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Interior Damping in an Anisotropic Materials and Constructions

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Abstract. **The design represents a generality of parts. The part consist of microelements. Physical features of microelements, type of connections between parts determine many characteristics of a design, including dissipation of energy. For their estimation consider cyclic loading of conditional microelement and typical connection between parts of constructions. In result is defined the equation of contour of a loop of a hysteresis and calculate her area at axial loading and a bend. The microelement consists of two ideally elastic particles, disjointed an elasto-plastic stratum. Orientation of a stratum has random character. At loading elastic parts of a microelement tend to move along a stratum. Displacements occur at some ultimate load when shear stress in a stratum reach yield strength. Values of yield limit of different microelements may differ. Therefore examination of character of loading of a device as a whole is necessary conduct with statistical methods. It is supposed, that allocations of an angle, defining orientation of a stratum of a microelement and of yield strength of layers, are the uniforms.**

Key words: **hysteresis, loops, description, formation, synthesis**

I. Introduction

The construction represents a generality of links. The link may be monolithic or composite. Monolithic devices consist of microelements. Physical singularities of microelements, character of their interaction among themselves, type of connections between links determine many performances of a construction, including dissipative. For their estimate we shall consider cyclical loading a conditional microcell and standard junction of links.

Let's assume, that the microelement consists of two ideally elastic particles, disjointed an elasto-plastic stratum. Orientation of a stratum has random character. At loading elastic parts of a microelement tend to move along a stratum. Displacements occur at some ultimate load when shear stress in a stratum reach yield strength. Values of yield limit of different microelements may differ. Therefore examination of character of loading of a device as a whole is necessary conduct with statistical methods. It is supposed, that allocations of an angle, defining orientation of a stratum of a microelement and of yield strength of layers, are the uniform.

II. Cyclical loading a microcell

Let's consider more in detail cyclical loading of a microelement. From an operation of an exterior loading $\boldsymbol{Q}$ in each microelement there is shifting force $\boldsymbol{S}$ which is directed along a stratum. Magnitude of $\boldsymbol{S}$ depends on intensity of exterior loading and from orientation of a stratum in relation to a direction of a principal vector of exterior forces (Fig.1). If a position of a stratum to determine an angle $\boldsymbol{\alpha}$ between an exterior normal $\boldsymbol{n}$ to a platform and a direction of a force line

$$S = Q \sin \alpha. \qquad (1)$$

At some limiting value $\boldsymbol{S = T}$ plastic flow of a stratum occurs. The value of an exterior loading corresponding to this state, term as yield strength. For each of microcells it depends not only on mechanical properties of a stratum, but also from his position in relation to an axis of loading. In a stratum located normally to a force line $\boldsymbol{\sin \alpha = 0}$, limiting states do not arise and such device appears ideally elastic. Disposing microelement definitely, it is possible to receive materials rather elastic, as well as materials with strongly expressed plastic properties.

Let’s consider behavior of a microelement at cyclical loading. The analytical relation of ratio between a loading and a strain at the first loading a microcell looks like

$$\boldsymbol{Q_1} = \left\{ \begin{matrix} \kappa\Delta, & \kappa\Delta \le Q_T, \\ Q_T, & \kappa\Delta > Q_T, \end{matrix} \right\}, \qquad (2)$$

where

$\boldsymbol{\kappa = \kappa_{сл} + \kappa_{ч}}$ - the summary rigidity of a device;

$\boldsymbol{\kappa_{сл}, \kappa_{ч}}$ - rigidities of a stratum and parts of a microcell;

Δ - a linear deformation, i.e. mutual migration of end platforms;

$\boldsymbol{Q_T}$ - yield strength.

If the loading has reached maximum magnitude and has transferred to decrease the relation between force and strains becomes

$$\boldsymbol{Q_2} = \left\{ \begin{matrix} \kappa\Delta - (\kappa A - Q_T), & \kappa A - 2Q_T \le \kappa\Delta < \kappa A, \\ -Q_T, & -\kappa A < \kappa\Delta < \kappa A - 2Q_T, \end{matrix} \right\}, \qquad (3)$$

where $\boldsymbol{A}$ - amplitude of a strain.

