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Analysis of a Cavitation for the ESS Target Model

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***Abstract* - In the present work some aspects of cavitation influence on the European Spallation Source (ESS) liquid metal target are investigated. The analysis is based on a calculation of the bubbles' radius change for bubbles moving along a changing pressure field.**

The shape of pressure field is obtained from numerical investigation of fluid flow in the ESS model. A simplified pressure model is used to find how a bubble's radius depends on pressure and temperature. The main goal of this work is to determine conditions (such as bubble initial radius, temperature, pressure) leading to bubble's growth or collapse. In all cases "stability" curves (curves dividing area of bubble growth from area of bubble collapse in dependence of some parameters) are constructed.

***Keywords* –cavitation, target model**

I. Equations

The dynamics of a radius of bubbles was analysed using the Rayleight-Plesset equation in case of so called "inertial controlled" behaviour of vapour bubble (no any other gases inside bubbles) in viscous fluid [1]:

$$R\frac{d^2R}{dt^2}+\frac{3}{2}\left(\frac{dR}{dt}\right)^2+\frac{2\sigma}{\rho R}+\frac{4\mu}{\rho R}\frac{dR}{dt}=\frac{p_s}{\rho}-\frac{p(t)}{\rho}, \qquad (1)$$

where $R(t)$ is the radius of a bubble, σ is the surface tension, μ is the dynamic viscosity and ρ is the density of a fluid; p_s is the saturated vapour pressure and $p(t)$ is the pressure in fluid outside a bubble, t denominates time.

In this case the temperature in the fluid and in the bubble considered equal, no mass transition between a bubble and the outer fluid exists. The surface tension, dynamic viscosity, density and saturated vapour pressure are dependent on temperature but are constant at fixed temperature value. For this approximation the temperature is considered uniform. It is applicable because – as results show – in case of bubble collapsing the lifetime of bubbles is about milliseconds so the temperature does not change significantly. In case of bubble growth the temperature along the path of a bubble is also growing, and these obstacles cause bubble growth.

The initial conditions for (1) are conditions of bubble stable state: the initial radius is R_0, and velocity of bubble wall dR/dt is zero.

The 3 sizes of bubbles were analysed: with the radius 1 mkm (10^{-6} m), 10 mkm (10^{-5} m) and 100 mkm (10^{-4} m). The values for physical parameters of fluid and vapour are taken according to working temperature values for different fluids used in ESS target.

The equation (1) was solved numerically using Wolfram MATHEMATICA 5.2 package. The provided command NDSolve was used to calculate the radius of a bubble and velocity of the bubble wall. As solution method the Automatic method was used. The NDSolve command also provides possibility to use some other methods such as implicit and explicit Runge-Kutta and Adams methods, but comparison of solutions shows that difference between obtained results is about 10^{-12} m, so the Automatic method was used.

II. The ESS Target Model

The model of ESS target is shown on Fig. 1. The calculated pressure distribution is shown on Fig. 2.

To model the pressure field in the target the following ideas was used. First, according to the calculations made for proposed target configuration the pressure in the front part of the model in some regions changes from value of approx. 10^5 Pa to $-2.5\cdot10^4$ Pa very fast, so pressure profile can be approximated by the curve like shown on Fig. 3. This curve can represent as a pressure in a region of a flow as a pressure along whole model depending on time scale. This curve was constructed in the following way. The basic value is the pressure at the inlet (point 1 on curve). In our case it was taken 10^5 Pa. The pressure value at point 2 (termed in this analysis "1st turn-point") is equal to 95% of the pressure at inlet ($9.5\cdot10^4$ Pa). At the 3rd point (termed "middle point") the pressure is 25% of the inlet pressure ($2.5\cdot10^4$ Pa). At the 4th point ("2nd turn-point") pressure is 55% of the inlet pressure ($5.5\cdot10^4$ Pa), and at the 5th point ("half of end-path") – 52.5 % ($5.25\cdot10^4$ Pa). The pressure at the outlet is equal to 50% of the pressure at inlet and its value is $5\cdot10^4$ Pa. To construct the curve shown on figure an interpolation had been used taking into account pressure values at points and 1st derivatives, and also assuming that pressure changes almost linearly between points.

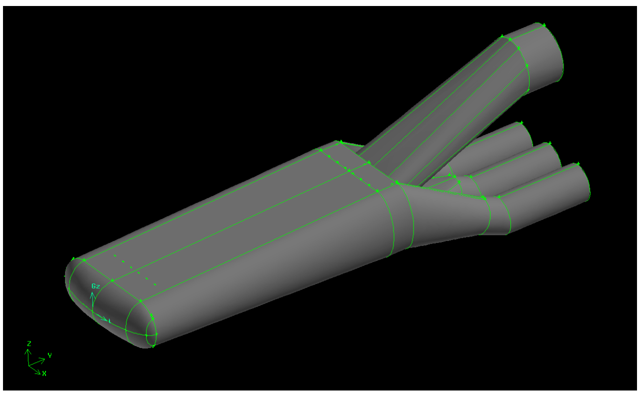


Fig. 1. The ESS target

The time scale for this pressure distribution is calculated for the whole model. The average fluid velocity 1.5 m/s (corresponds to the flux $15\cdot10^{-3}$ m^3/s and the area of the cross-section of 100 cm^2). The length of model is 1 m. Full path is 2 m. So, maximum time is approximately 1.3 sec. Turn-points of a flow are taken at 0.9 and 1.1 m far from the inlet.

