Scientific Journal of Riga Technical University
Transport and Engineering. Intelligent Transport Systems

2010
Volume 34

Estimation of Fatigue Durability Dispersion in Operation by Results of Tests and Loading Measurements

Vladislav P. Turko, *Riga Science Experimental Center “Aviatest LNK” Ltd.*

Abstract. **Change of the mechanism of accumulation of fatigue damages leads to sectional-linear description S-N curve view and change of functions of long life time distribution that have two-parametric bi-exponential distribution with logarithmic life time. The experimental estimations of durability mean and root-mean-square deviation under test condition are used as scale and position parameters of bi-exponential distribution. One of the advantages of this distribution is the smallest values of probabilities on the left tie of density of distribution compared with known ones.**

Keywords: **changing of damage accumulation, distribution function parameters estimating, fatigue S-N curve, two-parametric bi-exponential distribution with logarithmic life time.**

I. INTRODUCTION

Forecasting of design’s service life is impossible without the estimation of life time dispersion in the work operation. To make this estimation is possible by results of life tests. But the conditions of the tests are distinct from the operational ones.

Usually it is accepted that the kind of the life time distribution function is the same through the every loading levels and differs only in parameters of the chosen kind of the function. In design practice this parameters are estimated by results of fatigue tests. After that there are supposed (under the known form of S-N curve) to be determined the parameters of the distribution function of the same kind for great values of life time (more than 1 000 mln cycles). Thus frequently the function of distribution are chosen only from reasons of good accordance with the experimental data, without consideration the various nature of variability of experimental data at different loading levels, caused by various mechanisms of occurrence and accumulation of fatigue damages or other reasons.

In practice it is accepted that the best way to describe the life time distribution is to use the logarithmic-normal distribution:

$$F(N) = \Phi\{\frac{\lg N - a}{b}\} . \quad (1)$$

where
N – number of load cycles,
Φ {•} –Laplass function,
a - expectation mean or position parameter,
b - root-mean-square deviation or scale parameter.

or Weibull distribution:

$$F(N) = 1 - \exp[-\frac{(N-a)^m}{b}] \quad (2)$$

where
N – number of load cycles,
a - position parameter,
b - scale parameter,
m – mode parameter.

These distributions are received from different phenomenological models: the influence of a lot of insignificant factors on accumulation of fatigue damages and the hypothesis of a weakest-link respectively [1, 2]. Other models have not received the wide practice, basically because of the necessity determinate the plenty of parameters that considerably reduce the accuracy of their estimation.

II. ACCUMULATION OF FATIGUE DAMAGES DIFFERS THROUGH THE LOADING LEVELS

However both earlier researches [3, 4] and some modern works [5] show that the mechanism of fatigue damages accumulation strongly differs through the loading level, so it is necessary to describe the process of destruction by other phenomenological models in desired predicted area of long life times, which, unfortunately, cannot be confirmed experimentally due to its high duration and expensiveness. About the distinction of mechanisms of fatigue failure revealed during the fatigue tests it was marked in [6]. Moreover, even at one loading level various mechanisms of fatigue failure of objects were found out. In this case the distribution of durability is characterized by the mix of functions of distributions, usually no more than two distributions. This mix distribution depends, as considered in [7], from initial quality of researched object and can be represented as:

$$F(N) = \chi F_1(N) + (1-\chi) F_2(N) \quad (3)$$

where
$F_i(N)$ – the function of object durability distribution, which fatigue failure occurs on i- mechanism,
χ - a share of the objects which have collapsed on the first mechanism.

However such mixes of distributions should be absent practically due to increased quality assurance of production. In this case the presence of such a mix of distributions (3) shows that on this level of loadings one mechanism of fatigue failure may transitive to another. Let's call this point of transition as **Noi.** – the area of durability close to **Noi** cycles, where the failure mechanism is changed – "point of changing."

The changing from one mechanism of fatigue failure to another is accompanied by breaks of S-N curves, a multimodality of the distribution density, non-monotonous change of some parameters (a dispersion of fatigue durability, the functions of intensity, etc.). No one of any of the accepted fatigue models cannot explain and describe these phenomena.

