Joining Pointwise Geological Data to Interpolation Grids if the Data Search Area is a Circle

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Abstract **– Pointwise geological data σ_{in} are used as initial information for creating hydrogeological models (HM). Interpolation methods are applied to create σ-maps on (xy)–grids of HM. The maps represent geometrical and physical features of geological layers. If solutions of boundary field problems are used as interpolation results then σ_{in} represent boundary conditions. They must be joined to nodes of the interpolation grid. In this paper the case is considered when the local data search region presents a circle. Features of this approach are investigated if the circle radius changes from zero to two plane steps of a uniform (xy)–grid.**

Keywords **- data interpolation, hydrogeological models**

I. Introduction

To simplify explanations, the normalized uniform (x, y) – grid is considered where the plane approximation steps hx=hx=1.0. For the node p0, the local normalized search area is the 2×2 square enclosed by the eight neighbouring nodes p1, p2, … p8 (see Fig. 1). The node p0 represents the origin of the local $\bar{x} = x_0 - x$, $\bar{y} = y_0 - y$ local coordinate system where x0 , y0 and x, y are coordinates of the node p0 and the datum point σin , accordingly.

In the publication [1], three forms of data search regions were considered (a square, a circle, an area enclosed by arcs of hyperbola). In this publication, the region of the circle form is considered for the case when the total search area is enlarged to the 4×4 square. The local search area is defined by the expression:

$$r = \sqrt{\bar{x}^2 + \bar{y}^2}\ , \quad 2 \geq r \geq 0 \tag{1}$$

where r is the circle radius.

In Fig. 1, two quadrants corresponding to the cases r =1.0 and $r = \sqrt{2}$ are shown. If r =1.0, four types of data "give out" G–regions L1, L2, L3, L4 exist and features of these regions have been investigated if $0 \leq r \leq 1.0$ [1]. The numbers enclosed into brackets that follow the symbol Li of Fig. 1 point out indices p of nodes where data are sent. For example, L4(7,4,0,3) shows that data from this L4–region are sent to nodes p_7, p_4, p_0, p_3. If $r \geq$ 1.0 then much more complex distributions of the G–regions appear (see Fig. 1, the case $r = \sqrt{2}$). Any G–region is enclosed by arcs of circles (four circles if r =1.0). If r >1.0, the number of interfering circles enlarges and the number of possible different Li regions also increases. For the case $r = \sqrt{2}$ example, the regions L4, L5, L6, L7 exist (interference of 14 circles). In order to simplify graphical images of Li distributions, some groups of Li are united into more general G–regions with a common figure i. For the example $r = \sqrt{2}$, the L5, L6, L7 regions conditionally constitute the L6 region (see Fig. 2, $r = \sqrt{2}$). Objectives of the research are to examine images and sizes of general Li–regions, if $2 \geq r \geq 0$.

II. Images of General G–Regions Within the 1×1 Area

It is much simpler to consider images of the general G–regions within the 1×1 area, where the sizes Li of the general regions meet the formula:

$$\sum_{i=0}^{9} Li = 1.0 \tag{2}$$

where i=0 presents the L0–region size unreachable for data search (r<0.707); i=9 corresponds to the general L9 area size which appears when $r > \sqrt{2}$.

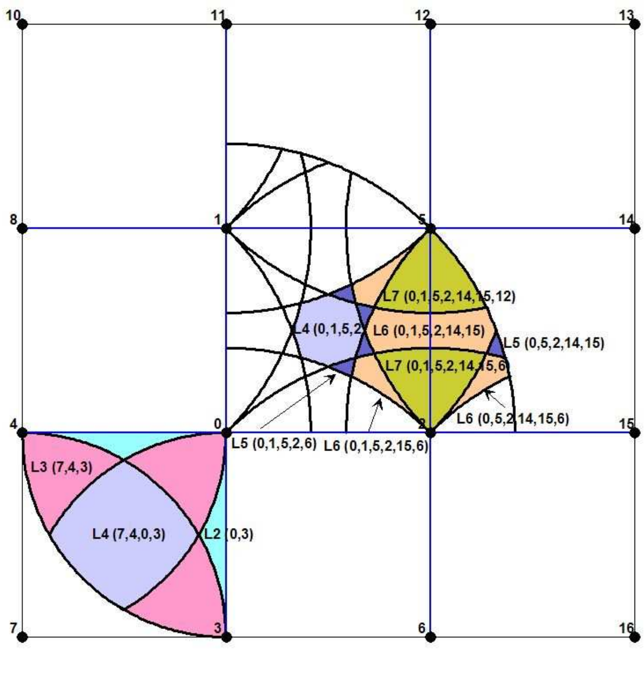

Fig. 1. Distributions of Li regions if r=1.0 (3 rd quadrant) and $r = \sqrt{2}$ (1 st quadrant)

