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Double Conductor Line above a Two-Layer Medium with Varying Electric Conductivity and Magnetic Permeability

Valentina Koliskina, *Riga Technical University*, Inta Volodko, *Riga Technical University*

***Abstract* – In the present paper the change in impedance per unit length of a double conductor line formed by two parallel wires with alternating current is calculated. The double conductor line is located above a two-layer medium. The magnetic permeability and electric conductivity of the upper layer are exponential functions of the vertical coordinate. The properties of the lower layer are assumed to be constant. The solution is obtained in closed form by means of the Fourier sine and cosine integral transform and is expressed in terms of improper integral containing Bessel functions.**

Keywords – eddy current testing, electric conductivity, Fourier transform, magnetic permeability

I. INTRODUCTION

Analytical solutions of eddy current testing problems for multilayer media with constant electric conductivity and magnetic permeability are well-known [1]. In some engineering applications (examples include surface hardening and decarbonization [2], [3]) the properties of a conducting medium are not constant. Experiments [2] indicated that there exists a thin upper layer with reduced magnetic permeability where the magnetic permeability is an exponential function of depth.

Two basic methods are usually used for the analysis of eddy-current problems in a conducting medium with varying properties. The idea of the first method is as follows. A conducting medium with varying electric conductivity and magnetic permeability is replaced by a multilayer medium with large number of relatively thin layers where the properties of each layer are assumed to be constant. Up to 50 layers with constant electric conductivity and magnetic permeability are used in [4] in order to represent the changes in the electric conductivity and magnetic permeability in the vertical direction. The second approach is based on closed-form solutions of the problem. The idea is to approximate the electric conductivity and magnetic permeability by means of relatively simple analytical expressions (containing parameters) with the hope that for some values of the parameters the problem can be solved analytically by means of known special functions (examples can be found in [5]-[8]). In particular, the solution found in [5] is generalized in [8] for the case where both parameters, that is, the electric conductivity and magnetic permeability, are exponential functions of the vertical coordinate.

In the present paper analytical solution is obtained for the case where a double conductor line with alternating current is located above a two-layer conducting medium. The electric conductivity and magnetic permeability of the upper layer are exponential functions of the vertical coordinate. The paper generalizes the results of [9] for the case of a two-layer medium. The problem is solved by means of the Fourier sine and cosine integral transform method. The solution is obtained in closed form in terms of improper integrals containing Bessel functions of a complex argument.

II. MATHEMATICAL FORMULATION OF THE PROBLEM

Eddy current testing method is widely used in engineering applications. Different types of eddy current probes can be used for inspection purposes. The choice of the form of an eddy current probe depends on a particular problem. It is shown in [10] that if the ratio of the sides of a rectangular frame with alternating current is 1:4 or smaller then the rectangular probe can be approximated by a double conductor line.

Consider two parallel infinitely long wires with current located above a two-layer conducting medium (see Fig. 1).

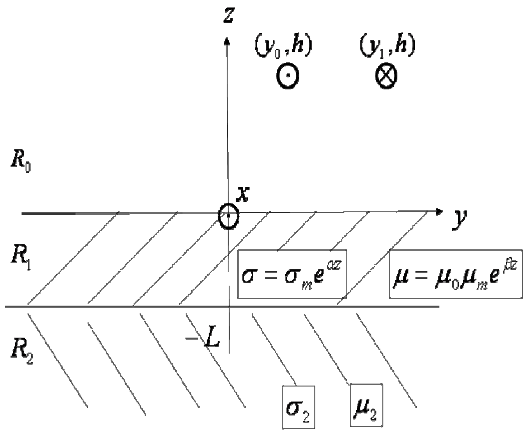

Fig. 1. Two parallel infinitely long wires above a two-layer conducting medium.

