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A Problem of Arrangement of Service Stations on the Given Territory

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Abstract. **A number of vital practical problems require the description of an arrangement of service stations on the considered territory. The problem of the analytical description of the available data on population allocation on the territory of Latvia is examined. The elaborated algorithm of the problem solution is based on the gradient method. The considered numerical examples show its efficiency. The author intends to apply the suggested approach to solving the practical arrangement problems.**

Keywords: **gradient method, population density, service stations, spatial arrangement.**

I. Introduction

Let us consider a real space *X* for that concrete *point* will be marked by *x*, for plane it is two-dimensional vector (it is available to consider another dimension too). *A distance* $l(x, x^*)$ is determined for points *x* и *x**, that satisfies usual conditional of distance axioms: $l(x,x) = 0$, $l(x, x^*) \ge 0$, $l(x, x^*) \le l(x, x') + l(x', x^*)$.

Some *objects* are arranged in the space (for example men, animals, sport facilities, stationers). Let us name as *x–object*, the object that is at the point *x*. The density of object arrangement is described by known density function $f(x) \ge 0$, so

$$\int_{x \in X} f(x)x = 1.$$

Some *service stations* must be arranged in the space, their number is *k*. It is necessary to determine those coordinates $x^{(1)}, x^{(2)}, \ldots, x^{(k)}$. If a *x*–object is serviced by *i*–th station then corresponding loss is equal to $g_x\left(x^{(i)}\right)$, for example $g_x\left(x^{(i)}\right) = g(x - x^{(i)})$. Let us call $g_x(\circ)$ as *loss function* and suppose that it is a symmetry according to zero ($g_x(x^{(i)}) = g_x(-x^{(i)})$) and convex (down).

All amount of service for the *x*–object is deviated between various service stations according to inverse proportion of the distances from the *x*–object and the station. Most precisely, a part of *x*–object service that belongs to the *i-th* station is

$$\delta_i(x) = \frac{\left(l\left(x, x^{(i)}\right)\right)^{-1}}{\sum_{j=1}^{k}\left(l\left(x, x^{(j)}\right)\right)^{-1}}. \tag{1}$$

Now a problem can be formulated as follows: to find coordinates $x^{(1)}, x^{(2)}, \ldots, x^{(k)}$ of station arrangement that minimizes the total loss:

$$D\left(x^{(1)}, x^{(2)}, \ldots, x^{(k)}\right) = \int \frac{1}{\sum_{i=1}^{k}\left(l\left(x, x^{(i)}\right)\right)^{-1}} \times$$

$$\times \sum_{i=1}^{k}\left(l\left(x, x^{(i)}\right)\right)^{-1} g_x(x^{(i)}) f(x) dx. \tag{2}$$

Initial problem statement has been given in article [1]. Density distribution function *f(x)* appliance for the case of population density distribution in Latvia has been described in [2]. Results obtained in previous articles are used for the solution of spatial arrangement problem to find out optimal locations of four service stations on Latvian territory. Station service type is not explicitly defined. Possible service types considered by author are sports facilities, car technical maintenance stations and others. The article is organized in the following way. At first gradient method is considered. Then we consider description of analytical dependencies and numerical example. The article ends by some conclusion remarks.

II. Gradient Optimization

Now we consider a case when space *X* is real plane $R^2 = (-\infty, \infty) \times (-\infty, \infty)$. Then the coordinates of an object are $x = \begin{pmatrix} x_1 & x_2 \end{pmatrix}^T$, coordinates of the *j*-st station are $x^{(j)} = \begin{pmatrix} x_1^{(j)} & x_2^{(j)} \end{pmatrix}^T$. We will use a gradient method [3] for the minimization of criteria (2). For that aim let us calculate a corresponding gradient. For a partial gradient we have the following expression:

$$\frac{\partial}{\partial x^{(j)}} D\left(x^{(1)}, x^{(2)}, \ldots, x^{(k)}\right) =$$

$$= \frac{\partial}{\partial x^{(j)}} \int \frac{1}{\sum_i \left(l\left(x, x^{(i)}\right)\right)^{-1}} \sum_{i=1}^{k} \left(l\left(x, x^{(i)}\right)\right)^{-1} g_x(x^{(i)}) f(x) dx =$$

