PREDICTION OF NETWORK TIME DELAY WITH POISSON DISTRIBUTION

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Keywords: network time delay, time series prediction, Poisson distribution, Matlab simulation

Abstract – Prediction of a network time delay is presented and discussed. Moving average, exponential moving average and linear least square prediction methods are applied. The accuracy of prediction is estimated with mean absolute error. It is assumed that network time delay have no predefined distribution and can come from Poisson probability distribution family with given range of lambda. Two types of autocorrelation are considered: zero autocorrelation and exponential autocorrelation. The simulation of prediction is realized in Matlab. The results of simulation are visualized and presented.

Introduction

In networked control system (NCS) actuators, sensors and controller are interconnected via a communication network. This interconnection makes it possible to reduce wiring costs and improves scalability of a system [1]. Main drawbacks of a network, which affect a control system dynamic, are transmission time delay and data lost. The properties of network time delay are depended from type of network. In random access networks time delay is variable. The reasons of variability are queues, collisions and change of transmission path. Focus of the paper is on prediction of a random time delay.

In literature different approaches of time delay prediction are used. Average (A), moving average (MA) and autoregressive integrated moving average (ARIMA) models are applied in [2], autoregressive (AR) model in [3] and neural networks (NN) in [4]. Two main approaches for experiment setup are applied: Internet connection between two hops with *rtt* time prediction and time delay generating from predefined distribution in simulating platforms, such as Matlab. In first case, the training sample of time delay was obtained by *ping* command. In second case, time delay training sample normally is generated from Normal, Poisson, Gamma and Weibull distributions with predefined parameters. Then for training sample of both approaches choose parameters of predictors.

There are two main drawbacks of above mentioned method. The parameters of predictors are dependent not only from time delay distribution, but also from autocorrelation, because the next time delay prediction is based on previous values. And the form of time delay distribution can variate, if object is mobile or if new objects are integrated in to the network.

In this paper the simulation of prediction of the random network time delay is realized. The samples from Poisson distribution family with zero and exponential autocorrelation functions are used. For the prediction of time delay are applied moving average (MA), exponentially moving average (EMA) and least linear squares (LLS) predictors.

Task of the paper is an estimation of accuracy of time series predictors at different time delay distribution and autocorrelation: $error = f(n, l, R_k)$.

System description

The simple system with a sender from one side and a receiver from second side of the network is investigated. The sender periodically sends packets to receiver via communication network. The receiver receive packet $i = (1,2,...,N)$ with time delay t_i (Fig. 1.).

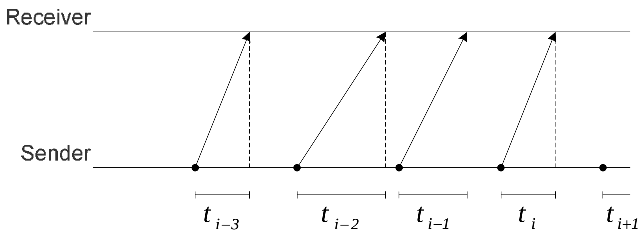

Fig. 1. Communication diagram

Time delay model

Assume that the time delay is a random value with Poisson probability distribution:

$$f(t; l) = \frac{l^t e^{-l}}{t!}, \qquad (1)$$

where

t – time delay;

l – parameter.

Parameter l is a natural number from the range $l \in (1,2,...,50)$. Poisson probability distribution family with l increment at one axis is represented in Fig. 2.

