

# Economic Forecasts with Bayesian Autoregressive Distributed Lag Model: Choosing Optimal Prior in Economic Downturn

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**Abstract.** Bayesian inference requires an analyst to set priors. Setting the right prior is crucial for precise forecasts. This paper analyzes how optimal prior changes when an economy is hit by a recession. For this task, an autoregressive distributed lag model is chosen. The results show that a sharp economic slowdown changes the optimal prior in two directions. First, it changes the structure of the optimal weight prior, setting smaller weight on the lagged dependent variable compared to variables containing more recent information. Second, greater uncertainty brought by a rapid economic downturn requires more space for coefficient variation, which is set by the overall tightness parameter. It is shown that the optimal overall tightness parameter may increase to such an extent that Bayesian ADL becomes equivalent to frequentist ADL. The results may be used in other fields of science where it is necessary to estimate/predict a process using Bayesian inference.

**Keywords:** Bayesian inference, Bayesian autoregressive distributed lag model, forecasting, Litterman prior, optimal prior.

## I. INTRODUCTION

Bayesian inference requires an analyst to set a prior. Setting the right prior is crucial for precise forecasts. This paper analyzes how optimal Litterman prior changes when an economy is hit by a recession. By an 'optimal Litterman prior' in this paper I define Litterman hyperparameters that minimize the root mean squared error from one-period ahead forecasts.

Although the question about what hyperparameters to use has been addressed in a series of papers by, among others, Litterman and coauthors (Litterman (1979), Doan, Litterman and Sims (1984), Litterman (1986)) and LeSage and coauthors (LeSage and Magura (1991), LeSage and Pan (1995), LeSage and Krivelyova (1999)), the role of a business cycle on the optimal prior, to the best of my knowledge, has not been discussed. Thus, this paper analyzes how (if any) prior hyperparameters should be altered for the best one-period ahead forecasting performance when there is a switch in a phase of a business cycle. For this task, an autoregressive distributed lag model (ADL) is chosen. The prior is set up like in Litterman (1979). The model is solved by 'mixed estimation' set forth in Theil and Goldberger (1961). Latvia's gross domestic product (GDP) was found to be well suited for the analysis. The results show that a sharp economic slowdown changes the optimal prior in two directions.

First, a lagged dependent variable loses its dominance as the key explanatory variable and, instead, more current information contained in leading indicator-type variables is of

greater importance to improve forecasts. This changes the structure of the optimal weight prior, setting smaller weight on the lagged dependent variable compared to variables containing more recent information.

Second, greater uncertainty brought by a swift economic downturn requires more space for coefficient variation, which is set by the overall tightness parameter. Particularly, the results show that, in economic downturn, the optimal overall tightness parameter may increase to such an extent that Bayesian ADL becomes equivalent to frequentist ADL, which may imply that a greater uncertainty in an economy requires more skills from an analyst to set the right prior such that, during great economic uncertainty, one may become more comfortable using frequentist rather than Bayesian inference.

The paper is organized as follows. Section 2 describes the model and its estimation procedure. Section 3 presents the results from a case study. Finally, Section 4 concludes.

## II. METHODOLOGY

### A. The Model

Consider an autoregressive distributed lag model (ADL) of order  $(p, q)$ :

$$y_t = \sum_{m=1}^p \beta_m y_{t-m} + \sum_{n=0}^q \gamma'_n x_{t-n} + \xi'_t z_t + \varepsilon_t \quad (1)$$

where  $y_t$  is the dependent variable,  $x_t$  is a  $d \times 1$  vector of key explanatory variables  $x = [x_1 \ x_2 \ \dots \ x_d]$ ,  $z_t$  is (a vector of) other explanatory variable(s) potentially containing a constant, a dummy variable for an outlying effect, etc., and  $\varepsilon_t \sim N(0, \sigma^2)$ . The Bayesian prior is set to

$$\begin{aligned} \beta_m &: N(\mathbb{1}_{\{1\}}(m), \sigma_m^2) \\ \gamma_{i,n} &: N(0, \sigma_{i,n}^2) \end{aligned} \quad (2)$$

where  $\mathbb{1}_{\{1\}}()$  is an indicator function,  $m = 1, 2, \dots, p$ ,  $i = 1, 2, \dots, d$ , and  $n = 0, 1, \dots, q$ . The specification of the standard deviation of the prior is à la Doan, Litterman and Sims (1984):