As shown in [8] the two-parametric bi-exponential distribution with logarithmic life time (as the limit distributions of maxima extremes) suitable describes the life time under low loading or high reliability objects. One of the advantages of this distribution is the smallest values of probabilities on the left tie of density of distribution ("cycles cut-off" without using the third parameter) compared with known ones (see Fig.1.). One of kind of this distribution is follow:

$$F(N) = \exp[-\exp(-\frac{\lg N - \alpha}{\beta})] \qquad (4)$$

where

N – number of load cycles,

α – position parameter,

β – scale parameter,

Much smaller probabilities in the left tail of distribution and a little raised on the right branch of the distribution corresponds to so-called S-shaped curves of the distribution that are characteristic of objects which have increased reliability or which working at a low loading level. The logarithmic presentation of the fatigue durability allows describing the non-monotonic function of the intensity, especially characteristic for fatigue failures [1].

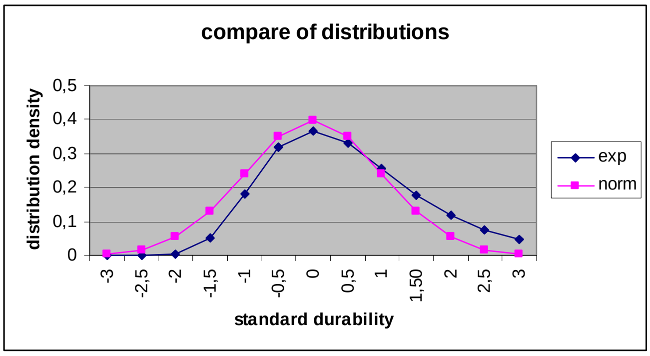

Fig. 1 The cycles "cut-off" effect for Bi-exponential density of distribution (smaller probabilities on the left tie of the distribution)

Take the supposition that one of the proofs of the various mechanism of accumulation of fatigue is the various values of S-N curve's parameter "**m**".

It is known, that the S-N curve is described by the expression:

$$\sigma^{m_{\text{ш}}} N = const \qquad (5)$$

or

$$m_i \lg \sigma + \lg N = const \qquad (6)$$

where

m_i – the inclination angle of the S-N curve's sectional-linear part numbered i,

σ – the loading level (stress, force, load etc.),

N – durability (number or cycles, life time etc).

More often the S-N curve (5) is described as linear function (6) with two parts (see Fig.2.) or as exponential mode (see Fig.3) [9].

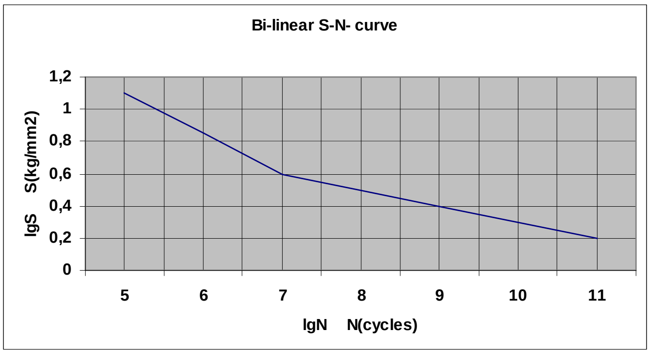

Fig. 2. Bi-linear S-N curve

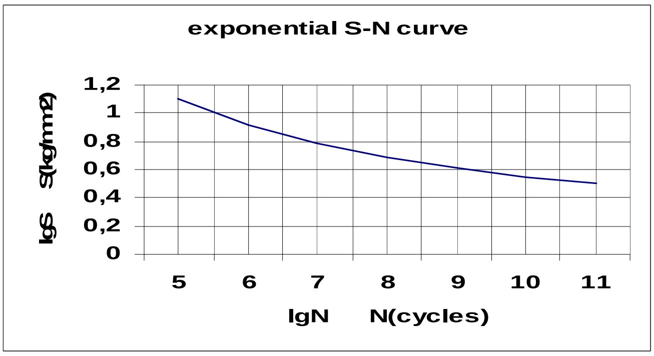

Fig. 3. Exponential S-N curve

However, both in the first and in the second case, different endurances areas (especially in its great values) are described usually with the extrapolation in area of great values by the same parameters. This, especially in logarithmic coordinates, leads to an appreciable error.

