RIGA TECHNICAL UNIVERSITY

Aleksandrs SISOJEVS

3D SCENE RENDERING

Summary of Doctoral Thesis

Rīga 2011

RIGA TECHNICAL UNIVERSITY
Faculty of Computer Science and Information Technology
Institute of Computer Control, Automation and Computer Engineering

Aleksandrs SISOJEVS
Student of the Doctoral Study Program "Image Processing and Computer Graphics, Computer Engineering and Computer Networks"

3D SCENE RENDERING

Summary of Doctoral Thesis

Supervisor
Dr.Habl.Sc.Ing., Professor
A.GLAZS

Rīga 2011

UDK 004.92(043.2)

Si 866 t

Sisojevs A. 3D scene rendering. Summary of Doctoral Thesis.-R.:RTU, 2011.-37 p.

Printed in accordance with the decision of the Board of Institute of Computer Control, Automation and Computer Engineering, Faculty of Computer Science and Information technology, Riga Technical University No 98 of 4th March, 2011

This work has been partly supported by the European Social Fund within the National Programme "Support for the carrying out doctoral study" programm's and post-doctoral researches" project "Support for the development of doctoral studies at Riga Technical University".

ISBN …….

DOCTORAL THESIS
SUBMITTED IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR A DEGREE OF THE DOCTOR OF ENGINEERING SCIENCES AT RIGA TECHNICAL UNIVERSITY

The defence of the thesis submitted for doctoral degree of computer systems will take place at an open session on October 10, 2011 in Meza Street 1/3, auditorium 202, Riga Technical University Faculty of Computer Science and Information Technology

OFFIICIAL REVIEWERS:

Professor, Dr.Hab.Sc.Ing. Yuri Merkuryev

Riga Technical University

Professor, Dr.Hab.Sc.Ing. Eugene Kopytov

Transport and Telecommunication Institute, Latvia

Associate professor, Dr.Sc.Ing. Vytenis Punys

Kaunas University of Technology, Lithuania

APPROVAL

I undersigned affirm that I have elaborated this doctoral thesis, which is submitted for doctoral degree in computer science at Riga Technical University. This doctoral thesis is not submitted in any other university for receiving a scientific degree.

Aleksandrs Sisojevs……………………………….. (Signature)

Date:……………………………………..

The doctoral thesis is written in Latvian and includes introduction, 4 sections and conclusions, 6 appendices and bibliography, in total 180 pages. Bibliography includes 129 references.

CONTENT

1. GENERAL WORK DESCRIPTION

1.1. Motivation of the Research

Visualization of the free - form three dimension (3D) objects (or rendering) is considered as one of the most actual problems in different areas, like, in medicine, engineering industry, design and others. There are tasks, where it is very important to provide real objects and/or projects 3D representation. To get solution for this task, we need visualization of the object that is based on objects 3D shape mathematical model.

In this case visualization of this 3D objects task is separating in two parts:

- The visualization of the free - form 3D objects mathematical model using visualization algorithm (or rendering);
- The visualization of the free - form 3D objects mathematical model foundation using mathematical description that is necessary for visualization.

One of the main problems in nowadays 3D computer graphic is acquiring the object in high quality. Classical polygon methods can't handle this task, because they are working only with interpolation, but not with the existing mathematical descriptions. In this case objects are described with a polygon mesh that is reducing the described objects quality. To get image with high quality, we need to use different methods of 3D visualization that are working with precise object descriptions and it is possible to precisely realize visual effects (lighting, texturing etc.) The only method of visualization that fits these demands is ray tracing. That's why the study object in the thesis is development and realization of methods that can be used to produce free - form objects and realize visual effects, using ray tracing.

To construct a 3D model, we need to describe a free – form surface that can be realized using parametrical free - form surface mathematical device that is based on polynomial spline. There are popular spline functions like Bezier surfaces and B-spline surface and their further development - NURBS (Non-Uniform Rational B-Spline) surfaces. In this case surface is constructed at specific range of parametric changes.

In the case of ray tracing we need to resolve an inverse task: find parametrical values in intersection point between the ray and the object's surface. In this case, the solution of this task can be solved as the visualization task of the parametrical free - form surface.

The actuality of doctoral thesis topic is connected with current goals to show the parametrical free - form surface modeling and visualization process. The visualization process consists of two tasks: get precise and high quality visualization image, and reduce time of object's generation.

As was mentioned before, the most effective visualization method of parametrical free - form surface is ray tracing. Ray-surface intersection point finding tasks fundamental mathematical motivation were published in 1982 [49]. In further years some tries were made to resolve this task for several parametrical free - form surface models. In 1990 [66], algorithm for direct searching ray – rational Bezier surface intersection point was proposed. In 2000 [62], algorithm for direct searching of ray NURBS surface intersection point was proposed. All other works (as an example [26] and [112]) are [66] or [62] combinations or modifications. The last publication [77] is dated by 2009. But existing algorithm have several disadvantages: the Nishita method [66] is not providing correct results in some special cases and Efremov modifications [26] do not provide a solution for this problem in general (for example, ray intersection between the surface and multiple parameter point). On the other hand, Martins method [62] is working with input data in vector form and this increases the number of calculations. Therefore, the first task is to find out the possibility to search for ray-parametrical surface intersection point in direct way with an algorithm that results in a high quality image, is easy in program realization, works with scalar data and is fast.

It should be noted that visualization of free - form surface is an actual task in technical design. Therefore, in the frame work of doctoral thesis a comparing analysis must be made with existing computer-aided design (CAD). In the frame of doctoral thesis existing CAD system are analyzed with the aim to evaluate visualization quality of free - form surface. Autodesk AutoCAD 2010 and Siemens Solid Edge ST3 computer-aided design systems are researched in this work. A mathematical model of real object (component of engineering industry) that is described as a rational free - form patch set was used for experiments.

The next part of the task is connected with the construction of 3D model. This work researches the possibilities of real objects modeling by using the following methods: local surface interpolation, global surface interpolation and surface approximation according to surface development with curves. The practical actuality is the construction of 3D model using free - form surfaces for more correct and precisely described real object. Also the actual task is volume estimation of the acquired model with high precision. In the frame of doctoral thesis a medical object was used as a practical example, the task of 3D medical object visualization arises from medical engineering problems and is connected with geometrical analysis of the 3D model, for example, volume estimation of the model.

1.2. The aim and tasks of the Doctoral thesis

The aim of the work is to develop mathematical methods and algorithm for high quality visualization of parametrical free - form surface (taking into account existing defects of visualization algorithms) and for geometrical modeling of the 3D real objects with possibility to calculate acquired model volume. The developed methods must be practically realized in software system and experiments must be conducted using the developed software.

To achieve the aim of doctoral thesis the following **work tasks** must be solved:

- Development of visualization method for parametrical free – form surface;
- Practical realization of visualization method for parametrical free - form surface;
- Mathematical methods of real 3D objects and volume estimation and development of algorithm and its practical realization;
- Conduction of experiments on developed visualization and modeling method, to show its practical usage possibilities.

Research methods

In this work for theoretical research 3D theory of computer graphics, theory of visualization (or rendering), theory of optimization, geometrical modeling, vector analysis are used.