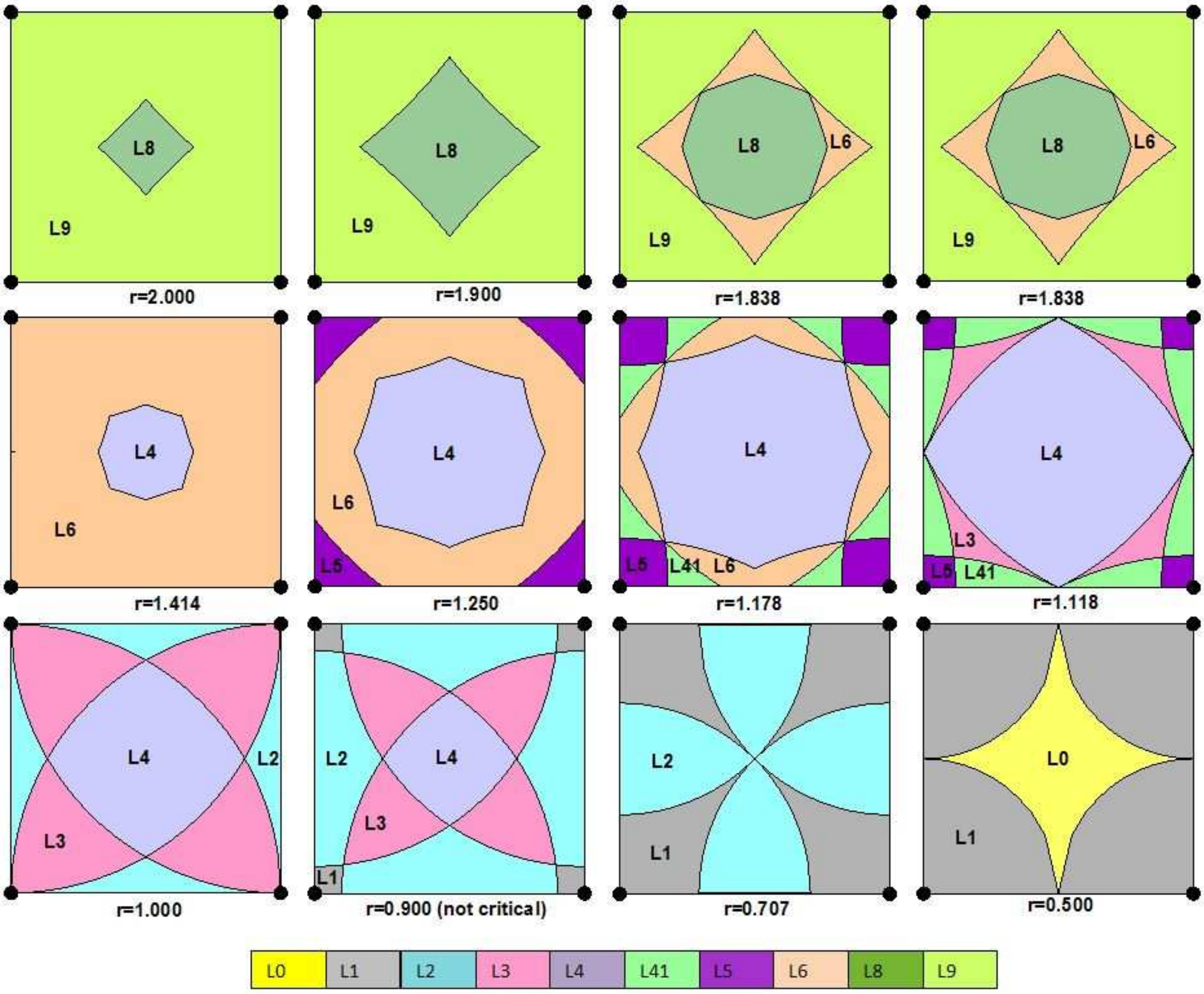

Fig.2. Images of general Li regions for some critical values of the radius *r*

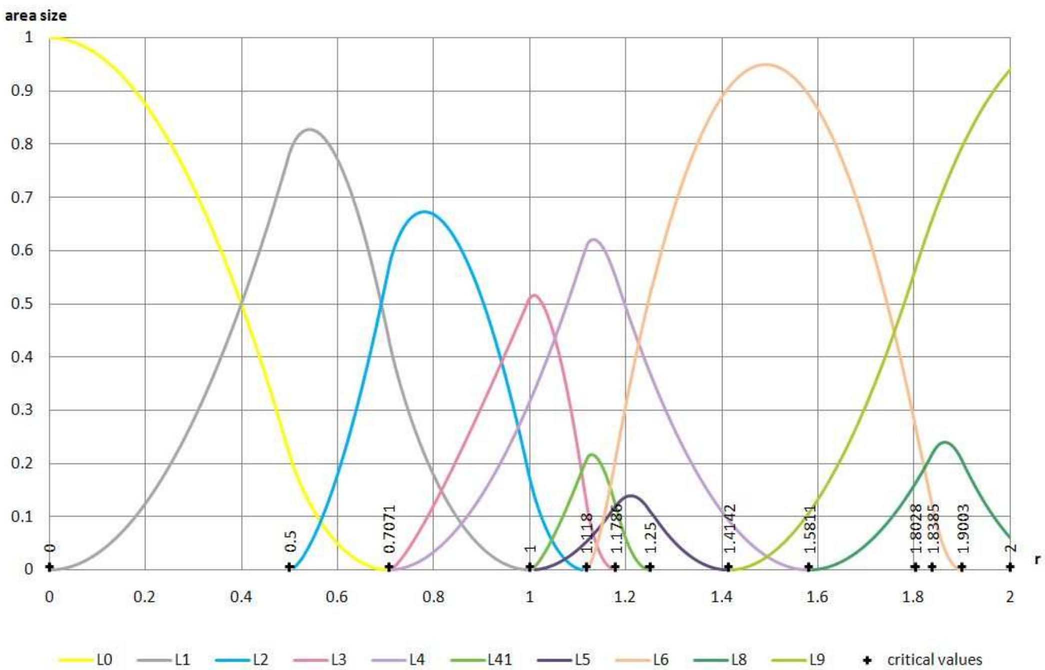

Fig. 3. Area sizes of general Li regions within the 1×1 area, versus the radius *r*

TABLE 1
GENERAL FORMULAS USED FOR COMPUTING AREAS OF L0, L1,… , L9 VERSUS *R*

Zone	L0	L1	L2	L3	L4	L41	L5	L6	L9	L8
zone 1 $0 \le r \le 0.5$	1-L1	πr^2								
zone 2 $05 \le r \le 0.7071$	1-L_Σ	πr^2-2L2	$\pi r^2 + A^1_{0.5}$							
zone 3 $0.7071 \le r \le 1.0$		B_{a1}	4-2L1- $A_{0.5}$	1- L_Σ	1+ $A^1_{0.5}$					
zone 4 $1.00 \le r \le 1.118$			$2A_{x1}$ -4-$A_{0.5}$	1- L_Σ	A_{x0}–0.5$A_{0.5}$	2(A_{x1}-4-$A_{0.5}$)	B_{a2} – 5			
