

Hierarchic Optimization of the Resources Allocation among the Units of the Logistic Centre

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Abstract. In this paper the problem of resources distribution in optimal way among the units of the Logistic centre was observed. This task could be solved by using the method of dynamic programming. Using the programming language Mathcad 14 there was created the special program which allows to make the corresponding calculations.

The solution of the real task for Logistic centre in Latvia is observed in this paper as the numerical sample of limited resource distribution between the units of the logistic centre in order to get the maximum profit.

Keywords: dynamic programming, hierarchic, logistic centre, optimization.

I. PROBLEM SETTING

One Latvian company is the producer and trader of some products. The significant part of the goods that is about 40% of total amount is imported from abroad. In the same time 60% of products has Latvian origin and is produced on the company's own production and with subcontractors. In order to minimize storage time and volume of produced goods and maximize speed and efficiency of goods deliveries to clients the supply chain was created by logistic service provider that was presented by company own Logistic centre. One of ideas of Logistic chain is to design together the facility and delivery

process to better reveal and support of customer purposes. Using the information technology and efficient flow of information between the members of the chain as well as understanding the business and needs of clients Logistic centre provides just in time deliveries of raw materials and ready-made goods. Activity of the company would not be successful without a good work of Logistics centre, which has the following hierarchic structure shown by the Fig.1.

The Logistic centre consists of two departments Logistic department and Production plant.

The logistic department of the company rents the warehouse of 600 square metres. There are 7 employees work in logistics: 3 persons are in the office and 4 are workers on the warehouse. The operational costs of the logistic department is for about 15 000 LVL per month. The tasks of the logistics department are to provide the delivery of raw materials for production units, to manage the warehouse of the ready made goods, to provide the delivery of ordered goods to clients in time and to control the quantity of goods on the store without it overloading. Own transport vehicles assist in distribution of goods and quick connection with production units.

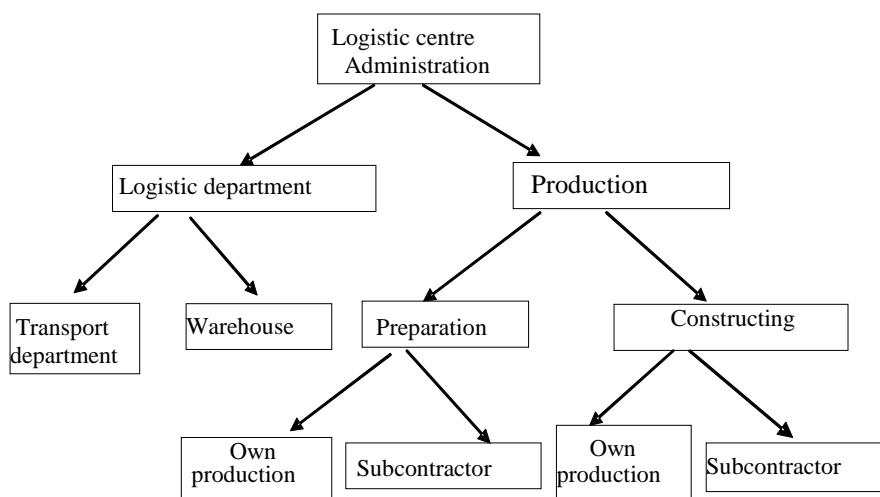


Fig. 1. The Hierarchic Structure

There are 70 employees are working in production process. Company owes 1000 square metres building and has own processing equipment. The current costs for production is about 45 000 LVL. The specific of the production industry is in it preliminarily planning. The production of raw materials has to be planned for about 11-12 months in advance before the delivery of ready-made goods to clients. From the other side ready goods are so named sensitive product. It means that goods realisation time is quite limited, so the lean time of the product should be especially as little as possible. The production order is made using the new season collection sales samples based on estimations of the clients ordered quantities. After receiving of production order the raw materials and accessories in necessary quantities have to be bought and delivered to the goods processing plant for the further preparation and constructing of the ready-made products.

