2010
Volume 34

Computation Model of Spatial Movement of the Car

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Abstract. **For the record of equalizations of motion inertial and constrained frames are used. Transition matrices and kinematics relations are set between the different frames of reference. As the generalized coordinates of cushion (spring up) body three linear moving of centre of gravity and three corner of turn in relation to the constrained coordinate axes is used. Position of noncushion (spring down) the masses described the vertical moving. Indignant motion is caused by the irregularity (unevenness) of road. Nonlinear resilient and dissipative forces in a saspension and tires are determined depending on their deformations. The design of casual indignations is conducted on the set spectral closeness. Motion of car on rough roads is accompanied the continuous vibrations of his construction.**

Keywords**: car, model, movement, over spring part, suspension, tire, energy dissipation**

I. Introduction

Movement of the car on rough roads is accompanied by continuous fluctuations of its design which render harmful influence on the driver, passengers, worsen operating conditions of units and aggregates. Fluctuations create additional dynamic loadings, accelerate the charge of a resource. Their intensity defines smoothness of a course of the car. Improvement of smoothness of a course is reached by giving of a design of the car of such properties which provide optimum dampferation and reduction in intensity of fluctuations. First of all this rational designing of a suspension bracket and selection of tires.

II. The Generalized Model and the Equations of Movement of the Car. Systems of Coordinates and Matrixes of Their Transformations

At studying the phenomena and processes in the nature, the decision of applied tasks in the most different areas of a science and technical equipment *mathematical models* of studied object are widely used. They allow to investigate analytically dynamic system, to determine its optimum parameters, to predict development of process. Thus mathematical modeling assumes use of modern means of calculations with their developed software.

At creation of dynamic model the car is considered in the form of the oscillatory system consisting of certain firm bodies, connected by elastic elements and damper devices. Disturbance of fluctuations occurs from various sources: roughness's of road, vibrations and not balanced the engine and units of transmission. But with concept of smoothness of a course the fluctuations caused by influence of road communicate only.

At dynamic calculations the basic interest represents behavior over spring (cushioning) mass. For record of the equations of movement three systems of coordinates are used.

The inertial system of coordinates $OXYZ$ is intended for record of the equations of movement of the center of mass C overspring weights. The beginning of this system O till the moment t_0 the first beginning of influence of disturbance is combined with the center of weights C of overspring parts. Further points O and C are separated. The system $OXYZ$ continues progress with speed $V_0(t_0)$. Axis OX is directed horizontally along a longitudinal axis back; the axis OY is perpendicular OX and also is directed on a vertical upwards; the axis OZ is perpendicular a longitudinal plane [1].

The connected central axes of coordinates $C\xi\eta\zeta$ are intended for record of the equations of rotary movement. Axes settle down along the main central axes of a body. At rotary movement they change the orientation in space (fig.1).

The forward moving axes $CX_0Y_0Z_0$ are entered for indication of corners of turn of the connected axes $C\xi\eta\zeta$. Their beginning also is combined with the center of weights C, and directions coincide with directions of corresponding axes of inertial system of coordinates. However, axes $CX_0Y_0Z_0$ inertial are not.

Current position of a system of the connected axes $C\xi\eta\zeta$ turning together with a body can be determined by means of three corners $\psi\upsilon\gamma$ of turn it relatively forward moving axes $CX_0Y_0Z_0$. Prior to the beginning of action of disturbance forward moving axes $CX_0Y_0Z_0$ and the connected axes $C\xi\eta\zeta$ are combined.

The first turn of a body together with the connected axes $C\xi\eta\zeta$ occurs on a corner of roving ψ with angular speed $\dot{\psi}\boldsymbol{j}$ around of an axis CY_0. The connected axes thus occupy intermediate position $C\xi_1\,\eta_0\,\zeta_1$ with single vectors i_1, j_0, k_1.

Matrix of transition from axes $C\xi_0\,\eta_0\,\zeta_0$ with single vectors i_0, j_0, k_0 to axes $C\xi_1\,\eta_0\,\zeta_1$ looks like

$$A_1 = \begin{vmatrix} \cos\psi & 0 & -\sin\psi \\ 0 & 1 & 0 \\ \sin\psi & 0 & \cos\psi \end{vmatrix} \qquad (1)$$

Similarly to be the second turn on a corner of galloping υ around of an axis $C\zeta_1$ with angular speed $\dot{\vartheta}\cdot\bar{k}_1$ and the third turn on a corner of cross-section rocking γ with angular speed $\dot{\gamma}_i \cdot \bar{i}_2$ around of an axis $C\xi_2$.