zone 5 $1.118 \le r \le 1.1786$				L5-C_{x2}	C^1_{x2}- $A_{0.5}$	A^1_{x1}–L3-L5	B_{a2} – 5	1- L_Σ		
zone 6 $1.1786 \le r \le 1.25$					B_{a2} - $A_{0.5}$	$A_{1.0}$ - C_{x2}	C_{x2}	1- L_Σ		
zone 7 $1.25 \le r \le 1.4142$					B_{a2} - $A_{0.5}$		A^1_{x1}	1- L_Σ		
zone 8 $1.4142 \le r \le 1.5811$					B_{a2} - $A_{0.5}$			1- L_Σ	A^1_{x1}	
zone 9 $1.5811 \le r \le 1.8028$								1- L_Σ	A^1_{x1}	B_{a3} - $A_{0.5}$
zone 10 $1.8028 \le r \le 1.8385$								1- L_Σ	A_{x3}	B_{a3} - $A_{0.5}$
zone 11 $1.8385 \le r \le 1.9003$								C_{x4}	A_{x3}	1- L_Σ
zone 12 $1.9003 \le r \le 2.0000$									A_{x3}	1-L9

L_Σ - sum of computed areas Li within a zone; for example: L0=1- L_Σ =1-L1-L2 (zone 2). Symbols of types *A, B, C* correspond to formulas of Table 2.