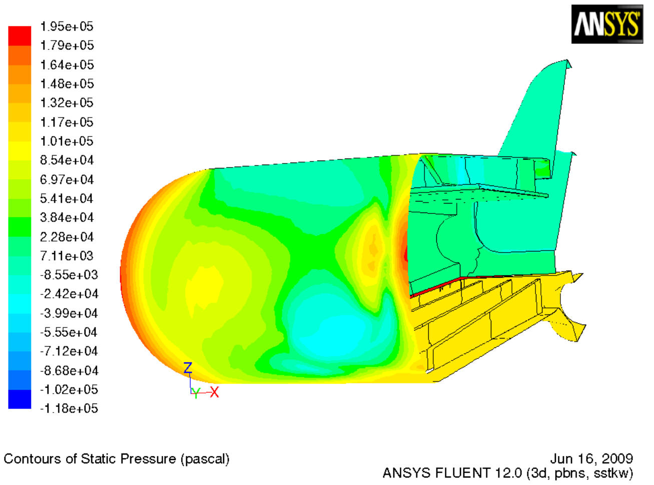


Fig. 2. The pressure distribution in the ESS target

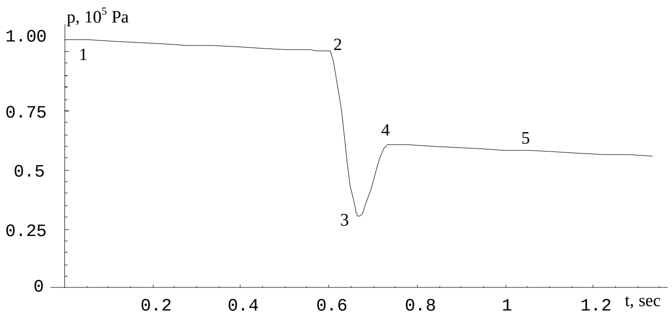


Fig. 3. The distribution of pressure in fluid used for calculation (p_{fluid}). Pressure is depending on time in seconds. Points: 1 – inlet, 2- 1st turn-point, 3 – middle-point, 4 – 2nd turn-point, 5 – half of end-path

This pressure profile can be easy modified. Also the additional pressure (positive or negative) can be applied to this profile, moving the curve up or down as necessary, but not changing the shape of profile. According to [1] and [2], the cavitation is possible in case of fluid pressure less than saturated vapour pressure at the concrete temperature. This means, that the pressure in fluid can be calculated in the following way (t represents time):

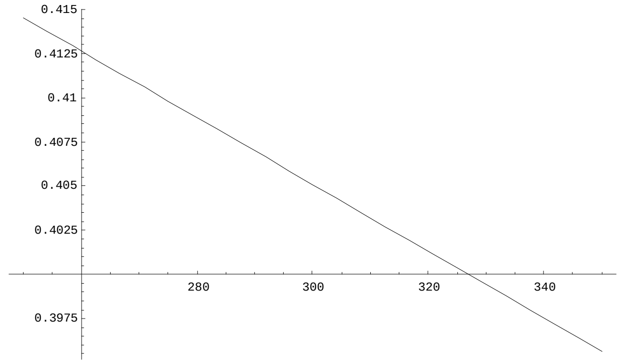
$$p_{fluid}(t) = p_m(t) + p_0 , \tag{2}$$

where p_m is the model pressure profile and p_0 is the additional pressure. Taking appropriate pressure p_0 we can obtain necessary pressure distribution relation to the saturated vapour pressure to check cases when cavitation is possible.

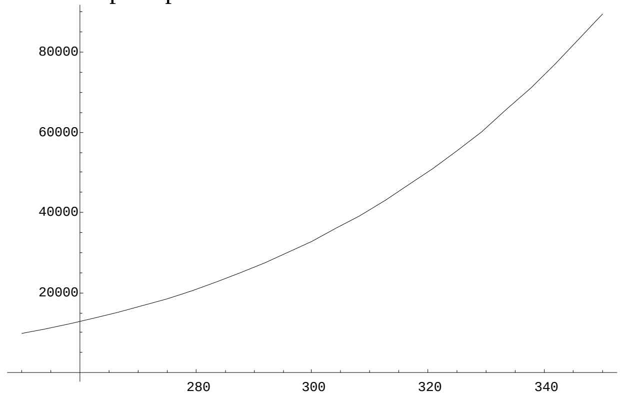
III. Physical Parameters for Hg Depending on Temperature in SI Units

Here the graphs for used physical parameters of Hg are presented.