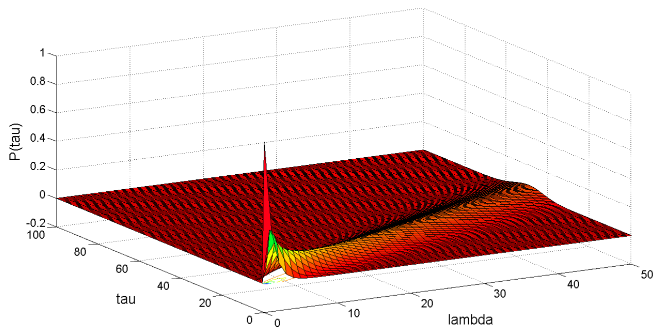

Fig. 2. Poisson probability mass function

Poisson probability is chosen because: it is one parameter distribution (only one coordinate axis is used) and widespread in queuing theory.
Also we have predefined autocorrelation. Autocorrelation coefficient is close to zero if time delay is random and is not equal to zero if time delay have correlation with itself. For example, in Ethernet networks generally autocorrelation of time delay have exponential form.
We assume that autocorrelation coefficient is zero in a. case and exponentially decrease in the b. case:

$$r_k^a = 0, \tag{2a}$$

$$r_k^b = 0.25^k. \tag{2b}$$

where
k – back shift operator.

Predictors

For time delay prediction moving average (MA), exponential moving average (EMA) and least linear square (LLS) methods are used [5].
Method of MA is based on calculating of the average value n latest measurements. Equation of next value prediction can be written as:

$$\hat{t}_{i+1} = \frac{\sum_{m=i-n+1}^{i} t_m}{n}. \tag{3}$$

In method of exponential moving average for measurements applies weighting factors which decrease exponentially. And as in previous method the prediction is based on n latest measurements of time delay. Equation can be written as:

$$\hat{t}_{i+1} = a t_i + (1-a)\hat{t}_i, \tag{4a}$$

$$a = \frac{2}{n+1}, \tag{4b}$$

where
a – parameter.

The third method is least linear squares. Equations for xy coordinate system for n previous y values can be written as:

$$\hat{y}_{i+1} = b_0 + b_1 x_{i+1}, \tag{5a}$$

$$b_0 = \bar{y} + b_1 \bar{x}, \tag{5b}$$

$$b_1 = \frac{\sum_{m=i-n+1}^{i} (x_m - \bar{x})(y_m - \bar{y})}{\sum_{m=i-n+1}^{i} (x_m - \bar{x})^2}, \tag{5c}$$

where
b_0, b_1 – parameters.

Value x is a packet number i and y is a time delay t.
By replacing y, x and $\bar{x}$ in equitation's (5a), (5b), (5c) with

$$y = t, \tag{6a}$$

$$x = i, \tag{6b}$$

$$\bar{x} = i - \frac{n-1}{2}, \tag{6c}$$

can get

$$\hat{t}_{i+1} = b_0 + (i+1)b_1, \tag{7a}$$

$$b_0 = \bar{t} + (i - \frac{n-1}{2})b_1, \tag{7b}$$

$$b_1 = \frac{\sum_{m=i-n+1}^{i} (i_m - (i - \frac{n-1}{2}))(t_m - \bar{t})}{\sum_{m=i-n+1}^{i} (i_m - (i - \frac{n-1}{2}))^2}. \tag{7c}$$

Prediction accuracy estimation

For the predictors accuracy estimation a mean absolute error (MAE) is calculated. Error calculation starts at step $i = n + 1$. General expression for the mean absolute error is:

$$MAE_i = \begin{cases} 0, & \text{if } i < n+1, \\ \dfrac{\sum_{m=n+1}^{i} |t_m - \hat{t}_m|}{i - n}, & \text{if } i >= n+1. \end{cases} \quad (8)$$

Simulation

The structure of simulation is shown at Fig. 3.

