Scientific Journal of Riga Technical University
Material Science and Applied Chemistry

2010

Volume 21

Method for Identification the Elastic Properties of Polymer Materiāls by Using Thin-Walled Cylindrical Specimens (TWCS Method)

Sergejs Gluhihs, *Riga Technical university*, Andrejs Kovalovs, *Riga Technical university*,
Aleksandrs Tishkunovs, *Riga Technical university*,Dace Cerpakovska, *Riga Technical university*,
Martins Kalnins, *Riga Technical university*

Abstract: **The method of determination of the elastic modulus is based on a solution to the problem of compression of a thin-walled circular cylindrical tube by two parallel planes. The contact problem is solved by the finite-element method (ANSYS). The deformation of a thin polymer shell is characterized by great displacements and relatively low elastic deformations in a large range of movement of parallel planes. This method was employed to calculate the elastic modulus for a series of experimental specimens consisting of five polymer compositions, whose elastic modulus have been found earlier via standard tension tests. The good correspondence between the results confirms the efficiency of the given method.**

Keywords: **ANSYS, POLYTEC, PET, paper-like cellulose fiber composite, vibration**

I INTRODUCTION

Standard methods for determining the elastic modulus of polymer materials are based on tension, compression and bending tests of specially prepared specimens. Contrary to steel, the linear elastic area of deformation of polymer materials in standard experiments is rather small. This is the reason for a noticeable measurement error in the results obtained by such methods.

The method suggested in this study is grounded on the solution of the problem of compression of a thin-walled circular cylindrical shell by two parallel planes with regard for the geometrical and physical nonlinearity. The account of nonlinear effects in determination of elastic modulus makes it possible to use a considerably greater range of the loading curve in the elastic region of deformation compared with that in standard methods of testing specimens for tension, compression, and bending. In this case, one can obtain more input data, thus increasing the accuracy of determining the elastic modulus. Nondestructive methods of determining the elastic modulus makes it possible to use these specimens for further experiments, depending on the time and temperature factors.

A similar problem was considered earlier by E. Lavendel and D. Dirba [1] in a geometrically nonlinear statement for a linear material obeying Hooke's law. The solution was obtained by using the variational method of minimization of potential energy on the assumption of the predominant role of bending energy over the energy of compression and shear. Two cases of contact interaction were considered, namely a contact in the central zone (elliptic form) and a contact in peripheral zones (dumbbell form). The deformed form of the ring was set by three and two geometrical parameters for the first and second cases, respectively. The loading diagrams were constructed for both possible forms of deformation. The transition boundary between the first and the second forms of deformation was determined by comparing the values of potential energy.

II PURPOSE AND STATEMENT OF THE PROBLEM

The purpose of the present investigation is to elaborate a method for determining the elastic modulus of a polymer material based on the solution of compression problem of circular shells by two planes by using the finite-element method in the finite element program ANSYS, taking into account the geometrical and physical nonlinearity.

Let us consider a thin homogeneous isotropic cylindrical shell of radius *R*, width *B*, and length *L* under the action of a force *P* that is transmitted through the undeformed upper plane as shown in Figure 1. The lower plane is immobile.

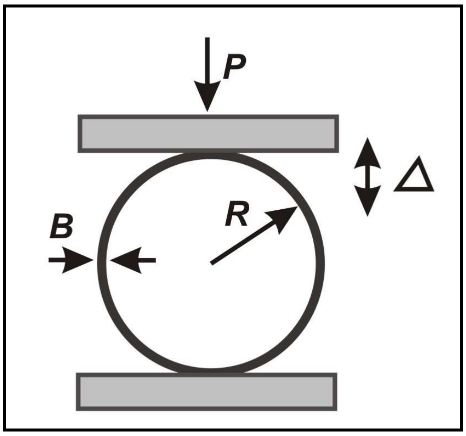

Fig. 1. Contact compression of a circular shell

The dependence of the force *P* on the displacement Δ (the relations between the force *P* and displacement Δ) is now called the loading diagram. According to [1], we introduce the dimensionless parameters of displacement α and load β

$$\alpha = \Delta/(2R)\,,\ \beta = PR^2/EJ\,, \qquad (1)$$

where Δ is the displacement of the upper plane, *E* is the elastic modulus of the material, and *J* is the moment of inertia of a rectangular cross section.