The alternating current in the wires at the points (y_0,h) and (y_1,h) is equal to $\pm I\exp(j\omega t)$, respectively, where I is the amplitude of the current. Regions $R_0=\{z>0\}$, $R_1=\{-L<z<0\}$ and $R_2=\{z<-L\}$ represent the upper half-space, the upper conducting layer and the lower half-space, respectively. The electric conductivity σ and magnetic permeability μ of region R_1 vary with depth in accordance with the formulas

$$\sigma = \sigma_m e^{\alpha z}\,,\ \mu = \mu_0 \mu_m e^{\beta z}\,, \tag{1}$$

where σ_m, μ_m, α, β are constants and μ_0 is the magnetic constant.

The vector potential has only one non-zero component in the x-direction in all three regions R_0, R_1 and R_2. In addition, the vector potential depends only on the coordinates y and z:

$$A_0 = A_0(y,z),\ \ A_1 = A_1(y,z),\ \ A_2 = A_2(y,z). \tag{2}$$

The functions A_0, A_1 and A_2 satisfy the following system of equations

$$\frac{\partial^2 A_0}{\partial y^2} + \frac{\partial^2 A_0}{\partial z^2} = -\mu_0 I \delta(y-y_0)\delta(z-h) + \mu_0 I \delta(y-y_1)\delta(z-h), \tag{3}$$

$$\frac{\partial^2 A_1}{\partial y^2} + \frac{\partial^2 A_1}{\partial z^2} - \beta \frac{\partial A_1}{\partial z} - j\omega\sigma_m \mu_0 \mu_m e^{(\alpha+\beta)z} A_1 = 0, \tag{4}$$

$$\frac{\partial^2 A_2}{\partial y^2} + \frac{\partial^2 A_2}{\partial z^2} - j\omega\sigma_2 \mu_0 \mu_2 A_2 = 0, \tag{5}$$

where $\delta(z)$ is the Dirac delta-function.
The boundary conditions are

$$A_0\,|_{z=0} = A_1\,|_{z=0}\,, \qquad \frac{\partial A_0}{\partial z}\,|_{z=0} = \frac{1}{\mu_m}\frac{\partial A_1}{\partial z}\,|_{z=0}\,, \tag{6}$$

$$A_1\,|_{z=-L} = A_2\,|_{z=-L}\,, \qquad \frac{\partial A_1}{\partial z}\,|_{z=-L} = \frac{\partial A_2}{\partial z}\,|_{z=-L}\,. \tag{7}$$

The functions A_0, A_1, A_2 and their partial derivatives with respect to y tend to zero at infinity:

$$A_0,\ A_1,\ A_2,\ \frac{\partial A_0}{\partial y},\ \frac{\partial A_1}{\partial y},\ \frac{\partial A_2}{\partial y} \to 0, \qquad \sqrt{y^2+z^2} \to \infty. \tag{8}$$

Problem (3)-(8) is solved in the next section by means of the Fourier cosine and sine integral transforms.

III. SOLUTION OF THE PROBLEM

It is convenient to represent the solution in each region as the sum of even and odd solutions (see [7]):

$$A_i(y,z) = A_{i\,\text{even}}(y,z) + A_{i\,\text{odd}}(y,z), \ \ i = 0,1,2 \tag{9}$$

The right-hand side of (3) is also represented as the sum of even and odd parts of the form

$$f(y,z) = f_{\text{even}}(y,z) + f_{\text{odd}}(y,z),$$

where

$$f_{\text{even}}(y,z) = f_{\text{odd}}(y,z) = \frac{\mu_0 I}{2}[\delta(y-y_1) - \delta(y-y_0)]\delta(z-h).$$

In order to find the even component of the solution, $A_i(y,z),\ i=0,1,2,$ the Fourier cosine transform of the form

$$\tilde{A}_i^{(c)}(\lambda,z) = \int_0^\infty A_{i\,\text{even}}(y,z)\cos\lambda y dy, \ \ i = 0,1,2 \tag{10}$$

is applied. In this case the right-hand side of (3) is replaced by $f_{even}(y,z)$.

