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Dynamics of Vibration Machine with Air Flow Excitation and Restrictions on Phase Coordinates

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Abstract **- The objective of presented article is to show possibilities of practical use of air or liquid flow in vibration engineering. Dynamics of vibration machine with constant air or liquid flow excitation is considered. In the first part vibration motion of the machine working head under constant air or liquid flow velocity excitation is investigated. The main idea is to find out optimal control law for variation of additional surface area of vibrating object within limits. The criterion of optimization is time required to move working head of the machine from initial position to end position. For solution of the high-speed problem the maximum principle is used. It is shown that optimal control action corresponds to the case of bound values of area limits. In the second part of this research restrictions on phase coordinates are taken into account. Examples on synthesis of real mechatronic systems are given.**

Keywords **- vibration machine, dynamics, motion, air flow excitation, optimal control, adaptive control, synthesis.**

I. INTRODUCTION

Dynamics of a vibration machine with one degree of freedom and constant air flow $\overline{V}_0$ excitation is investigated (see Fig. 1).

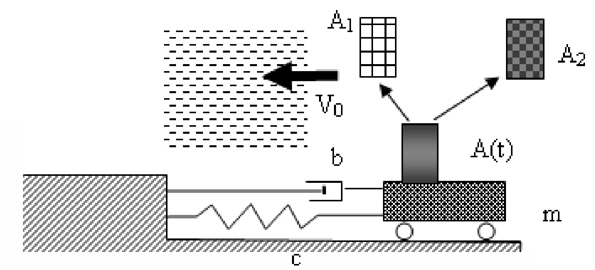

Fig. 1. Dynamic model of system with area *A*(*t*) control

System consists of working head with mass *m* connected by spring *c* and damper *b* to stationary base. The main idea is to find out optimal control law for variation of additional area *A(t)* of vibrating mass *m* within the following limits (1):

$$A_1 \le A(t) \le A_2, \tag{1}$$

where A_1 and A_2 are a lower and an upper levels of additional surface area of working head; *t* is a time.

The criterion of optimization is time *T* required to move working head *m* from initial position to end position. First of all to understand process of air (or fluid) excitation and optimal solution of control problem observe system without restrictions on phase coordinate (velocity).

For the system under study (see Fig. 1) the differential equation describing a motion of working head is as follows:

$$m\,\ddot{x} = -c\,x - b\,\dot{x} - u(t)\cdot(V_0 + \dot{x})^2, \tag{2}$$

where $u(t) = A(t)\cdot k$;
m is a mass;
$\ddot{x}$, $\dot{x}$ and *x* are an acceleration, velocity and displacement of the object;
c is a stiffness coefficient of spring; *b* is a damping coefficient;
V_0 is a constant velocity of wind flow;
A(t) is a law for area variation;
$u(t)$ is a control action; *k* is a constant.
It is necessary to determine the control action $u = u(t)$ for displacement of working head of the system (2) from the initial position $x(t_0)$ to the end position $x(t_1)$ during minimal time $K = T$ (here *K* is a criterion), if area *A(t)* has limits (1).

II. SOLUTION OF OPTIMAL CONTROL PROBLEM FOR SYSTEM WITH ONE DEGREE OF FREEDOM

For system excitation any time must be solved the high-speed problem [1 - 9]:

$$K = \int_{t_0}^{t_1} 1\cdot dt. \tag{3}$$

By assuming in (3) $t_0 = 0;\quad t_1 = T$, we have $K = T$.

Using substitutions $x_1 = x;\quad \dot{x}_1 = x_2$, equation (2) transforms to the following form:

$$\dot{x}_1 = x_2;\quad m\,\dot{x}_2 = -c\,x - b\,\dot{x} - u(t)\cdot(V_0 + \dot{x})^2.$$

In this case Hamiltonian takes the form (4) [1 – 3]:

$$H = \psi_0 + \psi_1 x_2 + \psi_2 \frac{1}{m}\cdot\left(-cx_1 - bx_2 - u(t)\cdot(V_0 + x_2)^2\right), \tag{4}$$

where $H = \psi \cdot X$.