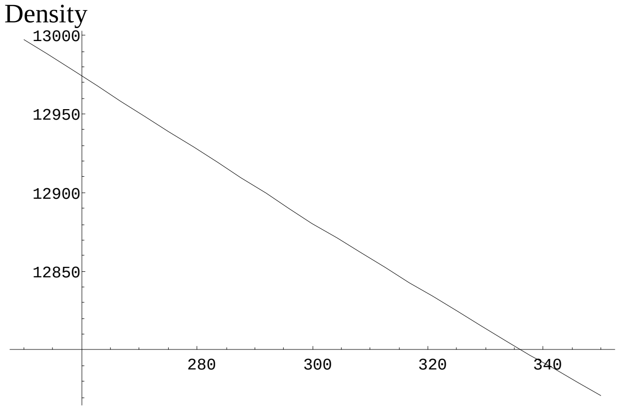
Surface tension



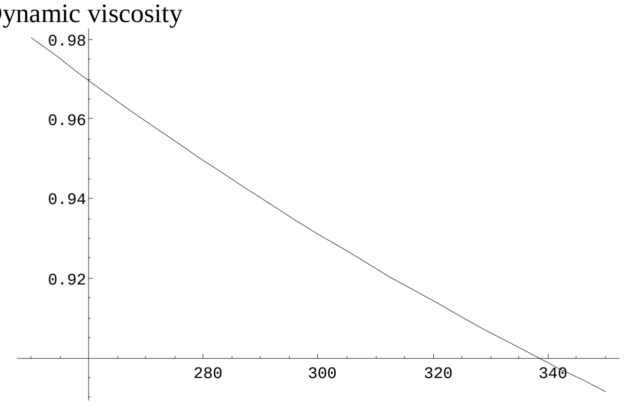
Saturated vapour pressure



Density



Dynamic viscosity



IV. The Results of Calculations

The following results are obtained for mercury Hg at the temperatures 250, 300 and 350 C. The additional pressure p_0 in (2) is taken to satisfy the condition at the inlet: $p_{fluid} \leq p_s$ at 250 C. So the relation between the pressure in fluid and the saturated vapour pressure at different temperatures is shown on Fig. 4.

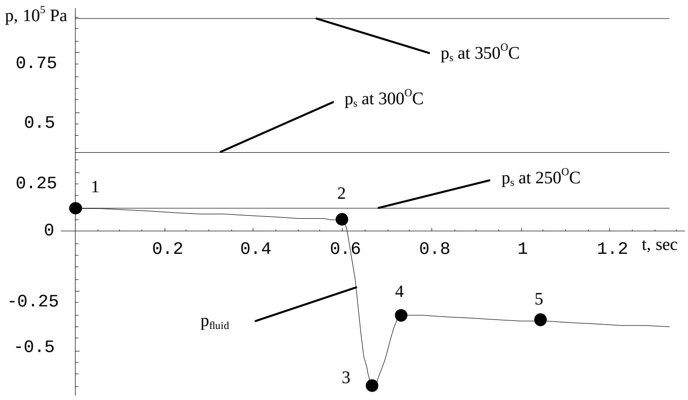
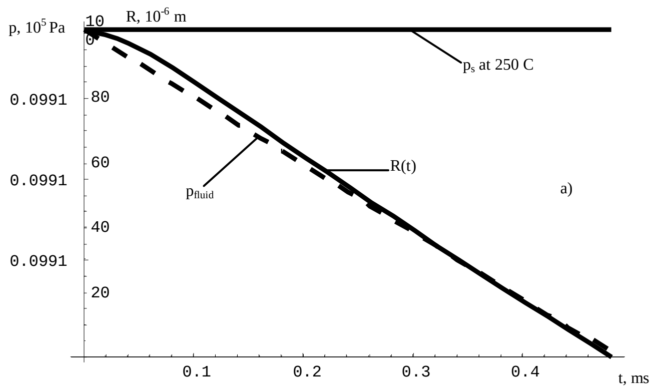
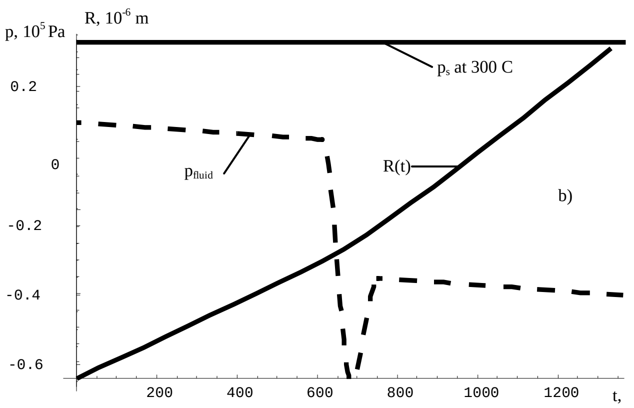


Fig. 4. The pressure in fluid and the saturated vapour pressure at 250, 300 and 350



Collapsing time = 0.481383 ms at temp= 250 C



Bubbles are growing at temp= 300 C

Fig. 4.1. Dependence of bubble radius on time at the inlet for different temperatures. a) for a collapsing bubble, b) for a growing bubble

As an example the two figures (4.1a and 4.1b) characterizing behaviour of bubbles are shown: bubbles of the equal initial radius 10^{-4} m at the inlet at temperatures 250 C and 300 C. For all other cases the depending of radius for growing and collapsing bubbles is similar. On these figures the *R(t)* represents the dependence of the bubble radius on time. For growing bubbles the scale is not shown because (as calculations show) the radius reaches values of 1-1.5 meters that is not possible in a model with cross-section of 100 cm^2. Such results just show that the type of cavitation changes or there are too many bubbles appears in fluid.

