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METHOD OF COMPLETE BIFURCATION GROUPS AND ITS APPLICATION IN NONLINEAR DYNAMICS

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Keywords: nonlinear dynamics, method of complete bifurcation groups, rare attractors, chaos, archetypal dynamical systems with one- and two-degrees-of-freedom

Abstract: A new approach for the global bifurcation analysis of strongly nonlinear dynamical systems is under consideration. The main idea of the approach is a concept of complete bifurcation groups and periodic branch continuation along stable and unstable solutions, named as a method of complete bifurcation groups (MCBG). In this paper it is shown that using MCBG allows to find new nonlinear effects and unknown before periodic (rare attractors) and chaotic regimes in archetypal dynamical systems with one- and two-degrees-of-freedom: bilinear, pendulum, rotor dynamics, with several equilibrium positions, negative damping.

Introduction

The method of complete bifurcation groups is worked out in the Institute of Mechanics of Riga Technical University by the scientific group working on Nonlinear Dynamics, Chaos, Catastrophes and Control. The method consists in direct numerical modeling of original nonlinear model, that is, without its simplification. Under the method of complete bifurcation groups we understand the complex of approaches to analysis of dynamic systems, which involves the following procedures: at fixed system parameters – searching of all periodic stable and unstable regimes and bifurcation subgroups with unstable periodic infinitiums (UPI) on plane of states, constructing of regimes' basins of attraction on plane of states; at varied system parameters – constructing of bifurcation diagrams (one parameter is varied) and bifurcation maps (two parameters are varied). Special importance in the method is the continuation of parameter solution (in one-parameter task) along a solution branch of definite regime (not along parameter) [16-19]. That allows to find new unknown before stable regimes in broadly used dynamical models of strongly nonlinear oscillatory systems [1-15].

In the present paper it will be demonstrated that using of the method of complete bifurcation groups allows to implement a global bifurcation analysis of strongly nonlinear oscillating and vibro-impact systems, and to find new nonlinear effects, unknown before periodic and chaotic regimes. This will be shown on typical nonlinear systems with one and two degrees-of-freedom: bilinear, pendulum, rotor dynamics, with several equilibrium positions, negative damping. New obtained results have practical importance in nonlinear vibroengineering: vibromoving, vibromixing, vibro-polishing, vibrowelding etc. All results were obtained numerically, using software NLO and SPRING, created by this paper authors [20-22].

Models and results

The first model under consideration is a model of valve system. The valve system has the simplest nonlinear model – bilinear restoring force, linear damping and harmonical excitation. The bilinear models are widely and long time used for dynamics investigation of many mechanical engineering systems, such as vibro-impact equipment, offshore structures, suspended bridges, air valves and switches. This model can be described by following differential equation

$$m\ddot{x} + b\dot{x} + f_1(x) = h_1 \cos \omega t, \qquad (1)$$

$$f_1(x) = \begin{cases} c_1 x & if\ x \le d \\ c_2 x - (c_2 - c_1)d & if\ x > d \end{cases}$$

where: m – mass; b – coefficient of linear dissipation; c_1, c_2 and d – stiffness coefficients and break point of bilinear restoring force; h_1, ω – amplitude and frequency of excitation.

Some important results for this model are represented in Figs.1-3. New rare regular (RA) and chaotic (ChA) attractors were found (Fig.1). These attractors exist in rather small range of parameter, but as it is shown in Fig.2, have rather big basins of attraction. Else, using MCBG we can see such phenomenon like two different bifurcation groups integrate into one bifurcation group (Fig.3).