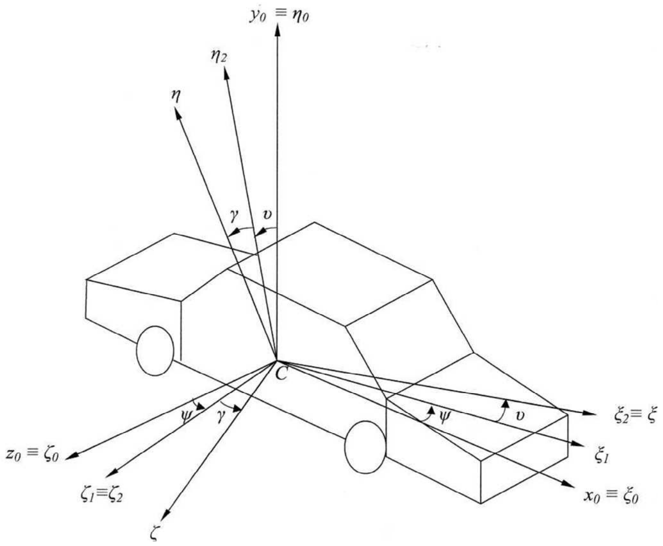

Fig.1. Axes of coordinates

$$A_2 = \begin{vmatrix} \cos\vartheta & \sin\vartheta & 0 \\ -\sin\vartheta & \cos\vartheta & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$A_3 = \begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & \sin\gamma \\ 0 & -\sin\gamma & \cos\gamma \end{vmatrix}$$

The full matrix of transition of coordinates of axes $C\xi_2\,\eta_2\,\zeta_2$ to axes $C\xi\eta\zeta$ looks like

$$A = A_1 \cdot A_2 \cdot A_3 \tag{2}$$

$$A = \mathrm{A}_1 \cdot A_2 \cdot A_3 = \begin{vmatrix} \cos\vartheta_i\cos\psi_i; & \sin\vartheta_i; & -\sin\psi_i\cos\vartheta_i; \\ \sin\gamma_i\sin\psi_i - \cos\gamma_i\cos\psi_i\sin\vartheta_i; & \cos\gamma_i\cos\vartheta_i; & \sin\gamma_i\cos\psi_i + \sin\psi_i\sin\vartheta_i\cos\gamma_i; \\ \cos\gamma_i\sin\psi_i + \sin\gamma_i\cos\psi_i\sin\vartheta_i; & -\sin\gamma_i\cos\vartheta_i; & \cos\gamma_i\cos\psi_i - \sin\psi_i\sin\vartheta_i\sin\gamma_i; \end{vmatrix}$$

By means of a matrix $[A]$ coordinates of any vector $\bar{a}_0$ set in system of axes $C\xi_0\eta_0\,\zeta_0$, can be calculated for system $C\xi\eta\zeta$ on a correlation

$$\{\bar{a}\} = [A] \cdot \{\bar{a}_0\} \tag{3}$$

Let's express by means of (2) and (3) vector of angular speed of a body $\bar{\omega}$ in the connected system of coordinates $C\xi\eta\zeta$. According to transformation, this vector $\bar{\omega}$ is the sum of three vectors corresponding angular speeds of roving, galloping and reeling

$$\bar{\omega} = \dot{\psi} \cdot \bar{j}_0 + \dot{\upsilon} \cdot \bar{k}_1 + \dot{\gamma} \cdot \bar{i}_2 \tag{4}$$