III. DETERMINATION OF PARAMETERS FOR ONE OF SECTIONAL-LINEAR S-N CURVE PART

In practice, for aluminium alloys, there applies a sectional-linear S-N curve with double logarithmic coordinates. We shall accept a hypothesis about the change of mechanism of fatigue accumulation (parameter **m**) at the following values of S-N curve durability.

Let's consider the following Table (values are shown as an example for one of details of helicopter Mi-2 unit) (see Fig.4.)

As it is stated above, it is inconvenient and expensive to determine experimentally a kind of the distribution function for long life time's values of durability, however it is possible to determine changing of a population (expectation) mean and a dispersion of the random variables in expression (6).

Then we show the determination of parameters for any of researched (constant values of **m**-parameter) areas resulted. (If it has to be shifted to another area and after that to the area of operational working loads it is necessary to repeat the resulted calculations, based on changing point **lgNo**.)

Let us assume that the kind of the S-N curves and distributions functions of the durability and the fatigue limits are the same assumed sectional-linear area. It is shown [8] that for extrapolation of tests results in the area of assumed loadings it is possible to take advantage of the following formula:

$$\begin{aligned}(\lg N_i - \lg N_{0i}) \lg^{m_i} \sigma_i &= \\ = (\lg N_j - \lg N_{0i}) \lg^{m_i} \sigma_j\end{aligned} \qquad (7)$$

where

$N_{i,,j}$ - design's life time under $\sigma_{i,,j}$ *loading*,

m_i, N_{0i} – S-N curve parameters, see Fig. 4 and Table I.

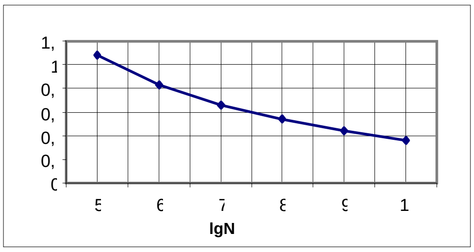

Fig. 4. Sectional-linear S-N curve

Let's set $\tau = \lg(N/N_0)$ and $S = \lg\sigma$ and take the logarithm (7):

$$\lg \tau_j = \lg \tau_i + m \lg S_i - m \lg S_j \qquad (8)$$

All random variables in (8) - are stochastic independent, therefore the following is true:

$$D\{\lg \tau_j\} = D\{\lg \tau_i\} + m^2 D\{\lg S_j\} \qquad (9)$$

where

D{•}- the variable dispersion operator.

Let's arrange the local linearization of lgS and lgτ (9) in the area of expectation mean E {•} [10]:

$$D\{\lg \tau_j\} = \frac{D\{\tau_j\}}{E^2\{\tau_j\}} = \operatorname{var}^2\{\tau_j\} \qquad (10)$$

$$D\{\lg \tau_i\} = \frac{D\{\tau_i\}}{E^2\{\tau_i\}} = \operatorname{var}^2\{\tau_i\} \qquad (11)$$

$$D\{\lg S_j\} = \frac{D\{S_j\}}{E^2\{S_j\}} = \frac{\lg^2 e D\{\sigma_j\}}{E^2\{S_j\} E^2\{\sigma_j\}} = \frac{\lg^2 e \operatorname{var}^2\{\sigma_j\}}{\lg^2 E\{\sigma_j\}} \qquad (12)$$

where

var{•} - the operator of variation coefficient,

e = 2,7183 - Napierian base.

Then use (10), (11) and (12) in (9):

$$\operatorname{var}^2\{\tau_j\} = \operatorname{var}^2\{\tau_j\} + \frac{m^2 \lg^2 e \operatorname{var}^2\{\sigma_j\}}{\lg^2 E\{\sigma_j\}} \qquad (13)$$

Apply the mean expectation operator to (8):

$$E\{\lg \tau_j\} = E\{\lg \tau_i\} + m \lg S_i - mE\{\lg S_j\} \qquad (14)$$

Changing the logarithmic operator in (14) with expectation mean operator:

$$\lg E\{\tau_j\} = \lg E\{\tau_i\} + m \lg \frac{S_i}{E\{S_j\}} \qquad (15)$$

TABLE I

THE SECTIONAL-LINEAR S-N CURVE DESCRIPTION AND PARAMETERS CHANGING

Range of durability, cycles c	Value of m parameter of S-N curve	A "point of changing" $\lg N_0$ when the mechanism of fatigue accumulation is changed	The distribution function of the durability
up 1 mln	4	1	Weibull (2)
from 1 mln up 10 mln	6	6	logarithmic-normal (1)
from 10 mln up 100 mln	8	7	logarithmic-normal (1)
from 100 mln up 1 000 mln	10	8	Bi-exponential (4)
from 1 000 mln	12	9	Bi-exponential (4)