The dashed line represents the pressure in fluid p_{fluid} from the moment of bubble born to the bubble collapse or to the moment of bubble leaving a model.

The solid line p_s represents the saturated vapour pressure at the corresponding temperature.

In case of bubble collapse the calculated time of bubble life is shown under the figure.

5. Analysis and Conclusions

As the figures above show, the small bubbles of radius 10^{-6} m collapse at all temperatures, bubbles of radius 10^{-5} m collapse at low temperatures and grow at high temperatures, and bubbles of radius 10^{-4} m grow at all temperatures. This can be explained by following.

The equation of vapour bubble equilibrium state is

$$p_{fluid} = p_s - 2\sigma/R, \qquad (3)$$

where p_{fluid} is the pressure in fluid, p_s is the saturated vapour pressure, σ is the surface tension and R is the radius of the bubble. In this formula saturated vapour pressure and surface tension are depending on temperature and with growth of temperature the first is growing, but the second – vice versa. For large bubbles the saturated pressure at low temperatures is approximately equal to the pressure caused by the surface tension force, and the exterior pressure in fluid causes collapse of a bubble. At high temperatures the situation changes and the saturated vapour pressure is sufficiently greater than the surface tension. In this situation vapour pressure pushes on bubble wall from inside and cause growth of a bubble if the exterior pressure is less than the vapour pressure. In case of small bubbles for all temperatures the saturated vapour pressure is weaker than the surface tension, so the exterior pressure in fluid causes bubble collapsing. In case of bubbles of a medium size at low temperatures the surface tension is greater than the saturated vapour pressure, so bubbles are collapsing, but at high temperatures – vice versa. At medium temperatures in some cases the 1st situation occurs, but in other cases – the 2nd, so results for this size of bubbles may be different.

The equilibrium state equation also shows the role of the pressure in fluid. If this pressure is less than saturated vapour pressure, the bubbles may grow. The larger is the difference then the larger is possibility of cavitation.

The tables below illustrate the results achieved. C means collapsing of a bubble, G – growing of a bubble.

TABLE I
RESULTS FOR HG

At t=250 C.

Radius, m	Inlet	1st turn-point	Middle point	2nd turn-point	Half of end-path
10^{-4}	C	C	G	G	G
10^{-5}	C	C	C	C	C
10^{-6}	C	C	C	C	C

At t=300 C.

Radius, m	Inlet	1st turn-point	Middle point	2nd turn-point	Half of end-path
10^{-4}	G	G	G	G	G
10^{-5}	C	C	G	C	C
10^{-6}	C	C	C	C	C

At t=350 C.

Radius, m	Inlet	1st turn-point	Middle point	2nd turn-point	Half of end-path
10^{-4}	G	G	G	G	G
10^{-5}	G	G	G	G	G
10^{-6}	C	C	C	C	C

The same results graphically are shown on Fig. 5. Bubbles with radius above corresponding line will grow, below – collapse.

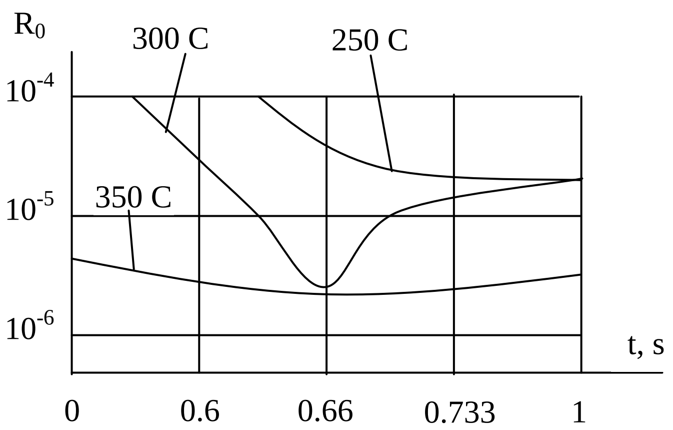


Fig. 5. Approximate curves of bubble stability

From these tables the role of the difference between the saturated vapour pressure and the pressure in fluid can be derived.

At t=250 C the pressure difference causes bubble growth only for large bubbles starting at the middle point of a model. From this point surface tension for large bubbles cannot compensate pressure difference, so bubbles are growing. For medium size and small bubbles the pressure difference not enough to enlarge bubbles.

At t=300 C the pressure difference overtakes the surface tension for large bubbles and at the middle point – for medium size bubbles.