Parameter ψ is determined by the following expression:

$$\psi = \begin{Bmatrix} \psi_0 \\ \psi_1 \\ \psi_2 \end{Bmatrix}; \; X = \begin{Bmatrix} 0 \\ x_2 \\ \frac{1}{m} \cdot [-cx_1 - bx_2 - u(t) \cdot (V_0 + x_2)^2] \end{Bmatrix}.$$

Scalar multiplication of two last vector functions ψ and X in any time (Hamiltonian *H* [3]) must be maximal [2 – 9]. To have such maximum, control action *u(t)* must be within the limits $u(t) = u_1; \quad u(t) = u_2$, depending only from the sign of function ψ_2 described by the expression (5) (see, for example, [3 – 6]):

$$\begin{aligned} & H = \max H, \\ & if \quad \psi_2 \cdot (-u(t) \cdot (V_0 + x_2)^2) = \max. \end{aligned} \tag{5}$$

Therefore, if $\psi_2 > 0$, the $u(t) = u_1$, and if $\psi_2 < 0$, the $u(t) = u_2$, where $u_1 = A_1 \cdot k$ and $u_2 = A_2 \cdot k$, see (1). Examples of very simple control action (with one and three switch points) are shown in Fig. 2 and 3.

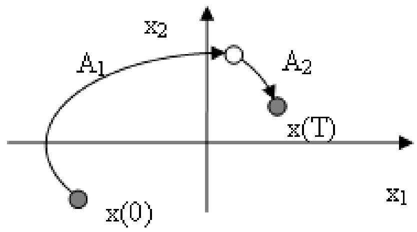

Fig. 2. Optimal control with one switch point

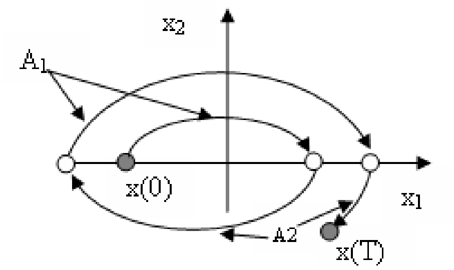

Fig. 3. Optimal control with three switch points, when $x_2 = 0$

How to find switch points (e.g., $\psi_2 > 0$ or $\psi_2 < 0$) here is not observed [3 – 9]. But the main conclusion of optimal control law may be formulated as follows: value of area *A* in any time must be on bounds of interval (1) – *A(t) = A₁* or A*(t)* = *A₂*. In real systems it allows to synthesize quasi optimal control actions (see, for example, [10 – 13]). Additionally it is necessary to mention that optimal control in time domain *u(t)* (like programming control) in real nonlinear systems without feed back often is unstable. Therefore in such cases a problem of synthesis of new real control systems includes itself a step of forming control like mixed function of phase coordinates and time $u(t) = u(x_1, x_2, t)$ (see, for example, [10 – 13]).

III. SOLUTION OF OPTIMAL CONTROL PROBLEM FOR SYSTEM WITH RESTRICTIONS ON PHASE COORDINATES

Solution of optimization problem for the system with restrictions on phase coordinates for engineering can be made by using the maximum principle of Pontryagin together with variation of parameters in phase plane [11 – 13]. Examples of restrictions and optimal trajectories are shown in Fig. 4 – 7.

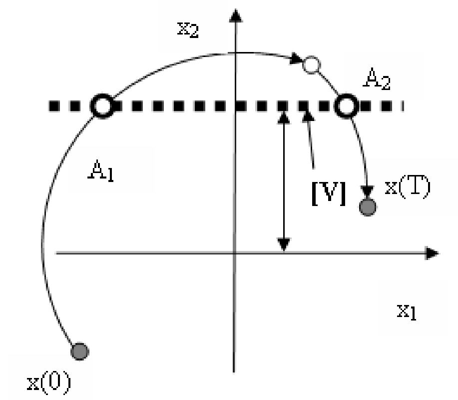

Fig. 4. Restriction on motion velocity

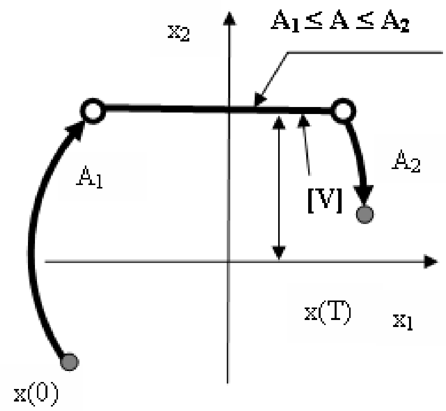

Fig. 5. Optimal trajectory for the case of restriction on velocity (phase coordinate) [13]

