ISSN 1691-5054

HEAT & POWER AND THERMAL PHYSICS

SILTUMENERĢĒTIKA UN SILTUMFIZIKA

2008-1

MODELLING OF HEAT TRANSFER THROUGH A MATERIAL WITH REGULAR DISTRIBUTED ELLIPTIC CAVITIES

SILTUMA PĀRNESES MODELĒŠANA MATERIĀLĀ AR REGULĀRI IZVIETOTIEM ELIPTISKIEM DOBUMIEM

Daiga Cepīte *Laboratory for Mathematical Modelling of Environmental and Technological Processes*
University of Latvia, Zeļļu street 8, LV-1002, Riga, Latvia,
assistant,
E-mail: daiga.cepite@lu.lv

Andris Jakovičs *Laboratory for Mathematical Modelling of Environmental and Technological Processes*
University of Latvia, Zeļļu street 8, LV-1002, Riga, Latvia,
leading researcher, Dr.phys.,
E-mail: Andris.Jakovics@lu.lv

Keywords: effective thermal conductivity, heat conduction, convection and radiation

Preface

Application of inclusions of gas due to its low thermal conductivity (e.g. air $\lambda \approx 0.026\ W/m \cdot K$) is a widely spread practice in industry of building structures which are expected to fulfil both high thermal insulation and high durability. Building blocks *Keraterm* (fig. 1) are an example of structures, in which thermal conductivity is reduced by arranging air inclusions. Air volume fraction of the block can not be increased too much to avoid the lack of the mechanical strength of highly porous structure. The most promising possibilities to minimize the thermal conductivity of the structure (in case of fixed air volume fraction) are connected with the possible variations in

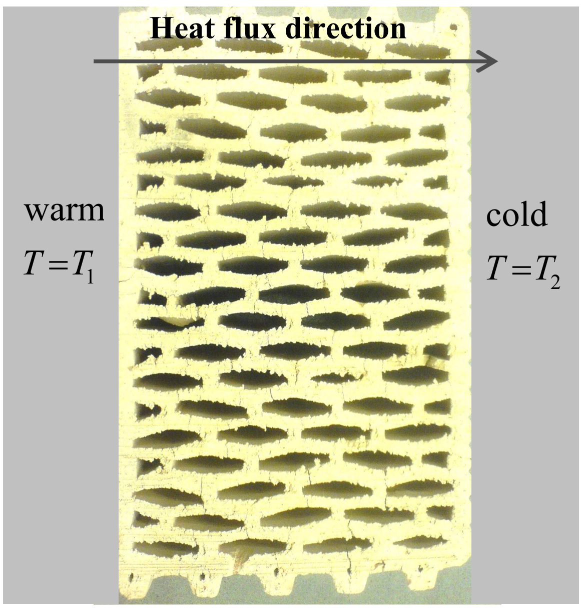


Fig. 1. Example of building block *Keraterm* (stock company *Lode*)

characteristic size and geometry of the cavities and their orientation towards the heat flux direction. Higher effective thermal conductivity λ_{eff} of structures is related to additional convective and radiation heat transfer processes inside the cavities. Convective heat transfer caused due to the thermal expansion of gases (e.g. for air $\beta \approx 0.004\, kg/m^3 \cdot K$) is a restrictive reason for increasing the characteristic size of cavities or using vertical layers of air as an insulator. In case of convection in-between two vertical glass panes it is well known that increasing the distance between panes beyond 18 mm does not lead to the decrease in effective thermal conductivity. In real life situations, intensity of convective heat transfer and radiation heat exchange is the largest in wintertime when the difference between the indoor and outdoor temperatures is maximal. It is worth to mention that humidity is another important factor, which reduces the thermal resistance of the structure, but impact of humidity has not been analysed in the present publication.

The aim of the present publication is to analyse the physical properties related to heat insulation of the composite material (close to that of *Keraterm*) which is formed of well conducting material $(\lambda > 0.25\ W/m \cdot K)$ and regularly placed air-filled cavities with elliptic cross-section. Heat transfer in the direction of the major and minor axes of the elliptic cavities has been analysed.

Mathematical model

Heat transfer equation is solved to analyse the impact of cavities on the effective thermal conductivity of the structure - heat conduction, thermal radiation and convection terms have been included.

Heat conduction and temperature distribution in opaque solid material is governed by Fourier's law [1]. In two-dimensional case:

$$\rho c_p \frac{dT}{dt} = \frac{\partial}{\partial x}\left(\lambda \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\lambda \frac{\partial T}{\partial y}\right), \tag{1}$$

where ρ - density, c_p - specific heat capacity.