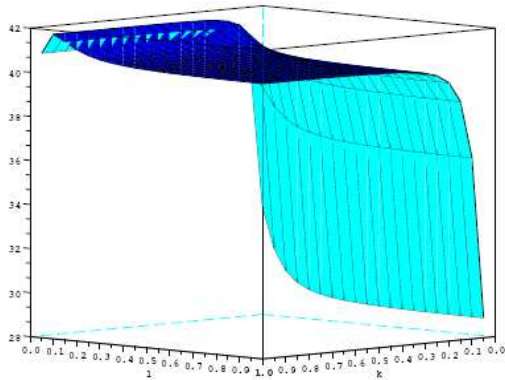
$$\begin{aligned} \sigma_m &= \theta k m^{-\phi} \\ \sigma_{i,n} &= \theta l (1+n)^{-\phi} \left( \frac{\hat{\sigma}_{u,i}}{\hat{\sigma}_{u,y}} \right) \end{aligned} \quad (3)$$

where  $\hat{\sigma}_{u,y}$  and  $\hat{\sigma}_{u,i}$  are the standard errors from a univariate autoregression involving  $y$  and  $x_i$ , respectively, so that  $\hat{\sigma}_{u,i} / \hat{\sigma}_{u,y}$  is a scaling factor that adjusts for varying

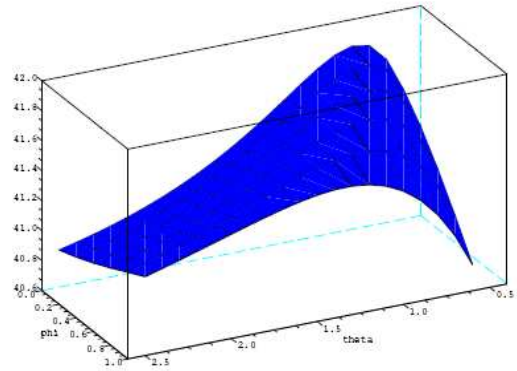


TABLE I  
A BRIEF COMPARISON OF SARMA, AR, FADL AND BADL.

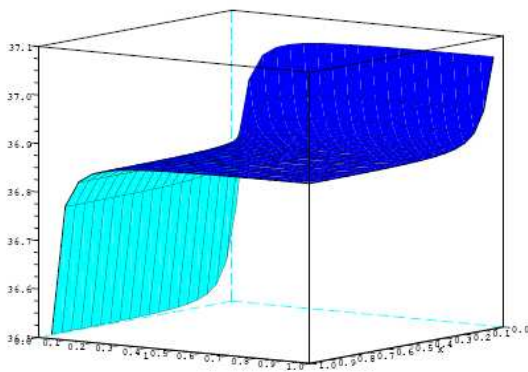
Model	RMSE	RMSE1sthalf	RMSE2ndhalf
SARMA(01)(01)	0.0328737	0.0160291	0.0436398
AR(1)	0.0275043	0.0194567	0.0336810
AR(2)	0.0263058	0.0203990	0.0311106
FADL(1,0)(D)	0.0277540	0.2011203	0.0330832
FADL(2,0)(D)	0.0289995	0.0272706	0.0306310
FADL(2,1)(D)	0.0253833	0.0196827	0.0300202
FADL(2,1)(E)	0.0257016	0.0216257	0.0292142
FADL(2,1)(D+E)	0.0247125	0.0220415	0.0271218
FADL(3,2)(D)	0.0260984	0.0216730	0.0298754
FADL(3,2)(E)	0.0257382	0.0217008	0.0292230
FADL(3,2)(D+E)	0.0253316	0.0251711	0.0254912
BADL(2,1)(D+E)(.95,.1,.8,0)	0.0239113	0.0196482	0.0275217
BADL(2,1)(D+E)(.05,1,2,0)	0.0264237	0.0258526	0.0269828
BADL(3,2)(D+E)(1,.35,.2,0)	0.0223288	0.0171109	0.0265400
BADL(3,2)(D+E)(.8,.25,.2,0)	0.0225414	0.0166686	0.0271732