And at t=350 C the pressure difference cause growth in any point not only for large size bubbles, but also for medium size bubbles. And only for small bubbles surface tension is enough to collapse bubbles.

According to the results obtained, the following recommendation can be made.

If it’s possible, it’s necessary to control the size of bubbles in a fluid, allowing only bubbles of sizes which collapsing at the chosen pressure in fluid.

Apply to the system the pressure that is greater than the saturated vapour pressure at temperature in the system. Additional calculations, which results doesn’t included in this report, show that if the fluid pressure in the system is greater than the saturated vapour pressure (the pressure at the inlet of the system was taken 10^5 Pa, but saturated vapour pressure of mercury at t=350 C is 89684.6 Pa), bubbles of all size observed are collapsing.

VI. ON CAVITATION IN PB AND PBBI EUTECTIC

The same analysis as above was made for lead Pb and lead-bismuth PbBi eutectic. The results obtained are shown in the tables. In case ob bubble collapse the time of collapse in milliseconds is written in the appropriate cells.

The temperatures observed are 350, 365 and 380 C. Data for physical parameters are taken from [3]. The pressure profile is the same, but the pressure at inlet is equal to the saturated vapour pressure at the lowest temperature for each fluid.

TABLE II
RESULTS FOR LEAD PB

At t=350 C.

Radius m	Inlet	1st turn-point	Middle point	2nd turn-point	Half of end-path
10^{-4}	C 0.095317	C 0.137002	G	G	G
10^{-5}	C 0.00302882	C 0.00310999	C 0.00650481	C 0.00414171	C 0.00423433
10^{-6}	C 0.000097292	C 0.0000975467	C 0.000101336	C 0.0000996588	C 0.0000997801

At t=365 C.

Radius m	Inlet	1st turn-point	Middle point	2nd turn-point	Half of end-path
10^{-4}	C 0.0954092	C 0.137412	G	G	G
10^{-5}	C 0.0030312	C 0.00311274	C 0.00655655	C 0.00414171	C 0.0042451
10^{-6}	C 0.0000973105	C 0.000097566	C 0.000101369	C 0.0000996854	C 0.0000998072

At t=380 C.

Radius m	Inlet	1st turn-point	Middle point	2nd turn-point	Half of end-path
10^{-4}	C 0.0955026	C 0.137829	G	G	G
10^{-5}	C 0.00303366	C 0.00311558	C 0.00661015	C 0.00416169	C 0.00425612
10^{-6}	C 0.0000973367	C 0.0000975931	C 0.00010141	C 0.0000997201	C 0.0000998424

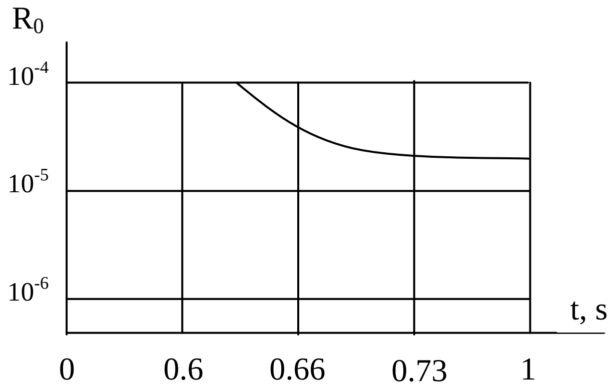


Fig. 6. Approximate curves of bubble stability (for all temperatures)

TABLE I

RESULTS FOR LEAD-BISMUTH EUTECTIC PBBI

At t=350 C.

Radius, m	Inlet	1st turn-point	Middle point	2nd turn-point	Half of end-path
10^{-4}	C 0.099708	C 0.154856	G	G	G
10^{-5}	C 0.00316384	C 0.00326029	C 0.00998045	C 0.00460066	C 0.00473431
10^{-6}	C 0.000101157	C 0.000101456	C 0.000105936	C 0.000103944	C 0.000104088

At t=365 C.

Radius m	Inlet	1st turn-point	Middle point	2nd turn-point	Half of end-path
10^{-4}	C 0.0997323	C 0.15517	G	G	G
10^{-5}	C 0.00316432	C 0.00326103	C 0.010122	C 0.00460769	C 0.00474229
10^{-6}	C 0.000101143	C 0.000101442	C 0.000105932	C 0.000103932	C 0.00010408

At t=380 C.