In case the element of media (liquid or gas) travel with the velocity $\boldsymbol{\upsilon}$ the convective derivative is taken on the left side of equation (1)

$$\rho c_p \left(\upsilon_x \frac{\partial T}{\partial x} + \upsilon_y \frac{\partial T}{\partial y}\right) = \frac{\partial}{\partial x}\left(\lambda \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\lambda \frac{\partial T}{\partial y}\right). \tag{2}$$

Velocity of the transient motion of incompressible gas $\boldsymbol{\upsilon}$ in the cavity is determined by Navier- Stokes equation in Boussinesq approximation

$$\begin{cases} \dfrac{\partial \boldsymbol{\upsilon}}{\partial t} + (\boldsymbol{\upsilon}\nabla)\boldsymbol{\upsilon} = -\dfrac{1}{\rho_l}\nabla p + \nu \nabla(\nabla \boldsymbol{\upsilon}) + \mathbf{f} - \mathbf{g} \\ \operatorname{div}\boldsymbol{\upsilon} = 0 \end{cases}, \tag{3}$$

where p - pressure, ρ_l - density of the liquid, ν - cinematic viscosity, $\mathbf{g}$ - acceleration due to gravity, thermal expansion coefficient $\beta = -\rho^{-1}(\partial\rho/\partial T)_p$, buoyancy force $\mathbf{f} = \beta(T - T_0)\mathbf{g}$, T_0 - reference temperature.

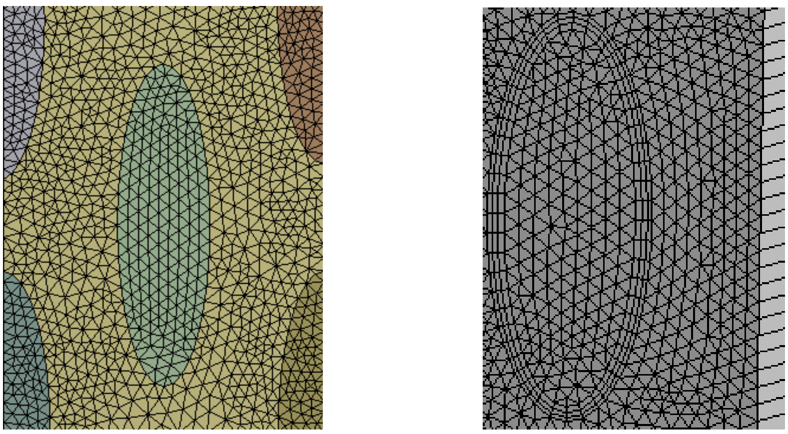
Discrete Transfer model [2] has been implemented for the radiation heat transfer modelling. Absorption coefficient $a = 0.01\,m^{-1}$ of the gas and emission coefficient $\varepsilon = 0.9$ on the gas-solid interface have been taken. Diffuse reflection from the interfaces and grey radiation model has been used. Radiation heat exchange is determined by equation

$$I(s) = \frac{\sigma_{StB} T^4}{\pi}\left(1 - e^{-as}\right) + I_0 e^{-as}, \quad (4)$$

where I - radiation intensity, s - distance from the point with I_0 radiation intensity, T - local temperature and σ_{StB} - Stefan-Boltzmann constant.

Steady state two-dimensional problem has been analysed in a rectangular domain (width Δl and height Δh) with low-conducting air-filled elliptic cavities. Infinite depth of the domain has been accomplished by applying symmetry conditions to front and back faces of the domain (fig. 1). Temperature difference $\Delta T = T_2 - T_1$ between the two vertical side walls has been applied. The resulting heat fluxes have been compared for different arrangements and different geometries of cavities. Projections of vector between the centres of two cavities have been chosen 2.08 and 2.8 cm, semi-major axis of the cavity $d_1 = 2.1\,cm$, semi-minor axis has been varied in the range $d_2 \in [0.37cm; 1.1cm]$ in order to compare situations with different volume ratio V_c/V, where V_c - volume of the cavities, V - total volume of the rectangular domain.

Thermal conductivity of the solid (like porous clay brick) has been set to $\lambda_1 = 0.3\ W/m \cdot K$, $c_p = 880\,J/kg \cdot K$. Physical properties of the material filling the cavities have been set according to air data at $T \approx 310\,K$: $\lambda_2(T) = 0.026\ W/m \cdot K$, dynamic viscosity - $1.92 \cdot 10^{-5}\ Pa \cdot s$, thermal expansion $\beta = 0.0043\,kg/m^3 \cdot K$, density $\rho(T) = 2.4812 - \beta\mathrm{T}\ \mathrm{kg/m^3}$ and specific heat capacity $c_p = 1004\,J/kg \cdot K$. Typical working temperatures are in the range $268\,K < T < 310\,K$ what match real conditions for building constructions.



a) b)

Fig. 2. Examples of mesh for the mathematical problem without (a) and with (b) buoyancy term. Due to the two-dimensional model only 1 element deep mesh has been used

ANSYS CFX [2] has been implemented for obtaining the steady state temperature and flow distribution. Examples of mesh fragments are shown in fig. 2; $1-5\cdot10^4$ elements have been used in total. Maximal residual below $5\cdot10^{-4}$ has been used as the main criteria for convergence of the model. Total heat imbalance has been monitored during the calculation process to ascertain that final solution corresponds to steady state situation.