The important task of the Logistics centre is to combine together two processes, each with own specific and demands: the production process and the goods realisation (sales) process. In our case the logistic department is the part of the both processes: raw materials supply for production and providing the delivery of goods to the final user for sales.

There is following sequence of the goods delivery.

Using the controlling computer system Microsoft Dynamics AX sales department creates the sales order with all necessary information about the client (name, address, contacts), type of

goods (model, colour, size, quantity) and delivery terms. After getting the sales order from clients and sales department warehouse employees collect, pack the goods and form the delivery order for the further transportation to clients.

One of the tasks for logistic department is to make the constant monitoring of open (not delivered) sales orders and make analysis of goods production volume. Based on the results of warehouse monitoring the production order for needed goods quantity is created for the processing plant and its subcontractors and estimation time of arrival on stock for each product is calculated.

In conditions of economic depression the volume of client orders decreased, that causes the stock overloading of goods and production volume decrease. Management of the company has to decrease the increasing costs without destroying the working process in the company. To find the right solution of the problem management decides to make the profit and loss analysis for various scenarios of company development in order to distribute available financial resource between the departments of the Logistic centre on the most efficient way.

Logistic centre consists of separate units. We consider the task of optimal distribution of limited resource between these units. The company has hierarchic structure, that other words means that each unit (besides of terminals) has two other units under his control, so in the end we get the treelike structure presented in the Fig.2.

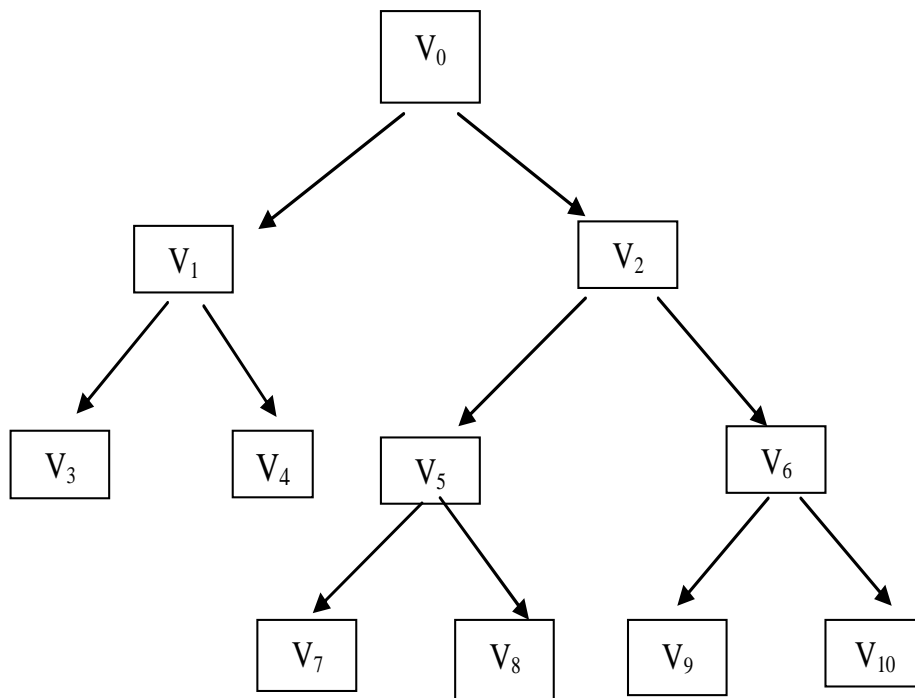


Fig. 2. The Treelike Structure

This structure is shown by the relation matrix T :

$$T := \begin{pmatrix} 1 & 3 & 5 & -1 & -1 & 7 & 9 & -1 & -1 & -1 & -1 \\ 2 & 4 & 6 & -1 & -1 & 8 & 10 & -1 & -1 & -1 & -1 \end{pmatrix}$$

To make the matrix understandable for the Mathcad 14 program it has to be transposed into one shown by Table I. Rows of the matrix mean company's units, but the columns –

number of the subordinate units. Column under number “0” shows number of unit, which is the left son, but column under number “1” presents the right son. Meaning of value -1 shows that there is no subordinate unit. So Table I corresponds to the treelike structure from Fig. 2.