$$= \int \frac{1}{\sum_i \left(l\left(x, x^{(i)}\right)\right)^{-1}} \times$$

$$\times \left(\begin{pmatrix} g_x(x^{(j)}) \left(-\left(l\left(x, x^{(i)}\right)\right)^{-2}\right) \frac{\partial}{\partial x^{(j)}} l\left(x, x^{(j)}\right) + \\ + \frac{1}{l\left(x, x^{(j)}\right)} \frac{\partial}{\partial x^{(j)}} g_x(x^{(j)}) \end{pmatrix} \right) f(x) dx -$$

$$- \int \left(\sum_i \left(l\left(x, x^{(i)}\right)\right)^{-1} \right)^{-2} \times$$

$$\times \begin{pmatrix} \left(-\left(l\left(x, x^{(j)}\right)\right)^{-2}\right) \frac{\partial}{\partial x^{(j)}} l\left(x, x^{(j)}\right) \times \\ \times \sum_i \left(l\left(x, x^{(i)}\right)\right)^{-1} g_x(x^{(i)}) \end{pmatrix} f(x) dx. \quad (3)$$

Now we are able to rewrite the gradient of D as

$$\nabla D(x^{(1)}, x^{(2)}, \ldots, x^{(k)}) =$$

$$= \left(\frac{\partial}{\partial x^{(1)}} D(x^{(1)}, x^{(2)}, \ldots, x^{(k)}), \ldots, \frac{\partial}{\partial x^{(k)}} D(x^{(1)}, x^{(2)}, \ldots, x^{(k)}) \right). \quad (4)$$

For two-dimensional case in formula (3) we have two-dimensional vector

$$\frac{\partial}{\partial x^{(j)}} D\left(x^{(1)}, x^{(2)}, \ldots, x^{(k)}\right) =$$

$$= \left(\frac{\partial}{\partial x_1^{(j)}} D\left(x^{(1)}, x^{(2)}, \ldots, x^{(k)}\right) \quad \frac{\partial}{\partial x_2^{(j)}} D\left(x^{(1)}, x^{(2)}, \ldots, x^{(k)}\right) \right).$$

For partial derivative (i = 1, 2) we have the following expression:

$$\frac{\partial}{\partial x_q^{(j)}} D\left(x^{(1)}, x^{(2)}, \ldots, x^{(k)}\right) =$$

$$= \frac{\partial}{\partial x_q^{(j)}} \int \frac{1}{\sum_i \left(l\left(x, x^{(i)}\right)\right)^{-1}} \sum_{i=1}^{k} \left(l\left(x, x^{(i)}\right)\right)^{-1} g_x(x^{(i)}) f(x) dx =$$

$$= \int \left(\sum_i \left(l\left(x, x^{(i)}\right)\right)^{-1} \right)^{-1} \times$$

$$\times \left(\begin{pmatrix} g_x(x^{(j)}) \left(-\left(l\left(x, x^{(j)}\right)\right)^{-2}\right) \frac{\partial}{\partial x_q^{(j)}} l\left(x, x^{(j)}\right) + \\ + \left(l\left(x, x^{(i)}\right)\right)^{-1} \frac{\partial}{\partial x_q^{(j)}} g_x(x^{(j)}) \end{pmatrix} \right) f(x) dx -$$

$$- \int \left(\sum_i \left(l\left(x, x^{(i)}\right)\right)^{-1} \right)^{-2} \times$$

$$\times \begin{pmatrix} \sum_i \left(l\left(x, x^{(i)}\right)\right)^{-1} g_x(x^{(i)}) \times \\ \times \left(-\left(l\left(x, x^{(j)}\right)\right)^{-2}\right) \frac{\partial}{\partial x_q^{(j)}} l\left(x, x^{(j)}\right) \end{pmatrix} f(x) dx. \quad (5)$$

Now instead of (4) we have the ($2 \times k$)-matrix of the partial derivatives

$$\nabla D(x^{(1)}, x^{(2)}, \ldots, x^{(k)}) =$$

$$= \begin{pmatrix} \frac{\partial}{\partial x_1^{(1)}} D(x^{(1)}, x^{(2)}, \ldots, x^{(k)}) & \ldots & \frac{\partial}{\partial x_1^{(k)}} D(x^{(1)}, x^{(2)}, \ldots, x^{(k)}) \\ \frac{\partial}{\partial x_2^{(1)}} D(x^{(1)}, x^{(2)}, \ldots, x^{(k)}) & \ldots & \frac{\partial}{\partial x_2^{(k)}} D(x^{(1)}, x^{(2)}, \ldots, x^{(k)}) \end{pmatrix}. \quad (6)$$