Verification of the model

It is well known that in case of solid rectangular domain with constant thermal conductivity λ fixed temperature difference between two opposite sides (adiabatic conditions on remainder two sides) leads to stationary linear temperature decline. In this situation a density of the heat flux q (W/m^2) is determined by Fourier's law $q=-\lambda\Delta T/\Delta l$, where ΔT is total temperature difference and Δl - width of the domain. In case equal thermal conductivity of solid and cavities has been set (as well as buoyancy and radiation have not been activated) theoretical predictions are identical to numerical results - difference between numerical and theoretical results does not exceed $0.1\,\%$ of the magnitude of the heat flux.

In case cavities with lower thermal conductivity are included in the system the resulting heat flux is expected to decrease. To characterise the structure effective thermal conductivity λ_{eff} Fourier's law can be rewritten as $Q=qS=-S\lambda_{eff}\Delta T/\Delta l$, where $Q(W)$ – total heat flux through the side wall of surface area S. Effective thermal conductivity depends on ratio λ_1/λ_2 as well as on V_1/V and geometry of the cavities, nevertheless $\lambda_2<\lambda_{eff}<\lambda_1$ (without buoyancy and radiation heat exchange). More accurate estimations can be done by obtaining effective thermal conductivity in two limiting cases – assuming that both materials form two parallel layers with volume ratio V_1/V and are placed parallel or perpendicularly to the outer temperature gradient (fig. 3). Analogy with parallel (bridge) or series connections of resistors can be used.

In addition to λ_{eff} each composite material can be characterised by thermal resistance $R=\Delta l/\lambda_{eff}$ and its inverse $U=1/R$, but the best choice for comparing structures with different thickness is to use λ_{eff}.

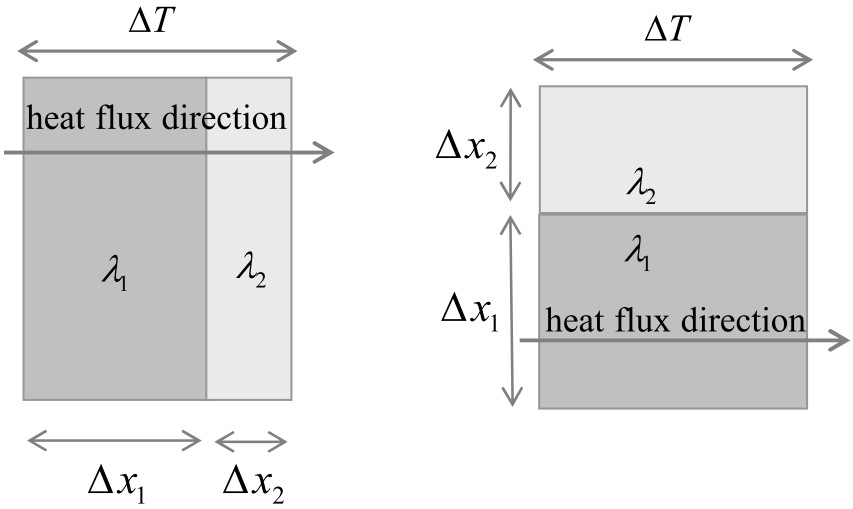
In case of perpendicular orientation towards heat flux direction (fig. 3a) thermal resistance is $R=\Delta x_1/\lambda_1+\Delta x_2/\lambda_2$, $K\cdot m^2/W$ and effective thermal conductivity is

$\lambda_{eff}=(\Delta x_1+\Delta x_2)/R=(\Delta x_1+\Delta x_2)/(\Delta x_1/\lambda_1+\Delta x_2/\lambda_2)$.

In case of parallel orientation:

$\lambda_{eff}=(\Delta x_1\lambda_1+\Delta x_2\lambda_2)/(\Delta x_1+\Delta x_2)$. These dependencies are shown in fig. 8 also.