1.3. Scientific novelty of Doctoral thesis

Scientific novelty of the doctoral thesis is**:**

1. Developed non-rational parametrical free - form surface visualization ray tracing mathematical methods and algorithms that use ray-surface intersection point searching task transformation to optimization task. For this task's solution new algorithm are proposed that consists of two parts: searching of previous parameters and specification of the parameters, according to methods of optimization. In contradiction to existing algorithms ([66], [62]), it results in reduced number of mathematical calculations and this, in turn, reduces the processing time of proposed method.
2. Developed non-rational parametrical free - form surface visualization ray tracing mathematical methods and algorithms, using ray-surface intersection point searching tasks numerical solution. In contradiction to Martin method [62] it results in reduced number of mathematical calculations and this, in turn, reduces the processing time of proposed method.
3. Developed geometrical modeling method of 3D real objects based on surface interpolation method. This method is based on Coons surface model.

4. Developed determination algorithm of 3D model volume, which is based on integral estimation method. In contradiction to existing algorithms, it is possible to calculate model volume with absolute exactness and reduce real object volume estimation error.
5. Developed geometrical modeling method of real 3D objects based on global surface interpolation method with the help of B-spline surface.
6. Developed real 3D objects geometrical modeling method based on surface development with parametrical curves. In contradiction to existing methods, the development of the surface is possible based on topological not orthogonal data structure.

Practical value of the work

Practical value of the work is the developed non-rational and rational visualization methods of parametric free - form s surface in ray-tracing and realized visualization system prototype. Effectiveness of developed visualization methods is shown by comparing with existing computer-aided design (CAD) systems on Autodesk AutoCAD 2010 and Siemens Solid Edge ST3 examples. 3D real object modeling method is developed that is approbated on medical object example. Geometrical modeling estimation 3D algorithm is developed in analytic form and realized in the 3D geometrical modeling system software. The implementation of the mentioned methods ensures high quality visualization of 3D objects, modeled by parametrical free - form surfaces set, and effective geometrical modeling of the 3D real objects.

1.4. The structure of the Doctoral thesis

The Doctoral thesis consist of introduction, 4 chapters, conclusions, 6 appendices and list of references. Work is written in Latvian.

In the introduction actuality of the topic is described, the aim and tasks of research are formulated, scientific methods used in the doctoral thesis are described, scientific novelty and practical value are estimated, and, also, the approbation characteristic of the work are shown.

In the first chapter of the Doctoral thesis “Visualization methods in nowadays and mathematical notation of free - form parametrical surface” a review is conducted with the aim to identify defects of the existing 3D geometrical modeling and visualization method and define the high quality visualization and 3D modeling method demands. Comparing analysis of visualization methods is made and description of free - form surface modeling and choice of the visualization method (ray tracing) is fortified.

In the second chapter of the Doctoral these “Visualization methods in nowadays and mathematical notation of free - form parametrical surface” existing algorithmic methods for free - form surface high quality visualization methods in ray tracing are reviewed, and critical compare analysis and identification defects of existing methods are made.

In the third chapter of the Doctoral thesis “Proposed 3D objects visualization methods, using parametrical free - form surface in ray tracing and modeling methods of 3D objects and analysis on medical object example” new method in free - form parametrical surface visualization is proposed, using ray tracing, and also methods of 3D geometrical model is proposed and proposed method analysis is described, which includes existing volume of 3D geometrical model estimation in analytic form. Modeling task with following analysis is reviewed on the medical object examples.

In the fourth chapter of the Doctoral thesis “Practical application of 3D objects parametrical free - form surfaces modeling and visualization methods” proposed visualization methods practical approbation is described. This chapter describes the obtained results comparing analysis with existing methods and computer-aided design (CAD) systems (on Autodesk AutoCAD 2010 and Siemens Solid Edge ST3 examples) and also modeling method practical approbation. The part of the main results and conclusions summarizes the results that are achieved in the work and proposes new tasks in the reviewed scientific direction.

2. SUMMARY OF THE DOCTORAL THESIS CHAPTERS

2.1. Modern visualization methods and mathematical notation of free - form parametric surface

In the first chapter of the doctoral thesis modern existing basic methods of visualization, mathematical notations of free - form surface and its mathematical models that are used in computer graphics, and also interpolation methods of free - form object surface were reviewed and analysed.

In the modern situation of 3D computer graphics the synthesis of the image effectiveness is in accordance with two directions:

- In the first case, the main role is attributed to fast operation. This approach includes polygonal graphics (objects approximate with a set of polygons, all algorithms and models are working already with plane polygon models, but not with precisely described models), device speed-up (graphical card instead of central processor is used to implement mathematical estimations), graphical libraries (OpenGL, DirectX, Java 3D). In this case scanning line visualization method is envisaged.
- In the second case, the approach is envisaged to achieve the highest precision and reality level of the image. This approach includes high definition visualization methods (physically-based lightening models, realistic texturing algorithms etc.). In this case visualization methods for ray tracing and radiosity method are envisaged.

In the first chapter of the doctoral thesis there are described and analyzed following visualization basic methods:

- Scanline rendering visualization method is intended for polygonal model visualization. This method is an example of the method that successfully uses features of image face raster to simplify the initial task and transfigure it into a range of simple tasks in smaller dimension space. Image in picture face can show as the set of horizontal pixel line.
- Radiosity method. Further development of polygonal graphical visualization method is connected with light distribution modelling. This method uses supreme element mathematics to stimulate diffused light distribution from surfaces to other scene objects. This is the method that tries to model situation where directly lighted objects' surfaces serve as indirect lightening source for other scene objects. Therefore, we can get realistic shadowing.
- Ray tracing. The main idea of an algorithm is following: follow a light ray or light flow from light source, register the light ray intersection with object's surfaces, ray refraction or reflection and its occurrence to the light ray. Because of its effectiveness, ray tracing is widely used in visualization technology.

Three dimensional or spatial surfaces are widely used in different mathematical objects' modelling, for example, in industrial objects development (automobiles, ships, planes, shoes, bottles, buildings etc.), they also have great meaning in describing physical phenomena and in visualization in natural sciences (geology, physics, medicine etc.). In 3D computer graphics the most popular surface description is parametrical description, because independently from axis the parametrical surfaces give the chance to easily perform cooperated fining changes with them. In the frame of the doctoral thesis there are described and analyzed 3 mathematical models of free - form parametrical surfaces that are used in computer graphics:

- Bezier surfaces – polynomial surface based on the global basis;
- B – spline surfaces – polynomial surface based on the local basis;
- NURBS surfaces that are rational expansion of B-spline surface.

In 3D computer graphics different object modelling is used employing surface approximation approach. Often there is a need to develop surface using interpolation approach. In the frame of the doctoral thesis two surface interpolation basic methods were described and analysed: local surface interpolation with the help of sculptural surface and global interpolation that is based on the parametrical surface.

Local surface interpolation method is based on bicubic Coons surfaces. Every bicubic Coons surface interpolates object fragment and as a result, the set of the Coons surfaces composes the sculptural surface of the object.

Global surface interpolation method is based on using B-spline surface. Taking into account the fact that B-splain surface is approximation surface on local basis (the change of one control point coordinate affects only a fragment of a surface and not the whole form of the surface) the task of interpolation can be formulated as a search for control points' area equivalent. Exactly this method is analysed in the doctoral thesis.

2.2. Modern methods for parametrical free - form surface high quality visualization

In the second chapter of the doctoral thesis there were described basic methods and the choice of free - form objects notations. The choice of ray tracing is showed and proven as a basic visualization method and free - form notation using analytical approach. In following sub chapters there are reviewed and analysed parametrical free - form surface visualization methods in ray tracing, its historical development, existing method restriction and disadvantages. Based on the results of algorithm analysis we can conclude that nowadays exist only two parametrical free - form surface visualization algorithms in ray tracing that are described in [66] and [62]. Also, Bezier surface is described in this algorithm combination [112] and in algorithm [66] modifications that are described in [26] and [27]. To perform comparative analysis of existing visualization algorithms, [66], [62], [112] and [26] proposed methods were reviewed more closely.