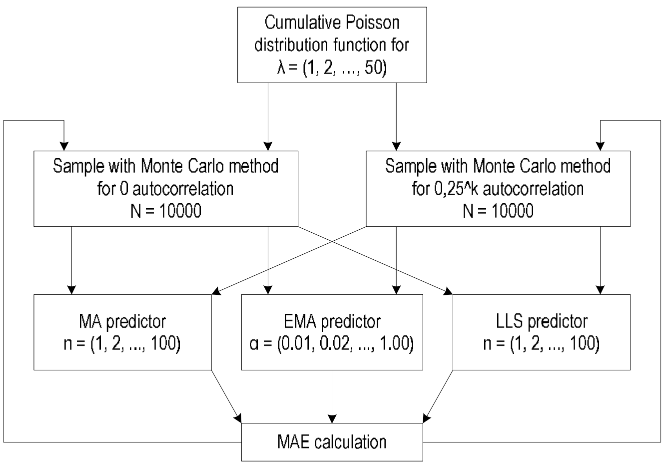

Fig. 3. Simulation structure

At start of simulation the values of cumulative probability distribution function for $l = 1$ are calculated. At the next step the samples with a. and b. case of autocorrelations are obtained by Monte Carlo method. Each sample size is $N = 10000$. The typical autocorrelation curves of samples are represented at Fig. 4. Then starting from $i = n$ value $\hat{t}_{i+1}$ is predicted with MA, EMA and LLS predictors. Prediction results are estimates with MAE. Simulation sequence for each l is repeated $Ne = 100$ times. After Ne is obtained all simulation steps are repeated for the next value.

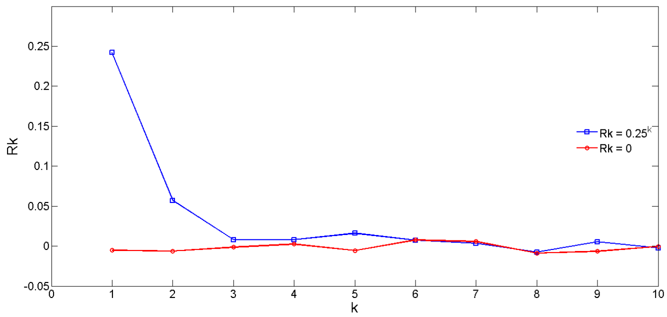

Fig. 4. Autocorrelation curves

Results

MAE contours of MA, EMA and LLS predictor are presented at Fig. 5. Left plot of a. case and right plot of b. case of autocorrelation. In b. case are reduced accuracy of MA predictor (exception for $n = (1,2)$), reduced accuracy of LLS and increased accuracy of EMA predictor. Dispersions of contours of all predictors are increased.