TABLE 2
FORMULAS CORRESPONDING TO SYMBOLS OF TYPES A, B, C USED IN TABLE 1

$A(t) = 2(t\sqrt{r^2 - t^2} + r^2 \arcsin(t/r)$ - general expression

Symbols of table 1	**Parameter**	**zones**
$A_{05} = 2A(0.5)$	$t = 0.5$	$r \ge 0.5$
$A^1_{05} = \pi r^2 - 2A(0.5)$		
$A_{x0} = A(x_0) - 2x_0 + 1$	$t = x_0 = 0.5\sqrt{4r^2 - 1}$	$r \ge 0.5$
$A_{1.0} = 4 + \pi r^2 - 2A(1.0)$	$t = 1.0$	$r \ge 1.0$
$A_{x1} = A(x_1) - 4x_1 + 4$	$t = x_1 = \sqrt{r^2 - 1}$	$r \ge 1.0$
$A^1_{x1} = A(x_1) - 4x_1 + 4 - A(1.0)$		
$C_{x2} = 4 + \pi r^2 - 16x_2 - 2(A(1 - x_2) - A(x_2))$	$t = x_2 = 0.5 - \sqrt{0.8r^2 - 1}$	$r \ge 1.25$
$C^1_{x2} = 1 + \pi r^2 - 16x_2 - 2(A(1 - x_2) - A(x_2))$		
$A_{x3} = A(1.5) + 6x_3 - 8 - A(x_3)$	$t = x_3 = \sqrt{r^2 - 2.25}$	$r \ge 1.5$
$C_{x4} = 2A(0.5) - 2A(1.5) + 12 - 24x_4 + 2A(1 + x_4) - 2A(1 - x_4)$	$t = x_4 = \sqrt{(9r^2 - 29.25)/13)}$	$r \ge 1.8072$
$B_{a1} = 4(a_1 - r^2 \arcsin(a_1 / r))$	$a_1 = 0.5(1 - \sqrt{2r^2 - 1})$	$1 \ge r \ge 1/\sqrt{2}$
$B_{a2} = 4(r^2 \arcsin(a_2 / r) - a_2) + 5$	$a_2 = 0.5(1 - \sqrt{2r^2 - 1} - 1)$	$r \ge 1.0$
$B_{a3} = 4(r^2 \arcsin((1 - a_3) / r) + a_3) + 1$	$a_3 = 0.5(3 - \sqrt{2r^2 - 1})$	$r \ge \sqrt{2.5}$
Auxiliary terms: $A(0.5) = 0.5(\sqrt{4r^2 - 1} + 2r^2 \arcsin(0.5 / r)$, $A(x_0) = 0.5\sqrt{4r^2 - 1} + 2r^2 \arcsin(\sqrt{4r^2 - 1} / 2r)$, $A(1.0) = 2(\sqrt{r^2 - 1} + r^2 \arcsin(1 / r))$, $A(x_1) = 2(\sqrt{r^2 - 1} + r^2 \arcsin(\sqrt{r^2 - 1} / r)$, $A(1.5) = 3\sqrt{r^2 - 2.25} + 2r^2 \arcsin(1.5 / r)$, $C(x_2) = 4((1 - x_2)\sqrt{r^2 - (1 - x_2)^2} - x_2\sqrt{r^2 - x_2^2} + r^2 \arcsin((1 - x_2) / r) - r^2 \arcsin(x^2 / r))$, $A(x_3) = 3\sqrt{r^2 - 2.25} + 2r^2 \arcsin(\sqrt{r^2 - 2.25} / r)$, $A(1 + x_4) = 2((1 + x_4)\sqrt{r^2 - (1 + x_4)^2} + r^2 \arcsin((1 + x_4) / r))$, $A(1 - x_4) = 2((1 - x_4)\sqrt{r^2 - (1 - x_4)^2} + r^2 \arcsin((1 - x_4) / r))$		

In Fig. 2, images of general Li–regions are shown for some critical values of *r*. Critical values are borders of zones where different set of general formulas must be used for computing area size of the Li–regions (Table 1).

Table 2 exposes the formulas corresponding to general symbols A, B, C used in Table 1.

As mentioned above, the general Li–regions may include a set of Li–regions, which figure *i* is smaller or larger than the general value of *i* (L5 and L7 in L6 when $r = \sqrt{2}$, Fig. 1). These extra regions may be interpreted as offsprings of the other Li–regions. For the example $r = \sqrt{2}$, the L5 and L7 regions are offsprings of L4 and L6 regions, accordingly. However, development of images for the offsprings are not traced. The only case explored is the L41 region which is the offspring of the L3 region. It exists, if $1.25 \geq r \geq 1$ (also see Table 1).