III ANALYSIS

A Numerical results

The finite-element solution is considered on the example of compression of a circular cylindrical shell with the following geometrical parameters: *R* = 62.5 mm, *B* = 1 mm, and *L* = 42 mm. The basic properties of the material are the Young's modulus *E* = 1200 MPa and Poisson ratio ν = 0.35.

The finite-element model constructed in the ANSYS program has the form of a quarter of a circular ring owing to symmetry. The boundary conditions correspond to the symmetry conditions. Since we consider thin-walled shells which can be subjected to great displacements and rotations, for different forms of elastic potentials, as a finite element we took a SHELL181 shell element, which corresponds to the requirements adopted. In the contact zone, a CONTA174 contact element is used. The elastic properties of the material were modeled by Hooke's relations and the Treloar [2] and Mooney–Rivlin [3] potentials. The problem is solved with account of friction in the contact zone. The solution of the nonlinear contact problem is performed by a stepwise procedure with respect to displacements and with an iteration refinement on each step. The loading diagram calculated by the finite-element method is shown in Figure 2.

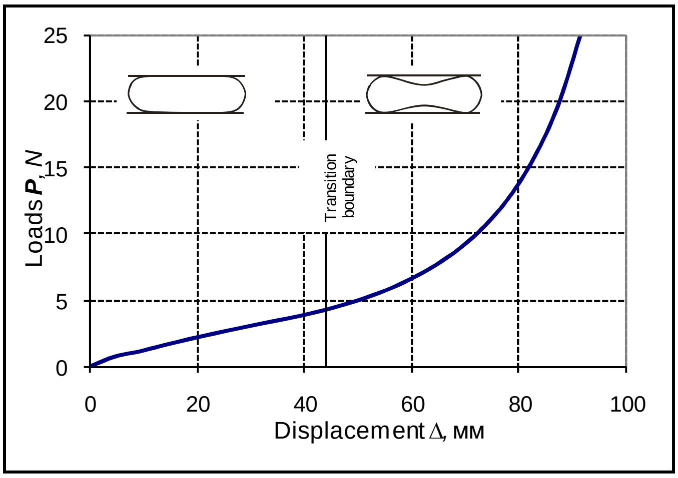

Fig. 2. Loading diagram

The transition boundary between the first (elliptic form with the central contact zone) and the second (dumbbell form with two peripheral contact zones) deformation forms was examined. According to the finite-element calculation, the transition from the first deformation form to the second one occurs after reaching 35% of the relative displacement of a compressive plane (Fig. 2). In [1], a 28.5% boundary is mentioned. The difference is explained by the more rough specification of deformation in that study compared with the finite-element method. Figure 3 shows the second deformation form with a peripheral contact zone. The influence of friction for a polymer–metal pair is quite insignificant, which is explained by the small relative contact zone between the shell and plate.

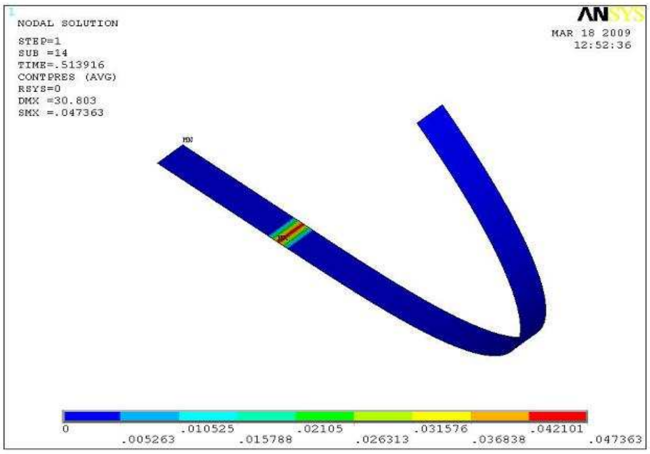

Fig. 3. Peripheral contact zone for an 8-form[dumbbell-form] deformation

B Experimental results

The solution obtained by finite element method was confirmed by testing the specimen in compression. The scheme of a test desk is shown in Figure 4.