Next two models under consideration are models of pendulum systems: one is 1 DOF model with linear restoring moment and with the periodically vibrating in both directions point of suspension (Fig.4a); another is 2 DOF model

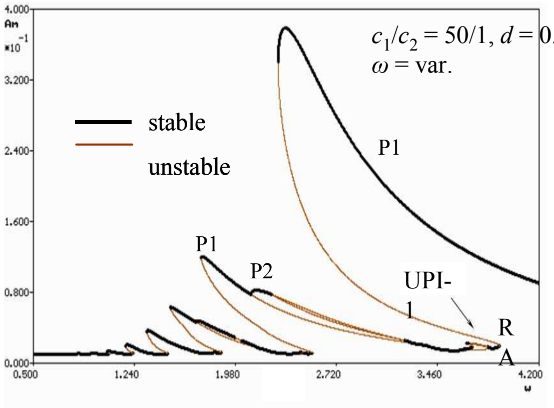

Fig. 1. Complicated bifurcation group 1T, which has rare chaotic attractor in parameter range marked UPI-1. Complete bifurcation diagram for valve system with bilinear elastic characteristic and linear dissipation at harmonic excitation [23]. Amplitude of oscillations Am of periodic regimes vs. frequency of excitation ω. Parameters: $m = 1$, $c_1 = 50$, $c_2 = 1$, $d = 0.01$, $b = 0.5$, $h_1 = 0.5$, $\varphi 0 = 0$, ω = var.

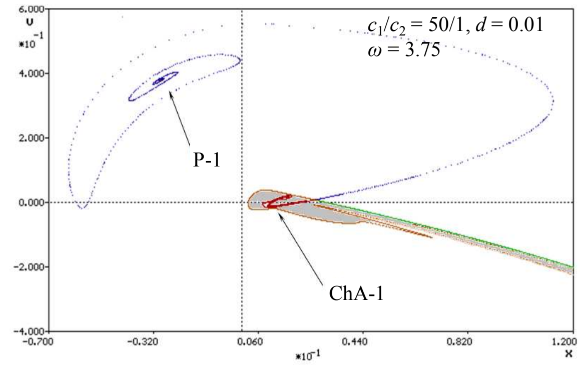

Fig. 2. Bassins of attraction of two attractors: regular resonant P1 and chaotic ChA-1. Cross-section of bifurcation diagram from Fig. 1 at $\omega = 3.75$ [23]. Valve system with bilinear elastic characteristic and linear dissipation at harmonic excitation. Parameters: $m = 1$, $c_1 = 50$, $c_2 = 1$, $d = 0.01$, $b = 0.5$, $h_1 = 0.5$, $\omega = 3.75$, $\varphi_0 = 0$

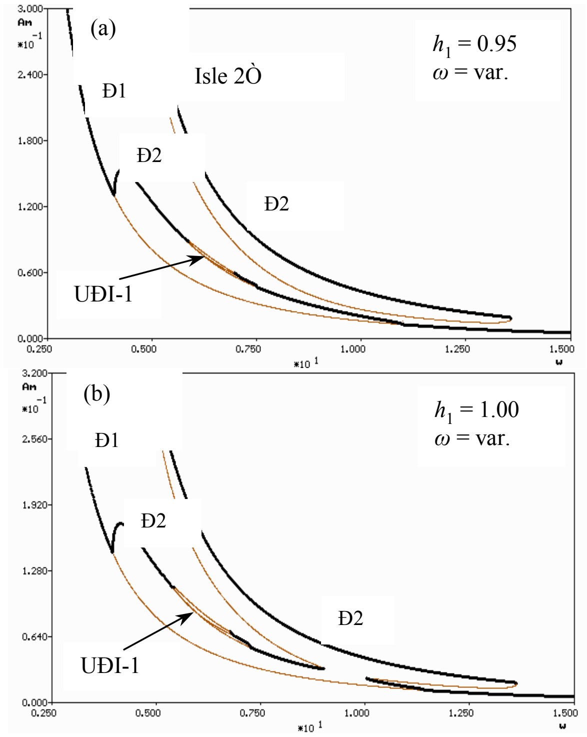

Fig. 3. Two different bifurcation groups, 1T and subharmonic isle 2T, integrate into one bifurcation group 1T. Complete bifurcation diagram for valve system with bilinear elastic characteristic and linear dissipation at harmonic excitation: amplitude of oscillations Am of periodic regimes vs. frequency of excitation ω. Parameters: $c_1 = 50$, $c_2 = 1$, $d = 0.01$, $b = 0.5$, $h_1 = 0.95$ (a) and $h_1 = 1.0$ (b), ω = var.