Applying the Fourier cosine transform (10) to problem (3)-(8) we obtain

$$\frac{d^2\tilde{A}_0^{(c)}}{dz^2} - \lambda^2 \tilde{A}_0^{(c)} = \frac{\mu_0 I}{2}(\cos\lambda y_1 - \cos\lambda y_0)\delta(z-h), \tag{11}$$

$$\frac{d^2\tilde{A}_1^{(c)}}{dz^2} - \beta\frac{d\tilde{A}_1^{(c)}}{dz} - \lambda^2\tilde{A}_1^{(c)} - j\omega\sigma_m\mu_0\mu_m e^{(\alpha+\beta)z}\tilde{A}_1^{(c)} = 0, \tag{12}$$

$$\frac{d^2\tilde{A}_2^{(c)}}{dz^2} - \lambda^2\tilde{A}_2^{(c)} - j\omega\sigma_2\mu_0\mu_2\tilde{A}_2^{(c)} = 0. \tag{13}$$

The boundary conditions are

$$\tilde{A}_0^{(c)}\,|_{z=0} = \tilde{A}_1^{(c)}\,|_{z=0}\,, \quad \frac{d\tilde{A}_0^{(c)}}{dz}\,|_{z=0} = \frac{1}{\mu_m}\frac{d\tilde{A}_1^{(c)}}{dz}\,|_{z=0}\,, \tag{14}$$

$$\tilde{A}_1^{(c)}\,|_{z=-L} = \tilde{A}_2^{(c)}\,|_{z=-L}\,, \quad \frac{d\tilde{A}_1^{(c)}}{dz}\,|_{z=-L} = \frac{d\tilde{A}_2^{(c)}}{dz}\,|_{z=-L}\,. \tag{15}$$

Consider two subregions of R_0, namely, $R_{00} = \{0 < z < h\}$ and $R_{01} = \{z > h\}$, respectively. The solutions in these regions are denoted by $\tilde{A}_{00}^{(c)}$ and $\tilde{A}_{01}^{(c)}$, respectively. The general solution to (11) in R_{01} has the form

$$\tilde{A}_{00}^{(c)}(\lambda,z) = C_1 e^{\lambda z} + C_2 e^{-\lambda z}. \tag{16}$$

The solution to (11) that is bounded as $z \to +\infty$ is

$$\tilde{A}_{01}^{(c)}(\lambda,z) = C_3 e^{-\lambda z} \tag{17}$$

The general solution to (12) in region R_1 can be written in the form (see [11]):

$$\tilde{A}_1^{(c)}(\lambda,z) = C_4 e^{\beta z/2} I_\nu\left(ce^{(\alpha+\beta)z/2}\right) + C_5 e^{\beta z/2} K_\nu\left(ce^{(\alpha+\beta)z/2}\right), \tag{18}$$

where $I_\nu(z)$ and $K_\nu(z)$ are the modified Bessel's functions of the first and second kind, respectively.

The parameters c and ν in (18) are defined as follows:

$$c = \frac{2\sqrt{j\omega\mu_0\mu_m\sigma_m}}{\alpha+\beta}, \quad \nu = \frac{\sqrt{\beta^2 + 4\lambda^2}}{\alpha+\beta}.$$

The solution to (13) that is bounded at $z \to -\infty$, is

$$\tilde{A}_2^{(c)}(\lambda,z) = C_6 e^{qz}, \tag{19}$$

where $q = \sqrt{\lambda^2 + j\omega\sigma_2\mu_0\mu_2}$.