T.Nishita, T.W.Sederberg, and M.Kakimoto [66] described methods crossing for problem solution in star – rational Bezier surface. This method was called Bezier clipping. This method can be classified as an algorithm partly based on division and partly as calculation method. After ray representation as crossing of two planes, ray – surface crossing problem can be projected from 4D space to 2D space. This method reduces the number of arithmetical operations which is necessary to perform de Casteljau subdivision with 50% in every subdivision iteration. Nishita highlighted that Bezier cutting idea can be successfully applied in cases when we need to resolve problem of cropped region estimation. At the same time Nishita noted that described method does not resolve some tasks, one of them being the frequent point search problem and the instability of the method in some surface special cases.

W. Martin et al. [62] proposed a method for reverse NURBS surface visualization in ray tracing. Using node vector processing for generating hierarchic structure of limited space, as a result tree depth declines in comparison with other subdivision methods. The idea is to use handling of node vector so that after NURBS surface transformation Bezier surface set is developed. This surface is wide enough and has narrow limiting box. Bezier surfaces do not use large volumes of memory and are used only for limiting box hierarchic structure construction. The advantage of this method is that achieved plane surface is a good starting condition for Newton's iterative method. NURBS surface calculation scheme that is proposed in the work is based on node vector handling. Unfortunately, algorithm of NURBS surface calculation works slower than Bezier surface calculation. Newton method requires to calculate surface point and two partial derivations for every iteration to get quadric convergence. It is better to divide the initial NURBS surface on Bezier surface during initial processing. This requires extra memory volume to save every surface separately, but it allows to speed-up the calculation significantly.

S.W. Wang, Z.C. Shih and R.C. Chang [112] proposed an algorithm that combines Bezier cutting algorithm and Newton iterative algorithm in order to create effective method for ray coherence application. The first intersection point of the running ray with the Bezier surface is calculated using Bezier iterative algorithm. All following intersection points in the same pixel row are calculated using Newton iterative algorithm. The last calculated intersection point is used as previous result for the following intersection point. The device for barrier detection is used to check-up whether the intersection point that is calculated using Newton iterative algorithm is the last point. When Newton's method is not achieving convergences, then Bezier cutting is used as a replacement for calculating the intersection point.

A. Efremov, V. Havran and H.P. Seidel [26] and [27] proposed the method for NURBS surface visualization in ray tracing using following method: object's every NURBS surface is transformed into equivalent rational set of Bezier surface and exactly this set is mapped. To solve rational Bezier surface problem Bezier cutting method, that is described in [66], is used. And, also [26] and [27] proposes some modifications that improve the activity and effectiveness of Bezier cutting method.

Achieved results are following: existing parametrical free - form surface visualization algorithms in ray tracing were reviewed and analysed, restrictions and disadvantages of these algorithms were defined. Analysis of an existing algorithm shows that:

- The method of Nishita [66] works only with Bezier surfaces, implementation of the method is relatively difficult, the work of method is unstable in some special cases;
- The method of Martin [62] works with NURBS surfaces (therefore, this method uses vector data that requires longer estimation time;
- The method of Wong [112] is not a separate method, but it is rather the combination of Nishita [66] and Martin [62] methods. Therefore, Wong's method inherited some restrictions and disadvantages from Nishita and Martins methods: the method works only with Bezier surfaces, the implementation of the approach is rather complicated, large volume of calculations and long calculation time;
- The method of Efremov [26] is an effort to improve some Nishita method's [66] restrictions and disadvantages: proposed modifications give a chance to visualize NURBS surface in indirect way, direct visualization procedure works only with Bezier surfaces, the approach is rather complicated at implementation level, large volume of calculations and long calculation time;

The main conclusions are: restrictions and disadvantages that we discovered during comparative analysis require us to embark on a new task – develop a method for Bezier, B-spline and NURBS surfaces for high quality visualization in ray tracing. Developed visualization methods should correspond to following requirements:

- The time of image visualization of developed methods should be shorter than the time of existing image visualization methods.

2.3. There are proposed 3D objects' visualization methods using parametrical free - form surfaces in ray tracing and 3D objects' modelling methods and analysis of the examples of medical objects

In the third chapter of doctoral thesis there are developed parametrical free - form surface visualization methods in ray tracing, and there are developed 3D geometrical methods of real objects' modelling.

Non-rational (Bezier and B-spline) and rational (NURBS) high quality visualization methods of parametrical free - form surface in ray tracing are developed in this chapter. Methods of 3D geometrical modelling of real objects are also developed in this chapter reposed on method of global and local interpolation, as well as analytical algorithm for 3D geometrical model volume determination and methods for real objects' 3D geometrical modelling, reposed on surface development with the help of parametrical curve help, were developed;

Bezier surface visualization for ray tracing. In the frame of doctoral thesis there was described new proposed method for Bezier surface visualization using ray tracing method. Ray–surface ray intersection point can be described with the help of a non-linear equation system. To find the unknown parameter value in the equation system we should review the case when ray $r(t)$ coincides with coordinate axis (for example, with Oz). In this case equation system transforms into the following equation system:

$$\begin{cases} S_X(u,v)=0 \\ S_Y(u,v)=0 \\ S_Z(u,v)=r_Z(t) \end{cases} \quad (1)$$

The solution of the equations system (1)(determination of the root) can be transformed into solution of optimization task. To find u and v parameters only 2 first equations of the system (1) are used, therefore, minimization function can be described in the following way:

$$g(u,v)=[S_X(u,v)]^2+[S_Y(u,v)]^2 \to \min_{u,v} \quad (2)$$

Function $g(u,v)$ optimization (minimization) task can be divided into two parts: preliminary searching and optimization of u and v parameters (determination of the root). Let's review these two parts separately.

1) Preliminary searching is based on Monte Carlo method. Searching procedure consists of three steps:

Step 1: At first we should generate random point $(u_i;v_i)$, $i \in [1;N]$, set **M**.

Step 2: From the set of points we should choose one point (u^*, v^*) where the value of the function $g(u,v)$ is minimal.

Step 3: In order to produce the following optimization from the set **M** we should choose point subset **M**. The points $(u;v) \in$ **M'** are chosen based on following inequality:

$$g(u_k,v_k)-g(u^*,v^*) \le \varepsilon_0, k \in [0,N] \quad (3)$$

where: u_k, v_k – k point coordinates; u^*, v^* - point coordinates that give the minimal value to the function $g(u_i,v_i)$; ε_0 - given accuracy; N+1 – random set of points.

Further optimization of the function $g(u,v)$ is carried out based on points from the subset (**M'**). Every point from subset should be optimized .

2) Optimization of parameters (determination of the root). Gradient method [44] is chosen to conduct the optimization. This method is chosen for two reasons: gradient method gives enough convergence speed and this method is enough simple for programming implementation. Changes of parameters u and v values change at every iteration step are calculated in following way:

$$\begin{cases} u_{i+1}=u_i-h\cdot\dfrac{\partial g(u_i,v_i)}{\partial u} \\ v_{i+1}=v_i-h\cdot\dfrac{\partial g(u_i,v_i)}{\partial v} \end{cases} \quad (4)$$

where: h is weight coefficient.