In case of elliptic cavities parallel to heat flux direction (volume ratio of air $V_1/V=\eta=\Delta x_1/(\Delta x_1+\Delta x_2)$) λ_{eff} is larger than $(\Delta x_1+\Delta x_2)/(\Delta x_1/\lambda_1+\Delta x_2/\lambda_2)$. In case only thermal conduction takes place it could not exceed $(\Delta x_1\lambda_1+\Delta x_2\lambda_2)/(\Delta x_1+\Delta x_2)$; in general case it can not be larger than maximal of λ_1 and λ_2.



a)

b)

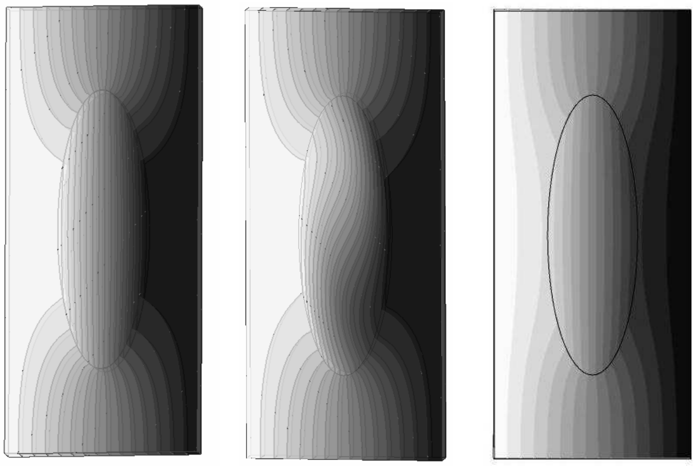


Fig. 3. Two layered structure. Layers are perpendicular (a) and parallel (b) to the heat flux direction

Density of the radiation heat flux q_{rad} between two parallel plates with temperatures T_1 and T_2 and surface emission coefficients ε_1 and ε_2

$$q_{rad} = \sigma_{StB}\left(T_1^4 - T_2^4\right) / \left(1/\varepsilon_1 + 1/\varepsilon_2 - 1\right), \tag{5}$$

can be taken as a measure to take or not to take radiation heat transfer into account in the model. Taking of values $T_1 = 293\,K$, $T_2 = 294\,K$, $\varepsilon_1 = \varepsilon_2 = 0.9$ leads to $q_{star} \approx 4.7\ W/m^2$. Typical magnitude of conductive heat transfer in building structures in winter-time is in the range $5 - 20\,W/m^2$, consequently for accurate calculations radiation heat transfer should be included.

Analysis of results

The first situation analysed is a structure which is formed of one elliptic inclusion with lower thermal conductivity ($\lambda_1/\lambda_2 \approx 10$) that is oriented perpendicularly to heat flux direction (fig. 4).

Heat flux avoids the direction of lower thermal conductivity, therefore contours curve around the inclusion and the highest temperature gradients arise inside the cavity. On both sides of the cavity regions arise with the temperatures close to the temperature fixed on the boundary (fig.4a).

In case only conductive heat transfer takes place the effective thermal conductivity is lower for perpendicularly oriented cavity ($\lambda_{eff\,\perp} = 0.17\,W/m\cdot K$) than for cavity which is oriented parallel to heat flux direction ($\lambda_{eff\ \parallel} = 0.22\, W/m\cdot K$) by the same volume fraction $\eta = V_c/V = 0.22$ (fig. 4a and 5a).

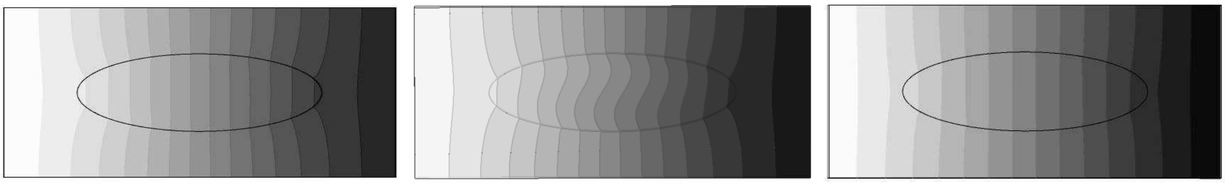
In case buoyancy and radiation heat transfer is included in the model, effective thermal conductivity increases. Results show that in case of average horizontal temperature gradient $\partial T/dl \approx 1.43\, K/cm$ the maximal velocity of convective motion is 1.6 cm/s in case cavity is oriented perpendicularly to heat flux (fig. 4b) and 1.1 cm/s in case cavity is oriented

perpendicularly to heat flux (fig. 4b) and 1.1 cm/s in case cavity

is oriented parallel to heat flux direction (fig. 5b). The larger the velocity of convective air motion, the larger the increase in effective thermal conductivity. Taking the convective heat transfer into account, effective thermal conductivity is lower if cavities are oriented perpendicularly to heat flux direction: $\lambda_{eff\ \parallel}$ = 0.23 $W/m\cdot K$ (fig. 5b) and $\lambda_{eff\ \perp} = 0.18$ $W/m\cdot K$ (fig. 4b). In both cases air velocity ($\upsilon < 2\, cm/s$) is low enough to assume laminar flow (typical Reynolds number $\mathrm{Re} = \upsilon_r\, l/\nu \approx 50$).