The vector potential is continuous at $z = h$:

$$\tilde{A}_{01}^{(c)}\,|_{z=h} = \tilde{A}_{00}^{(c)}\,|_{z=h} \quad . \tag{20}$$

Integrating (11) with respect to z from $z-\varepsilon$ to $z+\varepsilon$ and taking the limit as $\varepsilon \to +0$ we obtain

$$\frac{d\tilde{A}_{01}^{(c)}}{dz}\,|_{z=h} - \frac{d\tilde{A}_{00}^{(c)}}{dz}\,|_{z=h} = \frac{\mu_0 I}{2}(\cos\lambda y_1 - \cos\lambda y_0). \tag{21}$$

The values of the constants $C_1, C_2, ..., C_6$ in (17)-(19) are obtained from the boundary conditions (14), (15), (20) and (21). In particular, the values of the constants C_1 and C_2 are:

$$C_1 = -\frac{\mu_0 I}{4\lambda}(\cos\lambda y_1 - \cos\lambda y_0)e^{-\lambda h},$$

$$C_2 = \frac{\mu_0 I}{4\lambda} F e^{-\lambda h}(\cos\lambda y_1 - \cos\lambda y_0),$$

where

$$F = F_1 / F_2,$$

$$F_1 = G + \lambda\mu_m[DK_\nu(c) - I_\nu(c)],$$

$$F_2 = G - \lambda\mu_m[DK_\nu(c) - I_\nu(c)],$$

$$G = \frac{\beta}{2} I_\nu(c) + c\frac{(\alpha+\beta)}{2} I_\nu{}'(c) -$$
$$- D\left[\frac{\beta}{2} K_\nu(c) + c\frac{(\alpha+\beta)}{2} K_\nu{}'(c)\right],$$

$$D = D_1 / D_2,$$

$$D_1 = (2q-\beta)I_\nu\left(ce^{-(\alpha+\beta)L/2}\right) -$$
$$- c(\alpha+\beta)e^{-(\alpha+\beta)L/2} I_\nu{}'\left(ce^{-(\alpha+\beta)L/2}\right),$$

$$D_2 = (2q-\beta)K_\nu\left(ce^{-(\alpha+\beta)L/2}\right) -$$
$$- c(\alpha+\beta)e^{-(\alpha+\beta)L/2} K_\nu{}'\left(ce^{-(\alpha+\beta)L/2}\right).$$

Applying the inverse Fourier cosine transform of the form

$$A_{i,even}(y,z) = \frac{2}{\pi}\int_0^\infty \tilde{A}_i^{(c)}(\lambda, z)\cos\lambda y d\lambda, \quad i = 0,1,2.$$

to (16)-(19) we obtain the solution $A_{i\,\text{even}}(y,z), i = 0,1,2$ for the even component of the vector potential in regions R_0, R_1 and R_2.

The solution in region R_0 can be written in the form

$$A_{0\,\text{even}}(y,z) = A_{0\,\text{even}}^{\text{free}}(\lambda, z) + A_{0\,\text{even}}^{\text{ind}}(\lambda, z), \tag{22}$$

where

$$A_{0\,\text{even}}^{free}(y,z) =$$
$$= \frac{\mu_0 I}{2\pi}\int_0^\infty (\cos\lambda y_0 - \cos\lambda y_1)e^{-\lambda|z-h|}\cos\lambda y \frac{d\lambda}{\lambda}, \tag{23}$$

$$A_{0\,\text{even}}^{\text{ind}}(y,z) = \frac{2}{\pi}\int_0^\infty C_2 e^{-\lambda z}\cos\lambda y \frac{d\lambda}{\lambda}. \tag{24}$$

The first term on the right-hand side of (22) represents the even component of the vector potential in an unbounded free space while the second term is equal to the even component of the induced vector potential in the double conductor line due to presence of the conducting two-layer medium.