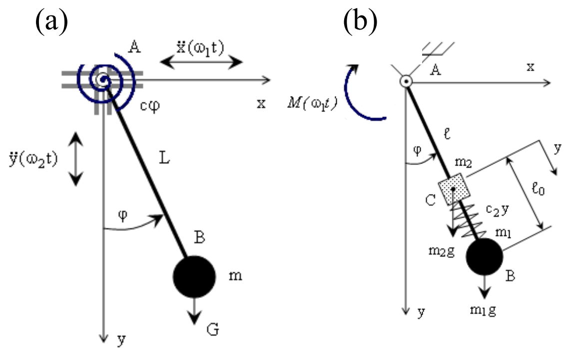

Fig. 4. Models of pendulum systems: a) with linear restoring moment and with the periodically vibrating point of suspension in both directions (see Eq.2, results in Fig.5-7); b) with external periodically excited moment (see Eq.3, results in Fig.8, 9).

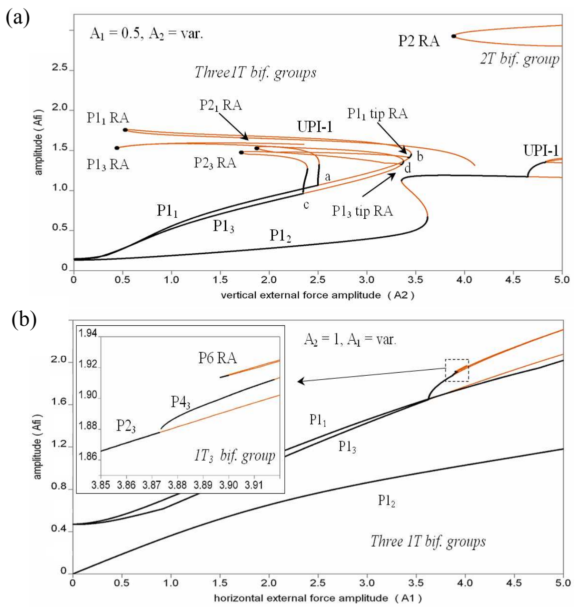

Fig. 5. Bifurcation diagrams of driven damped pendulum (see Fig.4a) with rare attractors. System has linear restoring moment and the periodically vibrating in both directions point of suspension. There are three 1T and one 2T (in Fig.5a) or one 6T (in Fig.5b) bifurcation groups with several symmetry breakings, period doublings, folds and tip type rare attractors. a) Amplitudes of oscillations vs. vertical external force amplitude A_2 (A_1 = 0.5, A_2 = var.); b) amplitudes of oscillations versus horizontal external force amplitude A_1 (A_2 = 1, A_1 = var.). Parameters: $m = 1$, $L = 1$, $b = 0.2$, $c = 1$, $\mu = 9.81$, $\omega = 1.5$, A_1 = var., A_2 = var. (see Eq.2).

with the external periodically excited moment (Fig.4b).

The first model is described by following equation

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where: m – pendulum mass; L – pendulum length; μ – gravitational constant; φ – angle of rotation; $\dot{\varphi}$ –angular velocity, $\dot{\varphi} = d\varphi / dt$; $b\dot{\varphi}$ - linear damping moment; b – damping coefficient; $c\varphi$ – linear restoring moment; c – stiffnes coefficient; $\ddot{x}(\omega_1 t) = -A_2 \omega^2 \cos \omega t$, $\ddot{y}(\omega_2 t) = -A_1 \omega^2 \sin \omega t$ - suspension point acceleration in horizontal and vertical direction due to external axcitation.