Preliminary search speed-up. Often in practice it is enough to find just the nearest surface's nearest intersection point. Hence, it is proposed to develop preliminary value map in order to find preliminary parameters a u and v value in each pixel. The map is composed of surface data that is coded in RGB channels. Red channel includes surface number. Mathematical relation can be described in following way:

$$Nr = R-1 \quad (5)$$

where: Nr is surface number, which is changing in diapason from [0; 254];

R is value of red channel, which is changing in diapason from [1; 255].

In case if R=0, we can say that in this pixel ray does not cross any point. Taking into account 24 bits image coding in RGB color system we can say that surface number is changing in diapason from [0; 254];

Green and blue channels consist of congener gradient texture that is on peace surface. Let's say, that color value in every channel is a whole number in diapason [0; 225]. Color value corner control points can be given in the way it is described in the first table.

Table 1

Corner control points color value

Control point	Color value
$P_{0,0}$	$(R;0;0)$
$P_{n,0}$	$(R;255;0)$
$P_{0,m}$	$(R;0;255)$
$P_{n,m}$	$(R;255;255)$

Other color values in gradient texture interpolate evenly and are put on the surface. The next task is to read data from the card. Input data in this case is R, G and B color value in every separate pixel. In this case we can find the preliminary value of parameters in following way:

$$\begin{cases} u_0 = \dfrac{1}{255} \cdot G \\ v_0 = \dfrac{1}{255} \cdot B \end{cases} \tag{6}$$

where: u_0, v_0 is the starting value of the parameter in the pixel; G is pixel intensity in green channel; B is pixel intensity in green channel.

B-spline surface visualization for ray tracing. In corresponding subchapter of the doctoral thesis new method for visualization of B-spline surface using the method of ray tracing is proposed and described. Visualization method is analogical to Bezier surface visualization method with only small changes that are connected with features of B-spline function. As it was mentioned before, the problem of surface-ray intersection point search can transformed into optimization task Function $g(u,v)$ (that is described with equation (2)) optimization (minimization) task in the case of the B-spline surface also is divided in two parts: Preliminary search with development of Preliminary value map and optimization of the u and v parameters (root searching) using the fastest descent method.

1) Preliminary search with preliminary development of value map. This task in whole is matching with analogical task in visualization of the Bezier surface, but there are several factors that are connected with the parameter diapason in B-spline surfaces changes in range from $u \in [u_{\min}; u_{\max}]$ and $v \in [v_{\min}; v_{\max}]$. Map is produced from surface data that is coded in RGB channels.

As well as in Bezier surface case, if R=0 we can say, that in this pixel ray does not cross any surface. Green and blue channels consist of congener procedure in gradient texture that is put on peace surface. Color value depends on u and v parameters and this can be calculated in following way:

$$\begin{cases} G = Round\left[255 \cdot \dfrac{(u - u_{\min})}{(u_{\max} - u_{\min})}\right] \\ B = Round\left[255 \cdot \dfrac{(v - v_{\min})}{(v_{\max} - v_{\min})}\right] \end{cases} \tag{7}$$

The next task is to read the data the card. Input data in this case is R, G and B color values in every separate pixel. Surface number is calculated using equation (5). Initial parameters value calculation differs from Bezier surface because of other parameter value diapasons. This can be calculated using following equation:

$$\begin{cases} u_0 = \frac{1}{255} \cdot (u_{\max} - u_{\min}) \cdot G + u_{\min} \\ v_0 = \frac{1}{255} \cdot (v_{\max} - v_{\min}) \cdot B + v_{\min} \end{cases} \tag{8}$$

where: u_0, v_0 is the initial value of parameters in the pixel; $u_{\min}$, $v_{\min}$ is minimal value of parameters in B-spline surface in pixel; $u_{\max}$, $v_{\max}$ is maximal value of parameters in B-spline surface in pixel; G-value of the green channel; B- value of the blue channel.

2) Optimization of the parameters (determination of the root). Method that is described in subchapter about Bezier surface visualization is quite effective. At the same time, this method requires large number of estimations in B-spline case. Mostly, it is connected with the fact that gradient calculation was performed at each iteration, and B-spline basis is not global. Now it is possible to use the gradient value (gradient direction) in running iteration that is achieved from advanced iteration. This method is known in literature as the fastest descent method [44]. Let's rewrite iteration step (equation (4)) in following way:

$$\begin{cases} u_0 = \frac{1}{255} \cdot (u_{\max} - u_{\min}) \cdot G + u_{\min} \\ v_0 = \frac{1}{255} \cdot (v_{\max} - v_{\min}) \cdot B + v_{\min} \end{cases} \tag{8}$$

where: u_{i+1}, v_{i+1} – parameter value in $(i+1)$ – iteration; u_i, v_i – parameter value in i iteration; h_i - weight coefficient in iteration step; Δu_i, Δv_i – changes of the parameters in iteration step.

Now let's look at algorithm activity by steps:

Step1. The initial point of the iteration, let's indicate it as (u_i,v_i). We should calculate changes of the parameters of matrix element values Δu_i and Δv_i.

Step2. Calculate new value of parameter u_{i+1} and v_{i+1} using equation (9). Then we need to calculate value of the function $g(u_{i+1}, v_{i+1})$, in order to evaluate new gained result. At this stage it is required to check-up whether iteration procedure should be prosecuted or it should be stopped.

Step3. In case if $g(u_{i+1}, v_{i+1}) < g(u_i, v_i)$ then we need to repeat second step without changing Δu_i and Δv_i, but in opposite situation we should go to Step4.

Step4. During the iteration situations when $g(u_{i+1}, v_{i+1}) > g(u_i, v_i)$ occurred. In this case we should reduce the weight coefficient h. During experiment it was noticed that the best result is given by weight coefficient h reduction by coefficient, which is inverse to "golden cut". After calculating new weight coefficient we should go back to Step1.

The method uses 3 conditions when iterative work should be terminated.

NURBS surface visualization for ray tracing. The corresponding subchapter of the doctoral thesis describes and proposes new method in visualization of NURBS surface using ray tracing method. Method is based on root determination for task solution, using projection of the surface on projection plane; Newton and Kramer methods combination is used for calculation. Visualization method is partly analogical to Bezier and B-spline surfaces visualization method and is divided into three parts: projection on $\boldsymbol{R}^2$ space (projection plane), preliminary search with Preliminary value map development, search for parameters u and v (root determination). Let's review these steps separately:

Projection on $\boldsymbol{R}^2$ space. Usually calculation of parameters is envisaged for $\boldsymbol{R}^3$ space in case of irrational surface and for $\boldsymbol{R}^4$ space in case of rational surface. Woodword [114], Nishita [66] showed that this problem can be simplified by projection on R^2 space. This means that the amount of arithmetical operations in calculation of rational surfaces is reduced to 25% (exactly this approach was showed by Nishita [66] in rational division of Bezier surfaces). But this approach is suitable also for NURBS surfaces in ray tracing. In the case of NURBS surface, ray and surface intersection point searching task is transformed into the non-linear equation system solution. In this case in order to find the value of parameters u and v

from equation system, that describes surface-star intersection, it is enough to use two equations:

$$\begin{cases} S_X^*(u,v) - x_R = 0 \\ S_Y^*(u,v) - y_R = 0 \end{cases} \tag{10}$$

where: $S_X^*(u,v)$, $S_Y^*(u,v)$ –surface equations in projection plane; x_R, y_R – ray projection point in projection plane.

Solution of equation system (10) is divided in two steps: Preliminary search that includes Preliminary value map development [86] and specification of the parameters.