The odd components of the solution, $A_{i,odd}(y,z), i = 0,1,2,$ are determined by means of the Fourier sine transform

$$\tilde{A}_i^{(s)}(\lambda, z) = \int_0^\infty A_{i\,\text{odd}}(y,z)\sin\lambda y dy, \; i = 0,1,2, \tag{25}$$

and the inverse Fourier sine transform

$$A_{i\,\text{odd}}(y,z) = \frac{2}{\pi}\int_0^\infty \tilde{A}_i^{(s)}(\lambda, z)\sin\lambda y d\lambda, \; i = 0,1,2. \tag{26}$$

It can be shown that the induced component of the vector potential in region R_0 can be written in the form

$$A_0^{ind}(y,z) = \frac{\mu_0 I}{2\pi}\int_0^\infty F e^{-\lambda z}[\cos\lambda(y-y_1) - \cos\lambda(y-y_0)]\frac{d\lambda}{\lambda}. \tag{27}$$

The change in impedance per unit length of the double conductor line is computed by means of the formula

$$Z_{\text{per unit length}}^{\text{ind}} = \frac{j\omega}{I}\oint_C A_0^{\text{ind}}(y,z)dx. \tag{28}$$

Substituting (27) into (28) and evaluating the integral we obtain

$$Z_{\text{per unit length}}^{\text{ind}} = \frac{\mu_0\omega}{\pi} Z, \tag{29}$$

where

$$Z = j\int_0^\infty \tilde{F} e^{-2\delta s}(\cos\delta s - 1)\frac{ds}{s}, \tag{30}$$

$$\tilde{F} = \tilde{F}_1 / \tilde{F}_2,$$

$$\tilde{F}_1 = \frac{\eta}{2} I_\nu(\tilde{c}) + \tilde{c}\frac{(\xi+\eta)}{2} I_\nu{}'(\tilde{c}) -$$
$$- \tilde{D}\left[\frac{\eta}{2} K_\nu(\tilde{c}) + \tilde{c}\frac{(\xi+\eta)}{2} K_\nu{}'(\tilde{c})\right] + s\mu_m[\tilde{D}K_\nu(\tilde{c}) - I_\nu(\tilde{c})],$$

$$\tilde{F}_2 = \frac{\eta}{2} I_\nu(\tilde{c}) + \tilde{c}\,\frac{(\xi+\eta)}{2} I_\nu{}'(\tilde{c}) -$$
$$-\tilde{D}\left[\frac{\eta}{2} K_\nu(\tilde{c}) + \tilde{c}\,\frac{(\xi+\eta)}{2} K_\nu{}'(\tilde{c})\right] - s\mu_m[\tilde{D}K_\nu(\tilde{c}) - I_\nu(\tilde{c})],$$

$$\tilde{D} = \tilde{D}_1 / \tilde{D}_2,$$

$$\tilde{D}_1 = (2\tilde{q}-\eta) I_\nu\left(\tilde{c}e^{-(\xi+\eta)\tilde{L}/2}\right) -$$
$$-\tilde{c}(\xi+\eta)e^{-(\xi+\eta)\tilde{L}/2} I_\nu{}'\left(\tilde{c}e^{-(\xi+\eta)\tilde{L}/2}\right),$$

$$\tilde{D}_2 = (2\tilde{q}-\eta) K_\nu\left(\tilde{c}e^{-(\xi+\eta)\tilde{L}/2}\right) -$$
$$-\tilde{c}(\xi+\eta)e^{-(\xi+\eta)\tilde{L}/2} K_\nu{}'\left(\tilde{c}e^{-(\xi+\eta)\tilde{L}/2}\right).$$

The parameters in (30) are defined as follows:

$$d = y_1 - y_0, \quad \delta = \frac{h}{d}, \quad \xi = \alpha d, \quad \eta = \beta d, \quad \tilde{L} = L/d,$$
$$\gamma = d\sqrt{\omega\sigma_m\mu_0\mu_m}, \quad \gamma_1 = d\sqrt{e^{-(\xi+\eta)\tilde{L}}}, \quad \tilde{c} = \frac{2\gamma\sqrt{j}}{\xi+\eta},$$
$$\tilde{q} = \sqrt{s^2 + j\gamma_1^2}, \quad \nu = \frac{\sqrt{\eta^2+4s^2}}{\xi+\eta}.$$

The change in impedance given by formula (30) is computed for different values of the parameters of the problem using package "Mathematica" since it allows one to calculate improper integrals and evaluate modified Bessel functions of variable order and complex argument. The results of calculations are presented in Fig. 2.