At repeated loading when the loading again increases, this relation becomes

$$Q_3 = \left\{ \begin{matrix} \kappa\Delta + (\kappa A - Q_T), & -\kappa A \le \kappa\Delta < -\kappa\Delta + 2Q_T, \\ +Q_T, & -\kappa A + 2Q_T < \kappa\Delta. \end{matrix} \right\} \qquad (4)$$

If a cycle of a modification of a loading reserved, the relation *(Δ - Q)* looks like a loop, the graph of her is shown in figure 2. The energy, dispelled for a cycle, is determined by the formula

$$\psi = 4\, Q_T \left(A - \frac{Q_T}{k} \right). \qquad (5)$$

At $Q_T = 0{,}5Ak$ the maxima of dissipation of an energy is ensured.

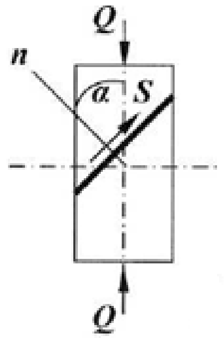

Fig. 1. Microelement

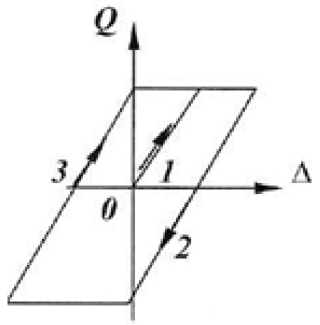

Fig. 2. Loop of loading

III. CYCLICAL TENSION AND COMPRESSION

If to consider a body from a material of a granular structure in conditions of a tension or compression the phenomenon of yield in microelements will arise not simultaneously because of difference of orientation of platforms of slip. The diagram of loading of combination of microelements at the homogeneous tension may be constructed only on the statistical basis.

Let's assume for simplicity, that the corner ***α*** **,** formed by an exterior normal to a platform of slip and a line of an operation of exterior force ***Q*** from a microelement to a microelement would vary within the limits $\boldsymbol{0 \le 2 \le \pi}$ **.** For simplicity allocation ***α*** in these boundaries we would shall accept the uniform, i.e. the structure of a material is those, that the probability of orientation of a stratum is identical to any of directions. We shall assume also, that mechanical performances of a stratum remain constant for all microelements of combination. Yield strength of a separate device would be determined under formula

$$Q_T = T / \sin\alpha. \qquad (6)$$

Allocation Q_T, if platforms of slip are located chaotically and allocation of corners *α* uniformly, is determined by a density under the formula

$$p(Q_T) = \frac{2\mathrm{T}}{\pi\, \mathrm{Q_T}} \frac{1}{\sqrt{\mathrm{Q_T^2} - \mathrm{T}^2}}, \qquad (7)$$

$\boldsymbol{p(Q_T)}$ displays, as the full loading is perceived by plastic and elastic links.

Now, using a hypothesis of flat sections, it is simply to install relation between a loading and a strain

$$P_1 = \int_{\mathrm{T}}^{\mathrm{k\nabla+T}} \mathrm{Q_T}\, p(Q_T)\, \mathrm{dQ_T} + \mathrm{k}\Delta \int_{\mathrm{k\nabla}}^{\infty} \mathrm{p}(\mathrm{Q_T})\, \mathrm{dQ_T}\,. \qquad (8)$$

This ratio establishes relation between force and a strain at loading a no deformed body, i.e. when initial stresses are equal to zero. If to take into account, that $p(Q_T)$ are determined under the formula (7), then

$$P_1 = \frac{2T}{\pi} \ln\left(\frac{k\Delta}{T} + 1 + \sqrt{\left(\frac{k\Delta}{T} + 1\right)^2 - 1} \right) + $$

$$+ \frac{2k\Delta}{\pi} \left(\frac{\pi}{2} - \arccos\frac{T}{k\Delta} \right). \qquad (9)$$