TABLE I
THE RELATION MATRIX T

	0	1
0	1	2
1	3	4
2	5	6
3	-1	-1
4	-1	-1
5	7	8
6	9	10
7	-1	-1
8	-1	-1
9	-1	-1
10	-1	-1

There are eleven numbers of concerned nodes, $n = 11$. There are various variants of the development from l to n_i for each i unit. Each j ($j = 1, 2, \dots, n_i$) variant assigns the following parameters: 1) concerned the i -th unit's profit $M_{i,j,0}$ from realization of the given variant j . It is significant, that profit depends on not the resources of the present unit only, but on the resources given to the subordinate units, too; 2) resource $M_{i,j,1}$ needed for the first subordinate unit (left son); 3) resource $M_{i,j,2}$ for the present unit; 4) resource $M_{i,j,3}$ needed for the second subordinate unit (right son).

Data for each i unit given by matrix M_i in the same way as it is shown for matrix M_0 bellow in the Table II. Matrix rows correspond to offered variants for present unit. Each row has above mentioned parameters for present variant.

TABLE II
0-NODE INDICES

$$M_0 := \begin{pmatrix} 0 & 0 & 0 & 0 \\ 6 & 19 & 3 & 48 \\ 2.5 & 9.1 & 0 & 37 \\ 4.3 & 26 & 2 & 36 \end{pmatrix}$$

The task is to distribute the resource between units in such way that the total profit would be maximal.

II. METHOD OF DYNAMIC PROGRAMMING

This task is solving by method of *dynamic programming*, created by Richard Bellman. The concept of dynamic programming means that the large problem is broken into incremental sub-problems, where each sub-problem can be solved and the solution is optimal. As a result, by using a

formula the final solution can be generated without being required to alter the previously solved sub-problems or recalculating some parts of the algorithm.

The dynamic programming supposes to make a decision step by step. In our case it means to take a decision for each separate unit consequently. Let's take the moment of time when the decision should be made for item v , if available resource is equal to r . This resource must be distributed among the item v (father) and all his offspring. Let we have k various solutions for unit v .

Then the Bellman function $F(v, r)$ gives a maximal profit in this case. The value of $F(v, r)$ means a maximum profit which could be got from unit v and all his offspring for the resource r .

These calculations should be used starting from the end units and come to the tree root.

Every time Bellman functions are calculated for the units which have sons (subordinate units) with already calculated Bellman function values. The end units have no sons (the condition $T_{v,0} + T_{v,1} = -2$ takes place for the item number v), and the whole resource is given to this unit. Therefore to apply the j -th project for such v item, it is necessary to satisfy condition (1)

$$Mv_{j,2} \leq r. \quad (1)$$

Among projects for the v -th item, the project j^* is chosen for which

$$j^* = \arg \{ j=1, \dots, n_i: \max \{ Mv_{j,0}: Mv_{j,2} \leq r \} \}. \quad (2)$$

For the rest units $v=1,2,\dots,10$ the condition

$$Mv_{j,1} + Mv_{j,2} + Mv_{j,3} \leq r \quad (3)$$

should be satisfied.

Now among all projects for the v -th item, the project j^* is chosen for which

$$j^* = \arg \{ j=1, \dots, n_i: \max \{ Mv_{j,0} + F(v0, Mv_{j,1}) + F(v1, Mv_{j,3}): Mv_{j,1} + Mv_{j,2} + Mv_{j,3} \leq r \} \} \quad (4)$$

where

$v0 = T_{v,0}$ - is the number of the left son of v ;

$v1 = T_{v,1}$ - is the right son of v .

As the results Bellman functions $F(v, r)$ will be calculated for all units and amounts of given resources $r = 0, 1, \dots, R$. Above mentioned procedure is named as *inverse algorithm* (*bottom-up*) of *dynamic programming*.