For the optimization we use the two-stage procedure. At the first stage the component-wise (coordinate-wise) optimization is used as follows. During the j-th iteration (j = 1, 2, …, k) function (2) is minimized with respect to both coordinates of the j-st station $x^{(j)} = \left(x_1^{(j)} \quad x_2^{(j)}\right)$, at the same time other coordinates do not change. According to the gradient method we move along the gradient with respect to $\left(x_1^{(j)} \quad x_2^{(j)}\right)$, recalculating the one continually. At the second stage we work with the full gradient (6).

III. DESCRIPTION OF ANALYTICAL DEPENDENCIES

Population density on the given territory has to be described at first. Reasonably the population of big cities and districts to be examined separately.

Let $\imath$ – be an district index, $\imath$ =1,2,…,w, where w – is a number of the considered districts. Let's use following designations:

$SR_\imath$ – is the territory of the $\imath$-th district (it's point $(x,y) \in SR_\imath$);

$\xi_\imath(x,y)$ – is indicator's function of the $\imath$-th district:

$$\xi_\imath(x,y)=\begin{cases} 1, \text{ if } (x,y) \in SR_\imath, \\ 0, \text{ otherwise}, \end{cases}$$

$\hat{H}_\imath$ – population of the $\imath$-th district,

$P_\imath$ – square of the $\imath$-th district, km^2,

$h_\imath$ – is population's density for the $\imath$-th district. It is calculated by the formula $h_\imath = \hat{H}_\imath / P_\imath$,

We assume that every district is represented by a circle. The circle radius is calculated judging by a condition, that the square of the circle and district coincide.

For the considered case function $\xi_\imath$ (x,y) is described as following

$$\xi_\imath(x,y)=\begin{cases} 1, \text{ if } & (y_\imath - r_\imath < y < y_\imath + r_\imath) \wedge \\ & \wedge \left[\begin{array}{l} x_\imath - \sqrt{(r_\imath)^2 - (y-y_\imath)^2} < \\ < x < x_\imath + \sqrt{(r_\imath)^2 - (y-y_\imath)^2} \end{array}\right], \\ 0, \text{ otherwise}, \end{cases}$$

$x_\imath, y_\imath$, - coordinates of the centre of the $\imath$-th district

$r_\imath$ – radius of the $\imath$-th district $r_\imath$, calculated by the formula

$$r_\imath = \sqrt{P_\imath / \pi}\,. \tag{7}$$

Now population density in point (x,y) is described as following function

$$f^{(r)}(x,y) = \sum_{i=1}^{w} h_\imath \xi_\imath(x,y)\,. \tag{8}$$

Let γ – be a city's index, γ=1,2,…,m, where m – where m is a number of considered big cities. Let's denote:

SP_γ - is the territory of the к γ-th city;

$\psi_\gamma(x,y)$ – is an indicator's function of the γ-th city:

$$\psi_\gamma(x,y)=\begin{cases} 1, \text{ if } (x,y) \in SP_\gamma, \\ 0, \text{ otherwise}, \end{cases}$$

Indicator's function $\psi_\gamma(x,y)$ is described as following

$$\psi_\gamma(x,y)=\begin{cases} 1, \text{ if } & (y_\gamma - r_\gamma < y < y_\gamma + r_\gamma) \wedge \\ & \wedge \left[\begin{array}{l} x_\gamma - \sqrt{(r_\gamma)^2 - (y-y_\gamma)^2} < \\ < x < x_\gamma + \sqrt{(r_\gamma)^2 - (y-y_\gamma)^2} \end{array}\right], \\ 0, \text{ otherwise}, \end{cases}$$

x_γ, y_γ, - coordinates of the centre of the γ-th city

r_γ – radius of the γ–th city, calculated by the formula

$$r_\gamma = \sqrt{P_\gamma / \pi}\,,$$

where P_γ – is square of the γ–th city.