Radius m	Inlet	1st turn-point	Middle point	2nd turn-point	Half of end-path
10^{-4}	C 0.099757	C 0.155489	G	G	G
10^{-5}	C 0.00316485	C 0.00326181	C 0.0102722	C 0.00461485	C 0.00475042
10^{-6}	C 0.000101133	C 0.000101432	C 0.000105933	C 0.000103932	C 0.000104076

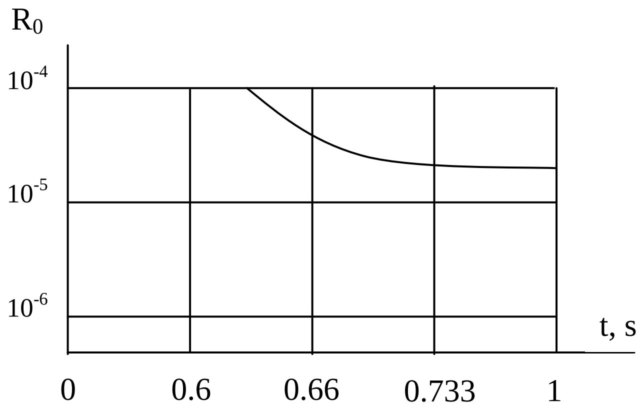


Fig. 7. Approximate curves of bubble stability (for all temperatures)

These results can be described in the same way as in case of mercury. Recommendations also are the same.

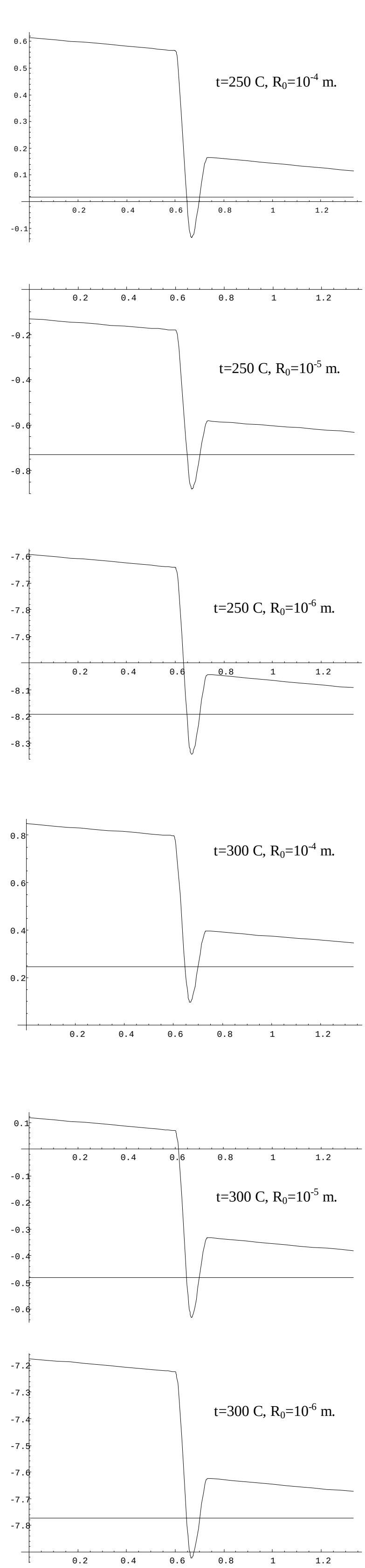
VII. ON BEHAVIOUR OF A VAPOUR BUBBLE CLOSE TO THE POINT OF EQUILIBRIUM STATE

To test behaviour of a bubble in case of high pressure gradients the 2 modifications of pressure profile was made.

The first modification does not affect the shape of profile shown on Fig. 3, but the additional pressure p_0 in (2) is chosen in the following way: the pressure at the inlet is such that pressure at the middle of the profile part between point 3 and 4 is equal to the $p_s - 2\sigma/R_0$, where R_0 is an initial radius of bubble. In this case a bubble is in an equilibrium state. For the different temperatures and the different radius of bubbles the p_0 also will be different. The graphs of these profiles are shown on Fig. 8.

Bubbles of desired radius are starting at the left cross-point of pressure profile and line $p_s - 2\sigma/R_0$. As calculations show bubbles of all sizes and at all temperatures are growing.

The second modification changes the shape of profile: the pressure at the middle point 3 will be now equal to the -25% ($-0.25\cdot10^5$ Pa) of the pressure at the inlet. In addition the first modification will be used. The pressure profile for bubbles of radius 10^{-4} m and temperature 350 C is shown on Fig. 9.



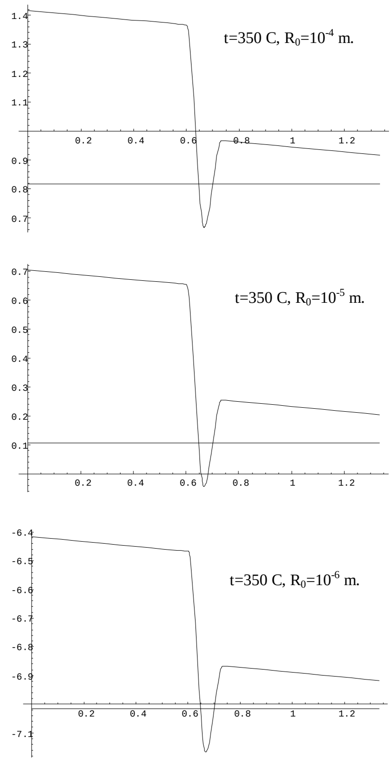


Fig. 8. Pressure in fluid for different temperatures and radius of a bubbles. Pressure in 10^5 Pa (vertical axis), time in seconds (horizontal axis). Horizontal lines shows a level of $p_s - 2\sigma/R_0$