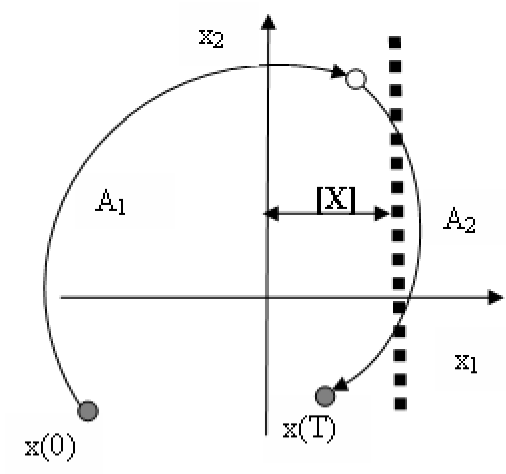

Fig. 6. Scheme of restriction on displacement

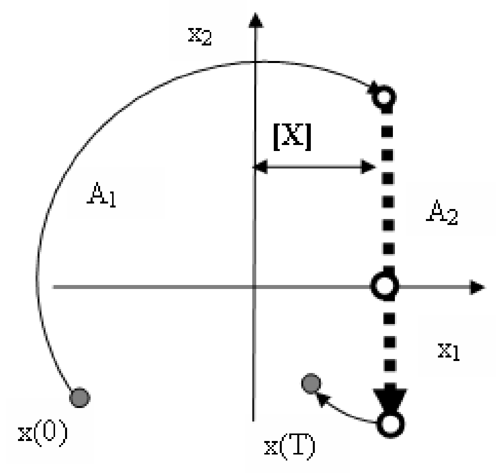

Fig. 7. Form of optimal trajectory for the case of limit on displacement with collision on obstacle [13]

IV. SYNTHESIS OF REAL CONTROL ACTION

For realizing of optimal control actions (in general case) system of one degree of freedom needs a feedback system with two adapters: one – for displacement measurement and another – for velocity measurement. There is a simple case of control existing with only one adapter when motion changes directions, as shown in Fig. 3 [13]. It means that control action is similar to negative dry friction and switch points are along zero velocity line. In that case equation of motion for large velocity $|V_0| \geq |\dot{x}|$ and dry friction is as follows (6):

$$m \cdot \ddot{x} = -c \cdot x - b \cdot \dot{x} - F \cdot sign(\dot{x}) + U(\dot{x}), \qquad (6)$$

where

$$U(\dot{x}) = -\left[k \cdot (V_0 + \dot{x})^2 \cdot A_1 \cdot \frac{1 + sign(\dot{x})}{2}\right] - \left[k \cdot (V_0 + \dot{x})^2 \cdot A_2 \cdot \frac{1 - sign(\dot{x})}{2}\right];$$

m - mass; c, b, F, k, V_0 – constants.

Examples of modelling are shown in Fig. 8 – 13.

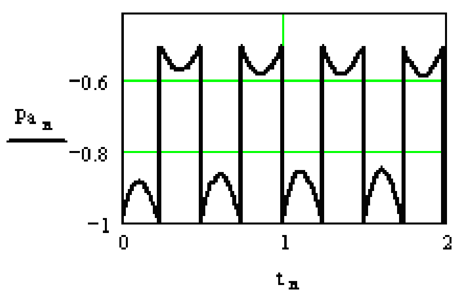

Fig. 8. Full control action (6) $Pa_n = U(\dot{x})$ in time t_n domain (SI system)

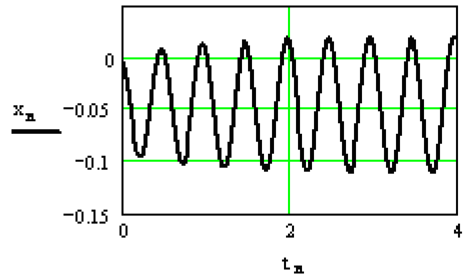

Fig. 9. Displacement x_n in time t_n domain (SI system)