Investigation results for model from Fig.4a are represented in Figs.5-7. The model has three different bifurcation groups 1T. Two of these groups are topologically similar and have rare attractors P1$_1$ RA and P1$_3$ RA (Fig.5a). Some cross-sections of bifurcation diagrams from Fig.5 are represented in Figs.6,7. Each 1T group has its own UPI (Unstable Periodic Infinitium) with corresponding chaotic attractors.

Equations of motion for the mathematical pendulum with additional linear mass moving along pendulum length (Fig. 4b)

$$\begin{cases} \left[m_1 l^2 + m_2 (l - l_0 + y)^2\right]\ddot{\varphi} + b_1\dot{\varphi} + 2m_2 (l - l_0 + y)\dot{\varphi}\dot{y} + \\ + m_1 \mu l \sin\varphi + m_2 \mu (l - l_0 + y)\sin\varphi = M(\omega_1 t) \\ m_2 \ddot{y} + b_2 \dot{y} + c_2 y - m_2 (l - l_0 + y)\dot{\varphi}^2 + m_2 \mu (1 - \cos\varphi) = 0 \end{cases} \qquad (3)$$

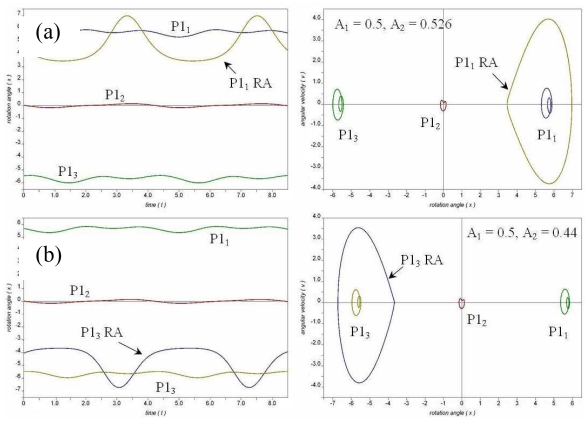

Fig. 6. Coexistence of P1 stable solutions and P1 RA rare attractors (see Fig.5a): a) time histories and phase trajectories for cross-section A_2 = 0.526; b) for cross-section A_2 = 0.44

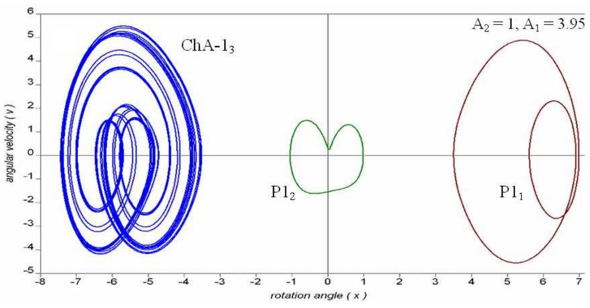

Fig. 7. Phase trajectories for coexisting periodic regimes $P1_1$, $P1_2$ and ChA-1_3 chaotic attractor for cross-section $A_1 = 3.95$ (see Fig.5b)

where φ – angle of rotation, read-out from a vertical line; $\dot{\varphi}$ – angular velocity; y – displacement of the additional mass, read-out from a quiescent state; $\dot{y}$ – velocity of the second mass; m_1 – mass of the pendulum, l – length of the pendulum; m_2 – second mass, l_0 – quiescent state of the second mass; μ – gravitational constant; $b_1\dot{\varphi}$, $b_2\dot{y}$ – linear dissipative forces (moments) of the pendulum and the second mass; b_1, b_2 – damping coefficients; c_2y – linear restoring force of the second mass; c_2 – stiffness coefficient; $M(\omega_1 t) = h_1 \cos\omega_1 t$ – external periodically excited moment; h_1 and ω_1 – amplitude and frequency of excitation.