Preliminary search stage includes development of Preliminary value map. This task corresponds to analogical task in B-spline surface visualization.

<u>Specification of the parameters (root determination).</u> One way to solve non-linear equations system (10) is to use Newton method [99]. In the frame of doctoral thesis there was proposed iterative procedure with the step that can be described in following way:

$$\begin{bmatrix} u_{i+1} \\ v_{i+1} \end{bmatrix} = \begin{bmatrix} u_i \\ v_i \end{bmatrix} + \frac{w(u,v)}{M(u,v)} \cdot \begin{bmatrix} M_U(u,v) \\ M_V(u,v) \end{bmatrix} \tag{11}$$

where:

$$M(u,v) = \det \begin{bmatrix} x^*(u,v) & \frac{\partial x^*(u,v)}{\partial u} & \frac{\partial x^*(u,v)}{\partial v} \\ y^*(u,v) & \frac{\partial y^*(u,v)}{\partial u} & \frac{\partial y^*(u,v)}{\partial v} \\ w(u,v) & \frac{\partial w(u,v)}{\partial u} & \frac{\partial w(u,v)}{\partial v} \end{bmatrix} \tag{12}$$

and

$$M_U(u,v) = \det \begin{bmatrix} x^*(u,v) & x_R & \frac{\partial x^*(u,v)}{\partial v} \\ y^*(u,v) & y_R & \frac{\partial y^*(u,v)}{\partial v} \\ w(u,v) & 1 & \frac{\partial w(u,v)}{\partial v} \end{bmatrix}; \; M_V(u,v) = \det \begin{bmatrix} x^*(u,v) & \frac{\partial x^*(u,v)}{\partial u} & x_R \\ y^*(u,v) & \frac{\partial y^*(u,v)}{\partial u} & y_R \\ w(u,v) & \frac{\partial w(u,v)}{\partial u} & 1 \end{bmatrix} \tag{13}$$

3D objects geometrical modelling using local interpolation (with Coons and Bezier surfaces) and volume estimation. Corresponding subchapter of the doctoral thesis proposes new 3D object geometrical modelling method based on local surface interpolation. Proposed method is based on interpolation model development . Model development is based on Coons surface basis. In order to visualize the object with the help of computer graphics standard resources (for example, graphical library OpenGL) or with the help of existing algorithm. Coon's surface model is transformed into Bezier surface model.

Let's review a case when we need to develop objects 3D mathematical model, but we know only control points on the surface. In this case surface should comply with some criteria: surface should go through all points, surface should be perpendicular to advanced curve vectors and surface should touch advanced task touching vectors.

Therefore surface interpolation problem arises. Let's suppose that we need to develop 3D object from input data that can be described as data array (matrix). In this case it is evident that it is necessarily to develop object sculpture surface, where every element (which describes some surface fragment) interpolates object part between proximal control points. In this case task is formulated in the following way:

- Develop mathematical model for object surface interpolation so that each element (every patch) satisfies before mentioned criteria.

Input data for mathematical models development is the control point data array with topologic orthogonal structure where one side pair makes a closed contour that is similar to side surface of the cylinder. In this case visualization task can be described as surface data array development. Bicubic Coons surface model is used in the work to describe surfaces. The Coons surface matrix form is described in 1.3.1.subchapter of the doctoral thesis. Taking into account control point data array in one surface part we can calculate as Coons surface. In this case control point data array for one Coons surface can be calculated in this way:

$$\left[P_C^{i1,j1}\right]=\begin{bmatrix} P_{i,j} & P_{i,j+1} & P_{i,j}^{v} & P_{i,j+1}^{v} \\ P_{i+1,j} & P_{i+1,j+1} & P_{i+1,j}^{v} & P_{i+1,j+1}^{v} \\ P_{i,j}^{u} & P_{i,j+1}^{u} & P_{i,j}^{uv} & P_{i,j+1}^{uv} \\ P_{i+1,j}^{u} & P_{i+1,j+1}^{u} & P_{i+1,j}^{uv} & P_{i+1,j+1}^{uv} \end{bmatrix} \tag{14}$$

where: $P_{i,j}$ – control points in surface part corners (points from input data array);

$P_{i,j}^{u}$, $P_{i,j}^{v}$ – tangent vectors in parametrical directions *u* and *v*; $P_{i,j}^{uv}$ – twist vectors.

Tangent vectors can be calculated in this way:

$$P_{i,j}^{u}=\frac{1}{2}\cdot\left(P_{i,j+1}-P_{i,j-1}\right) \text{ and } P_{i,j}^{v}=\frac{1}{2}\cdot\left(P_{i+1,j}-P_{i-1,j}\right) \tag{15}$$

this turn, twist vector can be calculated using equation:

$$P_{i,j}^{uv}=P_{i,j}^{u}\times P_{i,j}^{v} \tag{16}$$

In order to visualize 3D object with the help of computer graphics standard resources or with the help of an existing algorithm, we need to transform Coons surface to the Bezier surface. Equivalent Bezier surface control point data array can be calculated in this way:

$$\left[P_B^{i1,j1}\right]=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 1/3 & 0 \\ 0 & 1 & 0 & -1/3 \\ 0 & 1 & 0 & 0 \end{bmatrix}\cdot\begin{bmatrix} P_{i,j} & P_{i,j+1} & P_{i,j}^{v} & P_{i,j+1}^{v} \\ P_{i+1,j} & P_{i+1,j+1} & P_{i+1,j}^{v} & P_{i+1,j+1}^{v} \\ P_{i,j}^{u} & P_{i,j+1}^{u} & P_{i,j}^{uv} & P_{i,j+1}^{uv} \\ P_{i+1,j}^{u} & P_{i+1,j+1}^{u} & P_{i+1,j}^{uv} & P_{i+1,j+1}^{uv} \end{bmatrix}\cdot\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1/3 & 0 & 0 \\ 0 & 0 & -1/3 & 0 \end{bmatrix} \tag{17}$$

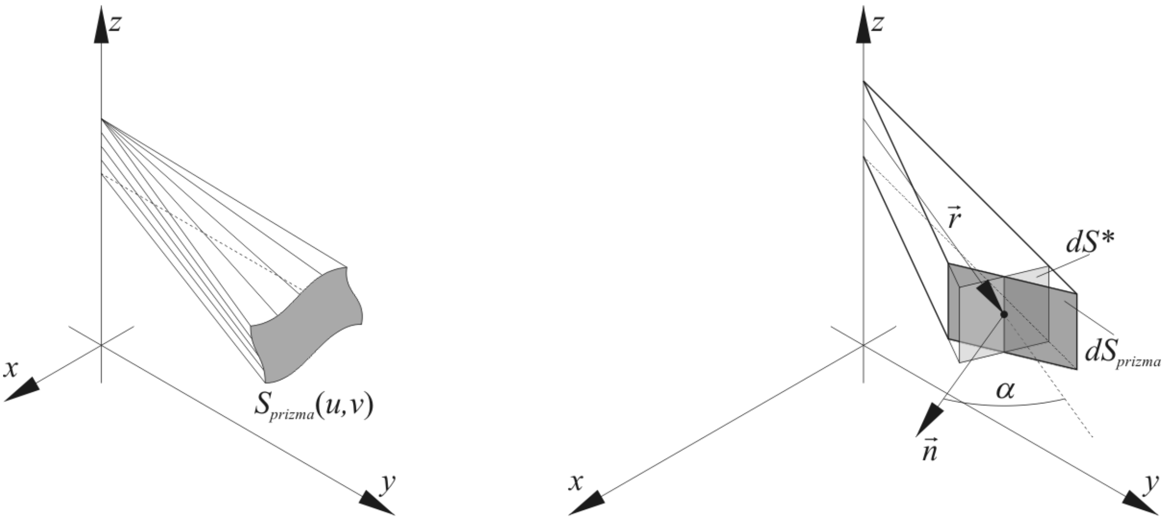
As a result we acquire Bezier surface data array of object's mathematical model. This method gives opportunity to use computer graphics standard resources. Standard graphical library – OpenGL [126] and, also visualization with ray tracing help is used in this work. Visualization method with ray tracing is described in the 3.1.subchapter of the doctoral thesis.