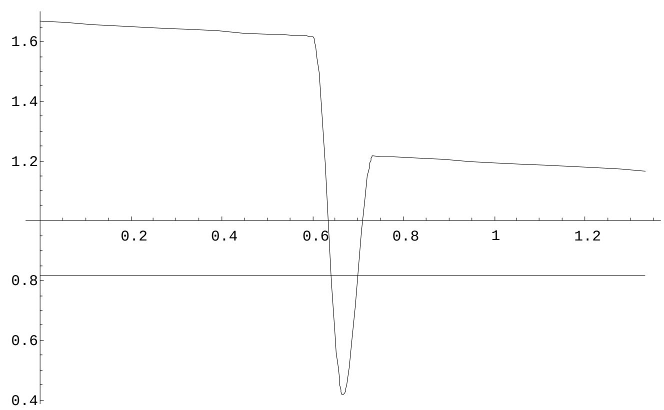


Fig. 9. Modified pressure profile at 350 C. Pressure in 10^5 Pa (vertical axis), time in seconds (horizontal axis). Horizontal lines show the level of $p_s - 2\sigma/R_0$

Bubbles start at the same point as above. Obtained results show that bubbles of small (10^{-6} m and 10^{-5} m) grow, but bubbles of large radius (10^{-4} m) collapse. But this doesn't mean that if small or medium size bubbles reach the size of large bubble, a growing changes to a collapsing. In this case the initial conditions are different – the initial velocity of bubble wall in case of growing small or medium bubble is not zero as in case of large bubbles for which the initial velocity is equal to zero (equilibrium state). In case of growing bubbles the inertia of a growing bubble wall prevents collapse of bubble.

In both cases if a bubble appears in the zone with pressure above the line $p_s - 2\sigma/R_0$, it will collapse. So to avoid cavitation it is necessary to hold minimal pressure in fluid greater than the $p_s - 2\sigma/R_0$. The dependences of minimal pressure in fluid to dump a cavitation on a temperature and on a radius of bubbles are shown on Fig. 10. Below the lines the cavitation exists, above – cavitation is dumped.

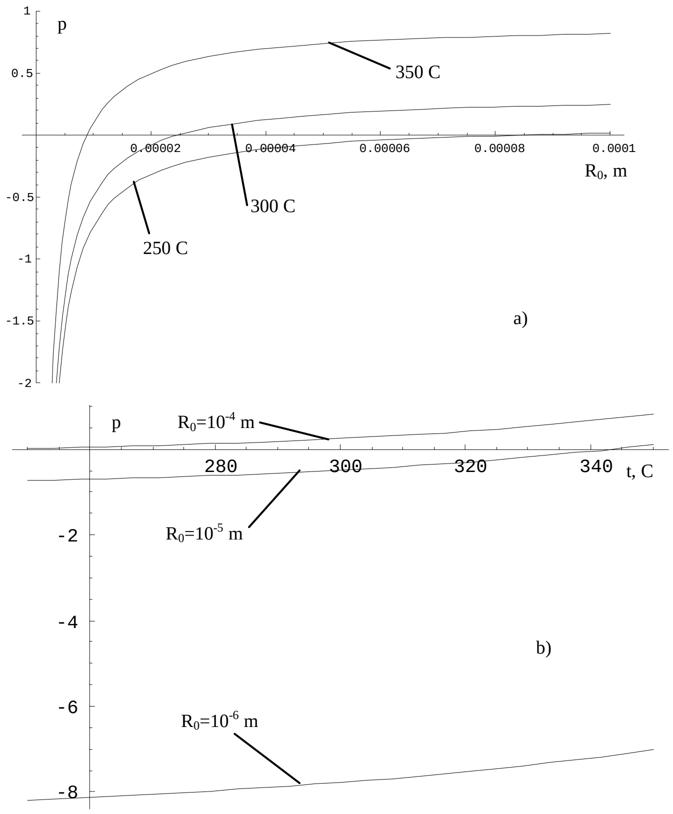


Fig. 10. The dependence of minimal pressure in fluid to dump a cavitation: a) on radius of bubbles for different temperatures; b) on a temperature for different radius of bubbles. Pressure in 10^5 Pa (vertical axis).

VIII. THE MINIMAL REQUIRED PRESSURE TO DUMP CAVITATION

The curves shown on the Fig. 10 are constructed using value of initial radius of bubbles. This value is not predictable, so it is necessary to choose a pressure in fluid using only physical parameters of a fluid. According to the theory [1], [2] and numerical experiments, the minimal required pressure in a fluid to dump cavitation must be greater than the saturated vapour pressure at desired temperature. The dependence of this on temperature for mercury Hg is shown on Fig. 11.

