THEORETICAL ANALYSIS OF THE METHOD OF SCOUR DEVELOPMENT IN TIME DURING THE FLOOD.
IZSKALOJUMU RAŠANĀS PLŪDU LAIKĀ METODES TEORĒTISKĀ ANALĪZE.

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Key words: scour, time, depth

Introduction

The contraction of the flow by bridge crossings on plain rivers changes the flow structure and leads to scour at bridge constructions. The failure of bridges as a result of scour at piers and/or abutments causes considerable damages and high expenses.

In our investigation, we used a differential equation of equilibrium of the bed sediment movement for clear water and elaborated a method for calculating the scour development with time at the abutments during the flood [1] and [2]. The method was confirmed by experimental data [1].

Melville [3] shows empirical dependence of the scour depth on the depth and intensity of the flow, the type of foundation, shape, size, alignment and channel geometry Radices et al. [4] presents an empirical analysis of the relations between the scour depth and the control parameters of the flow, river bed, and abutment.

A theoretical analysis of the method elaborated allowed us to estimate the influence of hydraulics and river-bed parameters on the scour at bridge abutments in the flood. The relative depth of scour depends on the contraction of the flow, relative grain size of the river bed and its distribution over the depth, kinetic parameter of the flow, relative depth and velocity of the flow, Froude number in relation to the slope, unsteadiness of the flow, time of scour, and duration of the flood. According to the earlier investigations, the scour at bridge abutments depends also on the shape of abutments and the angle of flow crossing. The dependence of the relative scour depth at the abutments on the above- mentioned parameters are presented graphically.

Method

The differential equation of equilibrium for the bed sediment movement in the conditions of clear water is

$$\frac{dw}{dt} = Q_s \,. \qquad (1)$$

Here, according to laboratory tests, the volume of the scour hole $w = 1/6\, \pi m^2 h_s^3$, t = the time, and Q_s = the sediment discharge out of the scour hole.

The left-hand part of Eq. (1) can be written as

$$\frac{dw}{dt} = \frac{1}{2}\pi m^2 h_s^2 \frac{dh_s}{dt} = a h_s^2 \frac{dh_s}{dt}\,, \qquad (2)$$

where h_s = the depth of scour and m = the steepness of the scour hole. The sediment discharge was determined by the Levi formula [5]

$$Q_s = AB \cdot V_l^4 \,, \qquad (3)$$

where B = mh_s is the width of a scour hole, V_l = the local velocity at the abutment, and A = a parameter in the Levi formula [5].

The sediment discharge during the scour development is found from the formula

$$Q_s = A\ mh_s \cdot V_{lt}^4 = b\frac{h_s}{k\left(1+\frac{h_s}{2h_f}\right)^4}\ , \qquad (4)$$

where $b = Am \cdot V_l^4$, k = the coefficient of changes in the discharge owing to the scour at the abutment, and h_f = the depth of water in the floodplain.

The hydraulic characteristics, the contraction flow rate, the velocities V_o and V_l, the grain size in different layers of the bed, the sediment discharge, as well as the depth and width of the scour varied during the flood.

Allowing for formulas (2) and (4), differential equation (1) can be presented as

$$ah_s^2\frac{dh_s}{dt} = b\frac{h_s}{k\left(1+\frac{h_s}{2h_f}\right)^4}\ . \qquad (5)$$

After integration, we obtain

$$N_i = \frac{t_i}{4D_i h_f^2} + N_{i-1}\,, \qquad (6)$$

where $N_i = 1/6x_i^6 - 1/5x_i^5$ and t_i = the time interval.

According to the method suggested, the hydrograph was divided in time steps, and each step was divided into time intervals. We performed calculations for each step of the hydrograph, so that to estimate the influence of the flow unsteadiness during the flood, but, in each time step, we assumed that the flow was steady.

Using the graph $N = f(x)$ for calculated N_i, we find x_i and the scour depth at the end of the time interval:

$$h_s = 2h_f(x-1)\,. \qquad (7)$$

To determine the development of scour depth during the flood, the hydrograph was divided into time steps with duration of 1 or 2 days, and each time step was divided into time intervals up to several hours or less. In laboratory tests, the time steps were divided into 20 time intervals. For each time step, the following parameters must be found: h_f – depth of water in the floodplain, Q/Q_b – rate of contraction of the flow, Δh – maximum backwater, d – grain size, H – thickness of the bed layer with d, and γ – specific weight of the bed material. As a result, we have V_l, V_o, A, D, N_i, N_{i-1}, and h_s at the end of time intervals and, finally, at the end of time step. For the next time step, the flow parameters were changed due to the flood and scour developed in the previous time step. The method is confirmed by experimental data [1] and [2].