or raising (15) to the power based 10

$$E\{\tau_j\} = E\{\tau_i\}(\frac{\lg \sigma_i}{\lg E\{\sigma_j\}})^m = E\{\tau_i\}\eta_\sigma^m \quad (16)$$

where:

$$\eta_\sigma = \frac{\lg \sigma_i}{\lg E\{\sigma_j\}} \quad (17)$$

- the load safety factor

Expressions (13) and (16) define the life time distribution function parameters of the design in operation through known loading parameters, measured in flight, and by results of tests.

IV. PARAMETERS OF LIFE TIME FUNCTION DISTRIBUTION

Life time distribution function for designs in operation conditions looks like this [8]:

$$F(\tau) = \exp[-\exp(-\frac{\tau - \alpha}{\beta})] \quad (18)$$

where
$\tau = \lg N$
N - number of load cycles,
$a = E\{\tau\} - 0,5772\beta$ (19)
- position parameter,

$$\beta = \frac{\sqrt{6 D\{\tau\}}}{\pi} \quad (20)$$

- scale parameter.
$\pi = 3,14\ldots$

Then parameter β (20) describing the dispersion of the design's life time in operation, can be appreciated by results of tests and load measurements in operation from the ratio (13) and (16).

$$\beta = \mathrm{var}\{\tau_i\}E\{\tau_i\}\frac{\sqrt{6}}{\pi} \quad (21)$$

So we know the parameters of position and scale and may determine the reciprocal distribution of life time under operational conditions.

V. DETERMINATION OF CYCLE'S SAFETY FACTOR FOR WORK LOADS OPERATION CONDITIONS

Now, knowing the dispersion of fatigue durability at operational loadings (21), it is possible to determine the necessary test time of the researched design part or to approve the safe operation life time, confirmed with some assurance.
Setting normative probability of destruction $\mathbf{P_{norm}}$, having **n** objects tested and observing the minimal operating time received at tests $\mathbf{N_{min}}$, we may to address to ratio, offered in [11]:

$$P_{norm} = \int_{-\infty}^{+\infty} n[1 - F_0(\lg N_{min})]^{n-1} \int_{-\infty}^{\lg N_{min} - \lg \eta_n} dF(\lg N)\, dF_0(\lg N_{min}) \quad (22)$$

where
P_{norm} – normative object destruction probability,
n - the quantity of the objects tested,
N_{min}, - the minimal test life time,
η_N – cycle's safety factor in accordance with normative object destruction probability.

Take (18) and (8) to (22), transit the changing variable

$$y = \exp[-\exp(-\frac{\lg N - \alpha}{\beta})] \quad (23)$$

and note the following

$$\nu = \exp(\frac{\ln \eta_N}{\beta M}) \quad (24)$$

where
M = 2,303 - the transition from natural to decimal logarithm factor,

have

$$P_{norm} = n\int_0^1 (1-y)^{n-1} y^\nu dy = \frac{\Gamma(n+1)\Gamma(\nu+1)}{\Gamma(n+\nu+1)} \quad (25)$$

where
Γ (•) – gamma-function
y - changing variable (23)

Taking the known ratios [12], we will have:

$$P_{norm}\prod_{i=0}^{n-1}(\frac{\nu}{n-1} + 1) = 1 \quad (26)$$

where
Π (•) – multiply operator.

Then the cycle's safety factor is determined with (24):

$$\eta_N = \exp(2,303\,\beta \ln \nu) \quad (27)$$

where
β - is determined with (21)
ν - is determined with (24)

Thus, as the operation loads $\boldsymbol{\sigma}_{\mathbf{work}}$, the test loads $\boldsymbol{\sigma}_{\mathbf{j}}$, given with some assurance (25), **n** tested objects and the minimal test life time $\mathbf{N}_{\mathbf{min\ test}}$, the S-N curve mode (7) [13], exceedance of test load under operation ones (17), the kind of distribution function (4) or (18) with its parameters (19) and (21) are known now it is need to resulted the minimal test life time to equivalent one at work loadings $\mathbf{N}_{\mathbf{min\ work}}$ Using the same S-N curved and (7) we determine the dispersion of fatigue work durability $\boldsymbol{\beta}$ and set a level of safety (probability of failure) $\mathbf{P}_{\mathbf{norm}}$. From (26) and (24) we determine cycles safety factor $\boldsymbol{\eta}_{\mathbf{N}}$.
So the safety life time under work conditions **R** is

$$R = \frac{N_{\min\ work}}{\eta_N} \qquad (28)$$

The expression (28) is based to determinate the safety life time if test results are known.