Let's spread out each of vectors-component of expression (4) in directions of coordinate axes $C\xi_0\eta_0\,\zeta_0$

$$\begin{aligned} \dot{\psi} \cdot \bar{j}_0 &= 0 \cdot \bar{i}_0 + \dot{\psi} \cdot \bar{j}_0 + 0 \cdot \bar{k}_0 \\ \dot{\vartheta} \cdot \bar{k}_1 &= \dot{\vartheta}\sin\psi \cdot \bar{i}_0 + 0 \cdot \bar{j}_0 + \dot{\vartheta}\cos\psi \cdot \bar{k}_0 \\ \dot{\gamma} \cdot \bar{i}_2 &= \dot{\gamma}\cos\vartheta \cdot \cos\psi \cdot \bar{i}_0 + \dot{\gamma}\sin\vartheta \cdot \bar{j}_0 - \\ &\quad - \dot{\gamma}\cos\vartheta \cdot \sin\psi \cdot \bar{k}_0 \end{aligned} \tag{5}$$

On the other hand, the vector of angular speed $\bar{\omega}$ can be expressed through projections to coordinate axes $C\xi_0\eta_0\,\zeta_0$. Considering (5), we receive

$$\begin{aligned} \bar{\omega}_0 &= \omega_{\xi 0} \cdot \bar{i}_0 + \omega_{\eta 0} \cdot \bar{j}_0 + \omega_{\zeta 0} \cdot \bar{k}_0 = \\ &= \left(\dot{\vartheta}\sin\psi + \dot{\gamma}\cos\vartheta \cdot \cos\psi\right)\bar{i}_0 + \left(\dot{\psi} + \dot{\gamma}\sin\vartheta\right)\bar{j}_0 + \\ &\quad + \left(\dot{\vartheta}\cos\psi - \dot{\gamma}\cos\vartheta \cdot \sin\psi\right)\bar{k}_0. \end{aligned} \tag{6}$$

Projections of a vector of angular speed to the connected axes of coordinates $C\xi\eta\zeta$ calculate as matrix production $\{\bar{\omega}\} = [A] \cdot \{\bar{\omega}_0\}$, or

$$\begin{Bmatrix} \omega_\xi \\ \omega_\eta \\ \omega_\zeta \end{Bmatrix} = [A] \begin{Bmatrix} \omega_{\xi 0} \\ \omega_{\eta 0} \\ \omega_{\zeta 0} \end{Bmatrix} = \begin{Bmatrix} \dot{\psi}\sin\vartheta + \dot{\gamma} \\ \dot{\psi}\cos\vartheta \cdot \cos\gamma + \dot{\vartheta}\sin\gamma \\ \dot{\vartheta}\cos\gamma - \dot{\psi}\cos\vartheta \cdot \sin\gamma \end{Bmatrix} \tag{7}$$

The kinematic parities defining projections of angular speed $\omega_\xi\ \omega_\eta\ \omega_\zeta$ on connected axes of coordinates $C\xi\eta\zeta$ through corners ψ, galloping υ, cross-section rocking γ and their derivatives are as a result received

$$\begin{aligned} \omega_\xi &= \dot{\psi}\sin\vartheta + \dot{\gamma} \\ \omega_\eta &= \dot{\psi}\cos\vartheta \cdot \cos\gamma + \dot{\vartheta}\sin\gamma \\ \omega_\zeta &= \dot{\vartheta}\cos\gamma - \dot{\psi}\cos\vartheta \cdot \sin\gamma. \end{aligned} \tag{8}$$

Kinematic parities (8) in the further be required in the form of, resolved relatively $\dot{\upsilon}, \dot{\gamma}, \dot{\psi}$

$$\dot{\vartheta} = \omega_{\eta} \sin\gamma + \omega_{\zeta} \cos\gamma$$

$$\dot{\psi} = \frac{1}{\cos\vartheta}\left(\omega_{\eta}\cos\gamma - \omega_{\zeta}\sin\gamma\right) \tag{9}$$

$$\dot{\gamma} = \omega_{\xi} - tg\vartheta\left(\omega_{\eta}\cos\gamma - \omega_{\zeta}\sin\gamma\right)$$

Correlations (8), (9) it is possible to receive from geometrical reasons. For this purpose it is necessary at each of three described turns to make designing vectors of angular speeds on a three of intermediate axes of coordinates received at turn.