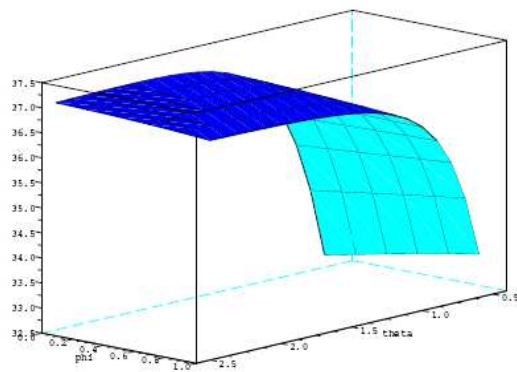
(a) Full sample. Optimal  $k = .95$  and optimal  $l = .1$ .



(b) Full sample. Optimal  $\theta = .9$  and optimal  $\phi = 0$ .

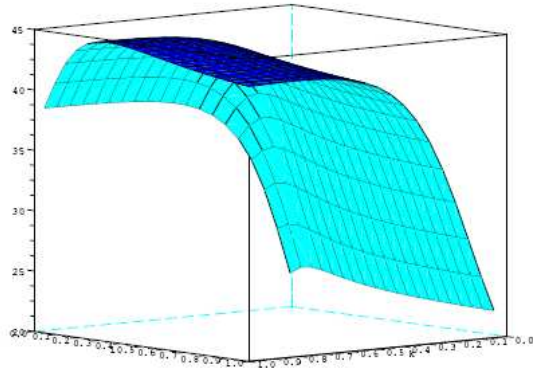


(c) Second half of the sample. Optimal  $k = .05$  and optimal  $l = 1$ .

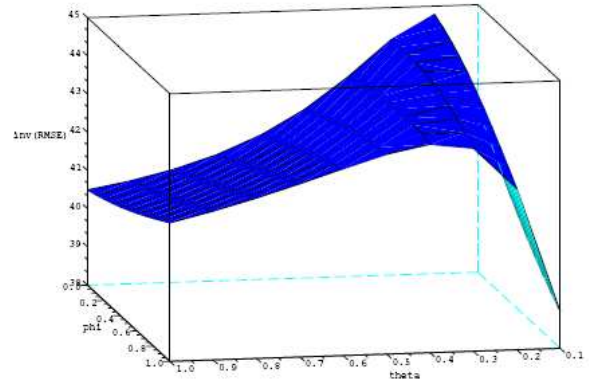


(d) Second half of the sample. Optimal  $\theta = 2$  and optimal  $\phi = 0$ .

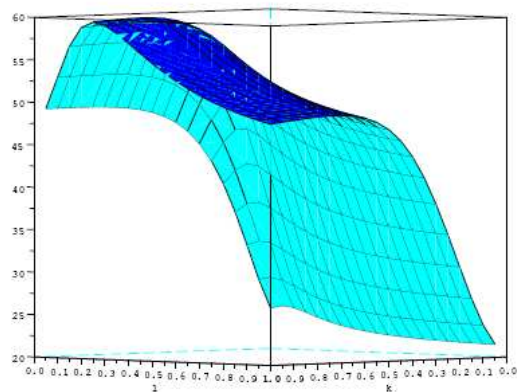
Fig. 1. Results from grid search for optimal prior for BADL(2,1)(D+E). Fig. 1a and 1b represent a full sample, whereas Fig. 1c and 1d represent the second half of the sample. The figures on the left (1a and 1c) show  $RMSE^{-1}$  (z axis) as a function of a weight vector  $(k,l)$  (x and y axis, respectively) at the RMSE-minimizing  $\theta$  and  $\phi$ . The figures on the right (1b and 1d) show  $RMSE^{-1}$  (z axis) as a function of  $\theta$  and  $\phi$  (x and y axis, respectively) at the RMSE-minimizing weight vector.