Direct algorithm (*top-down*) gives the optimal quantities of assignment resources for each unit v . It is realised in opposite direction of the above mentioned – from the root to the end units, each time moving from farther to one of his sons.

Optimal solution for root (unit with number 0) corresponds to the project j^* that satisfies the equality

$$F(0, R) = M0_{j^*,0} + F(1, M0_{j^*,1}) + F(2, M0_{j^*,3}),$$

$$MO_{j^*,1} + MO_{j^*,2} + MO_{j^*,3} \leq R, \quad (5)$$

because for unit number 0 we have all resource: $r^*(0)=R$
 In addition we fix the resource assigned for the sons

$$r(T_{0,0}) = MO_{j^*,0}, \quad r(T_{0,1}) = MO_{j^*,2} \quad (6)$$

The next step is to go to one of the sons v , knowing the resources given to them. In case the unit v has resource r and he has the son $v0 = T_{v,0}$ (left son) and the son $v1 = T_{v,1}$ (right son), then the optimal solution r^* for unit v will be found as the project j^* that satisfies the equality

$$F(v,r) = Mv_{j^*,0} + F(v0, Mv_{j^*,1}) + F(v1, Mv_{j^*,3}),$$

$$Mv_{j^*,1} + Mv_{j^*,2} + Mv_{j^*,3} \leq R. \quad (7)$$

The resources assigned for the sons are following:

$$r(T_{v,0}) = Mv_{j^*,0}, \quad r(T_{v,1}) = Mv_{j^*,2} \quad (8)$$

Direct algorithm is finished when all the points (units) are calculated till that ones who have no sons.

III. COMPUTER REALIZATION

There are following information about the computer programs using for realization of above mentioned Bellman algorithm.

Programming language we use is the wide known mathematical package Mathcad 14.

We create the united matrix M for using it in Mathcad programme:

$$M^T = [M0^T \ M1^T \ \dots \ M10^T]^T \quad (9)$$

Now we have the problem to find information concerning the concrete unit. To solve it we create the special Mathcad-program *Locman*. Then for an integer i *Locman* gives a number of matrix M row, that is the first row with information about the matrix Mi .

The programme *Father(v)* gives a vector with 2 components, one of which is -1. The second component is an integer one shows the number of unit, which is the father for unit v . The component location shows which son is the present unit: if it is on the top position then given unit is the left son, but in case it is located bellow given unit is the right son.

Let us consider an example:

$$Father(9) = \begin{pmatrix} 6 \\ -1 \end{pmatrix}$$

Here the father of the unit number 9 is the unit number 6 and the unit number 9 is the left son of the unit number 6.

$$Father(10) = \begin{pmatrix} -1 \\ 6 \end{pmatrix}$$

The father of the unit number 10 is the unit number 6 and the unit number 10 is the right son of the unit number 6.

The inverse algorithm of dynamic programming is realised by program *F(v,r)*, which gives Bellman function value for unit v , which has got the certain resource r . As the result we get the value of maximum profit and the number of the optimal solution from matrix M .

The direct algorithm is realized by program *GlobD*. In other words *GlobD* gives the matrix of optimal decisions for all units with resource equal R . Matrix rows related to the units and contain the optimal value for this unit. Here, r_i is the given for i unit, d_i – optimal decision for i unit.

IV. NUMERICAL RESULTS

For our example we have the following numeric data mentioned in Table III.

TABLE III
 NODES INDICES

$$M1 := \begin{pmatrix} 0 & 11 & 3 & 5 \\ 0.2 & 3.5 & 2 & 4 \\ 1.6 & 16 & 4 & 6 \end{pmatrix} \quad M2 := \begin{pmatrix} 6 & 14 & 2 & 32 \\ 2.3 & 6 & 0 & 31 \\ 2.7 & 9 & 2 & 24 \end{pmatrix}$$