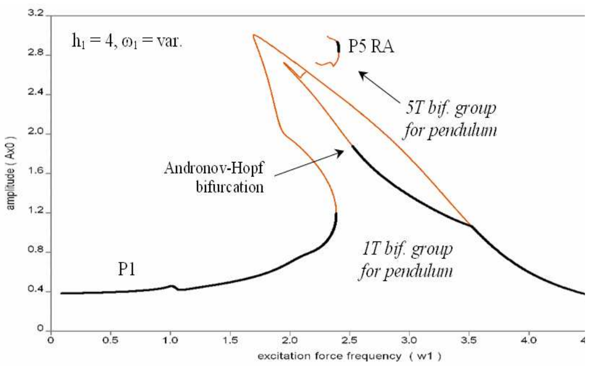

Fig. 8. Bifurcation diagram of 1T and 5T bifurcation groups with several symmetry breakings, period doublings, Andronov-Hopf bifurcations and tip type rare attractor P5 RA for pendulum system (see Fig.4b) with two degrees of freedom and the external periodically excited moment. Pendulum oscillation amplitudes vs. excitation frequency ω_1. Parameters: $m_1 = 1$, $m_2 = 0.1$, $l = 1$, $l_0 = 0.25$, $b_1 = 0.2$, $b_2 = 0.1$, $c_2 = 2$, $\mu = 10$, $h_1 = 4$, $\omega_1 =$ var. (see Eq.3).

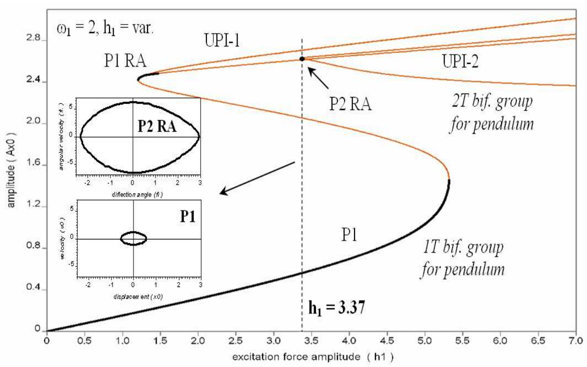

Fig. 9. Bifurcation diagram of 1T and 2T bifurcation groups with several symmetry breakings, period doublings, folds and tip type rare attractors for the pendulum system (see Fig.4b) with two degrees of freedom and the external periodically excited moment. Pendulum oscillation amplitudes vs. excitation amplitude h_1. Parameters: $m_1 = 1$, $m_2 = 0.1$, $l = 1$, $l_0 = 0.25$, $b_1 = 0.2$, $b_2 = 0.1$, $c_2 = 2$, $\mu = 10$, $\omega_1 = 2$, $h_1 =$ var. (see Eq.3).

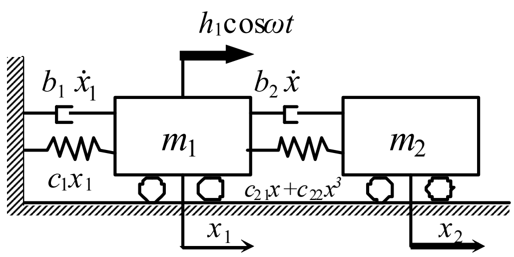

Fig. 10. Physical model of two body chained system

Results for this system are represented in Figs.8, 9. These Figs. show that method of complete bifurcation groups allows to find new unnoticed before regimes also in the system with two degrees-of-freedom. Thus, the application of the method of complete bifurcation groups for global analysis of forced oscillations is also possible for systems with several degrees-of-freedom.

The next model where unusual phenomenon is discovered is a model of chained system with two degrees-of-freedom with three equilibrium positions of second mass, linear dissipations, and harmonic excitation (see Fig.10).

Equations of motion are

$$\begin{cases} m_1\ddot{x}_1 + b_1\dot{x}_1 + c_1x_1 - b_2\dot{x} - c_{21}x - c_{22}x^3 = h_1\cos(\omega t + \varphi_0) \\ m_2\ddot{x}_2 + b_2\dot{x} + c_{21}x + c_{22}x^3 = 0 \end{cases} \quad (4)$$

where x_1, x_2 – generalized coordinates ($x = x_2 - x_1$); m_1, m_2 – mass of oscillating bodies; b_1, b_2 – linear dissipation coefficients; c_1 – stiffness coefficient of the first linear elastic constrain; c_{21}, c_{22} - stiffness coefficients of the second nonlinear elastic constrain; h_1, ω, φ_0 – amplitude, frequency and phase of excitation.