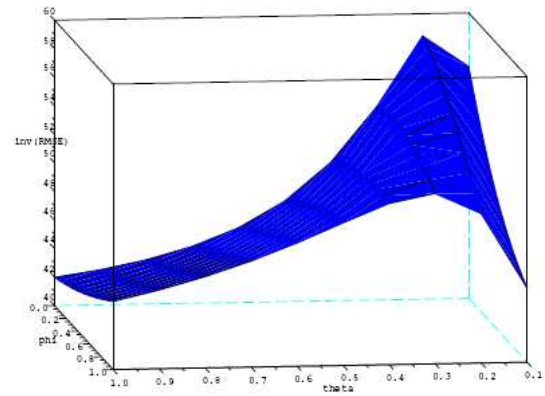
(a) Full sample. Optimal  $k = 1$  and optimal  $l = .35$ .



(b) Full sample. Optimal  $\theta = .2$  and optimal  $\phi = 0$ .



(c) First half of the sample. Optimal  $k = .8$  and optimal  $l = .25$ .



(d) First half of the sample. Optimal  $\theta = .2$  and optimal  $\phi = 0$ .

Fig. 2. Results from grid search for optimal prior for BADL(3,2)(D+E). Fig. 2a and 2b represent a full sample, whereas Fig. 2c and 2d represent the first half of the sample. The figures on the left (2a and 2c) show RMSE-1 (z axis) as a function of a weight vector ( $k, l$ ) (x and y axis, respectively) at the RMSE-minimizing theta and phi. The figures on the right (2b and 2d) show RMSE-1 (z axis) as a function of theta and phi (x and y axis, respectively) at the RMSE-minimizing weight vector.

The least RMSE in each column is framed. It can be seen that Bayesian ADL models compare well with other models. It can also be seen that the BADL(3,2) models give the most precise one-period ahead forecasts for the whole sample as well as for the first half of the sample among all the ADL models considered, but they are outperformed by FADL for the second half of the model. This observation suggests that the optimal Bayesian prior might be different for the first half of the model (smooth positive growth) compared to the second half of the sample when there is a rapid economic downturn. We check this hypothesis further by employing grid search for the optimal prior.

#### B. Search for optimal priors

First, the grid search is performed for BADL(2,1)(D+E). The weight vector  $[k \ l]$  is 2-dimensional, one element,  $k$ , for the dependent variable and one,  $l$ , for a single explanatory variable  $x$ , both ranging from .05 to 1 with step size .05. The

overall tightness,  $\theta$ , is set to range from .6 to 2.5 with step .1, and the lag decay,  $\phi$ , from 0 to 1 with step .2. So, the grid size is  $20 \times 20 \times 20 \times 6$  containing overall 48000 prior combinations for each one-period ahead forecast with sample size ranging from 17 to 51. The minimum RMSE for the whole sample is attained at the coordinate  $[19 \ 2 \ 3 \ 1]$  with the corresponding values  $[k \ l \ \theta \ \phi] = [.95 \ .1 \ .8 \ 0]$  with a boundary value at  $\phi = 0$ . The boundary for  $\phi$  can not be decreased further since negative values would presume lags of a higher order be more informative which is counterintuitive. Figures 1a and 1b show the inverse of the RMSE as a function of the prior for the whole sample.

Figure 1a shows the inverse of the RMSE as a function of the weight vector (the x and y axes represent  $k$  and  $l$ , respectively) given the rest of parameters,  $\theta$  and  $\phi$ , at their RMSE-minimizing values. It can be seen that the values of  $k$  have the major impact on the RMSE with acceptable range

about (4,1), otherwise the RMSE increases substantially. On the contrary, values of  $l$  have less influence on the RMSE given  $k$ , nonetheless, a peak is evident at  $l = .1$  for all acceptable values of  $k$ .

Similarly, Figure 1b shows the inverse of the RMSE as a function of  $\theta$  and  $\phi$  (representing x and y axes, respectively) given the RMSE-minimizing weight vector. It can be seen that the values of both  $\theta$  and  $\phi$  have a nontrivial impact on RMSE at its optimum with the maximizing values .8 and 0, respectively. The maximizing value of  $\phi = 0$  might be due to the small number of lags, which is one for each RHS variable in this model.