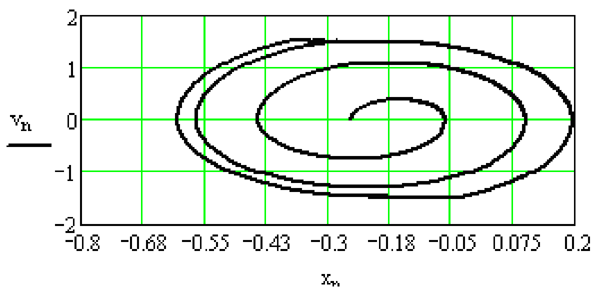

Fig. 10. Motion in a phase plane ($x = x_n$; $\dot{x} = V_n$) for the case of symmetrical velocity restrictions

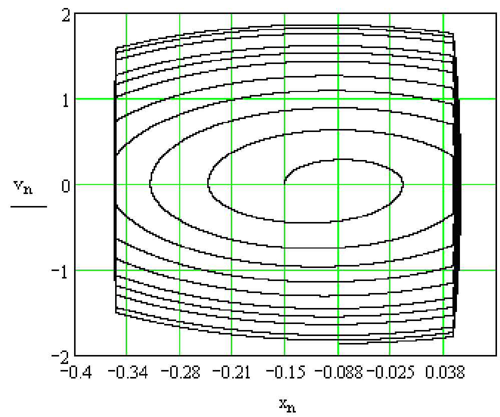

Fig. 11. Motion in a phase plane for the case of double-sided restrictions on displacement

An attempt to find more, than one limit cycle, was made during the investigation of complicated system with nonlinear cubic elastic force and dry friction. Motion of this system is described by the differential equation (7):

$$m \cdot \ddot{x} = -c \cdot x^3 - b \cdot \dot{x} - F \cdot sign(\dot{x}) - \left[k \cdot (V_0 + \dot{x})^2 \cdot A_2 \cdot \frac{1 - sign(\dot{x})}{2}\right] - \left[k \cdot (V_0 + \dot{x})^2 \cdot A_1 \cdot \frac{1 + sign(\dot{x})}{2}\right]. \qquad (7)$$

It is shown by the mathematical simulation, that there can be more than one limit cycle in the system (7) under non-periodical excitation (e.g. constant velocity V_0 of air or water flow). Typical examples are presented in Fig. 12 and 13. Both cycles are separated by different initial conditions. Therefore only one periodic cycle, corresponding to limitation on motion velocity or displacement (see Fig. 10 and 11), can be used in synthesis.

It is shown that adaptive systems are very stable because air excitation and damping forces depend on velocity in the second power.