III. THE EQUATIONS OF MOVEMENT OF THE CAR

At recording of the equations of movement over spring and not over spring masses are separated from its elastic-histereze connections which action is replaced with system of power factors. The equations of forward movement of the center of weights C are writing down in projections to inertial axes of coordinates ***0XYZ*** and look like:

. for over spring mass

$$M\ddot{X}_c = \Sigma R_x$$

$$M\ddot{Y}_c = \Sigma R_y \tag{10}$$

$$M\ddot{Z}_c = \Sigma R_z$$

. for not over spring masses

$$m_i \ddot{Y}_i = F_i^{el} + F_i^{a\text{м}} - F_i^{pn}, \tag{11}$$

at $Y_c > Y_i$

where $\bar{R} = R_x\bar{i} + R_y\bar{j} + R_z\bar{k}$ - the main vector of all forces acting on overspring weight;

F_i^{el}, F_i^{am}, F_i^{pn}, F_i^{fr} - forces of elasticity of springs, compression of amortization, compression of tires, dry friction of dempers, acting on *i-th* not overspring weights, ($i=1, 2 \dots n$, n - number of wheels).

The equations of rotary movement for overspring mass are writing down in projections to the connected axes of coordinates $C\xi\eta\zeta$ and look like

$$J_{\xi}\frac{d\omega_{\xi}}{dt} + \left(J_{\zeta} - J_{\eta}\right)\omega_{\eta}\varpi_{\zeta} = M_{C\xi}$$

$$J_{\eta}\frac{d\omega_{\eta}}{dt} + \left(J_{\xi} - J_{\zeta}\right)\omega_{\zeta}\varpi_{\xi} = M_{C\eta} \tag{12}$$

$$J_{\zeta}\frac{d\omega_{\zeta}}{dt} + \left(J_{\eta} - J_{\xi}\right)\omega_{\eta}\varpi_{\xi} = M_{C\zeta}$$

where

J_{ξ}, J_{η}, J_{ζ} - the main central moments of inertia;

ω_{ξ}, ω_{η}, ω_{ζ} - projections of a vector of angular speed $\bar{\omega}$ to the connected axes;

$M_{c\xi}$, $M_{c\eta}$, $M_{c\zeta}$ - projections to the connected axes of a vector of the main moment $\bar{M}_c = M_{c\xi}\bar{i} + M_{c\eta}\bar{j} + M_{c\zeta}\bar{k}$ of the forces, acting on overspring mass M, relatively her center of weights.

The equations of movement not overspring masses are writing down in projections to inertial axes *0XYZ.* Thus rotary components of movement and movement along axes *0X, 0Z* are not considered

$$\ddot{y}_1^p m_k^p = F_r^{p.l.} + F_{am}^{p.l.} - F_{pn}^{p.l.} - k\dot{y}_1^p$$

$$\ddot{y}_2^p m_k^p = F_r^{p.pr.} + F_{am}^{p.pr.} - F_{pn}^{p.pr.} - k\dot{y}_2^p$$

$$\ddot{y}_3^z m_k^z = F_r^{z.l.} + F_{am}^{z.l.} - F_{pn}^{z.l.} - k\dot{y}_3^z$$

$$\ddot{y}_4^z m_k^z = F_r^{z.pr.} + F_{am}^{z.pr.} - F_{pn}^{z.pr.} - k\dot{y}_4^z$$

where $F_r^{p.l.}, F_r^{p.pr.}, F_r^{z.l.}, F_r^{z.pr.}$ - forces wringing out of front and back springs;

$F_{am}^{p.l.}, F_{am}^{p.pr.}, F_{am}^{z.l.}, F_{am}^{z.pr.}$ - forces wringing out of front and back amortizators, shock absorbers;

$F_{pn}^{p.pr.}, F_{pn}^{p.l.}, F_{pn}^{z.pr.}, F_{pn}^{z.l.}$ - forces wringing out of front and back tires.

The burries of road cause wringing out of tires, which are determined dy expressions:

deformations of the springs

$$\Delta_{r2}^{p} = y_{a2} - y_2^p, \qquad \Delta_{r1}^{p} = y_{a1} - y_1^p,$$

$$\Delta_{r3}^{l} = y_{b1} - y_3^z, \qquad \Delta_{r4}^{l} = y_{b2} - y_4^z.$$

velocity of pression of the amortizators

$$\dot{\Delta}_1^{l.p} = \dot{y}_{a1} - \dot{y}_1^p, \qquad \dot{\Delta}_2^{pr.p} = \dot{y}_{a2} - \dot{y}_2^p,$$

$$\dot{\Delta}_3^{l.z} = \dot{y}_{b1} - \dot{y}_3^z, \qquad \dot{\Delta}_4^{pr.z} = \dot{y}_{b2} - \dot{y}_4^z.$$

Forces of pression of the springs and tires are determined with taking into account their hysteresis properties. Simpler than all hysteresis can be described by means of poligona's loops. Equations of skeletal line of loop can be got, using the amplitudel law of dispersion of energy.