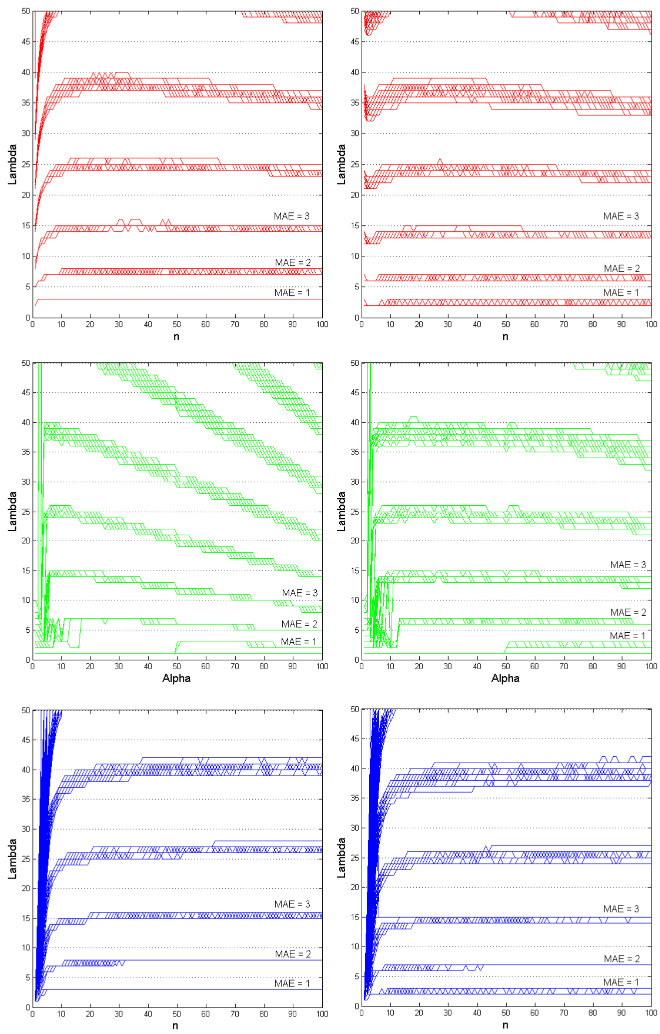

Fig. 5. Simulation results

Mean error curves of MA, EMA and LLS predictors for all l is shown at Fig. 6. The accuracy of EMA predictor in the range $4 < n < 20$ is reduced. Predictor obviously improve prediction accuracy with n increasing in the range $20 < n < 100$. Probably this tendency will continues for $n > 100$. Behavior of MA and LLS predictors accuracy is very similar: accuracy decrease in the range $4 < n < 12$ and $8 < n < 18$ respectively.

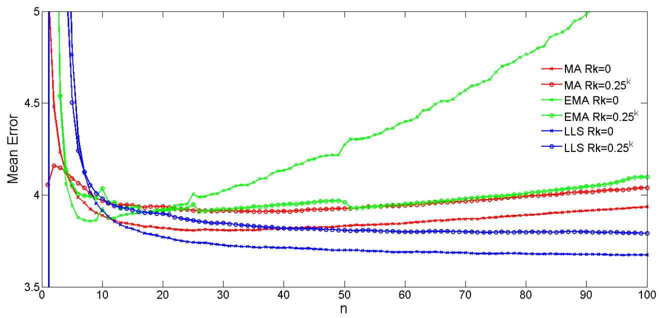

Fig. 6. Mean error curves

Conclusion

Prediction accuracy of network time delay with similar probability distribution can be different due to autocorrelation. It is necessary to consider autocorrelation at predictor and its parameter choosing. Accuracy of MA class predictors increasing then weighting coefficients become close to autocorrelation coefficients. Accuracy of LLS predictor at weak autocorrelation is better.

Future research in this direction is to found a bound at which MA class predictors more accurate than LLS predictor. To define an correlation between distribution moments (expectation, variance, skewness, kurtosis) and prediction accuracy.

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Renads Safins (B'03, M'05) received the B.S. degree in engineering science in computer control and computer science from the Riga Technical University in 2003 and M.S. degree in engineering science in computer control and computer networks.

He is currently working toward the Ph.D. degree in the department of technology of computer networks and systems, the Riga Technical University. Research topic is networked control over IP networks.

Renads Safins. Tīkla aizkaves laika ar sadalījumu pēc Puasona likuma prognozēšana

Darbs veltīts gadījuma mainīga prognozēšanas jautājumam. Izmantotas laika rindu prognozēšanas metodes – slīdošais vidējais, eksponenciāls slīdošais vidējais un mazāko kvadrātu metode. Prognozēšanas precizitāte tiek novērtēta ar vidējo absolūto kļūdu. Ir pieņemts, ka gadījuma mainīgam nav definēta varbūtības sadalījuma un tas var nākt no Puasona sadalījuma ģimenes ar dotu lambada parametra diapazonu. Aplūkoti divi autokorelāciju gadījumi: nulles un eksponenciāla autokorelacija. Prognozēšanas simulācija realizēta Matlab programmā. Rezultāti vizualizēti un piedāvāti darbā.

Ренад Сафин. Прогнозирование времени сетевой задержки распределённой по закону Пуассона

Работа посвещена вопросу прогнозирования случайной величины. Использованы методы прогнозирования временных рядов – скользящего среднего, экспоненциального скользящего среднего и метода наименьших квадратов. Точность прогноза оценивается с помощью абсолютной средней ошибки. Принимается что случайная величина не имеет определённого распределения и может относиться к семейству распределения вероятностей Пуассона с заданным диапазоном лямбда. Рассмотрены два случая автокореляции: нулевая и экспоненциальная. Симуляция прогнозирования реализована в программе Matlab. Результаты симуляции визуализированы и представлены в работе.