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ABSTRACTS

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USING OF THE VARIABLE SEPARATION METHOD FOR NAVIER-STOKES EQUATIONS IN HYDRODYNAMICS

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As it is known from theory the Navier-Stokes nonlinear partial differential equations for hydrodynamics has no common solution. In some cases it is possible to find exact analytical solution of these equations, but the number of such results is respectively low. The presented work is dedicated to the attempt to find possibilities to transform partial differential equations to the ordinary differential equation (or system of equations) by the method of variable separation.

Usually the same goal is achieved by self-similar approach where a new variable is constructed from real variables. In some cases a self-similar equation can be interpreted as equation obtained by variable separation. But the pure variable separation method in hydrodynamics is used very rarely.

The Navier-Stokes equations

$$(\mathbf{V} \cdot \nabla)\mathbf{V} = -\frac{1}{\rho}\,\mathrm{grad}(p) + \nu\nabla^2\mathbf{V}.$$

$$\mathrm{div}(\mathbf{V}) = 0$$

are considered in 4 most popular co-ordinate systems; Cartesian, polar (2D case of cylindrical co-ordinate system), cylindrical (in axisymmetrical case) and spherical (also in axisymmetrical case). The equations will be studied without boundary condition - the main goal of this investigation is to find all possibilities to transform these equations and only after that to formulate concrete problems.

The method of equations transformation is the following: basing on the second equation the stream function is introduced, and then this function is written as the multiplication of two functions, each of them is depending only on one variable. In the common case a type of these terms is not specified and is found in the solution process, but for polar co-ordinate system is necessary to study not only common case, but also functions in exponential form.

The appropriate ordinary differential equations or system of equations are obtained for all co-ordinate systems considered. In some cases solutions of these equations also are found: 2 solutions in Cartesian co-ordinates and 3 - in polar co-ordinates. In addition, the equations were studied using self-similar approach to find solutions which can be interpreted as solution with separated variables. The obtained results also are shown.

For practical needs often use the modification of Navier-Stokes equations - so-called boundary layer approximation. This approximation in Cartesians and cylindrical co-ordinate systems differs from exact equations. The same analysis as above was made also for this approximation too.