Loops describe with expressions

$$F(\Delta,\dot{\Delta}) = \begin{cases} C'\Delta + Tsign\ (\dot{\Delta}), \\ C''\Delta + Tsign\ (\dot{\Delta}) - \Delta_1(C'' - C') \end{cases}$$

Are obtain a differential equation

$$\frac{df}{dx} = \frac{f}{x} - \frac{dA}{4x\,dx}. \qquad (13)$$

This ratio installs connection between an equation by skeletal curve *f(x)* and relation dispersed for a cycle of loading of energy from amplitude of a cycle *A(x).* Knowing **relation** *A(x)* and initial rigidity of a system c_0, it is possible by taking advantage ratio (13), to receive an equation *f(x)* to generate outlines of a closed loop of a hysteresis.

Let's define a decrement of oscillations δ as the relation of energy dispersed for a cycle of loading, to the double energy of elastic deformation *W,* and we shall accept, that $W = fx/2$.

Then $$\delta = \frac{A}{2W} = \frac{A}{fx}, \quad A = \delta f x. \qquad (14)$$

We differentiate (14) on ***x:***

$$\frac{dA}{dx} = \delta' f x + f' x\delta + f\delta. \qquad (15)$$

Substituting expression (15) in (13), we obtain a differential equation:

$$\frac{f'}{f} = \frac{4 - \delta' x - \delta}{(4+\delta)x}.$$

Equation of a skeletal line and formation of an outline of a closed loop of a hysteresis. The offered above power method of identification of a line of initial loading together with a Masing principle is convenient for a construction of closed loops of a mechanical hysteresis. Amplitude relation of scattering of energy *A(x)* and initial rigidity c_0 are determined from static and dynamic tests. The integration of an equation (13) with allowance for of relation *A(x),* obtained from experiment, determines an equation *f(x)* of a line jf initial deformation. The outlines of a closed loop of a hysteresis, according to a Masing principle, are map of a skeletal line in the double scale. In activities [2,3] is underlined, that for many hysteresis systems the amplitude relation *A(x)* may be recorded as a degree function. However, such approximation unsufficiently completely reflects possible versions of scattering of energy in designs, in particular, does not describe the laws of scattering at a polygonal hysteresis. In this connection for *A(x)* suggest more general format of representation, in which is supposed availability of two sites:

At $0 < x < a$, $A(x) = 0;$

at $x > a$, $A(x) = b\,(x\text{-}a) + k\,(x\text{-}a)^n$, (15)

where *b, k, n* - factors dependent on a type of a design and its operating time ; and *a* - the value of generalized movement, after which reaching begins scattering energy.

The solution of an equation (11) for the first stage gives that $f = c\ x_0$, at $0 < x < a$. Thus initial site of a skeletal line is a section direct, i.e. linear elastic characteristic. At the second stage the equation (11) after a substitution in it of expression *dA/dx* gains a kind:

$$\frac{df}{d(x-a)} = \frac{f - b/4}{x-a} - \frac{nk}{4}(x-a)^{n-2}, \qquad (x \ge a). \quad (16)$$

The solution of this differential equation is the following expression

$$f = \frac{b}{4} + c_1(x-a) - \frac{nk}{4(n-2)}(x-a)^{n-1}, \qquad (17)$$

circumscribing a skeletal curve at the second stage. Constants of an integration c_0, c_1 on a sense are stiffness factors in the beginning each from sites. Various combinations of factors *a, b, k* in relation *A(x)* reduce to a realization of closed loops of a hysteresis of the various form.