The relation between a loading and a strain essentially differs from linear, especially in the field of small strains. Further are considered effects of cyclical loading. Therefore is necessary define of relation of maximum displacement $A = \boldsymbol{\Delta_{max}}$ from force P_{max}

$$P_1 = \frac{2T}{\pi} \ln\left(\frac{kA}{T} + 1 + \sqrt{\left(\frac{kA}{T} + 1\right)^2 - 1} \right) + $$

$$+ \frac{2kA}{\pi} \left(\frac{\pi}{2} - \arccos\frac{T}{kA} \right), \qquad (10)$$

where ***A*** - amplitude of a strain.

Decreasing of a loading after reaching a maxima reduces to other relation between $\boldsymbol{\Delta}$ and ***P*** **.** The part of the elementary microelements from a plastic state begins to cross in elastic, some appear in a plastic state, however, the direction of flow and force in stratums change the sign. Microelements with the highest yield strength, as well as at the previous stage, remain in an elastic state. The share of participation of these groups of microelements in perception of an exterior loading approximately can be estimated with the help of the formula

$$P_2 = -\int_T^{\frac{k(A-\Delta)}{2}+T} Q_T\, p(Q_T)dQ_T + \int_{\frac{k(A-\Delta)}{2}+T}^{kA+T} [Q_T - k(A-\Delta)]p(Q_T)dQ_t + $$

$$+k\Delta\int_{kA}^{\infty} p(Q_T)dQ_T. \qquad (11)$$

However, if to take into account (7), ***Δ*** and P_2 appear the bound among themselves a particular ratio

$$P_2 = \frac{2T}{\pi}\ln\frac{4T\left(kA+T+\sqrt{k^2A^2+2kAT}\right)}{\left[k(A-\Delta)+2T+\sqrt{k^2(A-\Delta)^2+4k(A-\Delta)T}\right]} -$$

$$-\frac{2k(A-\Delta)}{\pi}\times\left[arcCos\frac{T}{k\Delta+T} - arcCos\frac{2T}{k(A-\Delta)+2T}\right] +$$

$$+\frac{2k\Delta}{\pi}\left(\frac{\pi}{2} - arcCos\frac{T}{kA}\right). \qquad (12)$$

At $\Delta = A$ last expression coincides with (10), and if $\Delta = -A$**,** that $P_2 = -P_{1max}$. In case of repeated loading relation *(**Δ** - P)* looks like

$$P_3 = \int_T^{\frac{k(A+\Delta)}{2}+T} Q_T\, p(Q_T)dQ_T - \int_{\frac{k(A+\Delta)}{2}+T}^{kA+T} [Q_T - k(A+\Delta)]p(Q_T)dQ_t + $$

$$+k\Delta\int_{kA}^{\infty} p(Q_T)dQ_T, \qquad (13)$$

or

$$P_3 = \frac{2T}{\pi}\ln\frac{\left[k(A+\Delta)+2T+\sqrt{k^2(A+\Delta)^2+2kT(A+\Delta)}\right]^2}{\left[kA+T+\sqrt{k^2A^2+2kAT}\right]4T} +$$

$$+\frac{2k(A+\Delta)}{\pi}\times\left[arcCos\frac{T}{k\Delta+T} - arcCos\frac{2T}{k(A+\Delta)+2T}\right] +$$

$$+\frac{2k\Delta}{\pi}\left(\frac{\pi}{2} - arcCos\frac{T}{kA}\right). \qquad (14)$$

As well as in the previous case at $\boldsymbol{\Delta = -A}$ or $\boldsymbol{\Delta = A}$ expression for $\boldsymbol{P_3}$ coincides with limiting values P_{1max} and P_{1min} .