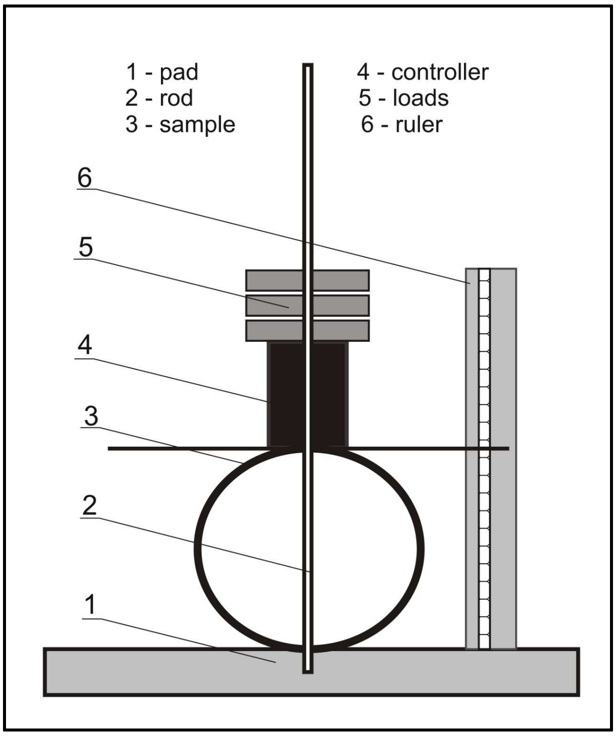

Fig. 4. Test desk for compression of a thin circular shell.

The basic advantage of this procedure is the simplicity of tests carried out by compressing a circular cylindrical shell by two parallel planes. This can be done on a simple test desk with guiding and compressing planes, a rule, and weights. The admissible (not exceeding 5%) error in determining the elastic modulus is ensured by measuring seven to eight sampling points on the force-displacement diagram in the range 0.2-0.8 of relative displacements. The accuracy of the method is achieved by using the nonlinear range of the elastic loading diagram of bending of a circular shell, which contains more information on the specimen behaviour than the linear section of tension diagrams in classical tension, compression, and bending tests.

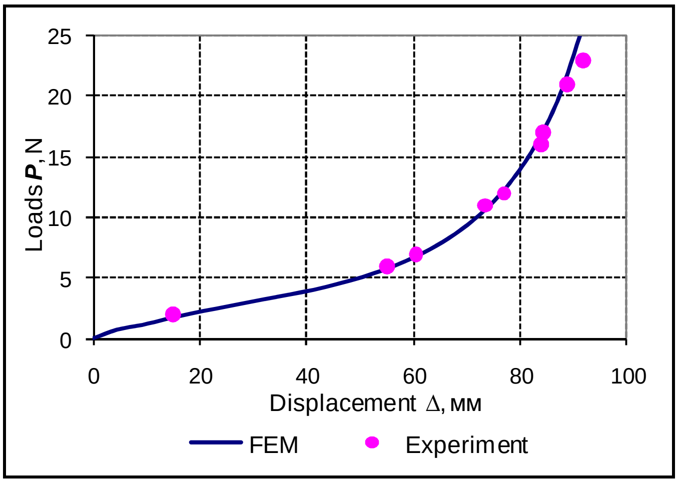

Fig. 5. A comparison between the results

A comparison between the results obtained by the finite-element calculation and experiment is presented in Figure 5. The difference between the calculated and experimental data in the range of relative displacements 0-75% is within the limits of 5%.

C Definition of the elastic modulus

According to previous research, we will consider a number of 30 thin circular shells with the following parameters: R = 20 mm, L = 50 mm, and B = 0.05-0.25 mm (step = 0.05 mm), E = 500, 1000-5000 MPa (step = 1000 MPa) and Poisson ratio ν= 0.35.