In case, when $a\neq0$, $b\neq0$, $k\neq0$ closed loops have till two curvilinear and straight-line inclined sites. If $a=0$, the inclined lateral sites become vertical. At $a=0$, $k=0$, but $b\neq0$ closed loops are transformed in bilinear with vertical lateral segments, if $a\neq0$, $b\neq 0$, but $k= 0$ - in bilinear with inclined sites. At last at *a=b, =0* but $k\neq0$ outlines of a closed loop are curvilinear. If thus of zero rigidity of a design c_0, c_1 in different directions of initial loading are various, for example, because of availability of a crack, the closed loop becomes asymmetrical, gaining the shape of a banana. Included in expression (15) amplitude laws of scattering of energy parameters *a, b, k, n* vary in accordance with accumulation with a design in maintenance of not desroying damages. The being available experimental data allow approximately to set ranges of change of these parameters from a moment of a beginning of maintenance of a design change the law of scattering of energy. It varies results in change of parameters and before failure.In initial stage the design works as a monolith, it the rigidity is maximum, scattering of energy it is not enough, closed loop of a hysteresis vary narrow. In accordance with a wear of fixed connections reduction rigities

of components and joints happens, transformation of closed loops of a hysteresis. It can call strong influence on dynamic loading of a design.
For definition of angular coordinates ϑ, ψ, γ of a body it is possible to use kinematic ratio (9). As a result rotary movement is described by six differential equations of the first order. Reduction of the equations of movement with simultaneous increase in their order is possible. For this purpose we differentiate ratio (8). We shall receive

$$\begin{aligned}
&\frac{dw_\xi}{dt} = \ddot{\psi}\sin\upsilon + \dot{\psi}\dot{\upsilon}\cos\upsilon;\\
&\frac{dw_\eta}{dt} = \ddot{\psi}\cos\upsilon\cdot\cos\gamma - \dot{\psi}\dot{\upsilon}\sin\upsilon\cdot\cos\gamma - \dot{\psi}\dot{\gamma}\cos\upsilon\cdot\sin\gamma + \ddot{\upsilon}\sin\gamma + \dot{\upsilon}\dot{\gamma}\cos\gamma;\\
&\frac{dw_\zeta}{dt} = \ddot{\upsilon}\cos\gamma - \dot{\upsilon}\dot{\gamma}\sin\gamma - \ddot{\psi}\cos\upsilon\cdot\sin\gamma + \dot{\psi}\dot{\upsilon}\sin\gamma - \dot{\psi}\dot{\gamma}\cos\upsilon\cdot\cos\gamma.
\end{aligned} \tag{16}$$

The system (12) will be transformed to system of the differential equations of the second order

$$\begin{aligned}
&J_\xi\ddot{\psi}\cdot\sin\upsilon + J_\xi\dot{\psi}\cdot\dot{\upsilon}\cdot\cos\upsilon + J_\xi\ddot{\gamma} + \left(J_\zeta - J_\eta\right)\left(\dot{\psi}\cdot\cos\upsilon\cdot\cos\gamma + \dot{\upsilon}\sin\gamma\right)\left(\dot{\upsilon}\cdot\cos\gamma - \dot{\psi}\cdot\cos\upsilon\cdot\sin\gamma\right) = M_{c\xi},\\
&J_\eta(\ddot{\psi}\cdot\cos\upsilon\cos\gamma - \dot{\psi}\cdot\dot{\upsilon}\cdot\sin\upsilon\cos\gamma - \dot{\psi}\dot{\gamma}\cos\vartheta\sin\gamma + \ddot{\vartheta}\sin\gamma - \dot{\vartheta}\dot{\gamma}\cos\gamma + \left(J_\xi - J_\zeta\right)(\dot{\psi}\cdot\cos\upsilon\cdot\cos\gamma\cdot\dot{\upsilon}\cdot\cos\gamma -\\
&\dot{\upsilon}\sin\gamma\cdot\dot{\psi}\cdot\cos\upsilon\cdot\sin\gamma - \dot{\psi}^2\cos^2\upsilon\cdot\sin\gamma\cdot\cos\gamma + \dot{\upsilon}^2\cdot\sin\gamma\cdot\cos\gamma) = M_{c\eta},\\
&J_\zeta(\ddot{\psi}\cdot\cos\gamma - \dot{\vartheta}\dot{\gamma}\sin\gamma - \ddot{\vartheta}\cos\vartheta\sin\gamma + \dot{\psi}\dot{\gamma}\cos\vartheta\cos\gamma) + \left(J_\eta - J_\xi\right)\cdot\\
&\left(\dot{\upsilon}\cdot\dot{\psi}\cdot\cos\upsilon\cdot\cos^2\gamma + \sin\gamma\cdot\cos\gamma(\dot{\upsilon}^2 - \dot{\psi}^2\cdot\cos^2\upsilon)\right) = M_{c\zeta},
\end{aligned}$$