3D free - form objects volume estimation. One of the most actual problems in object analysis is volume estimation. Algorithm is described and proposed in the doctoral thesis that allows to calculate object's volume using integral counting. Input data for this algorithm work is 3D object that is described with the help of the Bezier surface set. Based on described before method Bezier surface set can be described as data array of the surface $S(u,v)$. Let's think that object volume is a sum of curvilinear prism volume:

$$V_{obj}=\sum V_{prizma} \tag{18}$$

where: V_{obj} – object volume; V_{prizma} – curvilinear prism volume.

To calculate the volume of curvilinear prism see the example that is shown in Fig.1.a).



a) b)

Figure 1. a) example of curvilinear prism and b) example of differential volume detection.

Differential volume dV_{prizma}. in Fig. 1b) can be calculated in the following way:

$$dV_{prizma} = \frac{1}{2} \cdot |\vec{r}| \cdot \cos\alpha \cdot dS_{prizma} \tag{19}$$

where: $\vec{r}$ – perpendicular vector from axis Oz to surface point; α – angle between vector $\vec{r}$ and normal $\vec{n}$ to dS_{prizma}

To solve this differential equation, let's integrate both parts from (19) with surface integral. Therefore, we will get:

$$V_{prizma} = \frac{1}{2} \cdot \iint\limits_{(S_{prizma})} |\vec{r}| \cdot \cos\alpha \cdot dS_{prizma} \tag{20}$$

To solve integral (20), let's review some mathematical features. To solve first type's surface integral task a transformation to double integral is used. Consider that in the case of Bezier surface parameters u and v change in diapason from [0;1]. By taking into account this feature and other features that are described in the work, we can get the final equation of integral calculation of the curvilinear prism volume:

$$V_{prizma} = \frac{1}{2} \cdot \int_0^1 \int_0^1 \left(A \cdot S_X(u,v) + B \cdot S_Y(u,v) \right) du dv \tag{21}$$

where:

$$A = \begin{vmatrix} \dfrac{\partial S_Y(u,v)}{\partial u} & \dfrac{\partial S_Z(u,v)}{\partial u} \\ \dfrac{\partial S_Y(u,v)}{\partial v} & \dfrac{\partial S_Z(u,v)}{\partial v} \end{vmatrix} \text{ and } B = \begin{vmatrix} \dfrac{\partial S_Z(u,v)}{\partial u} & \dfrac{\partial S_X(u,v)}{\partial u} \\ \dfrac{\partial S_Z(u,v)}{\partial v} & \dfrac{\partial S_X(u,v)}{\partial v} \end{vmatrix} \tag{22}$$

Taking into account the difficulty of the integral expression, practically to calculate integral (21) graphical integration is used for convenience. Let's divide u diapason to n_2 parts and parameter v diapason to m_2 parts. In this case, curvilinear prism volume can be approximately calculated using following equation:

$$V_{prizma} \approx \frac{1}{2 \cdot n_2 \cdot m_2} \cdot \sum_{i2=0}^{n_2-1} \sum_{j2=0}^{m_2-1} A(u_{i2}, v_{j2}) \cdot S_X(u_{i2}, v_{j2}) + B(u_{i2}, v_{j2}) \cdot S_Y(u_{i2}, v_{j2}) \tag{23}$$

where: u_{i2}, v_{j2} – discreet values of the parameter;

n_2, m_2 – value of the steps in parametrical directions u and v.

Discreet values of the parameters can be calculated in following way:

$$u_{i2} = \frac{i2}{n_2 - 1} \text{ and } v_{i2} = \frac{j2}{m_2 - 1} \tag{24}$$

To calculate integral (21) value with absolute accuracy in the case of the Bezier surface, let's rewrite integral in following direction:

$$
\begin{aligned}
&V_{prizma} = \frac{1}{2} \cdot \sum_{i1=0}^{n} \sum_{j1=0}^{m} \sum_{i2=0}^{n} \sum_{j2=0}^{m} \sum_{i3=0}^{n} \sum_{j3=0}^{m} \Big[X_{i1,j1} \cdot Y_{i2,j2} \cdot Z_{i3,j3} \cdot \\
&\left(\left(\int_0^1 B_{i1,n}(u) \cdot \frac{dB_{i2,n}(u)}{du} \cdot B_{i3,n}(u) du \right) \cdot \left(\int_0^1 B_{j1,m}(v) \cdot B_{j2,m}(v) \cdot \frac{dB_{j3,m}(v)}{dv} dv \right) - \right. \\
&- \left(\int_0^1 B_{i1,n}(u) \cdot B_{i2,n}(u) \cdot \frac{dB_{i3,n}(u)}{du} du \right) \cdot \left(\int_0^1 B_{j1,m}(v) \cdot \frac{dB_{j2,m}(v)}{dv} \cdot B_{j3,m}(v) dv \right) + \\
&+ \left(\int_0^1 B_{i1,n}(u) \cdot B_{i2,n}(u) \cdot \frac{dB_{i3,n}(u)}{du} du \right) \cdot \left(\int_0^1 \frac{dB_{j1,m}(v)}{dv} \cdot B_{j2,m}(v) \cdot B_{j3,m}(v) dv \right) - \\
&\left. - \left(\int_0^1 \frac{dB_{i1,n}(u)}{du} \cdot B_{i2,n}(u) \cdot B_{i3,n}(u) du \right) \cdot \left(\int_0^1 B_{j1,m}(v) \cdot B_{j2,m}(v) \cdot \frac{dB_{j3,m}(v)}{dv} dv \right) \right) \Big]
\end{aligned} \tag{25}
$$

Mathematical proof of this equation is described in 3.4.subchapter of this doctoral thesis. In case if in under integral function is Bernshtein polynomial and is its derivative product (like in equation (25)) it is convenient to use the following equation to calculate the specific integral:

$$
\begin{aligned}
&V_{prizma} = \frac{1}{2} \cdot \sum_{i1=0}^{n} \sum_{j1=0}^{m} \sum_{i2=0}^{n} \sum_{j2=0}^{m} \sum_{i3=0}^{n} \sum_{j3=0}^{m} \Bigg[X_{i1,j1} \cdot Y_{i2,j2} \cdot Z_{i3,j3} \cdot \binom{n}{i1} \cdot \binom{m}{j1} \cdot \binom{n}{i2} \cdot \binom{m}{j2} \cdot \binom{n}{i3} \cdot \binom{m}{j3} \cdot \\
&\big(T(\Sigma i, n, i2) \cdot T(\Sigma j, m, j3) - T(\Sigma i, n, i3) \cdot T(\Sigma j, m, j2) + \\
&+ T(\Sigma i, n, i3) \cdot T(\Sigma j, m, j1) - T(\Sigma i, n, i1) \cdot T(\Sigma j, m, j3) \big) \Bigg]
\end{aligned} \tag{26}
$$

where function $T(\Sigma i, n, i2)$ can be calculated in following way:

$$
T(\Sigma i, n, i2) = \begin{cases} -\frac{1}{3} & ja \quad \Sigma i = 0 \\ \sum\limits_{k=0}^{3 \cdot n - \Sigma i - 1} (-1)^k \cdot \binom{3 \cdot n - \Sigma i - 1}{k} \cdot \left(\frac{i2}{k + \Sigma i} - \frac{n}{k + \Sigma i + 1} \right) & ja \quad \Sigma i \in [1; 3 \cdot n - 1] \\ \frac{1}{3} & ja \quad \Sigma i = 3 \cdot n \end{cases} \tag{27}
$$

where: $\Sigma i = i1 + i2 + i3$.