It follows from considering the images of Fig. 2 that they bear significant alterations when the radius *r* changes. In Fig. 3, the graphs of area sizes for general Li regions are shown if $2 \geq r \geq 0$. On the r-axis, the thirteen critical r values are pointed out. Despite the fact that, within each zone, quite different formulas have been used, (Tables 1, 2) the graphs of Fig. 3 are smooth. Most of them have points of maximum and minimums. It follows from Fig. 3 that the L6 size dominates over the other ones if *r*~1.5. However, the generalized L6 region also contains regions of the types L5 and L7. For this reason an exact pattern of data distribution from the L6 region is not known. The other generalized regions L8, L9 also includes untraced "offspring" regions. The data release pattern is exactly known if $1 \geq r \geq 0$. The recommended value for local data search is $r \leq 0.99$ [1]. If $r \geq 1$, then the data spread out pattern becomes very complex.

CONCLUSIONS

Investigation of local data search for local interpolation purposes has been carried out if the data search region has a circle form. The graphical images and their area sizes of data "give out" G–regions have been obtained if the circle radius *r* has values $2 \geq r \geq 0$. If $r \geq 1$ then patterns of data spread out becomes very complex and this case cannot be recommended for the local data search. This regime may be of interest if it is used for other specific purposes (image processing, data smoothing et.c.)

REFERENCES

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A. Spalviņš, I. Lāce. Punktveida ģeoloģisko datu piesaiste interpolācijas režģim, ja datu meklēšanas apgabals ir aplis.
Punktveida ģeoloģiskos datus σ_{in} izmanto hidroģeoloģisko modeļu (HM) veidošanai. Lai iegūtu σ - kartes HM (xy) – režģiem, tiek izmantotas interpolācijas metodes. Šīs kartes apraksta ģeoloģisko slāņu fizikālās un ģeometriskās īpašības. Ja kā interpolācijas rezultātu izmanto lauku teorijas robežproblēmu atrisinājumu, tad σ_{in} tiek izmantoti kā pirmā veida robežnoteikumi. Tos ir jāpiesaista interpolācijas režģa mezgliem. Publikācijā analizēts gadījums, ja lokālais datu meklēšanas apgabals ir aplis. Šī gadījuma īpašības izpētītas, ja apļa rādiuss mainās no nulles līdz homogēna (xy) – režģa diviem plaknes soļiem. Noskaidrots, ka ja meklēšanas apļa rādiuss pārsniedz režģa plaknes soļa garumu, tad datu izmantošanas apgabalu formas kļūst ļoti komplicētas. Rakstā doti šo formu grafiskie attēli kritiskajām rādiusu vērtībām, ja rādiuss mainās robežās no nulles līdz diviem režģa plaknes soļiem. Tabulu veidā dotas formulas, kuras izmanto minēto formu laukuma aprēķiniem. Konstatēts, ka nav ieteicams lokālās interpolācijas veikšanai izmantot meklēšanas rādiusu, kurš ir lielāks par režģa soli.

А. Спалвинь, И. Лаце. Привязка точечных геологических данных к интерполяционным сеткам, если область поиска данных имеет форму круга.
Точечные геологические данные σ_{in} применяются для построения гидрогеологических моделей (ГМ). Путем обработки интерполяционными методами, получают σ - карты для (xy) – сеток ГМ. Эти карты характеризуют физические и геологические свойства горизонтов. Если как результат интерполяции используется решение краевых задач теории поля, то σ_{in} принимаются в качестве граничных условий первого рода, которые необходимо привязать к узлам интерполяционной сетки. В этой публикации анализируется случай, когда область локального поиска данных имеет форму круга. Свойства этого варианта исследуются для области изменения радиуса круга от нуля до двух шагов однородной (xy) сетки. Установлено, что если радиус круга поиска данных превышает длину одного шага сетки, то формы областей использования исходных данных очень усложняются. Приведены рисунки этих форм для критических значений радиуса поиска, если тот меняется от нуля до величины двух шагов сетки. В виде таблиц даны формулы, с помощью которых можно определить площадь этих форм. Сделан вывод, что не рекомендуется для выполнения локальной интерполяции применять радиус поиска превышающий один шаг сетки.