After re-grouping receive

$$\begin{aligned}
&J_\xi(\ddot{\psi}\sin\upsilon + \ddot{\gamma}) + \dot{\psi}\dot{\upsilon}\cos\upsilon\left[J_\xi + (J_\zeta - J_n)\cos^2\gamma\right] + \frac{(\dot{\upsilon}^2 - \dot{\psi}^2\cos^2\upsilon)\sin^2\gamma}{2}(J_\zeta - J_n) = M_{c\xi},\\
&J_\eta(\ddot{\psi}\cdot\cos\upsilon\cdot\cos\gamma + \ddot{\upsilon}\cdot\sin\gamma) + \left(J_\xi - J_\zeta - J_\eta\right) + \dot{\upsilon}\cdot\sin\upsilon\cdot\cos\gamma + \left(J_\xi - J_\zeta - J_\eta\right)\left(\dot{\upsilon}\cdot\cos\gamma - \dot{\psi}\cdot\cos\upsilon\cdot\sin\gamma\right)\dot{\gamma} -\\
&-\frac{J_\xi - J_\zeta}{2}\dot{\psi}^2\cdot\sin\gamma = M_{c\eta},\\
&J_\zeta(\ddot{\vartheta}\cdot\cos\gamma - \ddot{\psi}\cos\vartheta\cdot\sin\gamma) + \left(J_\eta + J_\zeta - J_\xi\right)\cdot\dot{\psi}\dot{\vartheta}\sin\vartheta\cdot\sin\gamma + \left(J_\eta + J_\zeta - J_\xi\right)(\dot{\upsilon}\dot{\gamma}\cdot\sin\gamma + \dot{\psi}\dot{\gamma}\cdot\cos\upsilon\cdot\cos\gamma) +\\
&+\left(\frac{J_\eta - J_\xi}{2}\right)\dot{\psi}^2\cdot\sin 2\vartheta = M_{c\xi}.
\end{aligned} \tag{17}$$

IV. Definition of forces

External disturbance from roughnesses of road is transferred the car through tires which compression is determined by expressions

$$\Delta Y_i^{пн} = Y_i^{пн} + h_i\sin\left(\omega_i t + \varphi_i\right), \tag{18}$$

where $h_i(t)$ - height of protuberance of roughnesses of the road, being stochastic function;

ω_i, φ_i - circular frequency and phase shift. Thus indignation with constant frequency is modelled.

Generally casual disturbance generates on spectral density of height of protuberance of roughnesses of the concrete road. The method of Monte-Carlo is used. Stochastic function height of protuberance of roughnesses calculate off under the formula

$$h_i(t) = \sum_{n=1}^{N} A_n \sqrt{u_n^2 + v_n^2} \cdot \sin\left(\omega n + \xi n\right), \qquad (19)$$

where

A_n, ω_n, ξ_n - amplitude, frequency and a phase of n-th harmonic;

u_n , γ_n - normally distributed numbers with a zero average and an individual dispersion;

N - number of harmonics of casual process.

Amplitudes A_n of harmonics select so that the area of a site of spectral density an interval of frequencies $\Delta\omega_n = \omega_{n-0,5} - \omega_{n+0,5}$ was equal to a corresponding dispersion

$$A_n = \sqrt{2Sn(\omega n)\Delta\omega n}.$$

Modelling of stochastic function $h_i(t)$ is reduced to manufacture of its discrete values at regular intervals of time. The size of a step Δt gets out of a condition that the harmonic of the maximum frequency has been presented by 8-10 points. At calculation of the radial forces of the tires F_i^{pn} is necessary take into account its histereze properties. This considers their dempferations abilities and raises reliability of dynamic calculations.

Size of compression of springs, shock-absorbers it is determined by the formula

$$\Delta y_i = y_0 - y_i \text{ , at } y_0 > y_i . \qquad (20)$$

CONCLUSION

In work is offered the model of the car with taking in account nonlinear and dissipative properties of a suspension bracket. The model can be used on a design stage of the car for optimization and estimations of a technical qualities.