a) b) c)

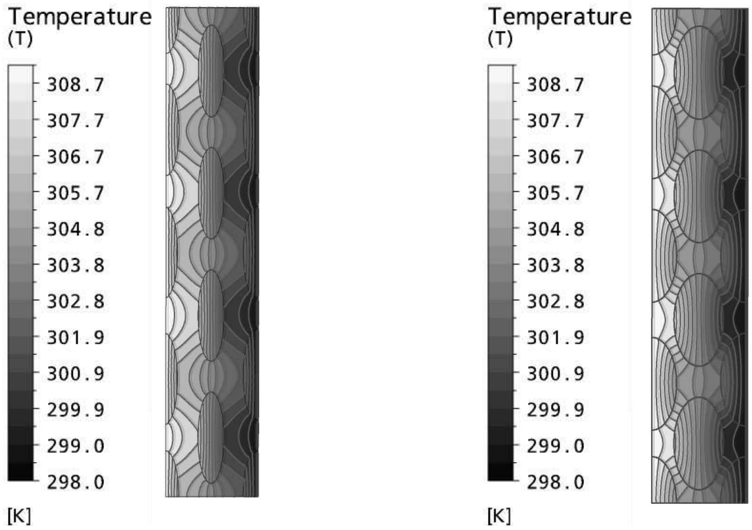
Fig. 4. Distribution of temperature in a structure with one cavity oriented perpendicularly to heat flux direction: a) non buoyant, b) buoyant, c) non-buoyant – with radiation heat transfer. The direction of gravity is downwards; lighter color corresponds to higher temperature



a) b) c)

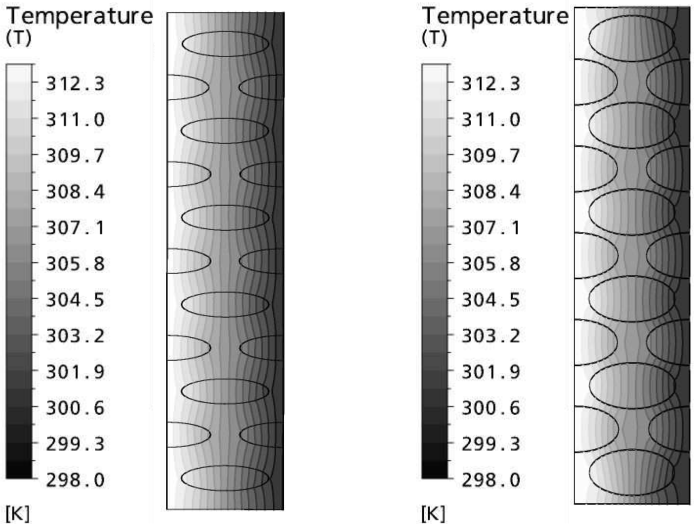
Fig. 5. Temperature distribution in a structure with one low conducting cavity parallel to heat flux direction: a) non-buoyant, b) buoyant, c) non-buoyant with radiation heat transfer. The direction of gravity is downwards; lighter colour corresponds to higher temperature

Numerical results show that the most important increase in effective thermal conductivity is achieved by radiation heat exchange mechanism - radiation heat transfer is much more important than convection in these cases. Resulting effective thermal conductivity due to conduction, convection and radiation is lower in case the cavity is perpendicularly oriented to the heat flux, $\lambda_{eff\ \parallel} = 0.26\, W/m\cdot K$ and $\lambda_{eff\ \perp} = 0.22\, W/m\cdot K$ respectively. For practical purpose, understanding of the role of heat transfer mechanisms and its impact on effective thermal conductivity in structures with many regularly placed cavities is essential - as well as the impact of cavity size and orientation on the effective thermal conductivity. For this purpose models shown in fig. 6 and fig. 7 with more layer cavities structure have been created. Cavities in proximal layers are displaced alternately, what reduces effective thermal conductivity of the structure and improves its mechanical strength. In this model dependence of effective thermal conductivity on geometry of the cavities has been analysed assuming conductive heat transfer only - this assumption give a possibility to estimate the maximal possible thermal resistance of those composites. Location of centres of the cavities has been maintained, but variation in semi minor axis has been done leading to η to be in range between 0.2 and 0.62. The same type of boundary conditions has been used.



a) b)