The populations densities of the cities calculated by formula $H_\gamma = \hat{H}_\gamma / P_\gamma$, where

$\hat{H}_\gamma$ – population of the γ-th city

The density of distribution of the population of γ -th city on territory will be described by two-dimensional normal density:

$$f^{(p)}(x,y) = \frac{H_\gamma}{2\pi\sigma_\gamma^2} \exp\left\{-\frac{1}{2\sigma_\gamma^2}\left[(x-x_\gamma)^2 + (y-y_\gamma)^2\right]\right\}, \tag{9}$$

where σ_j is a standard deviation for the density of distribution of the population calculated by the formula:

$$\sigma_\gamma = \rho r_\gamma,, \tag{10}$$

where ρ>0,5 – is a coefficient of city's attraction.

Definitive expression for population density of Latvia in a point (x, y) will be

$$f(x,y) = f^{(r)}(x,y) + \sum_{\gamma=1}^{m} \psi_\gamma(x,y) f^{(p)}(x,y)\,. \tag{11}$$

Further let us use the following distance function and loss function:

$$l(x,z) = \sqrt{(x_1 - z_1)^2 + (x_2 - z_2)^2}\,, \tag{12}$$

$$g(x,z) = |x_1 - z_1| + |x_2 - z_2|\,. \tag{13}$$

Then we have the following derivatives:

$$\frac{\partial}{\partial z_q} l(x,z) = -\frac{1}{\sqrt{(x_1 - z_1)^2 + (x_2 - z_2)^2}} (x_i - z_i), \quad (14)$$

$$\frac{\partial}{\partial z_q} g(x,z) = \frac{\partial}{\partial z_q} g\left(\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}\right) = \begin{cases} 1 \text{ if } x_i < z_i, \\ -1 \text{ otherwise} \end{cases} \quad (15)$$

Now we are able to use formula (5) for optimization.

IV. Numerical Example

Arrangement of 4 service stations across territory of Latvia has been taken an example. As we have mentioned before, possible station types are sport facilities, car technical maintenance stations and others. On the Tables I and II the data about the population and the square of Latvia on cities and districts is represented, as well as their coordinates. Data is offered to us by LR Centrālā statistikas pārvalde (www.csb.gov.lv). On the Figure 1 the map of population's density of Latvia is shown. Information source is Latvijas Pašvaldību savienība (LPS, http://www.lps.lv). The different colors correspond with different allocation in the given region.

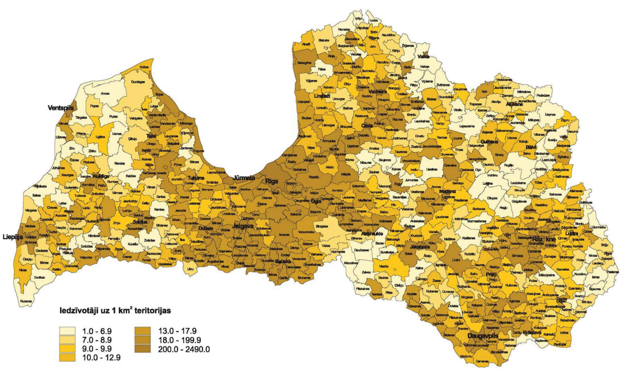

Fig.1. Population's density in cities and areas

TABLE I

Characteristics of Big Cities of Latvia

Index γ	City	Coordinate x_γ	Coordinate y_γ	Population, $\hat{H}_\gamma$	Square, P_γ, km²
1	Rīga	190	150	722 485	307
2	Daugavpils	340	30	108 091	73
3	Jelgava	170	110	66 051	60
4	Jūrmala	150	180	55 408	100
5	Liepāja	10	100	85 477	60
6	Rēzekne	390	100	36 345	18
7	Ventspils	50	200	43 544	55