The three graphs correspond to the cases $\eta = -1$, $\eta = -5$ and $\eta = -8$ (from left to right). The points (from top to bottom) correspond to the values of $\gamma = 1,2,...,10$. The other parameters are: $\xi = 0$, $\tilde{L} = 0.05$ and $\delta = 0.05$.

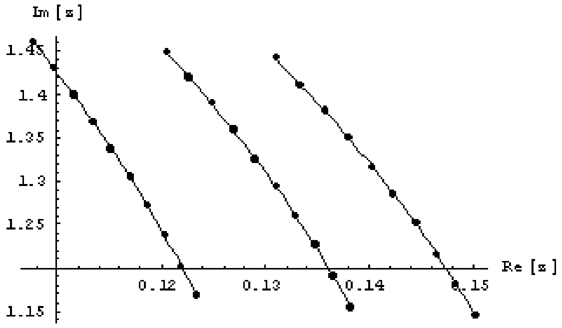

Fig. 2. The change in impedance (30) as a function of γ

IV. CONCLUSIONS

The formula for the change in impedance of a double conductor line located above a two-layer conducting medium is derived in the paper. The magnetic permeability and electric conductivity of the upper layer are exponential functions of the vertical coordinate. Analytical solution of the problem is obtained by means of the method of Fourier sine and cosine transforms. Computational results are presented for different values of the parameters of the problem. Calculations are performed with "Mathematica".

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REFERENCES

[1] C.V. Dodd and W.E. Deeds, "Analytical solutions to eddy-current probe-coil problems", Journal of Applied Physics, vol.39, pp. 2829-2838, 1968.

[2] C.-C. Tai, H.-C. Yang and Y-H. Liu, "Modeling the surface condition of ferromagnetic metal by the swept-frequency eddy current method", IEEE Transactions on Magnetics, vol.38, pp. 205-210, 2002.

[3] C.-C. Tai, "Characterization of coatings on magnetic metal using the swept-frequency eddy current method", Review of Scientific Instruments, vol.71, pp. 3161-3167, 2000.

[4] E. Uzal and J.H. Rose, "The impedance of eddy current probes above layered metals whose conductivity and permeability vary continuously", IEEE Transactions on Magnetics, vol.29, pp. 1869-1873, 1993.

[5] G.A. Kasimov and Yu.V. Kulaev, "An applied electromagnetic transducer above the object of inspection with the electrical and magnetic properties of the material changing with depth", The Soviet Journal of Nondestructive Testing, vol.14, pp. 550-552, 1978.

[6] E. Uzal, J.C. Moulder, S. Mitra, and J.H. Rose, "Impedance of coils over layered metals with continuously variable conductivity and permeability: Theory and experiment", Journal of Applied Physics, vol.74, pp. 2076-2089, 1993.

[7] M.Ya Antimirov, A.A. Kolyshkin, and R. Vaillancourt, *Mathematical models for eddy current testing*, CRM, Montreal, Canada, 1997.

[8] V. Koliskina and I. Volodko, "Impedance of a coil above a half-space with varying electric and magnetic properties", Scientific Proceedings of RTU, vol.51, pp. 44-48, 2009.

[9] V. Koliskina, "Double conductor line above a half-space with varying electric and magnetic properties", *Proceedings of the 10th international conference on nonlinear systems and wavelet analysis*, Sousse, Tunisia, May 2010, pp. 104 – 108.

[10] V.V Vlasov and V.A. Komarov, "Interaction of the magnetic field of a long single-turn loop with a conducting ferromagnetic cylinder", The Soviet Journal of Nondestructive Testing, vol.8, pp. 433-440, 1972.