Analysis of the Method

For analyzing the method, we have transformed formulas (6) and (7) to a form that shows clearly that they contain dimensionless parameters and characteristic of the flow.

Now, formula (7) has the form

$$N_i = \frac{2 t_i A \varphi^4 g^2}{\pi m K^4} \frac{\Delta h^2}{h_f^2} + N_{i-1} . \qquad (8)$$

Rotenburg [6] has found that the relative maximum backwater is a function of the following parameters:

$$\frac{\Delta h}{h_f} = f\left(\frac{Q}{Qb}; P_K; P_{Kb}; \frac{Fr}{i}; \frac{h}{h_f}\right), \qquad (9)$$

where Q/Q_b = the contraction of the flow, P_K = the kinetic parameter of the open flow, P_{Kb} = the kinetic parameter of the flow under the bridge, $Fr/i_0 = v^2/gLi_0$ = the Froude number in relation to the river slope, h/h_f = the relative depth of the flow, h = the average depth of the flow, and h_f = the depth of water in the floodplain.

In the general form, Eq. (8) can be written as

$$N_i = \frac{2 A_1 g^2}{\pi m} \frac{\varphi^4}{k^4} \frac{h^2}{h_f^2} \frac{1}{\left(\frac{d}{h_f}\right)^{0.25}} \cdot \qquad (10)$$

$$\cdot \left\{ \frac{P_K}{2}\left[\left(\frac{Q}{Q_b}\right)^2 - 1\right] + \frac{1}{2} P_{Kb} \sqrt{\frac{1}{Fr/i}}\left[\left(\frac{Q}{Q_b}\right)^2 + 1\right] + P_{Kb} \right\}^2 \cdot t_i + N_{i-1}$$

From Eq. (10), relative depth of scour is a function the next parameters:

$$\frac{h_s}{h_f} = 2(x-1) = f\left(\frac{Q}{Qb}; P_K; P_{Kb}; \frac{Fr}{i}; \frac{d}{h_f}; \frac{\beta V_0}{V_l}; \frac{h}{h_f}; t; N_{i-1}\right), \qquad (11)$$

where d/h_f = the relative grain size of the river bed and N_{i-1} = the scour formed during the previous time step.

The depth of scour at the abutments depends on the flow-crossing angle α (Richardson et al. [7]) and on the abutment shape m (Richardson and Davis [8]).

In the general form, the relative depth of scour is a function of the dimensionless parameters and time:

$$\frac{h_s}{h_f} = 2(x-1) = f\left(\frac{Q}{Qb}; \frac{v}{gh}; \frac{Fr}{i_0}; \frac{d}{h_f}; \frac{\beta V_0}{V_l}; \frac{h}{h_f}; t; N_{i-1}; m; \alpha; a; t_{fl}\right), \qquad (12)$$

where a = the unsteadiness of the flow and t_{fl} = the flood duration.

We present a graphical dependence of the relative scour depth on different flow and bed parameters. Figure 1 shows the relative depth of scour versus the contraction rate of the flow. With increase in the contraction of the flow, the relative depth of scour increases.

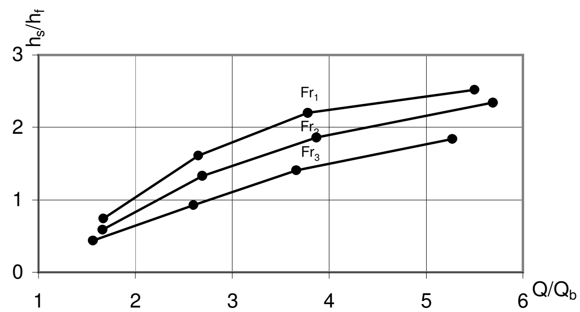

Fig. 1. Relative depth of scour versus the contraction rate of the flow.

The dependence of the relative depth of scour on the relative grain size is presented in Fig. 2. With increase in the relative grain size, the relative depth of scour reduces. In the case of a stratified bed, it is necessary to introduce the layer thickness H_d into the initial data, taking into account the grain size.