Now, calculating the minimum RMSE for the second half of the sample, the optimum value is attained at the coordinate [1 20 15 1] with the corresponding values [ $k$   $l$   $\theta$   $\phi$ ] = [.05 1 2 0] with three boundary values for  $k$ ,  $l$  and  $\phi$ . It can already be seen that the optimal prior weight is different compared to the full sample. Figures 1c and 1d show the inverse of the RMSE as a function of the prior for the second half of the sample. Figure 1c looks almost like the inverse of Figure 1a. Now, the RMSE is increasing with  $k$ , with an optimum at the lowest  $k$  considered; other values of  $k$  would significantly increase the RMSE at all levels of  $l$ , the latter being also critical for optimal RMSE with acceptable range about (.3,1), otherwise the forecast error increases substantially. This observation is in line with our hypothesis that, during sharp decline in the economy, explanatory variables containing most recent information are more important than the lagged dependent variable.

Figure 1d shows that, for the second half of the sample, the optimal tightness parameter is higher compared to the full sample, with acceptable values in about (1,2.5), otherwise the forecast error increases substantially. This observation is as expected since the model coefficients should be given more flexibility during a rapid change in an economy. For acceptable  $\theta$ , the values of lag decay parameter,  $\phi$ , is of less importance. The forecasting performance of BADL(2,1)(D+E) for the first half of the sample is not impressive and thus not presented here.

Having explored BADL(2,1)(D+E), we now check the results for BADL(3,2) (D+E) whose forecasting performance for all sample spaces considered, as it can be seen in Table 1, is promising. The grid space is formed by  $k$  and  $l$  being from .05 to 1 with step .05,  $\theta$  from .1 to 1 with step .1, and  $\phi$  from 0 to 1 with step .1. The coordinate for the least RMSE for full sample is [20 7 2 1] with the prior values [ $k$   $l$   $\theta$   $\phi$ ] = [1 .35 .2 0], showing some resemblance with the results for BADL(2,1)(D+E). The inverse RMSE for full sample around the optimal prior values is shown in Figures 2a and 2b. The behavior of the inverse RMSE around its optimal value is similar to that of BADL(2,1)(D+E).

We can see from Table 1 about the model's BADL(3,2)(D+E) comparatively competitive forecasting performance for the first half of the sample. Figures 2c and 2d show the inverse RMSE around its optimum as a function of prior parameters for the first half of the sample. We see that the results are similar to the results from a full sample with

optimal  $k = .8$ ,  $l = .25$ ,  $\theta = .2$  and  $\phi = 0$ . It can also be seen that  $l$  has more influence on the RMSE compared to the full sample, with lowest RMSE concentrating on the lowest part of  $l$  space.

Regarding the results for the second half of the sample, the coordinate of the optimal value is [20 20 10 1], with all values being at a boundary and suggesting a greater  $\theta$  (i.e., more flexibility for coefficient values). An extensive search for the optimal  $\theta$  resulted to its value around  $10^5$  with RMSE being the same as for FADL(3,2)(D+E) at least up to and including the 7<sup>th</sup> digit after a comma, shown in Table 1. The latter result might suggest that during a sharp decline in an economy one might wish to set the overall tightness parameter,  $\theta$ , so loose that one is more comfortable to use frequentist version of ADL.

#### IV. CONCLUSIONS

Bayesian inference requires an analyst to set priors. Setting the right prior is crucial for precise forecasts. This paper analyzes how optimal prior changes with business cycle, specifically, when an economy is hit by a recession. Latvia's GDP is well suited for this analysis. The results show that when economy is growing, the optimal overall tightness parameter is less than one, and the optimal weight vector sets a higher weight on a lagged dependent variable compared to other explanatory variables. However, a swift economic downturn changes the optimal prior considerably in two directions.

First, a lagged dependent variable loses its dominance as the key explanatory variable and, instead, more current information contained in leading indicator-type variables is of greater importance to improve forecasts. This changes the structure of the weight prior, setting smaller weight on the lagged dependent variable compared to variables containing more recent information.

Second, greater uncertainty brought by a rapid economic downturn requires more space for coefficient variation, which is set by the overall tightness parameter. Particularly, the results show that, in economic downturn, the optimal overall tightness parameter may increase to such an extent that Bayesian ADL becomes equivalent to frequentist ADL, which may imply that a greater uncertainty in an economy requires more skills from an analyst to set the right prior such that, during great economic uncertainty, one may become more comfortable using frequentist rather than Bayesian inference. This inference may be used likewise in other fields of science where it is necessary to estimate/predict a process using Bayesian inference.