In the Fig. 11 there is presented a unusual bifurcation group of sub-harmonic isle P2. Unusual is that all branches of solutions related to particular group are unstable, that is, at least one of indicators of stability (multiplier) is always out of unit circle borders. But varying system parameters, for example, mass of oscillating bodies, or linear dissipation coefficients, or particular stiffness coefficients, or excitation frequency, leads to appearance of stable periodic regimes (rare attractors), and as a result chaotic behavior, on previously unstable branches (Fig.12).

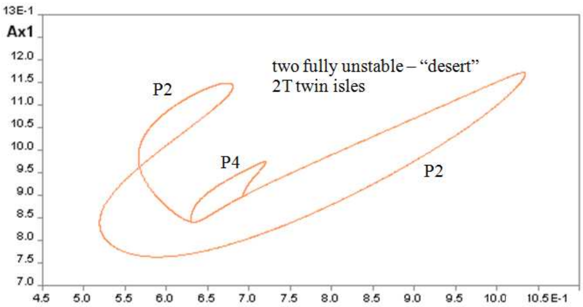

Fig. 11. New bifurcation group of completely unstable subharmonic twin isles 2T. Bifurcation diagrams for driven chain system with two degrees-of-freedom with three equilibrium positions of the second mass m_2 and linear dissipation at harmonic excitation [24]. Amplitude of oscillations of the first mass vs. h_1. Parameters: $m_1 = m_2 = 1$, $b_1 = b_2 = 0.2$, $c_1 = 1$, $c_{21} = -1$, $c_{22} = 1$, $\omega = 1$, $\varphi_0 = 0$, h_1 = var.

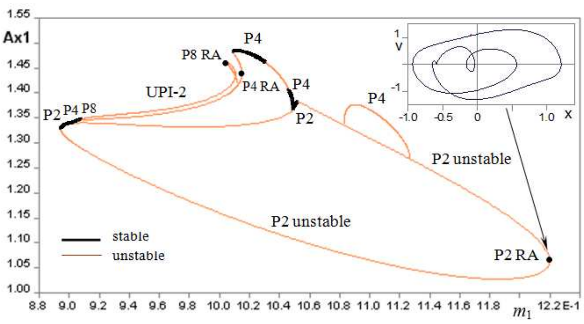

Fig. 12. Appearance of stable regimes – rare attractors – on the fully unstable subharmonic isles 2T (Fig. 11) at varying the first mass m_1. Bifurcation diagrams for driven chained system with two degrees-of-freedom with three equilibrium positions of the second mass m_2 and linear dissipation at harmonic excitation [24]. Amplitude of oscillations of the first mass vs. m_1. Parameters: $m_2 = 1$, $b_1 = b_2 = 0.2$, $c_1 = 1$, $c_{21} = -1$, $c_{22} = 1$, $h_1 = 1$, $\omega = 1$, $\varphi_0 = 0$, $k = 7$, m_1 = var.

Topologically subharmonic isles with chaotic behavior have form of closed shell for on two parameters plane 1DOF systems (Fig.13): narrow band (rare attractor) of stable period nT regime on shell border (near the fold bifurcation) and Unstable Periodic Infinitium inside the shell. It is typical, presence of narrow coat of chaotic attractors inside the shell.

The next case is piece-wise linear system with negative damping and harmonic excitation

$$\ddot{x} + F_1(x) + F_2(x,\dot{x}) = h_1\cos(\omega t + \varphi_0) \qquad (5)$$

where x – generalized coordinate; $F_1(x)$, $F_2(x, \dot{x})$ – nonlinear restoring and dissipative forces (see Fig.14); h_1, ω, φ_0 – amplitude, frequency and phase of excitation.