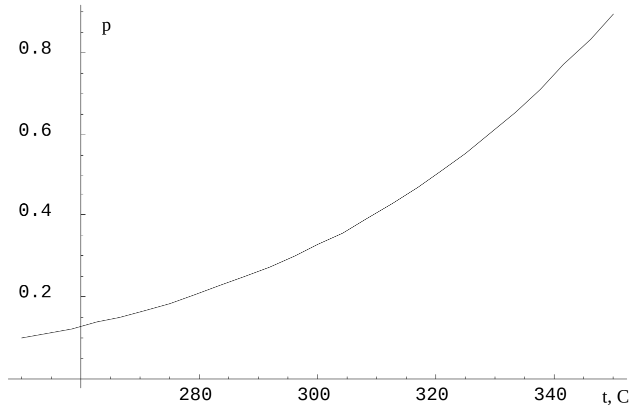


Fig. 11. The dependence of the minimal pressure in fluid on a temperature. Pressure in 10^5 Pa

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J. E. Freibergs, V. Kremeņeckis. Kavitācijas analīze ESS mērķa modelī
Šajā darbā tiek pētīti daži kavitācijas ietekmes aspekti uz ESS (European Spallation Source) šķidro metālu mērķi. Analīze ir balstīta uz burbuļu rādiusa izmaiņas aprēķināšanu, burbuļiem pārvietojoties mainīgajā spiediena laukā. Galvenie fizikālie parametri, kas ietekmē uz burbuļi, ir šķidruma tvaika spiediens burbulī, virsmas spraigums, šķidruma viskozitāte un spiediens šķidrumā. Tiek pētīta tikai tvaika kavitācija (burbulī ir tikai šķidruma tvaiki). Fizikālo parametru vērtības izvēlēti trim šķidrā metāla veidiem, kurus plāno izmanto mērķī: dzīvsudrabam, svinam un svina – bismuta sakausējumam. Katram šķidrumam ir raksturīgs savs darba temperatūru diapazons, kas ir no…. gan ar pašu šķidrumu fizikālām īpašībām, gan ar siltumu, kas izdalās mērķi daļiņu apstarojuma dēļ. Katram šķidrumam izvēlētas trīs temperatūras vērtības: darba diapazona robežas un viduspunkts. Spiediena lauka veids ir izvēlēts saskaņā ar šķidruma plūsmu modelī,

kas ir iegūts plūsmas hidrodinamikas skaitlisko aprēķinu rezultātā. Vienkāršots spiediena modelis tiek izmantots, lai noskaidrotu burbuļa rādiusa atkarību no spiediena un temperatūras. Tiek pētītas situācijas, kas atbilst burbuļu rāšanai dažādos modeļa punktos, un to kustība pa šķidruma plūsmu modelī. Galvenais šī darba mērķis ir noteikt nosacījumus (tādus kā burbuļa sākotnējais rādiuss, temperatūra, spiediens), kas ietekmē uz burbuļa augšanu vai sabrukumu. Visos gadījumos ir konstruētas „stabilitātes” līknes (līknes, kas atdala burbuļa augšanas apgabalu no burbuļa sabrukuma apgabalu atkarībā no dažādiem parametriem). Kā ilustrācijas uzzīmētas raksturīgas līknes, kas rada burbuļa rādiusa augšanu vai dilšanu. Tabulās visiem gadījumiem uzradīti burbuļu dzīvības laiki līdz sabrukšanai. Darbā arī noradīti kavitācijas vadīšanas iespējas.

Я. Э. Фрейбергс, В. Кременецкий. Анализ кавитации в модели мишени ESS

В данной статье рассматриваются некоторые аспекты влияние кавитации на жидкометаллическую модель мишени ESS (European Spallation Source). Анализ основан на расчете изменения радиуса пузырька при его движении в меняющемся поле давления. Основными физическими параметрами, влияющими на пузырек, являются давление паров жидкости внутри пузырька, сила поверхностного натяжения жидкости, вязкость жидкости и давление в жидкости. Изучается только паровая кавитация (пузырь наполнен только парами жидкости). Значения физических параметров выбраны для трех типов жидких металлов, использование которых планируется в мишени: ртути, свинца и сплава свинец - висмут. Для каждой из этих жидкостей характерен свой диапазон рабочих температур, обусловленный как физическими свойствами самих жидкостей, так и теплом, выделяемым при облучении модели частицами. Для каждой жидкости выбраны 3 значения температуры: границы рабочего диапазона и его середина. Форма поля давления выбрана в соответствии с потоком жидкости в мишени, полученного путем численного расчета гидродинамики модели. Упрощенная модель поля давления используется для определения радиуса пузырька в зависимости от давления и температуры. Рассматриваются ситуации, соответствующие появлению пузырьков в разных частях модели, и их движение вдоль потока жидкости в модели. Основной целью данной работы является выявление условий (например, начальный радиус пузыря, температура, давление), влияющих на рост пузыря или его схлопывание. Во всех случаях строятся кривые "устойчивости" (кривая разделяющая область роста пузыря от области схлопывания пузыря, в зависимости от различных параметров). В качестве иллюстраций приведены характерные кривые, показывающие рост или убывание радиуса пузырька. В таблицах для всех рассматриваемых случаев приведено время жизни пузырьков до их схлопывания. В работе указаны возможности управления кавитацией.