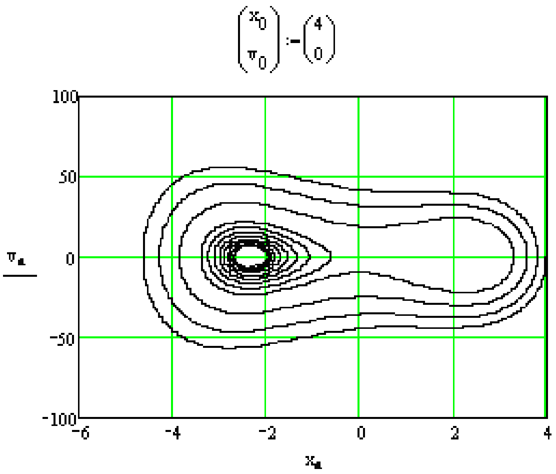

Fig. 12. Motion in phase plane for left side limit cycle

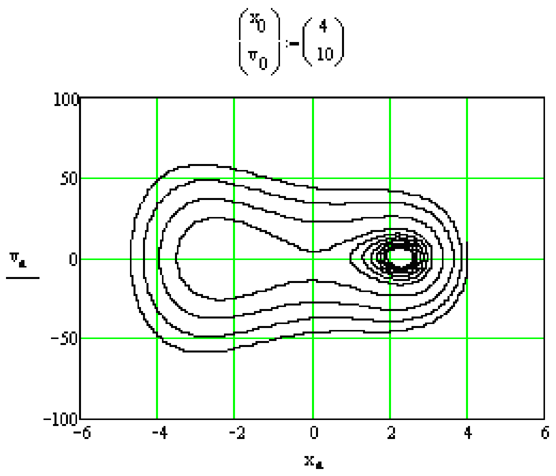

Fig. 13. Motion in phase plane for right side limit cycle

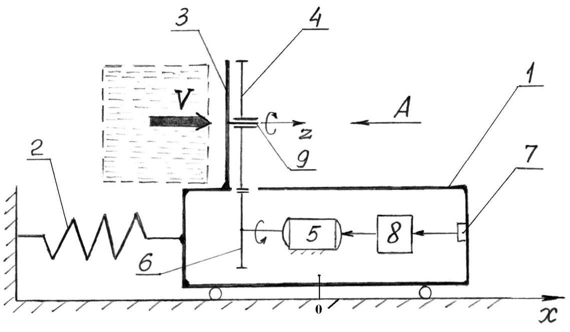

Fig. 14. Schematic diagram of vibration machine with air flow excitation and adaptive control

Schematic diagram of the synthesized mechatronic system with air flow excitation and adaptive control is shown in Fig. 14 [16]. Working head 1 of the vibration machine (see Fig. 14) is connected with stationary base through elastic element 2. Vibration excitation of the system is realized by the action with air flow *V* on special screen, which consists of two parts: stationary disk 3 and rotatable disk 4. Disk 4 has a possibility to rotate about axis *z* and thanks to this it can change position relative to disk 3. Rotation of disk 4 is realized with the aid of step motor 5 through gear transmission 6. Vibration transducer 7 generates impulse control signal proportional to the velocity $\dot{x}$ of working head 1. Control signal through pulse shaping unit 8 acts on step motor 5.

Variant of design of rotatable disk 4 is shown in Fig. 15. Both disks 3 and 4 have identical holes 11, uniformly distributed along a circle.

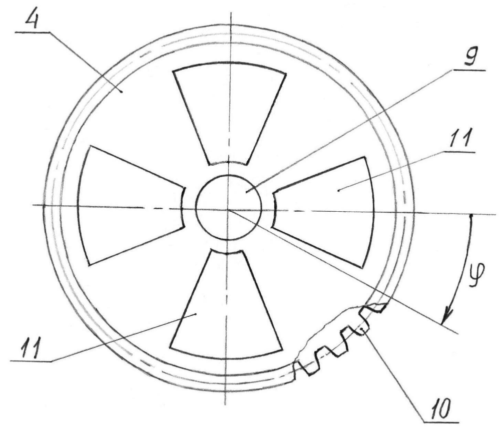

Fig. 15. Variant of design of rotatable disk

Holes 11 in disks 3 and 4 may be fair or unfair, and this is dependent on the value of turning angle φ of disk 4. If holes 11 in both disks are fair, then effective frontal surface area of the screen is minimal $A_{\min}$. But if holes 11 are unfair, then effective frontal surface area is maximal $A_{\max}$. Therefore it is possible to vary an effective frontal area *A* of the screen over a wide range.

Excitation force *P*, which acts on working head of the vibration machine, can be calculated by the formula:

$$P = k \cdot \rho \cdot A \cdot V_r^2 , \qquad (8)$$

where *k* is coefficient depending on geometrical form of the screen; ρ is air density; $V_r = (V - \dot{x})$ is relative velocity of air flow. The proposed control action ensures the maximal value $A_{\max}$ of the screen's effective surface during working head motion in air flow *V* direction, but the minimal value $A_{\min}$ – during working head motion in opposite direction to air flow. In accordance with the equation (8) excitation force *P* is changed in the same way as area *A*. And there is vary important, that changing of force *P* from P_{min} to $P_{\max}$ and on the contrary is realized in time moments of momentary stops of the working head (under the condition $\dot{x}$ = 0). Thanks to this efficiency of vibration machine can be sufficiently increased.

Operation of vibration devices with air flow excitation was investigated experimentally inside wind tunnel of the firm ARMFIELD (see Fig. 16).