In this system we observe rare attractors and bifurcations usual for system with several DOF: unstable fold, symmetri lost, Andronov-Hopf.

Rare attractors exist also in the nonlinear rotor dynamics [19]. In particular, in Fig.15 it is shown that RA are born on the unstable branch of the amplitude-frequency response when introduce small asymmetry into the parameters of a nonlinear suspension

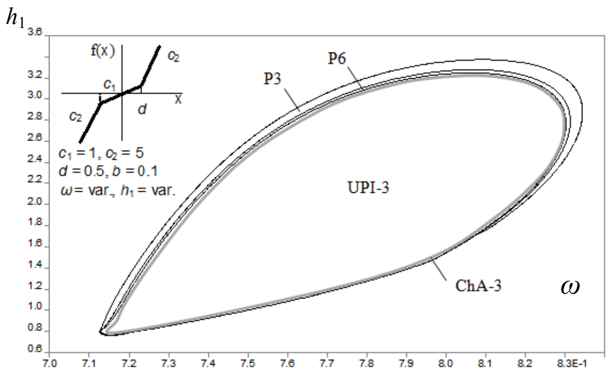

Fig. 13. Bifurcation map (ω-h_1) of subharmonic isle 3T with tip type rare attractors P3 and chaotic coat ChA-3. Trilinear symmetrical system with linear damping and harmonic excitation [25]. Parameters: $m = 1$, $c_1 = 1$, $c_2 = 5$, $d = 0.5$, $b = 0.1$, $\varphi_0 = 0$, ω = var., h_1= var

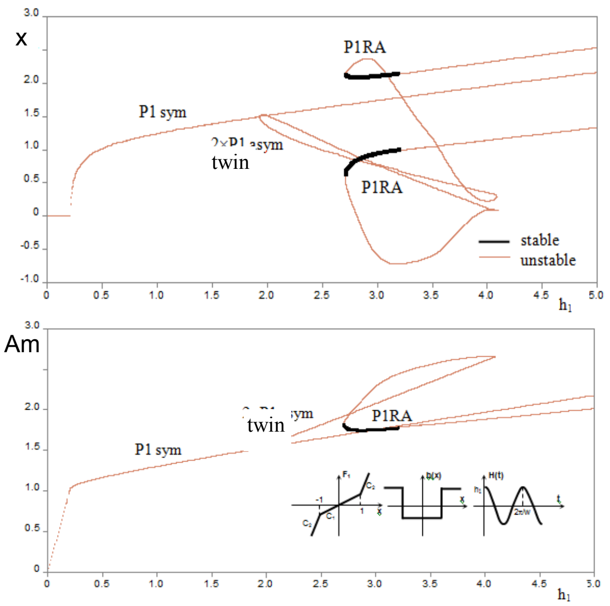

Fig. 14. Rare attractor, unstable symmetry lost and unstable fold bifurcation. Bifurcation diagram for piece-wise linear system with negative damping and harmonic excitation. a) fixed point coordinate vs. excitation amplitude h_1; b) amplitudes of oscillations vs. h_1. Parameters: $m = 1$, $c_1 = 1$, $c_2 = 9$, $d = 1$, $b_1 = 0.2$, $b_2 = -0.2$, $\varphi_0 = 0$, $\omega = 1$, $h_1 =$ var.

$$\begin{cases} \ddot{x} + (1+k)\left[c_1 x + c_3 x\left(x^2 + y^2\right)\right] + b\dot{x} = e\omega^2 \cos(\omega t) \\ \ddot{y} + c_1 y + c_3 y\left(x^2 + y^2\right) + b\dot{y} = e\omega^2 \sin(\omega t) \end{cases} \quad (6)$$

where x, y – generalized coordinates; b – linear dissipation coefficient; c_1, c_3 – stiffness coefficients of suspension; k – coefficient of asymmetry; e – eccentricity, ω – frequency of excitation.