The energy, dispelled at cyclical loading, is determined under the formula

$$\psi = \int_{-A}^{+A}(P_3 - P_2)d\Delta =$$

$$= \int_{-A}^{+A}\ln\frac{\left[k(A+\Delta)+2T+\sqrt{k^2(A+\Delta)^2+4kT(A+\Delta)}\right]^2\left[k(A-\Delta)+2T+\sqrt{k^2(A+\Delta)^2+4kT(A-\Delta)}\right]^2 d\Delta}{4T\left(kA+T+\sqrt{k^2A^2+2kAT}\right)} -$$

$$-\frac{2k}{\pi}\int_{-A}^{+A}\left[(A+\Delta)arcCos\frac{2T}{k(A+\Delta)+2T} - (A-\Delta)arcCos\frac{2T}{k(A-\Delta)+2T} + 2A\,arcCos\frac{T}{kA+T}\right]d\Delta. \qquad (15)$$

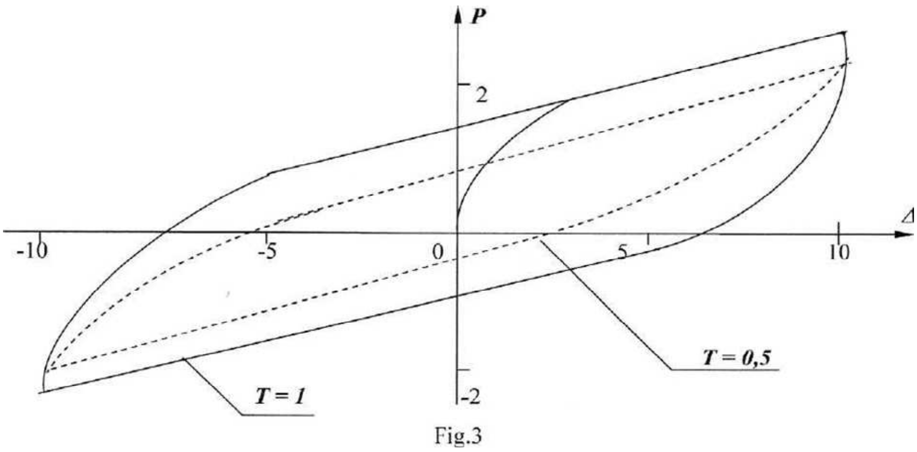

Fig.3. Loops of histeresis

In figure 3 calculated hysteresis loops for different values $T_1 = 0{,}5$ and $T_2 = 1$, in both cases $\kappa = 1, A = 10$ are shown.

IV. Cyclical bending

Let's select a single platform in cross-section of a prismatic rod. Then at the first loading the stress operating on this platform, may be defined under the formula

$$\sigma = \int_{\tau}^{\varepsilon E+\tau} \sigma_T p(\sigma_T) d\sigma_T + \varepsilon E \int_{\varepsilon E}^{R} p(\sigma_T) d\sigma_T, \quad (16)$$

where σ_T and E - yield strength and a modulus of elasticity of a microelement. Combination of microelements forms a material of a rod; τ - yield strength of an elastic-plastic stratum by which two elastic parts of a microelement are divided; ε - strain at a level of a position of a single platform; R – bending strength of a material.

Now it is possible to define a value of the moment of interior forces in a cross-section of a rod, for example, for rectangular section it is equal

$$M = b\int_{-h}^{+h} \sigma_y dy =$$

$$= \mathrm{b}\int_{-\mathrm{h}}^{+h}\left[\int_{\tau}^{\tau+\varepsilon E} \sigma_T p(\sigma_T) d\sigma_T + \varepsilon E \int_{\varepsilon E}^{R} p(\sigma_T) d\sigma_T\right] y dy \quad (17)$$

Using a hypothesis of flat sections for conditionally elastic material, it is possible to find a distribution of a strain on an altitude of cross-section of a rod $\boldsymbol{\varepsilon = \frac{\varepsilon_0}{h} y}$, where ε_0 - a strain in the point of section most deleted from a neutral axis; h, y - an altitude of section and coordinate of a point. Now it is visible, that in (17) integrand function appears depending exclusively from coordinate y. Similar expressions can be received for stages of a unloading and repeated loading.