This work has been supported by the European Regional Development Fund within the project „ Bezpilota aviācijas kompleksa izstrāde un lidaparātu industriālo prototipu izveide Latvijas tautsaimniecības uzdevumu risināšanai”.

REFERENCE

1. **Кордонский Х.Б., Фридман Я.Ф.** Некоторые вопросы вероятностного описания усталостной долговечности. *Заводская лаборатория*, №7, т.42, (1976), 829-847 стр.
2. **Вейбулл В.** *Усталостные испытания и анализ их результатов.* - Москва, Машиностроение, 1964., 276 стр.
3. **Шабалин В.И.** О разрыве в кривых усталости дуралюмина. *Доклады АН СССР*, - №4, т.122, (1958), 600-602 стр.
4. **Морозов Л.В**. Циклическая прочность и прогнозирование долговечности. *Второй Всесоюзный съезд по теории машин и механизмов,* 1984, сентябрь. – Одесса, Тезисы докладов, 1984, часть 1., 77 стр.
5. **Шанявский А.А**. *Моделирование усталостных разрушений металлов. Синергетика в авиации.*- Москва, Монография, 2007, 498 стр.
6. **У.А. Вуд.** Некоторые результаты исследования природы усталости металлов. В кн. Усталость и выносливость металлов. - Москва, Издательство иностранной литературы, 1963, 61-81 стр.
7. **Маслов Л.И., Кунавин С.А.** *Методические рекомендации по определению и прогнозированию характеристик циклической трещиностойкости конструкционных материалов.*- Москва, Институт металлургии им. А. А. Байкова, АН СССР, 1982 г., 86 стр.
8. **Турко В.П.** Модель усталостного разрушения при низких уровнях напряжений. *Международный симпозиум: Прочность материалов и элементов конструкций при звуковых и ультразвуковых частотах нагружения.* 1984, сентябрь. - Киев, Наукова Думка, Тезисы докладов международного симпозиума, 1984, 59 стр.
9. **Степнов М.Н., Гиацинтов Е.В.** *Усталость легких конструкционных сплавов.* - Москва, Машиностроение, 1973г, 317 стр.
10. **Крамер Г.** *Математические методы статистики.* - Москва, Мир, 1975., 648 стр.
11. *Вертолеты. Расчеты и проектирование.ч.2. Колебания и динамическая прочность.* Миль М.Н., Некрасов А.В. и пр. - Москва, Машиностроение, 1967, 424 с.
12. **Прудников А.П., Брычков Ю.А., Маричев О.И.** *Интегалы и ряды.* - Москва, Наука, 1981, 800 стр.
13. **Турко В.П.** Коэффициент запаса по циклам при низком уровне нагруженности. *Научно-техническая конференция: Прочность материалов и элементов конструкций при звуковых и ультразвуковых частотах нагружения.* Киев, 1992, октябрь.- Киев, Институт проблем прочности АН Украины, Тезисы докладов. 56 стр.

Vladislav P.Turko was born in Kislovodsk (Stavropol territory, Russian Federation).
Educational background: 1979.-1982. - RCAEI postgraduate study on the Faculty of aircraft repair and technology, 1975 The supreme state scientific - engineering courses of improvement of qualification on invention and patenting, 1968.-1974. - Riga Civil Aviation Engineers Institute, engineer-mechanics aircraft and engines maintenance, Riga, Latvia.
Operational experience: 2009 -… - deputy director (ISO-quality system); 1997 - 2009 Riga scientific - experimental centre “ Aviatest LNK ” Ltd, technical director. 1992-1997 RSEC “ Aviatest LNK ” Ltd, the senior scientific employee. 1982-1992 Riga branch GosNII GA, the scientific employee.
Awards: 2000 Open Stock Company " Tupolev " - «the Medal of academician A.N.Tupolev » for execution investigations on maintenance of safe operation of the plane TU – 154”