The results of calculating the compressive force for these shells for the level of relative compression 80% are presented in Table 1.

The forces obtained are converted into a relative loading β according to formula (1), as shown in Table 2.

As seen from the table, the results obtained for the dimensionless force are practically similar. The same results were also obtained for other relative displacements α. Thus, for the range of relative shell thicknesses B/R = 0.0025-0.0125, we can construct a unified loading diagram in the coordinates β–α (Fig. 5).

The universal loading diagram in dimensionless coordinates in the given range of thicknesses and elastic moduli is approximated by a 6-power polynomial:

$$\beta = 3650.8\cdot\alpha^6 - 7254.8\cdot\alpha^5 + \\ +5496.6\cdot\alpha^4 - 1879.5\cdot\alpha^3 + 272.7\cdot\alpha^2 - 0.7882\cdot\alpha \qquad (2)$$

Equation (2) allows us to solve the inverse problem of determination of the elastic modulus according to experimental points on the loading diagram. Using the test desk (Fig. 2), seven to eight points on the loading diagram were measured. The values were transformed into dimensionless coordinate's β–α. By comparing them with the unified loading curve in the form of Equation 2, we found the elastic modulus for each point. The final modulus gives averaging over all the points of measuring. The deviations of the elastic modulus at each point were calculated and the average error was evaluated. The error not exceeding 5% points to the reliability of the sought-for elastic modulus. The given algorithm allows us to automate the calculations and obtain the required elastic modulus with indication of an error.

TABLE 1.
COMPRESSIVE FORCE AT A RELATIVE COMPRESSION OF 80%

E (MPa)	P(N), (α=0.8)				
	B=0.25	B=0.50	B=0.15	B=0.10	B=0.05
500	3.5	1.8	0.75	0.22	0.028
1000	7.0	3.6	1.51	0.45	0.056
2000	14.0	7.2	3.02	0.89	0.112
3000	21.0	10.7	4.52	1.34	0.168
4000	27.9	14.3	6.03	1.79	0.223
5000	34.9	17.9	7.54	2.23	0.279

TABLE 2.
DIMENSIONLESS COMPRESSIVE FORCE AT A RELATIVE COMPRESSION OF 80%

E (MPa)	β, (α=0.8)				
	B=0.25	B=0.50	B=0.15	B=0.10	B=0.05
500	42.9	43.0	42.7	42.2	42.9
1000	42.9	43.0	43.0	43.2	42.9
2000	42.9	42.9	43.0	42.7	42.9
3000	42.9	42.9	42.9	42.9	42.9
4000	42.9	42.9	42.9	43.0	42.9
5000	42.9	42.9	42.9	42.8	42.9