For simplification of consequent calculations we shall assume, that the density function of angles of orientation of platforms of slip in microelements express the formula

$$q(\alpha) = \frac{ctg\alpha}{(R-\tau)\sin\alpha}, \quad (18)$$

then the density function of yield strength is determined under the formula

$$p(\sigma_T) = \frac{1}{R-\tau}, \quad (19)$$

i.e. she appears the uniform on all cut of stresses from minimum, relevant to yield strength $\boldsymbol{\tau}$ of a stratum on platforms, up to bending strength of material R.

Executing the operations ordered by expression (16) under condition of (19), for stresses in a stratum at a level $\boldsymbol{y}$ from a neutral line of a rod, we shall receive

$$\sigma_1 = \frac{\varepsilon E}{\tau - R}\left(\tau + R - \frac{\varepsilon E}{2}\right). \quad (20)$$

Further it is possible to define the moment of interior forces

$$M_1 = \frac{2bE\varepsilon_0 h^2}{R-\tau}\left(\frac{\tau+R}{3} - \frac{\varepsilon_0 E}{8}\right) \quad (21)$$

If the moment of external forces $\boldsymbol{M}$, having peaked, will change the sign of a derivative and will begin to be diminished, then the moment of interior forces on a site of a unloading will be determined so

$$M_2 = \frac{2bEh^2}{R-\tau}\left[\frac{3}{16}E\left(A_0^2 - \varepsilon_0^2\right) - \frac{EA_0\varepsilon_0}{8} + \frac{\varepsilon_0(\tau+R)}{3}\right]. \quad (22)$$

Here A_0 - an amplitude value of a strain of the most deleted point of cross-section of a rod. The moment in section at repeated loading is determined under the formula

$$M_3 = \frac{2bEh^2}{R-\tau}\left[\frac{3}{16}E\left(\varepsilon_0^2 - A_0^2\right) - \frac{EA_0\varepsilon_0}{8} + \frac{\varepsilon_0\left(\tau+R\right)}{3}\right]. \quad (23)$$

If now to take advantage of relation between strain and an angle of rotation of cross-section as $d\varphi = \frac{\varepsilon_0}{h}dS$, that the area of a hysteresis loop of an element of a rod with magnitude of a differential of an arc *dS* will be defined under the formula

$$d\phi = \frac{1}{h}\left[\int_{-A}^{+A}\left(M_2 - M_3\right)d\varepsilon_0\right]dS$$

or

$$d\phi = \frac{bE^2hA_0^3}{R-\tau}dS. \quad (24)$$

The area of a hysteresis loop of a rod in length l at pure curving is determined under the formula

$$\phi = \frac{E^2A_0^3}{R-\tau}bhl. \quad (25)$$

Last formula allows to estimate dissipative properties of a rod under bending. It is visible, that dissipation is proportional to a cube of amplitude of a strain and a volume. Simplicity of the shape of these relations is a corollary of an adopted *distribution law* $p(\sigma_T)$ *and they approximately reflect actual* properties of a material. However, it does not eliminate possibility of using of an offered procedure for an estimation of dissipative properties of devices of actual constructions at the solution of many practical problems.

More powerful radiant of dissipation of energy are junctions of links. It may be welded, bolt, rivet, press and other junctions. Shaping a hysteresis loop and definition of its square considered at cyclical loading a composite construction. The amplitude law of dissipation of energy is circumscribed. Laws of dissipation of energy from amplitude of loading may be linear, degree or combined. The differential equation linking the law of dissipation of energy with an equation of a line of initial loading of a hysteresis loop is offered. The equation allows describing a head circuit of a hysteresis loop if the law of dissipation is known. Thus principle Masing's is used. The law of dissipation of energy may be obtained during dynamic trials or assigned by analogy. Advantage of such mode of synthesis of hysteresis loops consists in refusal of thorough reviewing all stages of cyclical loading, introduction of simplifying hypotheses, at last, in use of the most important factor, defining a dynamic response - amounts of a dispersed energy. The analysis of cyclical loading of constructions establishes variety of aspects of hysteresis loops. For some kinds of junctions it is possible to specify the optimum parameters ensuring maximum dissipation of energy.