$$M3 := \begin{pmatrix} 4.6 & 0 & 11 & 0 \\ 4 & 0 & 4 & 0 \\ 7.3 & 0 & 16 & 0 \\ 40 & 0 & 71 & 0 \end{pmatrix} \quad M4 := \begin{pmatrix} -4.6 & 0 & 5 & 0 \\ -3.8 & 0 & 4 & 0 \\ -5.7 & 0 & 0.57 & 0 \\ 8 & 0 & 90 & 0 \end{pmatrix}$$

$$M5 := \begin{pmatrix} 3 & 6.3 & 0.5 & 7.2 \\ -1 & 3.8 & 1 & 1.28 \\ 1.9 & 4.3 & 1.5 & 3.6 \end{pmatrix} \quad M6 := \begin{pmatrix} 3 & 18.2 & 1.8 & 12.3 \\ 3.3 & 19.3 & 2.5 & 9.6 \\ 0.8 & 17.6 & 0.56 & 6 \end{pmatrix}$$

$$M7 := \begin{pmatrix} 2.1 & 0 & 6.25 & 0 \\ 0 & 0 & 3.8 & 0 \\ 1.9 & 0 & 4.27 & 0 \end{pmatrix} \quad M8 := \begin{pmatrix} 0.87 & 0 & 7.25 & 0 \\ -1 & 0 & 1.28 & 0 \\ 0 & 0 & 3.56 & 0 \end{pmatrix}$$

$$M9 := \begin{pmatrix} 1.74 & 0 & 18.25 & 0 \\ 2.32 & 0 & 19.3 & 0 \\ 0.8 & 0 & 17.6 & 0 \end{pmatrix} \quad M10 := \begin{pmatrix} 1.23 & 0 & 12.3 & 0 \\ 0.98 & 0 & 9.57 & 0 \\ 0 & 0 & 5.6 & 0 \end{pmatrix}$$

The hierarchic structure was presented before in Table I. The information about nodes indices is presented by Matrix $M0$ (see Table II) for 0-unit (root) and matrixes $M1$ - $M10$ for other units from the Table III.

Let us put the global available resource as $R:=64$. The created program gives the following results:

$$F(0,R) = 15.9.$$

One corresponds to the third solution for the item 0 (for the root): $j^* = 3$.

Gotten results for each unit are presented by matrix *OptD* (see Table IV). The matrix columns correspond to the item. The matrix has four rows. The zero row contains the number (v) of the items. The first row contains the value of the optimal resource (r^*) for the corresponding item, the second row contains the value of the Bellman function (F), the third row contains the number of optimal solution (j^*) from matrix *M*.

TABLE IV

THE OPTIMAL SOLUTIONS FOR RESOURCE OF 64(MATRIX OPTD)

v	0	1	2	3	4	5	6	7	8	9	10
r^*	64	26	36	16	6	9	24	0	0	0	0
$F(v,r)$	15.9	8.9	2.7	7.3	0	0	0	0	0	0	0
j^*	3	3	3	3	0	0	0	0	0	0	0

V. CONCLUSION

The problem of resources distribution in optimal way among the units of the Logistic centre was observed. The Logistic centre has the treelike structure. There are some various ways of development for each unit of the Logistic centre. The ways differ from each other by necessary resource and profit receiving. The task is solving by using the method of *dynamic programming*, created by Richard Bellman. During the work using the Mathcad 14 package the special program which helps to make the calculations was created.

Olga Girvica. Resursu sadalījuma starp loģistikas centra struktūrvienībām hierarhiskā optimizācija