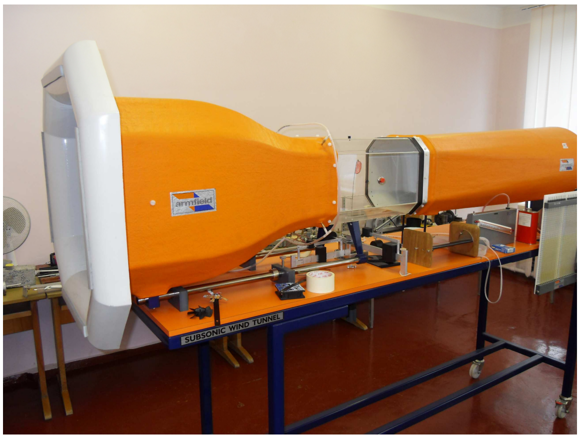

Fig. 16. Wind tunnel of the firm ARMFIELD

Parameters of subsonic wind tunnel were the following: length 2,98 m; width 0,8 m; height 1,83 m. Varying-speed motor driven unit downstream the working section permits continuous control of airspeed between 0 and 26 m/s. Experiments confirm the efficiency of airflow excitation.

V. CONCLUSION

Air or water flow can be used for excitation of objects motion in vibration engineering. Control of object's surface area interacting with air flow makes it possible to develop very efficient mechatronic systems. Algorithm of synthesis of strongly non-linear mechanical systems includes solution of optimization problem and allows designing of fundamentally new vibration devices. Adapters and controllers must be used for practical realization of such systems. For this purpose it is possible to apply very simple control actions, which have solutions with the use of sign functions.

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Jānis Vība, Vitālijs Beresņevičs, Lauris Štāls, Māris Eiduks, Edgars Kovals, Maarja Krūsmā. Vibromašīnas ar gaisa plūsmas ierosmi un ierobežojumiem uz fāžu koordinātēm dinamika

Raksta mērķis ir parādīt gaisa vai šķidruma plūsmas ierosmes praktiskā pielietojuma iespējas vibrotehnikā. Tiek analizēta vibromašīnas ar konstantu gaisa vai šķidruma plūsmas ierosmi dinamika. Raksta pirmā daļa veltīta mašīnas izpildorgāna vibrācijas kustības analīzei gadījumā, kad gaisa vai šķidruma plūsmas ātrums ir konstants lielums. Pamatideja ir atrast optimālo vadības likumu, pēc kura vajadzētu variēt izpildorgāna darba virsmas laukumu dotās robežās. Optimizācijas kritērijs ir laiks, kas nepieciešams, lai pārvietotu mašīnas izpildorgānu no sākuma pozīcijas līdz galējam stāvoklim. Ātrdarbības problēmas risināšanai izmantots maksimuma princips. Parādīts kā pie optimālas vadības darba laukumam jāpieņem robežvērtības. Pētījumu otrajā daļā ņemti vērā ierobežojumi uz fāžu koordinātēm. Aplūkoti reālo mehatronikas sistēmu sintēzes piemēri.

Янис Виба, Виталий Бересневич, Лаурис Шталс, Марис Эйдукс, Эдгарс Ковалс, Маарья Круусмаа. Динамика вибромашины с возбуждением от потока воздуха и при ограничениях на фазовые координаты

Цель статьи – показать возможности практического применения в вибротехнике вынуждающих воздействий, генерируемых потоком воздуха или жидкости. Анализируется динамика вибромашины при возбуждении от постоянного потока газа или жидкости. Первая часть статьи посвящена анализу вибрационного движения исполнительного органа машины в случае, когда скорость потока воздуха или жидкости постоянна. Основная идея заключается в нахождении оптимального закона управления, по которому целесообразно изменять рабочую площадь исполнительного органа машины. Критерий оптимизации – время, необходимое для перемещения исполнительного органа из начального в конечное положение. Для решения проблемы быстродействия использован принцип максимума. Показано, что при оптимальном управлении рабочая площадь должна принимать граничные значения. Во второй части исследований принимаются во внимание возможные ограничения на фазовые координаты. Рассмотрены примеры синтеза реальных систем мехатроники.