V. Differential equation of a line of initial loading hysteresis construction

For a conclusion of a ratio, interesting for us, we shall consider quasi-static loading of a system with one degree of freedom. Let's assume, that the system is allocated a set of drains of energy, which with increase of level of a load sequentially are uncovered. Let's enter the following labels: x - movement; f - non-linear elastic function of initial loading, sometimes called "by a skeletal line"; A - area of a closed loop of a hysteresis at a symmetrical cycle, i.e. quantity of energy dispersed for a cycle of loading; c_i - rigidity of a system on i a stage of increase of a load.

Let's consider the first phase of loading, to which there corresponds increase of a load. Right at the beginning of loading, while $0 \le x \le x_1$, the system remains linear, i.e. the scattering of energy is absent. With increase of a load, when $x \ge x_1$, in a system the first drain of energy is opened, occur mutual slipping of particles. The cyclical change of a load in this case results in formation of a four-coal closed loop of a hysteresis and scattering of energy.

It is possible to show, that thus

$$f = c_1x + x_1(c_0 - c_1);$$

$$A = 4x_0(c_0 - c_1)(x - x_0) = k_1(x - x_0), \quad (x_0 \le x \le x_1), \quad (26)$$

where c_0, c_1 - rigidity coefficient of a loaded system on an initial site $(0 \le x \le x_1)$ and on a site first slipping $(x_1 < x \le x_2)$; k_1 - constant of proportionality between the area A of a closed loop of a hysteresis and increment of movement $\Delta x_1 = x - x_1$ on a site first slipping.

Generalizing obtained outcomes for cyclical loading after a deployment i of a drain of energy, it is possible to record the following expressions for a skeletal line $f(x)$ and relations $A(x)$

$$f(x) = \sum_{1}^{i}(c_{i-1} - c_i)x_i + c_ix; \quad (27)$$

$$A(x) = A_{i-1} + 4(f_i - x_ic_i)(x - x_i) = A_{i-1} + k_i(x - x_i), \quad (x_i \le x \le x_{i+1}). \quad (28)$$

From a ratio (28) it is possible to receive expression for rigidity appropriate to a i - site of the skeletal broken line:

$$c_i = \frac{f_i}{x_i} - \frac{A(x) - A_{i-1}}{4x_i\,(x - x_i)}. \qquad (29)$$

Taking into account (29), that

$c_i = \frac{\Delta f_i}{\Delta x_i}$, $\Delta f_i = f(x) - f_i$,

$\Delta x_i = x - x_i$, $\Delta A_i = A(x) - A_{i-1}$,

expression (29) is write as

$$\frac{\Delta f_i}{\Delta x_i} = \frac{f_i}{x_i} - \frac{\Delta A_i}{4x_i \Delta x_i}. \qquad (30)$$

Passing in expression (30) to a limit at $\Delta x_I \rightarrow 0$, i.e., supposing, that the deployment of one drain of energy continuously follows other, we obtain a differential equation

$$\frac{df}{dx} = \frac{f}{x} - \frac{dA}{4x\,dx}. \qquad (31)$$

This ratio installs connection between an equation by skeletal curve ***f(x)*** and relation dispersed for a cycle of loading of energy from amplitude of a cycle ***A(x).*** Knowing relation ***A(x)*** and initial rigidity of a system c_0, it is possible by taking advantage ratio ***(11),*** to receive an equation ***f(x)*** to generate outlines of a closed loop of a hysteresis. Let's define a decrement of oscillations δ as the relation of energy dispersed for a cycle of loading, to the double energy of elastic deformation ***W,*** and we shall accept, that $W = f\,x / 2$. Then a system c_0, it is possible by taking advantage ratio (31), to receive an equation *f(x)* to generate outlines of a closed loop of a hysteresis. Let's define a decrement of oscillations δ as the relation of energy dispersed for a cycle of loading, to the double energy of elastic deformation *W,* and we shall accept, that $W = f\,x / 2$. Then