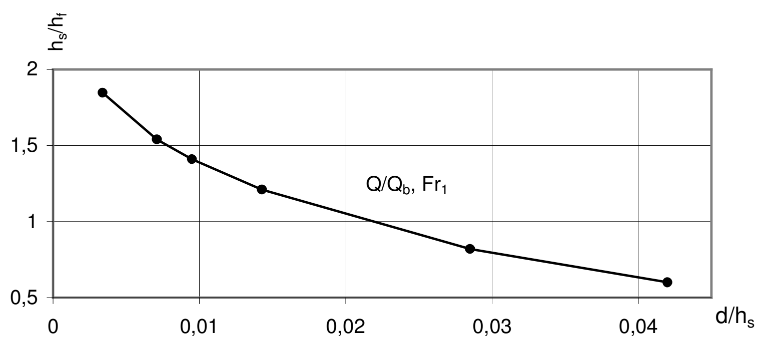

Fig. 2. Relative depth of scour versus the relative grain size.

The influence of the open-flow parameters, such as the kinetic parameter of the flow equal to v^2/gh_f and the Froude number relative to the slope $Fr/i_0 = v^2/gLi_0$, is shown in Figs. 3 and 4. The quieter the flow, the smaller the kinetic parameter of the flow and the smaller the scour depth.

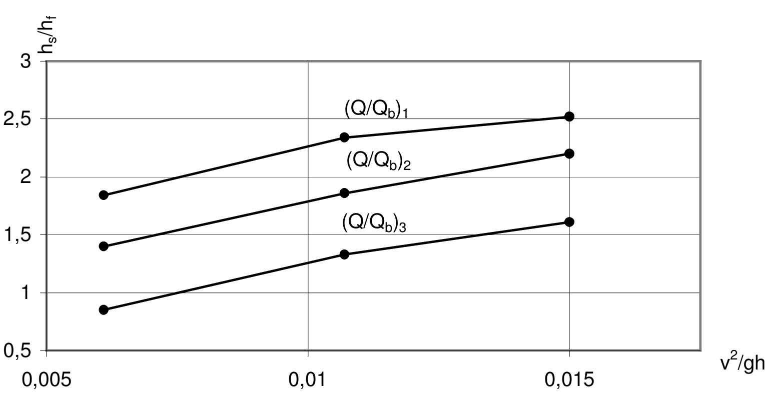

Fig. 3. Relative scour depth versus the kinetic parameter of the flow.

As seen from the figures, the smaller the ratio between the inertia and frictional forces, the smaller the value of Fr/i_o, and the greater the relative scour depth.

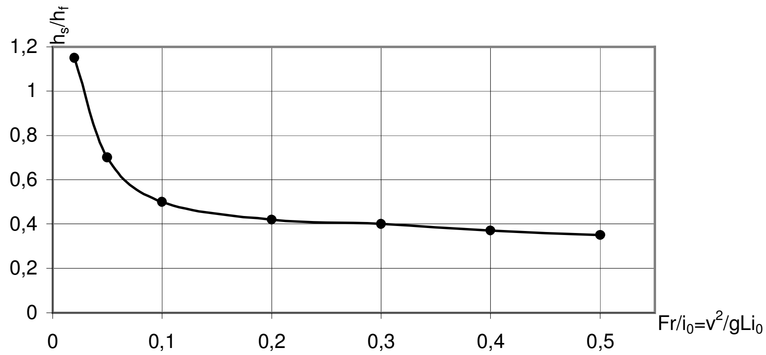

Fig. 4. Relative depth of scour versus the Froude number in relation to the slope.

With development of the scour depth, the ratio $\beta V_0/V_l$ increases. The greater the scour depth, the less the difference between the velocities V_0 and V_l (Fig. 5). At the beginning of the scour, the smaller value of $\beta V_0/V_l$ can be accompanied by the greater relative scour.