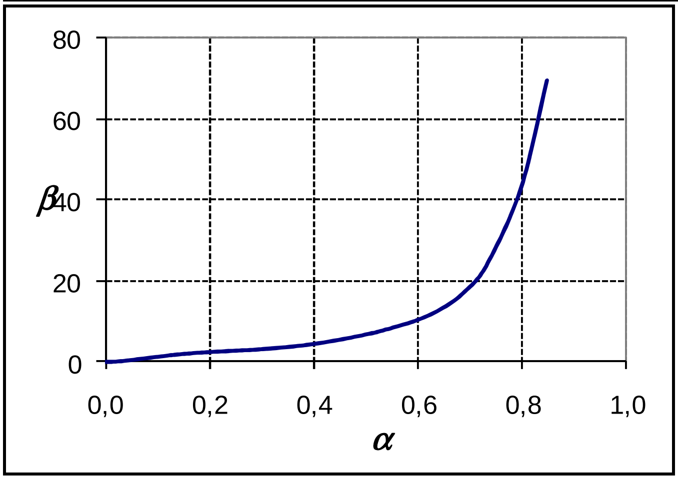

Fig. 5. A unified loading diagram of a thin shell

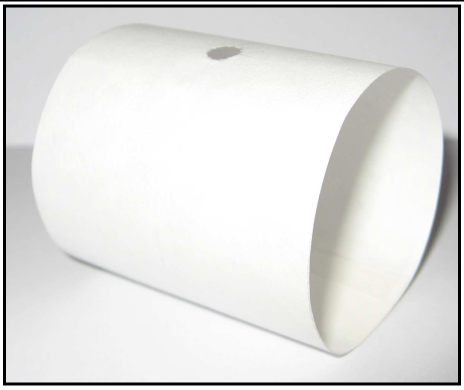

Fig. 6. Paper-like cellulose fiber composite

IV RESULTS AND DISCUSSION

Two numerical tests – tensile and modal test have been used to compare accuracy of TWCS method.

A Tensile Test

The method of determination of the elastic modulus of thin films discussed above was successfully approved on a series of various kinds of thin (b=0.06 – 0.17 mm) paper-like cellulose fiber composites with different density (Fig. 6).

About 140 mm long and 50 mm wide strip of the material was twisted around 40 mm diameter stencil. Ends of the strip were joined by scotch tape to make up the tube. Two 5 mm holes symmetrically were knocked out in the tube. Prepared sample was put on 5 mm steel rod (normally fixed in stable pad) and then progressively loaded by discrete loads (Fig. 2). Displacement was measured by use of simple ruler and magnifier with the accuracy ± 0.25 mm.

Five - six measurements were made in the relative displacements range 0.2 - 0.8. Obtained values of elastic modulus show acceptable correspondence with respective values got from tensile tests (Tab. 3).

TABLE 3.
TENSILE TEST RESULTS

Brand	Density, g/cm^2	Thickness, mkm	Tensile strength, MPa	Elongation at break, %	Elastic modulus, MPa	
					Tensile	TWCS
KP 90	0.67	137	53.9	2.3	5010	5040
SC 45	0.74	61	45.8	1.6	4780	4490
SC 115	0.79	145	41.1	1.9	4200	4300
CY 90	0.85	106	50.0	2.4	4700	4820
ML 150	0.91	166	53.5	2.5	4800	4860

B Modal Test

Three polyethylene terephtalate (PET) specimens (Fig.7) with following geometrical sizes: R = 37.7 mm, L = 36.4 mm, B = 0.44-0.46 mm was tested for vibration in order to measure the eigenfrequencies and the corresponding modes. The natural frequencies of the specimens were measured by a POLYTEC PSV-400-B Scanning Laser Vibrometer. Equipment consists of a PSV-I-400 LR optical scanning head equipped with high sensitivity vibrometer sensor (OFV-505), an OFV-5000 controller, PSV-E-400 junction box, an amplifier Bruel&Kjaer type 2732, and a computer system with data acquisition board and PSV Software. Software. Polytec Vibrometer operates on the Doppler principle, measuring the frequency shift of back-scattered laser light from a vibrating structure to determine its instantaneous velocity and displacement. The specimens have been excited by acoustic equipment (Fig. 8).

After modal test the elastic modules of specimens obtained by TWCS method. The difference between the values obtained for the first elastic modules does not exceed 3% (Tab. 4) Average value for each specimens are 4571 MPa, 4523 MPa and 4470 MPa respectively. The difference between the average values obtained for the three elastic modules does not exceed 3%, which confirms the accuracy of the method suggested.

The elastic modules received by TWCS method were verified by comparing the experimentally measured eigenfrequencies with numerical results from ANSYS program. The differences between experimental and numerical solution do not exceeding 6 % (Tab. 5).

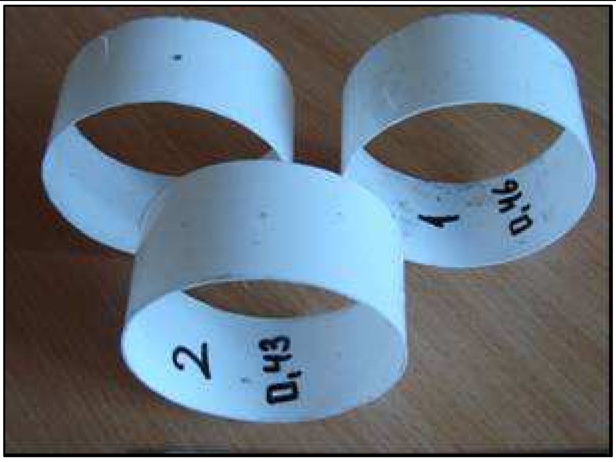

Fig. 7. PET specimens.

a) b)

Fig. 8. a) Polytec equipment, b) PET specimen.