Conclusion

It is shown that using of the method of complete bifurcation groups allows to implement a global bifurcation analysis of strongly nonlinear oscillating and vibro-impact systems, and to find new nonlinear effects, unknown before periodic and chaotic regimes. This was shown on typical nonlinear systems with one and two degrees-of-freedom: bilinear, pendulum, rotor dynamics, with several equilibrium positions, negative damping. Obtained results have practical importance in nonlinear vibroengineering: vibromoving, vibromixing, vibropolishing, vibrowelding etc.

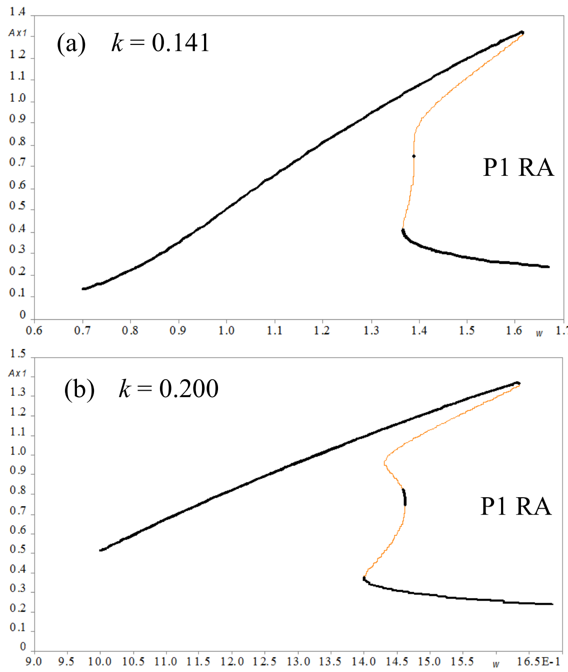

Fig. 15. The birth of the unusual period-1 rare attractor (P1 RA) on the 'unstable' branch of the driven damped slightly asymmetric rotor system with the cubic elastic nonlinearity. Parameters: $c_1 = 1$, $c_3 = 1$, $b = 0.2$, $\omega =$ var.; (a) $k = 0.141$, (b) $k = 0.200$ [19].

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M. Zakrževskis, R. Smirnova, I. Ščukins, V. Jevstigņejevs, V. Kugelevičs, V. Frolovs, A. Klokovs, E. Šilvāns. Pilno bifurkāciju grupu metode un tas pielietojums nelineārā dinamikā
Darbā ir aplūkota jauna pieeja būtiski nelineāro dinamisko sistēmu globālai bifurkāciju analīzei. Pieejas pamatideja balstās uz pilno bifurkāciju grupu koncepcijas un periodisko režīmu turpinājuma pa parametru gar stabiliem un nestabiliem risinājumiem, un saucas par pilno bifurkācijas grupu metodi (MPBG). Dotajā darbā parādīts, ka MPBG pielietojums ļauj atrast jaunus nelineāros efektus un nezināmus agrāk periodiskus (retie atraktori) un haotiskus režīmus tipveida dinamiskās sistēmās ar vienu un ar divām brīvības pakāpēm: bilineārās, svārstu, rotoru, ar dažiem līdzsvara stāvokļiem, sistēmās ar negatīvo berzi.

М. Закржевский, Р. Смирнова, И. Щукин, В. Евстигнеев, В. Кугелевич, В. Фролов, А. Клоков, Э. Шилван. Метод полных бифуркационных групп и его применение в нелинейной динамике
В статье рассматривается новый подход к глобальному бифуркационному анализу сильно нелинейных динамических систем. Основная идея подхода основывается на концепции полных бифуркационных групп и продолжении по параметру периодических режимов вдоль устойчивых и неустойчивых решений, названных методом полных бифуркационных групп (МПБГ). В данной статье показано, что применение МПБГ позволяет находить новые нелинейные эффекты и неизвестные ранее периодические (редкие аттракторы) и хаотические режимы в типовых динамических системах с одной и двумя степенями свободы: билинейной, маятниковой, роторной, с несколькими положениями равновесия, в системах с отрицательным трением.