$$\delta = \frac{A}{2W} = \frac{A}{f\,x}, \qquad A = \delta f x. \qquad (32)$$

Aftere differentiate (32) on *x:*

$$\frac{dA}{dx} = \delta' f\,x + f' x\delta + f\delta. \qquad (33)$$

Substituting expression (33) in (31), we obtain a differential equation:

$$\frac{f'}{f} = \frac{4 - \delta' x - \delta}{(4+\delta)x}. \qquad (34)$$

VI. Differential equation of a line of initial loading.

The offered above power method of identification of a line of initial loading together with a Masing principle is convenient for a construction of closed loops of a mechanical hysteresis. Amplitude relation of scattering of energy *A(x)* and initial rigidity c_0 are determined from static and dynamic tests. The integration of an (31), obtained from experiment, determines an equation *f(x)* of a line of initial deformation.

The outlines of a closed loop of a hysteresis, according to a Masing principle, are map of a skeletal line in the double scale. In activities is underlined, that for many hysteresis systems the amplitude relation *A(x)* maybe recorded as a degree function. However, such approximation no sufficiency completely reflects possible versions of scattering of energy in designs, in particular, does not describe the laws of scattering at a polygonal hysteresis. In this connection for *A(x)* suggest more general format of representation, in which is supposed availability of two sites:

$$\begin{aligned} &\text{at} \quad (0 < x < a), \quad A(x) = 0; \\ &\text{at} \quad (x > a), \quad A(x) = b\,(x-a) + k\,(x-a)^n, \end{aligned} \qquad (35)$$

where *b, k, n* - factors dependent on a type of a design and its operating time; and *a* - the value of generalized movement, after which reaching begins scattering energy.

The solution of an equation *(11)* for the first stage gives that $f = cx_0$, at $(0 < x < a)$. Thus initial site of a skeletal line is a section direct, i.e. linear elastic characteristic. At the second stage the equation (31) after a substitution in it of expression *dA/dx* gains a kind:

$$\frac{df}{d(x-a)} = \frac{f - b/4}{x-a} - \frac{nk}{4}(x-a)^{n-2}, \quad (x \geq a). \qquad (36)$$

The solution of this differential equation is the following expression

$$f = \frac{b}{4} + c_1(x-a) - \frac{nk}{4(n-2)}(x-a)^{n-1}, \qquad (37)$$

circumscribing a skeletal curve at the second stage. Constants of an integration c_0, c_1 on a sense are stiffness factors in the beginning each from sites. Various combinations of factors *a, b, k* in relation *A(x)* reduce to a realization of closed loops of a hysteresis of the various form. In case, when $a \neq 0$, $b \neq 0$, $k \neq 0$ closed loops have till two curvilinear and straight-line inclined sites.

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Fields of professional interests: Mechanics. Mechanical engineering. Dynamical loading and stability. Transport machine strength. Dynamic response and smoothness of a course of means of transport. Constructional and internal damping of transport machines. Pipeline transport. Computer analysis and calculations of transport machines constructions. Dynamic calculations, optimal designing and tests of means of transport.