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**Ginters Bušs. Ekonomikas prognozēšana ar Beiesa autoregresīvo sadalīto lagu modeli: optimālā priora izvēle ekonomikas lejupslīdes laikā**

Lai veiktu Beiesa analīzi, analītiķim ir nepieciešams noteikt priorus. Pareizo prioru noteikšana ir nozīmīga precīzu prognožu veikšanai. Šis raksts analizē, kā optimālais priors mainās līdz ar biznesa ciklu, konkrēti gadījumā, ja ekonomiku ir skārusi recesija. Latvijas IKP laikrinda ir piemērota šādai analīzei. Šim nolūkam tika izvēlēts autoregresīvais sadalīto lagu modelis. Rezultāti rāda, ka normālos apstākļos, kad kustības trajektorija ir labi paredzama, vispārējais ciešuma parametrs ir mazāks par vienu, un optimālā priora svārstību vektors piešķir lielāku svaru atkarīgajam novēlotajam mainīgajam nekā parējiem izskaidrojošajiem mainīgajiem. Strauja ekonomikas lejupslīde izmaina optimālo prioru, galvenokārt, divos veidos. Pirmkārt, atkarīgais novēlotais mainīgais zaudē savas izskaidrošanas spējas, līdz ar ko tiek izmainīta optimālā priora svārstību vektora struktūra, nosakot mazāku svaru atkarīgajam mainīgajam salīdzinot ar izskaidrojošajiem mainīgajiem, kas satur jaunāku informāciju. Otrkārt, straujas ekonomiskās lejupslīdes radīta lielāka neziņa par nākotni prasa lielāku brīvību koeficientu variācijai, ko nosaka vispārējais ciešuma parametrs. Rezultāti rāda, ka lielas neziņas apstākļos optimālais vispārējais ciešuma parametrs var pieaugt tādā mērā, ka Beiesa ADL kļūst vienāds ar frekventistu ADL, kas nozīmē, ka lielas neziņas apstākļos analītiķim ir nepieciešamas lielākas iemaņas Beiesa priora izvēlē, un ka analītiķim varētu būt vieglāk piemērot frekventistu metodi, nevis Beiesa metodi. Šo secinājumu var izmantot ne tikai ekonomikas nozares speciālisti, bet jebkuras nozares speciālisti, kas saskaras ar problemātiku prognozēt procesa vai objekta trajektoriju.

**Гинтерс Буш. Прогноз экономики с помощью Байесовской модели авторегрессионного распределенного лага: выбор оптимального априорного распределения в случае экономической рецессии**

Байесовский подход требует от аналитика выбора априорного распределения. Этот выбор имеет важное значение для точного прогноза. Эта статья анализирует, как оптимальные априорные распределения изменяются, когда экономика находится в рецессии. Для этой задачи, выбрана модель авторегрессионного распределенного лага. Валовой внутренний продукт Латвии хорошо подходит для проведения такого анализа. Результаты показывают, что, когда экономика растет, оптимальный общий параметр стесненности ниже одного, и оптимальный вектор веса определяющий больший вес на зависимую переменную по сравнению с другими переменными. Резкий экономический спад меняет оптимальных априорных распределения в двух направлениях. Во-первых, зависимая переменная теряет свою силу как главная объясняющая переменная, и потому меняется структура оптимального веса априорных распределения, определяющий меньший вес на зависимую переменную по сравнению с переменными, содержащий более новую информацию. Во-вторых, большая неопределенность, вносимая за счет быстрого экономического спада требует больше возможности для вариации коэффициентов, которых определяют общие параметры стесненности. Показано что, в экономической рецессии, общий параметр оптимальной стесненности может увеличиться до такой степени, что Байесовское АДЛ становится эквивалентной частотным АДЛ, что может означать, что степень неопределенности в экономике требует больше навыков аналитика для установки правого априора в такой степени, что во время беликой экономической неопределенности, можно стать более удобным использование классического а не Байесовского вывода. Этот вывод может быть использован также в других областях науки, где это необходимо прогнозирование процесса с использованием байесовского вывода