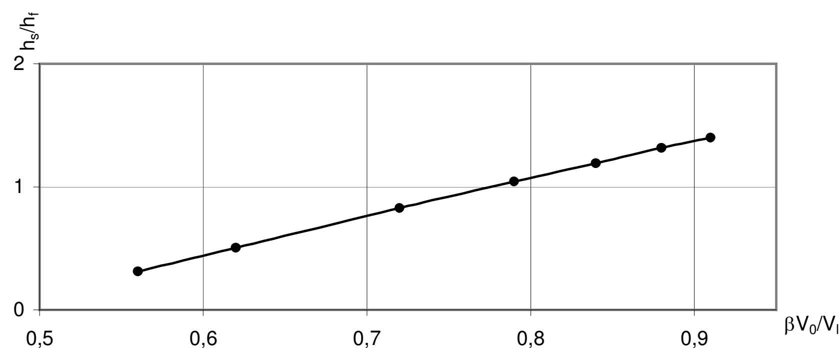

Fig. 5. Relative depth of scour versus $\beta V_0/V_1$.

The scour depth depends on the time of scour and the flood duration. Under steady flow conditions in tests and calculations, at the beginning, the depth of scour rapidly developed and then, with time, it increased more slowly, as shown in Fig. 6. The development of scour under unsteady flow is shown in Fig. 7; it has a different curve compared with that in the steady flow conditions.

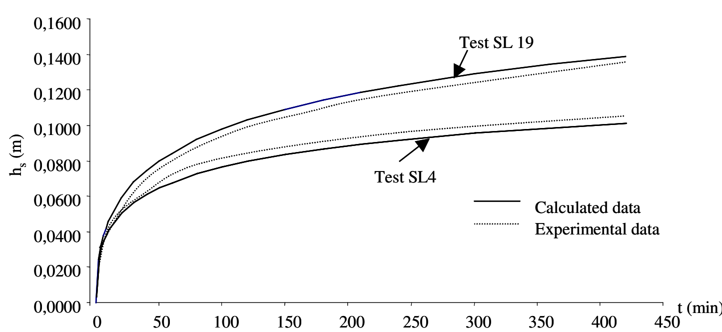

Fig. 6. Scour development with time for SL4 and SL19 tests under steady flow conditions.

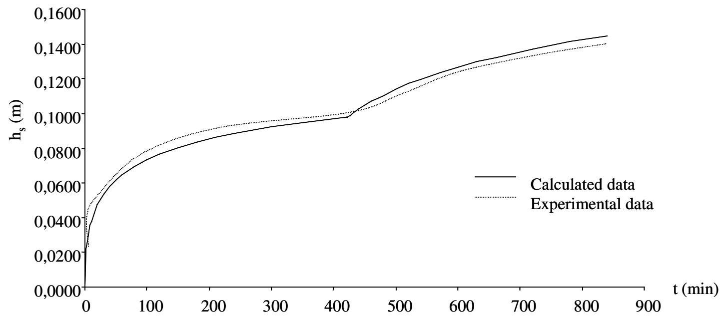

Fig. 7. Scour development with time for a TL1 test under unsteady flow conditions.

The development of scour was restricted by the duration of the flood. The scour ceases on the peak of the hydrograph or just after it. Figure 8 shows the depth of scour as a function of the flood duration. Contrary to experiments in hydraulics flumes, in the real flood, the time of scour development is restricted by duration of the flood, and therefore the depth of scour does not reach the equilibrium stage.

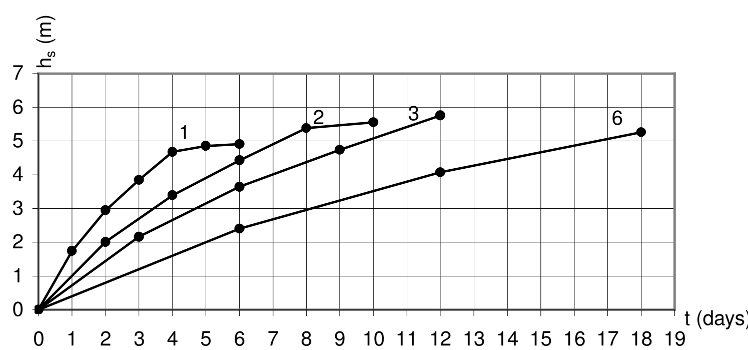

Fig. 8. Depth of scour development during the flood. Duration of time steps of the hydrograph 1, 2, 3, and 6 days.

Conclusions

A method for computing the depth of scour development with time at the abutments during the flood was elaborated. According to the theoretical analysis of this method, the relative depth of scour depended on the contraction of the flow, relative grain size of the river bed and its distribution over the depth, kinetic parameter of the flow, relative depth and velocity of the flow, Froude number in relation to the slope, unsteadiness of the flow, time of scour, and duration of the flood. According to the earlier investigations, the scour at bridge abutments also depends on the abutment shape and the flow crossing angle. The dependence of the relative depth of scour at the abutments on the above-mentioned parameters was shown graphically.