Anatolijs Kobcevs. Iekšējā dempferēšana anizotropajos materiālos un konstrukcijās
Konstrukcija ir posmu kopums. Viengabalainie posmi sastāv no mikroelementiem. Mikroelementu fiziskās īpašības, kā arī saišu raksturs starp tiem nosaka vairākus konstrukcijas raksturlielumus, tajā skaitā arī disipatīvus. To novērtēšanai tika apskatīts ciklisks nosacītā mikroelementa slogojums un tipiskās saites starp konstrukcijas posmiem. Rezultātā tika iegūts histerēzes cilpas kontūras vienādojums un aprēķināts tās laukums. Mikroelements sastāv no divām elastīgām daļām, kas ir savienotas ar elastīgi plastisku slāni. Slāņu orientācijai ir gadījuma raksturs. Saspiešanas gadījumā elastīgās daļas tiecās uz pārvietošanos slānī. Pārvietošanās sākas pie slodzes, kad tangenciālais spriegums sasniedz slāņa plūstamības robežu. Plūstamības robežas vērtība mikroelementiem var būt dažāda, tāpēc konstrukciju pētījumus jāveic ar statistiskām metodēm. Tiek pieņemts, ka leņķu sadalījums, kas nosaka mikroelementu un plūstamības robežu stāvokli, ir vienmērīgs. Kā aprēķinu kontrukcijas tiek apskatītas daudzslāņu konsolsijas. Cikliski slogojot šīs sijas, pakāpeniski rodas savstarpējās atsevišķu slāņu izslīdes. Tiek izjaukta deformēšanas linearitāte, un slogojuma skeletlīnija pāriet gabala lineārā formā. Tam atbilst poligonālā histerēzes cilpa. Palielinoties sijas slāņu skaitam, cilpas lūzumi nogludinās. Lai saņemtu skeletlīnijas vienādojumu, tika piedāvāts diferenciālvienādojums, kas izmanto enerģijas izkliedes amplitūdas likumu. Cilpas kontūras veidojas pēc Mazinga principa kā skeletlīnijas attēlojums divreiz lielākā mērogā. Piedāvāta enerģijas izkliedes amplitūdas likuma, kuru dažreiz var iegūt aprēķinu vai eksperimentālā ceļā, universālā pieraksta forma. Pamatojoties uz šo likumu, tiek veidotas skeletlīnijas un dažāda veida histerēzes cilpas kontūras konstrukciju sastāvdaļām.

Анатолий Кобцев. Внутреннее демпфирование в анизотропных материалах и конструкциях
Конструкции представляют собой общность звеньев. Монолитные звенья состоят из микроэлементов. Физические особенности микроэлементов, характер связей между ними определяют многие характеристики конструкций, в том числе и диссипативные. Для их оценки рассмотрено циклическое нагружение условного микроэлемента и типичных связей между звеньями конструкций. В результате получено уравнение контура петли гистерезиса и рассчитана её площадь при осевой нагрузке и изгибе. Микроэлемент состоит из двух упругих частей, связанных упруго-пластичным слоем Ориентация слоев имеет случайный характер. При сжатии упругие части стремятся сдвинуться по слою. Перемещения начинаются при нагрузке когда касательные напряжения достигают предела текучести слоя. Величины предела текучести у микроэлементов могут быть разными. Поэтому исследования конструкций необходимо проводить статистическими методами. Предполагается, что распределения углов, определяющих положения микроэлементов и пределов текучести слоев, являются равномерными. В качестве расчётных конструкций рассматриваются многослойные консольные балки. При их циклическом нагружении последовательно возникают взаимные проскальзывания отдельных слоёв. При этом нарушается линейность деформирования и скелетная линия нагружения принимает кусочно-линейную форму, Этому соответствует полигональная петля гистерезиса, При увеличении числа слоёв балки изломы петли сглаживаются и в пределе она трансформируется в гладкую с криволинейными очертаниями контура. Для получения уравнения скелетной линии предложено дифференциальное уравнение, использующее амплитудный закон рассеяния энергии. Контуры петли формируются на основе принципа Мазинга, как удвоенное в масштабе отображение скелетной линии. Предложена универсальная форма записи для амплитудного закона рассеяния энергии, который можно иногда получить расчётным или экспериментальным путём. На основании этого закона формируются скелетная линия и контуры петли гистерезиса разного вида для составных конструкций.