[11] A.D. Polyanin and V.F. Zaitsev, *Handbook of exact solutions for ordinary differential equations*, Chapman&Hall/CRC, Florida, USA, 2003.

Valentina Koliskina was born in Liepaja, Latvia in 1954. She received the diploma in applied mathematics from the Riga Technical University, Riga, Latvia in 1976. She is now a PhD student at the Department of Engineering Mathematics at the Riga Technical University. Her research interests include mathematical models of eddy current testing problems. She is a member of the Latvian mathematical society.
E-mail:

Inta Volodko was born in Kraslava, Latvia in 1965. She received the diploma in applied mathematics from the Latvian University, Riga, Latvia in 1988. She is now a professor and a head of Department at the Department of Engineering Mathematics at the Riga Technical University. Her research interests include mathematical models of eddy current testing problems. She is a member of the Board of the Latvian mathematical society and a member of Latvian Scientific Union.
E-mail:

Valentīna Koliškina, Inta Volodko. Divvadu līnija virs vadošas divslāņu vides ar mainīgu elektrisko vadāmību un magnētisko caurlaidību

Rakstā iegūta divvadu līnijas impedances izmaiņas formula, kad divvadu līnija atrodas virs vadošas divu slāņu vides. Divvadu līnija sastāv no diviem bezgalīgi gariem paralēliem vadiem ar mainīgu strāvu. Tiek pieņemts, ka vadošās vides augšējā slāņa parametri (elektriskā vadāmība un magnētiskā caurlaidība) ir eksponenciāli atkarīgas no vertikālās koordinātas. Vienādojumu sistēma vektorpotenciāla noteikšanai atrisināta analītiski, pielietojot Furjē integrālās transformācijas metodi. Parasts diferenciālvienādojums augšējā vadošā slānī ar mainīgiem elektriskiem un magnētiskiem parametriem ir atrisināts ar Beseļa funkciju palīdzību. Atrisinājums ir iegūts kā neīstais integrālis no funkcijas, kas satur kompleksa argumenta modificētās Beseļa funkcijas. Formula, pēc kuras var aprēķināt impedances izmaiņu uz divvadu līnijas vienu garuma vienību, iegūta, integrējot inducēto vektorpotenciālu pa divvadu līnijas kontūru. Skaitliskie aprēķini impedances izmaiņai atkarībā no uzdevuma parametriem iegūti, izmantojot programmu paketi „Mathematica". Iegūto analītisko atrisinājumu vektorampotenciālam vadošā vidē var vispārināt daudzslāņu vides gadījumam.

Валентина Колышкина, Инта Володко. Двухпроводная линия над проводящей двухслойной средой с переменной электропроводностью и магнитной проницаемостью

В статье получена формула для расчета изменения импеданса двухпроводной линии, расположенной над проводящей двухслойной средой. Двухпроводная линия состоит из двух бесконечно длинных параллельных проводов с переменным током. Предполагается, что параметры верхнего слоя проводящей среды (электропроводность и магнитная проницаемость) экспоненциально зависят от вертикальной координаты. Система уравнений для вектор-потенциала решена аналитически с помощью метода интегрального преобразования Фурье. Обыкновенное дифференциальное уравнение в верхнем проводящем слое с переменными электрическими и магнитными свойствами решено с использованием функций Бесселя. Решение для вектор-потенциала получено в виде несобственного интеграла, содержащего модифицированные функции Бесселя комплексного аргумента. Формула для расчета изменения импеданса на единицу длины двухпроводной линии получена интегрированием индуцированного вектор-потенциала по контуру двухпроводной линии. Проведены числовые расчеты изменения импеданса в зависимости от параметров задачи с использованием пакета „Mathematica". Полученное в статье аналитическое решение для вектор-потенциала в проводящей среде может быть обобщено на случай многослойной среды.