Vladislavs P. Turko. Prognozētais nogurumizturības vērtējums pēc testu un uzmērījumu iekraušanas rezultātiem
Tiek izvirzīta hipotēze, ka parametra izmaiņas noguruma līknes gabala lineāras attēlošanas gadījumā ir saistītas ar noguruma bojājumu uzkrāšanās mehānisma izmaiņām, kas noved pie ilgizturības sadalījuma un robežu funkciju veida izmaiņām atkarībā no apskatāmā objekta pielietošanas ekspluatācijas apgabala slogojuma līmeņa. Bet eksperimentāli noteikt sadalījuma funkcijas veidu ekspluatācijas slogojumu darba diapazonā nav iespējams un/vai ir diezgan grūti liela laika patēriņa un materiālieguldījumu dēļ. Paaugstinātā drošuma objektiem un/vai zemos slogojuma līmeņos tiek piedāvāts izmantot dubulto eksponenciālo sadalījumu ar ilgizturības logaritmisko aprēķinu, kas ir iegūts kā maksimumu sadalījums. Sadalījuma stāvokļa un novirzes parametru novērtēšanai tiek pielietotas vidējas un vidēji kvadrātiskas novirzes novērtējumi, kas tika iegūti izmēģinājumu laikā, neatkarīgi no ilgizturības sadalījuma veida izmēģinājumos. Tālāk sadalījuma novirzes un mēroga parametru pakāpeniskais aprēķins pēc noguruma līknes gabala lineārās funkcijas ekspluatācijas slogojumu apgabalā ļauj novērtēt apskatāmā objekta ilgizturību. Viena no dubultā eksponenciālā sadalījuma pielietošanas priekšrocībām ir iespējas aprakstīt jutības slieksni pa cikliem mazo kvantiļu jomā (garantētā resursa jomā) bez nepieciešamības ievest trešo parametru, kas būtiski paaugstina sadalījuma parametru, kā arī ilgizturības (resursa) novērtēšanas precizitāti, samazinoties kopējam izmēģinājumu apjomam. Tika parādīts, kā pēc izmēģinājumu rezultātiem un ekspluatācijā zināmiem statistiskiem slogojuma parametriem aprēķināt apskatāmā objekta resursu, nosakot rezerves koeficientu katrā ciklā un uzdodot tā bojājuma koeficientu.

Владислав П.Турко. Оценка разброса усталостной долговечности в эксплуатации по результатам испытаний и замера нагруженности
Показана возможность оценки разброса усталостной долговечности для кусочно-линейного отображения кривой усталости. Выдвигается гипотеза, что изменение параметра m в кусочно-линейном отображении кривой усталости связано с изменением механизма накопления усталостных повреждений, которое с необходимостью должно приводить к изменению вида функций распределения долговечности и пределов выносливости в зависимости от уровня нагруженности эксплуатационной области применения рассматриваемого объекта. Однако экспериментально определить вид функции распределения в рабочем диапазоне эксплуатационных нагрузок не представляется возможным и/или затруднительно в силу больших временных и материальных затрат. Предлагается для объектов повышенной надежности и/или при низких уровнях их нагруженности использовать двойное показательное распределение с логарифмическим исчислением долговечности, полученное как распределение максимумов. Для оценки параметров положения и сдвига распределения применяют оценки среднего и среднеквадратичного отклонения, полученные при испытаниях,

независимо от вида распределения долговечности при испытаниях. Далее поэтапный расчет параметров сдвига и масштаба распределения по линейно-кусочной функции кривой усталости в область эксплуатационных нагрузок позволяет оценить долговечность рассматриваемого объекта. Одним из преимуществ применения двойного показательного распределения является возможность описания порога чувствительности по циклам в области малых квантилей (области гарантированного ресурса) без необходимости введения третьего параметра, что значительно повышает точность оценки параметров распределения, а значит и долговечности (ресурса) при снижении объема испытаний. Показано, как по результатам испытаний и известных в эксплуатации статистических параметров нагруженности рассчитать ресурс рассматриваемого объекта, определяя коэффициент запаса по циклам и задавая требуемую вероятность его разрушения.