Fig. 6. Temperature distribution in a structure with cavities oriented perpendicularly to heat flux direction: a) $\eta = 0.34$, b) $\eta = 0.62$



a) b)

Fig. 7. Temperature distribution in a structure with cavities oriented parallel to heat flux direction:
a) $\eta = 0.33$, b) $\eta = 0.60$

In case of elliptic cavities with equal dimensions, it is not possible to increase magnitude of η too close to 1 due to wane of mechanical strength. Increase of η leads to increase of local temperature gradients in blocks what could be the reason for crack due to thermal tension. In fig.8 dependence of effective thermal conductivity on cavity volume ratio is shown as well as approximate dependence (continuous line) of effective thermal conductivity on air volume ratio in basic two layer model illustrated in fig. 3. Results of numerical models with elliptic cavities lie in-between results of two-layer model (in case only heat conduction is taken into account).

It was shown that in case of the block with one cavity (fig. 4 and 5) effective thermal conduction is lower if cavities are oriented perpendicularly to heat flux direction. Even larger is the difference in case of the two layer idealized model, but the low thermal conductivity predicted by two layer model can not be obtained in real life situations due to convection and radiation heat transfer that increase the thermal conductivity.

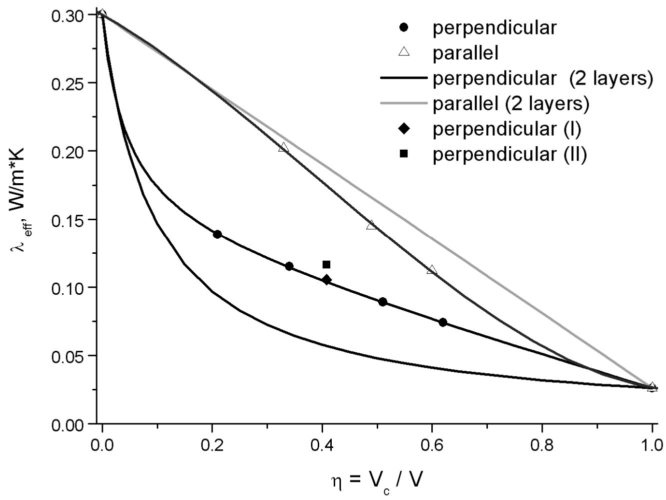
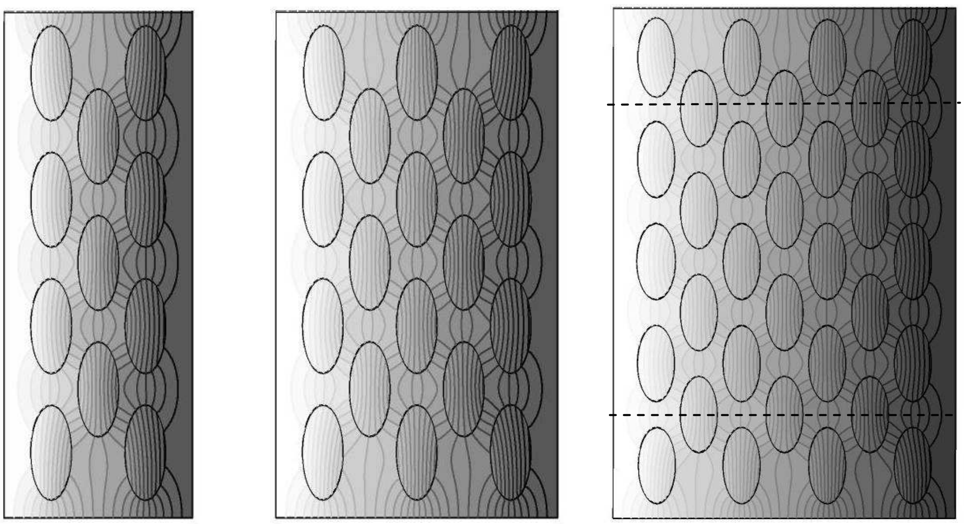


Fig. 8. Dependence of effective thermal conductivity on cavity volume ratio η



a) b) c)

Fig. 9. Temperature distribution in a structure with cavities with lower thermal conductivity oriented perpendicularly to heat flux direction. Lighter colour corresponds to higher temperature

To ascertain that the lack of cavities in the outer zones of the structure does not cause significant inaccuracy in calculation of effective thermal conductivity, additional calculation taking into account only middle zone (between 2 scattered lines in fig. 9c) has been done. Results of two methods of calculation (with (*I*) and without (*II*) including the outer layers) of effective thermal conductivity of the structure shown in fig. 9c are presented in fig. 8. It is evident that in case *I* good agreement with results of the model shown in fig. 6 has been obtained – it means that the effective thermal conductivity depends on cavity geometry and orientation towards the heat flux direction not on the numbers of layers of cavities in the calculation domain. Since the heat flux in outer zones is larger (due to lack of cavities with lower thermal conductivity) the effective thermal conductivity *II* is slightly larger than *I*.