TABLE II

Characteristics of Districts of Latvia

Index ι	District	Coordinate x_ι	Coordinate y_ι	Population, $\hat{H}_\iota$	Square, P_ι, km²
1	Aizkraukles rajons	270	110	40 116	2 567
2	Alūksnes rajons	360	210	24 483	2 245
3	Balvu rajons	390	170	27 245	2 381
4	Bauskas rajons	210	100	50 988	1 881
5	Cēsu rajons	270	180	56 614	2 973
6	Daugavpils rajons	340	40	39 496	2 526
7	Dobeles rajons	130	100	37 980	1 632
8	Gulbenes rajons	340	180	26 281	1 876
9	Jelgavas rajons	170	110	36 941	1 605
10	Jēkabpils rajons	300	90	52 593	2 997
11	Krāslavas rajons	400	50	33 313	2 288
12	Kuldīgas rajons	60	150	35 822	2 500
13	Liepājas rajons	30	110	43 849	3 593
14	Limbažu rajons	230	230	37 798	2 602
15	Ludzas rajons	420	110	31 305	2 412
16	Madonas rajons	330	140	42 918	3 349
17	Ogres rajons	240	130	64 060	1 843
18	Preiļu rajons	340	80	38 317	2 042
19	Rēzeknes rajons	380	100	40 442	2 809
20	Rīgas rajons	210	150	161119	3 132
21	Saldus rajons	50	110	36 735	2 182
22	Talsu rajons	100	200	46 680	2 748
23	Tukuma rajons	130	150	54 813	2 457
24	Valkas rajons	300	220	31 723	2 441
25	Valmieras rajons	260	240	58 328	2 373
26	Ventspils rajons	60	190	13 945	2 462

TABLE III

Population's Density, Radius and Standard Deviation for the Big Cities of Latvia

Index γ	City	Density, H_γ	Radius r_γ	σ_γ
1	Rīga	2353,37	9,89	6,92
2	Daugavpils	1480,70	4,82	3,37
3	Jelgava	1100,85	4,37	3,06
4	Jūrmala	554,08	5,64	3,95
5	Liepāja	1424,62	4,37	3,06
6	Rēzekne	2019,17	2,39	1,68
7	Ventspils	791,71	4,18	2,93

TABLE IV

POPULATION'S DENSITY AND RADIUS FOR DISTRICTS OF LATVIA

Index i	District	Density, h_i	Radius, r_i
1	Aizkraukles rajons	15,63	28,59
2	Alūksnes rajons	10,91	26,73
3	Balvu rajons	11,44	27,53
4	Bauskas rajons	27,11	24,47
5	Cēsu rajons	19,04	30,76
6	Daugavpils rajons	15,64	28,36
7	Dobeles rajons	23,27	22,79
8	Gulbenes rajons	14,01	24,44
9	Jelgavas rajons	23,02	22,60
10	Jēkabpils rajons	17,55	30,89
11	Krāslavas rajons	14,56	26,99
12	Kuldīgas rajons	14,33	28,21
13	Liepājas rajons	12,20	33,82
14	Limbažu rajons	14,53	28,78
15	Ludzas rajons	12,98	27,71
16	Madonas rajons	12,82	32,65
17	Ogres rajons	34,76	24,22
18	Preiļu rajons	18,76	25,50
19	Rēzeknes rajons	14,40	29,90
20	Rīgas rajons	51,44	31,57
21	Saldus rajons	16,84	26,35
22	Talsu rajons	16,99	29,58
23	Tukuma rajons	22,31	27,97
24	Valkas rajons	13,00	27,88
25	Valmieras rajons	24,58	27,48
26	Ventspils rajons	5,66	27,99

TABLE V

RESULTS OF SEQUENTIAL CYCLES FOR TWO-DIMENSIONAL CASE

Iteration number	1	2	3	4	5
$x_1^{(1)}$	0	11.92	34.71	54.71	61.55
$x_2^{(1)}$	0	15.13	48.65	90.07	154.01
$x_1^{(2)}$	100	129.36	192.70	268.86	227.77
$x_2^{(2)}$	130	137.31	151.17	126.99	158.92
$x_1^{(3)}$	60	74.61	90.53	108.52	159.42
$x_2^{(3)}$	60	86.32	148.23	144.93	159.22
$x_1^{(4)}$	190	277.72	285.86	314.59	333.28
$x_2^{(4)}$	150	138.48	140.57	122.17	125.65
D	$2.3 * 10^8$	$1.89 * 10^8$	$1.64 * 10^8$	$1.52 * 10^8$	$1.44 * 10^8$

Table V contains the results of sequential cycles of the optimization procedure.

From the table we can see how the gradient method improves the criteria value continually.