References

[1] Gjunsburgs, B., and Neilands, R. (2004). Local Velocity at Bridge Abutments on Plain Rivers. *Proceedings of Second International Conference on Fluvial Hydraulics, River Flow 2004, Naples*, *1: 443-448.*

[2] Neilands, R., and Gjunsburgs, B. (2001). Scour Development in Time near Abutments. *Scientific Proceedings of Riga Technical University, Riga, Vol. 2. No. 2, pp. 123-129.*

[3] Melville, B. W. (1997). Pier and Abutments Scour: Integrated Approach. *Journal of Hydraulic Engineering* 123 (2): 125–136.

[4] Radice, A., Franzetti, S., and Ballio, F. (2002). Local Scour at Bridge Abutments. *Proceedings of the International Conference on Fluvial Hydraulics, River Flow 2002, Louvian – La – Neuve 2*: 1059–1068.

[5] Леви И. И. (1969). Динамика русловых потоков. Гидрометеоиздат, Москва.

[6] Ротенбург, И. С., Вольнов, В. С. (1969). Примеры проектрования мостовых переходов. Высшая школа, Москва.

[7] Richardson, E. V., Simons, D. B., and Julien, P. Y. (1990). *Highways in the River Environment. FHWA–H*-190–016, Federal Highway Administration, U. S. Department of Transportation, Washington, DC.

[8] Richardson, E.V. and Davis, S. R. (1995). *Evaluating Scour at Bridges, 3d ed., Hydraulic Engineering Circular, No. 18, Publication NO. FHWA–IP*-90–017, Federal Highway Administration, U. S. Department of Transportation, Washington, DC.

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R.Neilands, B.Gjunsburgs un R.R. Neilands. Izskalojumu rašanās plūdu laikā metodes teorētiskā analīze.
Tilta pamatu vietējo izskalojumu rašanās plūdu laikā metodes teorētiskā analīze. Saskaņā ar metodes teorētisko analīzi, izskalojumu relatīvais dziļums ir atkarīgs no plūsmas sašaurinājuma, upes dibena grunts relatīvā graudu lieluma un to sadalījuma pa dziļumu, plūsmas kinemātiskajiem parametriem, relatīvā plūsmas ātruma un dziļuma, Frouda koeficienta attiecībā pret slīpumu, plūsmas svārstībām, izskalojumu un plūdu ilguma. Saskaņā ar iepriekšējiem pētījumiem, izskalojumi pie tiltu pamatiem ir arī atkarīgi no pamatu formas un plūsmas šķērsošanas leņķa. Pamatu izskalojumu relatīvā dziļuma atkarība no augstāk minētajiem parametriem ir parādīta grafiski.

R.Neilands, B.Gjunsburgs and R.R. Neilands. **Theoretical analysis of the method of scour development in time during the flood.**
Theoretical analysis of the method of local scour development in time during the flood of the abutments was presented. According to the theoretical analysis of this method, the relative depth of scour depended on the contraction of the flow, relative grain size of the river bed and its distribution over the depth, kinetic parameter of the flow, relative depth and velocity of the flow, Froude number in relation to the slope, unsteadiness of the flow, time of scour, and duration of the flood. According to the earlier investigations, the scour at bridge abutments also depends on the abutment shape and the flow-crossing angle. The dependence of the relative depth of scour at the abutments on the above-mentioned parameters was shown graphically.

Гюнсбург Б., Нейланд Р. и Нейланд Р.Р. **Теоретический анализ метода расчета размыва во времени в период паводка.**
В статье представлен теоретический анализ ранее предложенного метода расчета местного размыва у устоев мостов во времени в период паводка. Выполненый анализ позволяет установить зависимость относительной глубины размыва от меры стеснения потока, относительной крупности грунта и его распределения по глубине, параметра кинетичности потока, относительнои глубины и скорости нестесненного потока, числа Фруда отнесенного к продольному уклону потока, нестационарность потока, времени размыва и продолжительности паводка. Зависимости относительной глубины местного размыва у устоев мостов от названных выше параметров представлены графически в данной стате.