TABLE 4.
ELASTIC MODULUS FOR SPECIMEN 1

Δ, mm	P, N	α	β	E, Mpa	|Δ|, %
11.96	1.973	0.159	2.04	4648	1.7
16.36	2.373	0.217	2.54	4496	1.6
17.86	2.573	0.237	2.67	4638	1.5
22.36	3.073	0.297	3.08	4800	5.0
28.36	3.946	0.376	4.00	4750	3.9
30.86	4.346	0.409	4.57	4578	0.2
34.36	5.046	0.456	5.56	4369	4.4
42.86	7.946	0.568	8.92	4289	6.2
Average value				4571	3.1

TABLE 5.
EIGENFREQUENCIES FOR THE FIRST SPECIMEN

№	ANSYS, Hz	Experiment, Hz	|Δ|, %
1	78.6	76.0	3.4
2	141.3	143.0	1.2
3	222.3	227.0	2.1
4	356.2	-	-
5	426.5	434.0	1.7
6	603.0	-	-
7	690.0	650.0	6.2

V CONCLUSIONS

A theoretically and experimentally grounded method is suggested for determining the elastic modulus of a polymer material based on the solution of the problem of compression of circular shells by two planes with account of geometrical and physical nonlinearity. The values obtained for the elastic modulus show a good agreement with standard tensile tests of dumbbell specimens.

The simplicity of the experiment and the use of a wide range of the loading curve in the elastic region of deformation make it possible to obtain more initial data, which provides the higher accuracy in determining the elastic modulus.

The method enables the use of a "single specimen principle". For example, changes in the modulus caused by different processes affecting the structure and properties of polymer material, for example, different kinds of aging of polymer materials, absorption of liquids (water) and vapors by a certain material, and so on, can be studied on a solitary [on one and the same] specimen.

To study the temperature dependences of material elastic modules is also more convenient by using the method of TWCS than by traditional tensile tests.

ACKNOWLEDGEMENT:

This research work is supported by the Latvian Ministry of Education and Science, project No 7549.2 and No 7061.1

REFERENCES:

1. Lavendel E. and Dirba D. Problems of dynamics and strength. – Riga:Zvaigzne, 1970. – Vol. 20. - pp. 195-200.
2. Treloar L. Physics of Elasticity of Resin. - Moscow:IL, 1953-240 pp.
3. Rivlin R.S. Large elastic deformations of isotropic materials I-IV. – London: Phil. Trans. Roy. Soc., -1948. -Vol. A241. - pp. 459-525.

Sergejs Gluhihs, Dr. Sc.ing.
Riga Technical University, Institute of Materials and Structures
Address: Azenes 16/22, LV-1048, Riga, Latvia

Andrejs Kovalovs, M.Sc.ing.
Riga Technical University, Institute of Materials and Structures
Address: Azenes 16/22, LV-1048, Riga, Latvia

Aleksandrs Tishkunovs, students
Riga Technical University, Institute of Materials and Structures
Address: Azenes 16/22, LV-1048, Riga, Latvia

Dace Čerpakovska, M.Sc.ing.
Riga Technical University, Institute of Polymer Materials
Address: Azenes 14/24, LV-1048, Riga, Latvia

Martis Kalnins, Profesors, Dr.habil.sc.ing.
Riga Technical University, Institute of Polymer Materials
Address: Azenes 14/24, LV-1048, Riga, Latvia

