \le n \le 1.5$. Damping component is expressed as viscous damping part or as stiff-plastic part, damping force parameters can be determined based on classical impulse-momentum equation and the work-energy principal. In the contact force the equation's parameters may be identified if the coefficient of restitution e is known.

In order to analyze the impact successfully it is necessary to determine whether plastic deformation may occur in the impact. If impact velocities lower than propagation velocity of elastic wave across the body, i.e. $v_0 \le 10^{-5}\sqrt{E/\rho}$, there is no permanent indentation after impact. The advantage of Hertz contact force model with its damping representation over well known Kelvin-Voigt viscoelastic model is its nonlinearity, the first shows more realistic force-displacement diagram.

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Polukoško S., Sokolova S., Kononova O. Kontakta spēki vibro-triecienu sistēmās
Trieciena spēku noteikšana ir nepieciešama mašīnu un mehānismu elementu stiprības aprēķinos. Šajā darbā tiek aplūkots zema ātruma viendabīgu izotropu sfēriskas un cilindriskas formas ķermeņu taisns trieciens. Ķermeņiem ir tādas formas un izmēri, kuriem ir nozīme tikai lokālas deformācijas trieciena momentā. Trieciena parametru iepriekšējam aprēķinam ķermeņu sadursmes punktā vai pa līniju tiek izmantoti kontaktmehānikas vienādojumi un secinājumi. Nosakot spiedienu punktā vai uz kontakta līnijas, var novērtēt trieciena sākuma ātrumu, pie kura sākas ķermeņa plastiskās deformācijas. Tādiem ķermeņiem ir būtiskas lokālas deformācijas kontakta zonā. Viļņu efektu, kas rodas triecienā, var neņemt vērā. Ķermeņa kustību trieciena laikā var aprakstīt ar otrās kārtas diferenciālvienādojumu $m\bar{a} = -\bar{F}$ *, kur spēks* $\bar{F}$ *sastāv no elastīgās un disipācijas komponentes, kas ņem vērā ķermeņu reālās īpašības. Triecienā spēka elastīgās komponentes modelēšanai tiek izmantoti Herca kontaktu spēki sfēriskām un cilindriskām virsmām. Disipācijas komponentē, kas atkarīga no deformācijas ātruma, tiek ievērots enerģijas zudums triecienā. Galvenā uzmanība ir veltīta spēku modeļiem uz Herca teorijas bāzes. Ir veikts trieciena vienādojumu skaitlisks risinājums.*

Polukoshko S., Sokolova S., Kononova O. Contact Forces in Vibro-Impact Systems
The low-speed direct impact of homogeneous isotropic bodies of spherical and cylindrical forms is considered in this work. At collisions of such bodies in a point or on a line for the preliminary calculation of impact parameters the equations of contacts mechanics are used. When the pressure in contact point or on the contact line is defined it is possible to estimate initial velocity of impact, at which plastic deformations of body begin. The local deformations in the contact area are essential for such bodies. Effects of waves, caused by impact, may be neglected. Motion of body during impact it is possible to describe by differential equation $m\bar{a} = -\bar{F}$ *, where force* $\bar{F}$ *consists of elastic and dissipative components, taking into account the real properties of bodies. For the modeling the elastic component Hertz contact forces for spherical or cylindrical surface are used. Energy dissipation during impact is taken into account by damping components depending on deformation velocity. A few models of such forces are created on the basis of theory of Hertz. The numerical solution of the equations of impact for various forces is executed.*

Полукошко С., Соколова С., Кононова О. Контактные силы в виброударных системах
Определение сил, возникающих при ударе, необходимо для проведения прочностных расчётов элементов машин и механизмов. В данной работе рассматривается низкоскоростной прямой удар однородных изотропных тел сферической и цилиндрической формы. При столкновении таких тел в точке или по линии для предварительного расчёта параметров удара используются уравнения и выводы контактной механики. Определив давление в месте контакта, можно оценить начальную скорость удара, при которой начинаются пластические деформации тела. Для таких тел существенными являются местные деформации в зоне контакта. Волновыми эффектами, возникающими при ударе, можно пренебречь. Движение тела во время удара можно описать дифференциальным уравнением $m\bar{a} = -\bar{F}$ *, где сила* $\bar{F}$ *состоит из упругой и диссипативной составляющих, учитывающих реальные свойства тел. Для моделирования упругой составляющей используются контактные силы для сферических и цилиндрических поверхностей. Потеря энергии при ударе учитывается составляющей, зависящей от скорости деформации. Приводятся несколько моделей таких сил, построенных на основе теории Герца. Выполнено численное решение уравнений удара.*