3D objects geometrical modelling using global interpolation. The corresponding subchapter of this doctoral thesis proposes 3D objects geometrical modelling method that is based on global surface interpolation. Model development is based on B-spline surface. Proposed method is approbated in practical application in medicine, but this method can also be used in different other areas (science, techniques, medicine etc.).

As in previous subchapter this subchapter reviews the case when it is needed to develop 3D mathematical model of the object, but we know only control points on object's surface. In this case, surface should comply with some criteria: surface should go through all points, surface order in every parametrical direction is given before, and object is interpolated with one surface. Thereby, global surface interpolation task emerges. Let's suppose that input data is described as topological orthogonal control point data array (matrix). And the structure of this data should be transferred into topological cylindrical structure, linking one side with opposite located side. Exactly this kind of example is described in 3.3.subchapter and illustrated in Figure 3.3. In this image we can see object's carcass that is developed from control point data array. In this case visualization task can be described in the following way:

- We should develop B-spline surface that in one parametrical direction develops closed contour and complies with above mentioned conditions.

Global interpolation method using B-spline is described in 1.3.2.subchapter of the doctoral thesis, in case if surface is open in both parametrical directions.

Lets assume that initial data array size is *cr x pt*, where *cr* – row number and *pt* – column number. To use B-spline surface in interpolation task, it is comfortable to use periodical B-spline function in both directions. It is possible to choose surface order in every parametrical direction (described as *p* and *q*). Coefficients n and m can be calculated in the following way:

$$n = cr - 1 \text{ and } m = pt + q - 1 \tag{28}$$

In this case B-spline surface control point data array size is (n+1) x (m+1) and control point value interpolation can be calculated in the following way:

$$[S*] = \begin{array}{c} \overbrace{\hphantom{S_{0,0} \quad \cdots \quad S_{0,pt-1}}}^{pt \quad kolonnas} \overbrace{\hphantom{S_{0,0} \quad \cdots \quad S_{0,q-1}}}^{q \quad kolonnas} \\ \begin{bmatrix} S_{0,0} & \cdots & S_{0,pt-1} & S_{0,0} & \cdots & S_{0,q-1} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ S_{cr-1,0} & \cdots & S_{cr-1,pt-1} & S_{cr-1,0} & \cdots & S_{cr-1,q-1} \end{bmatrix} \end{array} \tag{29}$$

where: S_{ij} – discreet values of the parameters.

Let's rewrite matrix (29) to matrix – column way:

$$\begin{aligned} [S] = [S_{0,0} \quad \cdots \quad S_{0,pt-1} \quad S_{0,0} \quad \cdots \quad S_{0,q-1} \\ S_{1,0} \quad \cdots \quad S_{1,pt-1} \quad S_{1,0} \quad \cdots \quad S_{1,q-1} \quad \cdots \\ \cdots \quad S_{cr-1,0} \quad \cdots \quad S_{cr-1,q-1} \quad S_{cr-1,0} \quad \cdots \quad S_{cr-1,q-1}]^T \end{aligned} \tag{30}$$

Unknown control point data array can be described in the following way:

$$[P] = [P_{0,0} \quad \cdots \quad P_{0,m} \quad P_{1,0} \quad \cdots \quad P_{1,m} \quad \cdots \quad P_{n,0} \quad \cdots \quad P_{n,m}]^T \tag{31}$$

Coefficient data array element value can be calculated taking into account two conditions:

1. diapason of *u* and *v* parameters depends on number of control points and on chosen node vector in each parametrical direction;
2. in initial surface describing data structure that is reviewed in this subchapter, it is convenient to consider that parametric values in initial control points are divided evenly in every parametrical direction.

In reviewed example the size of initial point data array is similar to the size of B-spline control point data array. Based on this, matrix [*P*] value can be found using different methods. In practice we used Gauss – Jordan elimination method.

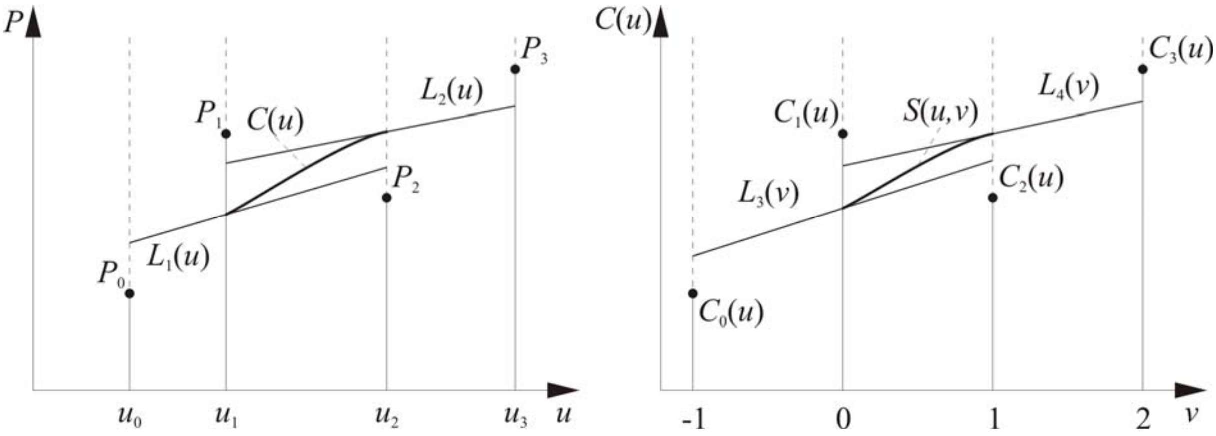
3D objects geometrical modelling using development of parametrical surface with curves. Corresponding subchapter the doctoral thesis proposes new method that is based on development of parametrical surface with approximation curves in every parametrical direction. Proposed method is approbated in practical application on the example of medicine objects, but method can be used in different other areas (science, techniques, medicine etc.). Let's review the case when is necessary to develop 3D object mathematical model, but we known only control points on object's surface. At the same time control points comply with following conditions:

1. In 3D space control points are described by layers;
2. Distance between layers is homogeneous;
3. Order of control points in every layer is pre-determined and creates closed contour;
4. Number of control points in every layer is not equivalent;
5. Number of control points in every layer is quiet large.

In this case the surface should comply with the following criteria: surface should come closer to control points and surface should insure the reconciliation of object's form.

If we take into account above mentioned conditions a problem of surface local approximation will arise. In this case it is evidently necessary to develop sculpture surface where every element approximates objects fragment between two proximal layers. Proposed method for parametrical surface development consists of two steps: curve set development in one parametrical direction and surface development based on gained curve set.