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Anatolijs Kobcevs. Telpiskas automobiļa kustības aprēķina modelis
Kustības vienādojumu pierakstīšanai tiek lietotas inercionālās un saistītās koordināšu sistēmas. Tiek uzstādītas pārejas un kinemātisko attiecību starp dažādām atskaites sistēmām matricas.
Kā atsperotā ķermeņa vispārējās koordinātes tiek izmantotas trīs smaguma centra lineārās pārvietošanās un trīs pagrieziena leņķi attiecībā pret saistītām koordinātu asīm. Pārejas no inercionālās koordinātu sistēmas uz ar automobili saistīto sistēmu matrica tiek noteikta kā triju starpmatricu vektoriālais atvasinājums. Katra no tām atbilst pagriezienam pa vienu no koordinātu leņķiem. Pirmais automobiļa pagrieziens kopā ar saistītām koordinātu asīm notiek pa pārvietošanās leņķi. Otrais pagrieziens notiek pa lēkāšanas leņķi un trešais – pa šūpošanās leņķi. Neatsperoto masu stāvokli apraksta vertikālā pārvietošanās. Perturbēto kustību izraisa ceļa nelīdzenumi. Nejaušo perturbāciju modelēšana tiek veikta pēc dotā spektrālā blīvuma. Nelineārie elastīgie un disaptīvie spēki automobiļa piekarē un riepās tiek noteikti atkarībā no to deformācijas. Cikliskā spēka atkarību no tā radošās kustības apraksta histerēzes cilpa. Cilpas forma var būt poligonāla vai līklīnijas. Histerēzes cilpu kontūras tiek formētas pēc Mazinga principa un ar skeletlīnijas sākumslodzes vienādojuma palīdzību. Automobiļa kustību pa nelīdzeniem ceļiem pavada nepārtrauktas tā konstrukcijas svārstības. Tas atstāj nelabvēlīgu iespaidu uz vadītāju, pasažieriem un pasliktina darba apstākļus. Svārstības rada papildus dinamiskās slodzes, samazina resursa kalpošanas laiku. To intensitāte nosaka automobiļa līdzenu gaitu. Lai uzlabotu gaitas līdzenumu, automobiļa konstrukcijai ir jābūt tādām īpašībām, kuras nodrošina optimālu svārstību slāpēšanu un svārstību intensitātes samazināšanu. Galvenokārt tā ir racionāla piekares projektēšana un riepu izvēle.

Анатолий Кобцев. Расчётная модель пространственного движения автомобиля
Для записи уравнений движения используются инерциальные и связанные системы координат. Установлены матрицы перехода и кинематические соотношения между различными системами отсчёта. В качестве обобщенных координат подрессоренного тела используются три линейных перемещения центра тяжести и три угла поворота относительно связанных координатных осей. Матрица перехода от инерциальной системы координат к связанной с автомобилем системе определяется как векторное произведение трёх промежуточных матриц. Каждая из них соответствует повороту на один координатный угол. Первый поворот автомобиля вместе со связанными координатными осями происходит на угол рысканья. Второй поворот совершается на угол галопирования и третий – на угол покачивания. Положение неподрессоренных масс описываются вертикальными перемещениями. Возмущенное движение вызывают неровности дороги. Моделирование случайных возмущений проводится по задаваемой спектральной плотности. Нелинейные упругие и диссипативные силы в подвеске и шинах определяются в зависимости от их деформаций. Циклическая зависимость силы от вызываемого ей перемещения имеет вид петли гистерезиса. Петли могут быть полигональными или иметь криволинейные очертания. Контуры петель гистерезиса формируются с помощью принципа Мазинга и уравнения скелетной линии начального нагружения. Движение автомобиля по неровным дорогам сопровождается непрерывными колебаниями его конструкции. Они оказывают вредное воздействие на водителя, пассажиров, ухудшают условия работы. Колебания создают дополнительные динамические нагрузки, ускоряют расход ресурса. Их интенсивность определяет плавность хода автомобиля. Улучшение плавности хода достигается приданием конструкции автомобиля таких свойств, которые обеспечивают оптимальное демпфирование и снижение интенсивности колебаний. В первую очередь, это рациональное проектирование подвески и подбор шин.