Lai līdz minimumam samazinātu produktu glabāšanas laiku noliktavās, kā arī tūlumu, ko tie aizņem, un maksimāli palielinātu preču piegādes ātrumu un efektivitāti, loģistikas pakalpojumu sniedzējs izstrādāja piegādes ķēdi, kuru prezentēja pašas kompānijas Loģistikas Centrs. Viens no Loģistikas ķēdes uzdevumiem ir vienlaikus izveidot gan iekārtas, gan piegādes procesu tā, lai tie palīdzētu labāk noteikt un apmierināt klientu vajadzības. Izņemot informācijas tehnoloģijas un efektīvu informācijas plūsmu starp ķēdes locekļiem, kā arī izprotot klientu uzņēmējdarbību un vajadzības, Loģistikas Centrs precīzi laikā piegādā izejmateriālus un gatavās preces. Ekonomiskās krīzes apstākļos pasūtījumu skaits no klientiem samazinās, un tas izraisa krājumā esošo preču pārslogojumu, kā arī ražošanas apjomu samazināšanos. Kompānijas vadībai jāsamazina pieaugošās izmaksas, neiznīcinot darba procesu kompānijā. Lai atrastu pareizo problēmas risinājumu, vadība nolēmj veikt peļņas un zaudējumu analīzi dažādiem kompānijas attīstības scenārijiem, lai visefektīvākajā veidā sadalītu pieejamos finanšu resursus starp Loģistikas Centra nodaļām.

Šajā rakstā ir aplūkota problēma, kas saistīta ar resursu optimālo sadalījumu starp loģistikas centra posmiem. Loģistikas centram ir kokveida struktūra. Katrai centra vienībai ir iespējami vairāki attīstības varianti, kas atšķiras viens no otra gan ar nepieciešamo resursu, gan ar iegūstamo peļņu. Loģistikas centra vadības uzdevums bija sadalīt finanšu resursus starp visām kompānijas apakšvienībām tādā veidā, lai būtu maksimāla peļņa. Uzdevums tiek risināts, izmantojot Ričarda Belmana izstrādāto dinamisko programmēšanu. Optimālai stratēģijai piemīt tāda īpašība, ka, lai arī kāds būtu sākotnējais stāvoklis un sākotnējais lēmums, turpmākajiem lēmumiem jāveido optimāla stratēģija par stāvokli, kāds radies sākotnējā lēmuma rezultātā. Izņemot programmatūras paketi Mathcad 14, darba gaitā tika izveidota speciāla programma, ar kuras palīdzību veikti aprēķini. Prezentētais skaitliskais piemērs, kā ierobežots resurss tiek sadalīts starp loģistikas centra apakšvienībām, lai iegūtā peļņa būtu maksimāli liela, ilustrē reāla uzdevuma risinājumu vienam loģistikas centram Latvijā.

Ольга Гирвица. Иерархическая оптимизация распределения ресурсов между подразделениями логистического центра

В условиях кризиса объемы заказов от клиентов снизились, что повлекло за собой переполнение склада и снижение объема производства. Появилась необходимость снижать расходы, при этом не нарушая рабочего механизма в компании. Для принятия правильного решения руководство Логистического центра решило проанализировать расходы связанные с содержанием собственного производства и логистики, а также получаемые от них доходы, для того чтобы наиболее эффективно и с максимальной прибылью распределить имеющийся ресурс между всеми подразделениями компании.

В данной статье рассмотрена проблема оптимального распределения ресурсов между звеньями логистического центра. Логистический центр имеет древовидную структуру. Для каждой единицы центра возможны несколько вариантов развития, отличающиеся друг от друга как необходимым ресурсом, так и получаемой прибылью. Поставленная задача руководства Логистического центра заключается в том, чтобы распределить ограниченный финансовый ресурс между всеми подразделениями компании таким образом, чтобы прибыль была максимальной.

Задача решается с помощью динамического программирования, разработанного Ричардом Беллманом. Оптимальная стратегия обладает тем свойством, что каковы бы ни были начальное состояние и начальное решение, последующие решения должны составлять оптимальную стратегию относительно состояния, полученного в результате начального решения. Используя программный пакет Mathcad 14, в процессе работы была создана специальная программа, с помощью которой проводились вычисления. Представленный численный пример распределения ограниченного ресурса между подразделениями логистического центра таким образом, чтобы полученная прибыль была максимальной, иллюстрирует решение реальной задачи для одного логистического центра в Латвии.

Using the dynamic programming method the real formulated task for distribution of necessary resource of 64 thousands lat among the units of the Logistic centre in Latvia is solved and the solution for getting of optimal profit were found for each unit.

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