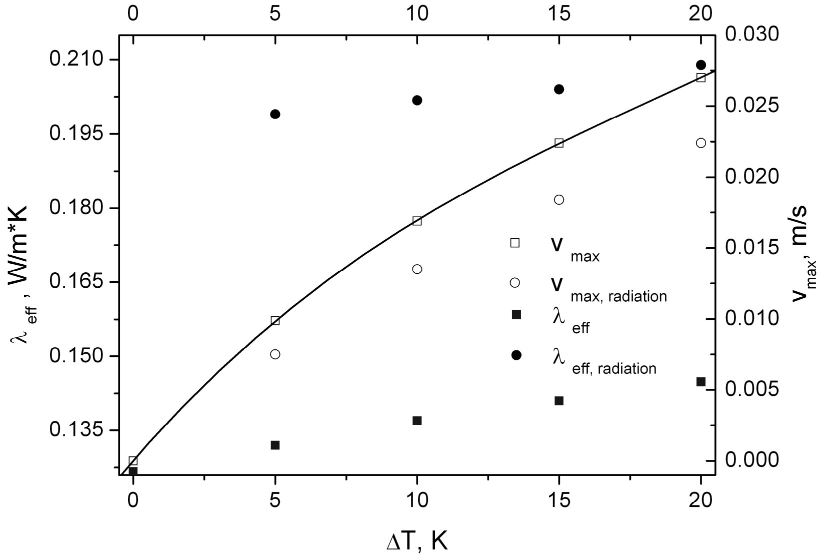
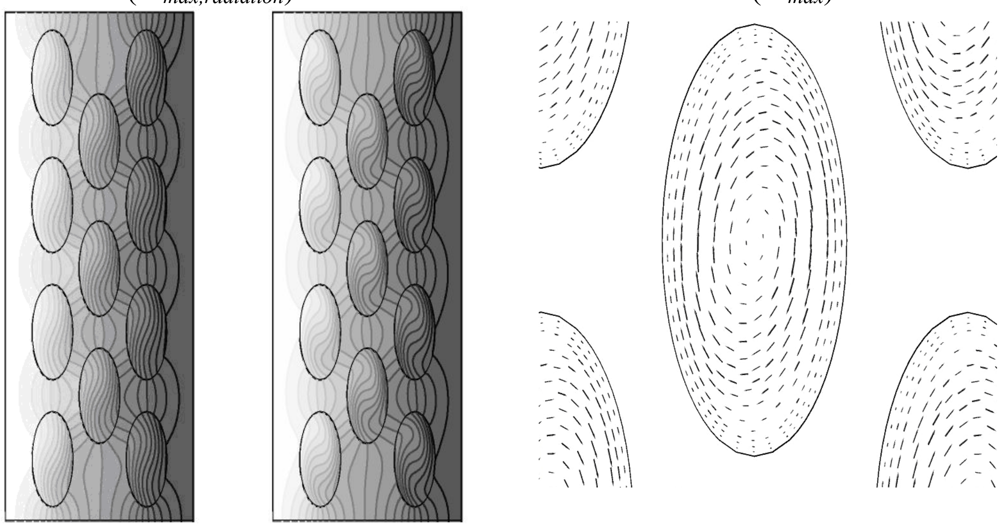


Fig. 10. Dependence of effective thermal conductivity and the maximal velocity of convective gas flow on temperature difference ΔT between vertical boundaries with

($\upsilon_{max,radiation}$) and without radiation heat transfer (υ_{max})



a) b) c)

Fig. 11. Distribution of temperature and velocity: a) $\Delta T = 5K$, b) $\Delta T = 20K$. Lighter colour corresponds to higher temperature; the direction of gravity is downwards

Model shown in fig. 9a was used to evaluate importance of convection in determining the effective thermal conductivity of the structure. In fig. 10 it is shown that enlargement of ΔT (temperature difference between two vertical boundaries which are in distance $8.3\,cm$) causes increase in the maximal velocity of convective flow as well as in the effective thermal conductivity of the structure. In case ΔT is close to zero and only molecular mechanism of heat transfer takes place then $\lambda_{eff} \approx 0.126\ W/m{\cdot}K$, in case $\Delta T = 20\,K$ $\lambda_{eff} \approx 0.142\ W/m{\cdot}K$ (fig. 11) – effective thermal conductivity has increased due to convective heat transfer.