Sergejs Gluhihs, Andrejs Kovaļovs, Aleksandrs Tiškunovs, Dace Čerpakovska, Mārtiņš Kalniņš. Polimēru elastīgo īpašību noteikšanas metode izmantojot cilindriskas formas plānsienu paraugus (TWCS metode).
Piedāvātā elastības moduļa noteikšanas metode balstās uz plānsienu cilindriska parauga (čaulas) deformēšanas uzdevuma risinājumu, saspiežot to starp divām paralēlām plaknēm, ievērojot ģeometrisko nelinearitāti. Kontakta uzdevums tika risināts ar galīgo elementu metodi (ANSYS). Metode paredz eksperimentāli noteikt cilindriska parauga saspiešanai nepieciešamā spēka P un atbilstoša pārvietojuma Δ sakarību. Ievesti bezdimensiju parametri: pārvietojumam $\alpha = D/(2R)$ un slodzei $\beta = PR^2 / EJ$, kur R ir cilindriskā parauga rādiuss, E – materiāla elastības modulis, J – taisnstūra šķērsgriezuma inerces moments. Konstruēta universālā bezdimensiju parametru sakarība $\beta(\alpha)$ – slogošanas diagramma - relatīvā čaulas biezuma B/R vērtībām 0.0025-0.0125 (B – biezums). Sakarība aproksimēta ar sestās pakāpes polinoma vienādojumu. Iegūtais vienādojums ļauj atrisināt apgrieztu uzdevumu nosakot materiāla elastības moduli, izejot no vairākiem eksperimentāliem punktiem uz slogošanas diagrammas. Mērījumiem iespējams izmantot vienkāršu ierīci. Plānsienu čaulas deformācija raksturojas ar lielu viegli mērāmu pārvietojumu un tam atbilstošu relatīvi nelielu elastīgo deformāciju plašā paralēlo plakņu gājiena diapazonā. Pieņemama elastības moduļa vērtības noteikšanas kļūda, kas nepārsniedz 5%, pierādīta veicot septiņus – astoņus mērījumus relatīvā pārvietojuma diapazonā 0.2 – 0.8. Metode izmantota vairāku polimērkompozītu elastības moduļa vērtības noteikšanai, kuru stiepes elastības modulis tika iepriekš noteikts veicot standarta stiepes eksperimentu. Pietiekami laba iegūto rezultātu sakrišana apstiprina izstrādātās metodes efektivitāti.

Сергей Глухих, Андрей Ковалев, Александр Тишкунов, Даце Черпаковска, Мартыньш Калныньш. Определение модуля упругости полимерных материалов на тонкостенных цилиндрических образцах (TWCS метод).
Предложенная методика основана на решении задачи деформирования тонкостенной круговой цилиндрической оболочки при сжатии двумя параллельными плоскостями с учетом геометрической нелинейности. Контактная задача решается методом конечных элементов с использованием программы (ANSYS). Метод предусматривает экспериментальное определение зависимости силы сжатия образца P и соответствующего перемещения Δ. Введены безразмерные параметры: для перемещения $\alpha = D/(2R)$ и нагружения $\beta = PR^2/EJ$, где R радиус цилиндрического образца, E – модуль эластичности материала, J – момент инерции прямоугольного сечения образца. Конструирована универсальная зависимость безразмерных характеристик. $\beta(\alpha)$ - диаграмма нагружения - для значений относительной толщины оболочки B/R 0,0025-0,0125 (B – толщина). Зависимость аппроксимирована полиномом шестой степени. Полученное уравнение позволяет решать обратную задачу определения значения модуля эластичности материала по нескольким экспериментальным точкам на диаграмме нагружения. Для измерений может быть использована простая установка. Деформация тонкостенной оболочки характеризуется большими легко измеряемыми перемещениями и относительно малыми значениями упругой деформации в широком диапазоне хода параллельных плоскостей. Приемлемая погрешность измерения значения модуля, не превышающая 5 %, доказана осуществлением семи – восьми замеров в диапазоне относительного перемещения 0,2 – 0,8. Метод использован для определения модуля эластичности нескольких полимерных композитов, значения модуля эластичности которых были определены при растяжения стандартными методами. Удовлетворительная сходимоть результатов подтверждает эффективность предложенной методики.