V. CONCLUSION

A problem of service station arrangement in spatial space is considered. The elaborated algorithm of the problem solution is based on the gradient method. The considered numerical examples show its efficiency. The author intends to apply the suggested approach to solving the practical arrangement problems.

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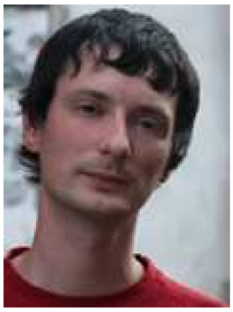

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Andrejs Kašurins. Apkalpojošo staciju dotajā teritorijā izvietošanas uzdevums

Daudziem praktiskiem uzdevumiem nepieciešams apskats par apkalpojošo staciju izvietojumu teritorijā. Sākotnēji dotais uzdevums tika aprakstīts darbā 'On a Problem of Spatial Arrangement of Service Stations'. Savukārt darbā 'Statistical description of a distribution of population's density on the Latvian territory' tika piedāvāta izvietošanas blīvuma aprakstīšanas procedūra, kas pielietota iedzīvotāju blīvuma izvietošanai Latvijas teritorijā. Tika apskatīts iedzīvotāju izvietošanas Latvijas teritorijā datu analītiskā apraksta uzdevums. Iegūta izteiksme, kas noteiktām Latvijas teritorijai koordinātām ļauj aprēķināt atbilstošu iedzīvotāju blīvumu. Lielu pilsētu un rajonu iedzīvotāju skaitu ir lietderīgi izskatīt atsevišķi. Dotā raksta aprēķina nosacījumos ir pieņemts katru rajonu aprakstīt ar apli. Apļa rādiusu aprēķina pēc nosacījumiem, ka apļa laukums sakrīt ar rajona laukumu, un viņu kopējā sakritība ir maksimāla. Tiek pieņemts, ka iedzīvotāju blīvums visā rajonā ir vienmērīgs. Pilsētas teritorijas iedzīvotāju blīvums aprakstīts ar vienmērīgu divdimensiju blīvumu. Dotajā rakstā divu citēto darbu rezultāti tiek izmantoti optimālā telpiskā četru apkalpošanas staciju izvietojuma Latvijas teritorijā uzdevumu risināšanā. Pie tam konkrēts apkalpošanas veids netiek konkretizēts. Tādā veidā autors kā apkalpošanas veidus apskata sporta celtnes, automobiļu tehniskās apkalpošanas stacijas u.c. Tika izstrādāts speciāls programmas aprīkojums Mathcad valodā. Izstrādātais programmu komplekss ļauj aprēķināt apkalpošanas staciju koordinātas Latvijas teritorijā. Optimizācijas procesā tiek izmantota gradienta metode. Eksperimenti rādīja, ka algoritms strādā veiksmīgi.

Андрей Кашурин. Задача размещения станций обслуживания на заданной территории

Многие важные практические задачи требуют описания распределения станций обслуживания на рассматриваемой территории. Первоначально данная задача была описана в работе 'On a Problem of Spatial Arrangement of Service Stations'. В работе 'Statistical description of a distribution of population's density on the Latvian territory' была предложена процедура описания плотности распределения применительно к распределению плотности населения на территории Латвии. Рассмотрена задача аналитического описания имеющихся данных по распределению населения по территории Латвии. Получено выражение, позволяющее для заданных координат территории Латвии вычислять соответствующую плотность населения. В данной статье результаты двух цитированных работ используются для решения задачи оптимального пространственного размещения четырёх станций обслуживания на территории Латвии. Целесообразно население крупных городов и районов рассматривать по отдельности. При наших расчетах условно описываем каждый район кругом. Радиус круга вычисляется из условия, что площадь круга и района совпадают, а их общая часть максимальна. Принимаем, что плотность населения района распределяется равномерно по его территории. Плотность распределения населения города по территории описана двумерной нормальной плотностью. Конкретный вид обслуживания при этом не конкретизируется. Так, автором рассматривались в качестве видов обслуживания спортивные сооружения, станции технического обслуживания автомобилей и пр. Разработано специальное программное обеспечение на языке Mathcad. Разработанный комплекс программ позволяет вычислять координаты станций обслуживания на территории Латвии. Во время оптимизации используется метод градиента. Эксперименты показали, что алгоритм работает успешно.