In its turn radiation heat transfer significantly enlarges effective thermal conductivity in the model shown in fig. 11. In case $\Delta T = 15\,K$ and radiation heat transfer has been taken into account the resulting value of the effective thermal conductivity is $\lambda_{eff} \approx 0.20\ W/m{\cdot}K$; in case radiation heat exchange has been excluded $\lambda_{eff} \approx 0.14\ W/m{\cdot}K$. Temperature difference inside cavity is diminished due to radiation heat transfer; that is the reason why intensity of convection characterised with the maximal velocity in fig. 10 is diminished as well.

Conclusions

Convection inside cavities ($\eta = 0.35$ and $\Delta T/\Delta l \approx 2.4\,K/cm$) is able to enlarge the effective thermal conductivity by 15%. In typical conditions for building blocks temperature drop does not exceed $\Delta T/\Delta l \approx 1\,K/cm$ and in this situation enlargement of the effective thermal conductivity due to convection heat transfer is lower ($\approx 5\ \%$). On the contrary enlargement of effective thermal conductivity due to radiation heat transfer is more significant: about 50% in case $\Delta T/\Delta l \approx 0.6\,K/cm$. Therefore, to simplify calculation process convective heat transfer can be ignored, but radiation heat transfer has to be included in the heat transfer process models of similar building structures. According to numerical results, structure with cavities oriented perpendicularly to heat flux direction possesses lower effective thermal conductivity than one with cavities oriented parallel to heat flux direction. It is a reason to consider the perpendicular direction in developing production of composite building materials in order to diminish heat losses from buildings.

Acquired range of effective thermal conductivity is in good agreement with that of *Keraterm 44* building blocks produced by stock company "*Lode*" what is in the range $\lambda_{eff} = 0.13 - 0.23\ W/m \cdot K$ [3]. It shows that obtained results can be used to analyse importance of different heat transfer mechanisms in building blocks with macroscopic cavities. This approach can be used effectively for optimisation of cavity geometry to develop structures with the highest thermal resistance for fixed value of $\eta = V_c/V$.

References

1. Incropera F. P., De Witt D. P. Fundamentals of heat and mass transfer. John Wiley & Sons, New York, 1990.
2. ANSYS CFX, Release 11.0 User's Guide.
3. Lodes būvniecības sistēma 2.izdevums, A/S „Lode" (in Latvian).

Acknowledgement:

Research has been supported by the *European Regional Development Fund (ERDF)*.

Cepīte D., Jakovičs A., Siltuma pārneses modelēšana materiālā ar regulāri izvietotiem eliptiskiem dobumiem

Pētījumā parādīts, kā, izmantojot lietišķās modelēšanas programmas, iespējams efektīvi prognozēt kompozītmateriāla ar anizotropām īpašībām efektīvo siltuma vadītspēju. Konkrēti tiek analizēta cieta materiāla ar eliptiskiem, ar gaisu pildītiem dobumiem siltuma vadītspējas atkarība no dobumu izmēra un orientācijas attiecībā pret siltuma plūsmu, kā arī konvekcijas un siltuma pārneses starojuma ceļā radītās vadītspējas izmaiņas.

Materiāla, kura parametri tuvi Keraterm blokiem un eliptisko dobumu garākā ass ir orientēta perpendikulāri siltuma plūsmai, siltuma vadītspēja ir būtiski mazāka nekā struktūrai, kur dobumu garākās asis orientētas paralēli siltuma plūsmai pie tās pašas dobumu tilpuma daļas materiālā.

Cepite D., Jakovics A. Modelling of a heat tranfer through the material with regular distributed elliptic cavities

It has been shown that the effective thermal conductivity of anisotropic composite material (well conducting media with regular cavities of the air) can be studied by numerical modelling. The influence of the orientation and size of the cavities on the effective thermal conductivity of the structure as well as the role of the convective and radiation heat transfer in the process has been examined.

It has been shown that effective thermal conductivity of the structure close to that of Keraterm is lower in case the cavities are oriented perpendicularly to the heat flux direction than in case the cavities are oriented parallelly to the heat flux direction.

Цепите Д., Якович А. Моделирование теплопереноса в материалах с регулярной структурой полостей

В иследовании показоно, что изпользование прикладных программ моделирования позволяет успешно прогнозировать эффективную теплопроводность композитных материалов с анизотропными свойствами. Конкретно анализируется зависимость теплопроводности твердого материала с воздушными полостями элиптического сечения от их размера и ориентации относительного теплового потока, а также анализируются изменения теплопроводности визваные тепловой конвекцией и излучением.

Теплопроводность материала, параметры которого подобны блокам Keraterm и длинные оси элиптических полостей ориентированы перпендикулярно тепловому потоку, существенно ниже теплопроводности структуры, где эти оси ориентированы параллельно тепловому потоку при той же обемной доле полостей в материале.