First step, curve set development consists of curve development task at every layer. The idea of the method is the approximation of curve stage. Input data is P_i set with corresponding parameter u_i value in every consistent point. At every layer parameter u changes in diapason [0; 1]. Stage approximation was commissioned with four row points and parametrical curve approximates the stage between two central points of four. This method can be called curved parabolic approximation. Fig. 2a) illustrates this situation.



a) b)

Figure 2. Curved parabolic approximation idea

Taking into account Fig. 2a). curved parabolic approximation is described in following way:

$$C(u)=\frac{u-u_2}{u_1-u_2}\cdot L_1(u)+\frac{u-u_1}{u_2-u_1}\cdot L_2(u) \tag{32}$$

where: $C(u)$ – curve stage at interval [u_1; u_2]; u_1, u_2 –values of parameters in control points; u –curve parameter.

Linear function can be calculated in the following way:

$$L_1(u)=a_1\cdot u+b_1 \text{ un } L_2(u)=a_2\cdot u+b_2 \tag{33}$$

Choice of linear coefficients must insure the minimal speed from control points to linear function. In this case in order to calculate linear coefficient values we can use the smaller quadrate method. This can be described with linear equation system:

$$\begin{bmatrix} \sum_{i=0}^{2} u_i^2 & \sum_{i=0}^{2} u_i \\ \sum_{i=0}^{2} u_i & 3 \end{bmatrix}\cdot\begin{bmatrix} a_1 \\ b_1 \end{bmatrix}=\begin{bmatrix} \sum_{i=0}^{2} P_i\cdot u_i \\ \sum_{i=0}^{2} P_i \end{bmatrix} \text{ and } \begin{bmatrix} \sum_{i=1}^{3} u_i^2 & \sum_{i=1}^{3} u_i \\ \sum_{i=1}^{3} u_i & 3 \end{bmatrix}\cdot\begin{bmatrix} a_2 \\ b_2 \end{bmatrix}=\begin{bmatrix} \sum_{i=2}^{3} P_i\cdot u_i \\ \sum_{i=2}^{3} P_i \end{bmatrix} \tag{34}$$

Matrix value of linear coefficient can be found using different methods. Taking into account that number of equations in each system is 2 in practice it is comfortable to use inverse matrix method:

$$\begin{bmatrix} a_1 \\ b_1 \end{bmatrix}=\begin{bmatrix} \sum_{i=0}^{2} u_i^2 & \sum_{i=0}^{2} u_i \\ \sum_{i=0}^{2} u_i & 3 \end{bmatrix}^{-1}\cdot\begin{bmatrix} \sum_{i=0}^{2} P_i\cdot u_i \\ \sum_{i=0}^{2} P_i \end{bmatrix} \text{ and } \begin{bmatrix} a_2 \\ b_2 \end{bmatrix}=\begin{bmatrix} \sum_{i=1}^{3} u_i^2 & \sum_{i=1}^{3} u_i \\ \sum_{i=1}^{3} u_i & 3 \end{bmatrix}^{-1}\cdot\begin{bmatrix} \sum_{i=2}^{3} P_i\cdot u_i \\ \sum_{i=2}^{3} P_i \end{bmatrix} \tag{35}$$

<u>*Second step*</u> is surface development using curve development in second parametrical direction; the curve set obtained in the first step is used as input data. This method is analogical to the first step. Taking into account that the distance between layers is similar it is possible to accept *v* parameter values generally, and than diapason between two central points will be the same [0; 1]. Fig. 2b) illustrates this situation in case of *u=const.* Taking into account Fig. 2b) descriptions, linear parabolic approximation is described in the following way:

$$S(u,v) = (1-v) \cdot L_3(v) + v \cdot L_4(v) \tag{36}$$

where: $S(u,v)$ –resulting surface; v – second surface parameter.

Linear functions $L(v)$ can be calculated in the following way:

$$L_3(v) = a_3 \cdot v + b_3 \text{ un } L_4(v) = a_4 \cdot v + b_4 \tag{37}$$

Linear coefficients should be chosen to ensure minimal speed from control points to linear function. In this case we can also use the smallest quadrate method in order to calculate linear coefficient values. Taking into account the even division of the parameter *v* it can be calculated in the following way using inverse matrix method:

$$\begin{bmatrix} a_3 \\ b_3 \end{bmatrix} = \frac{1}{6} \cdot \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} C_2(u) - C_0(u) \\ \sum_{i=0}^{2} C_i(u) \end{bmatrix} \text{ and } \begin{bmatrix} a_4 \\ b_4 \end{bmatrix} = \frac{1}{6} \cdot \begin{bmatrix} 3 & -3 \\ -3 & 5 \end{bmatrix} \cdot \begin{bmatrix} C_2(u) + 2 \cdot C_3(u) \\ \sum_{i=1}^{3} C_i(u) \end{bmatrix} \tag{38}$$

2.4. Practical application of 3D object parametrical free - form surface modelling and visualization method

The fourth chapter of this doctoral thesis is dedicated to the new visualization and modelling method's, that was proposed in chapter 3, practical realization and its experimental research and issue solving that is based on this thematic. Previously acquired theoretical results were practically realized with the help of software and by using Delphi programming environment and Object Pascal programming language, and graphical library OpenGL.

3D object that is modelled with Bezier surface visualization method to ray tracing approbation. Newly developed Bezier surface high quality visualization method was proposed in this doctoral thesis, this method is described in 3.1.subchapter of the doctoral thesis. In order to evaluate proposed method value in practical experiments there was made a comparative analysis with existing algorithms that are described in [66] and [62], and also [112] and [26]. All these methods were realized in the computer program form. Programming tool in experimental stage was programming environment Borland Delphi 7.0.

An approach of 1 ray on pixel was used for scene visualization. The size image size that was gained in the experiments was 640x480 pixels. During experiment 2 scenes were visualized: as the first scene a teapot on the plane rectangle background was visualized as experimental object; the second was a duck. All surfaces in experimental scene were described with the help of Bezier surface. Images gained with the help of proposed method are shown in Fig.3. Images gained with the help of other methods are shown in the Appendix 1 of the doctoral thesis.

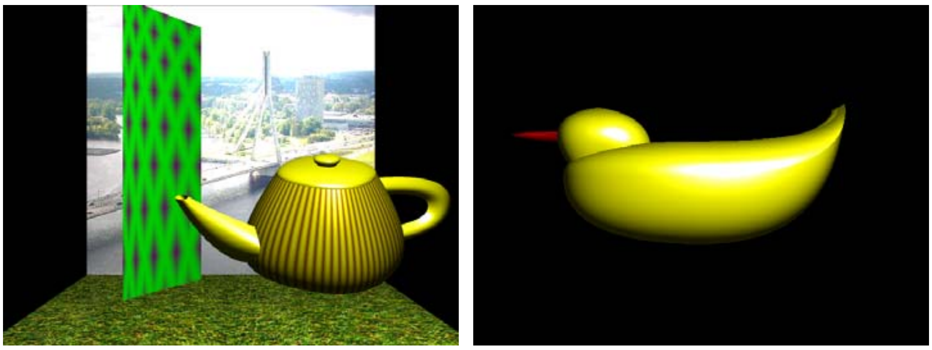


Figure 3. Scene that was achieved using proposed method.

The first criterion that was evaluated the proposed method is the quality of the gained image. As we can see from acquired data that is shown in the Appendix 1 of the doctoral thesis the quality of the proposed method is much better as gained images do not have deformations on surface borders. The second criterion is the visualization time of the image. Image visualization times in the experiment are shown in Table 2.

Table 2

Image visualization time

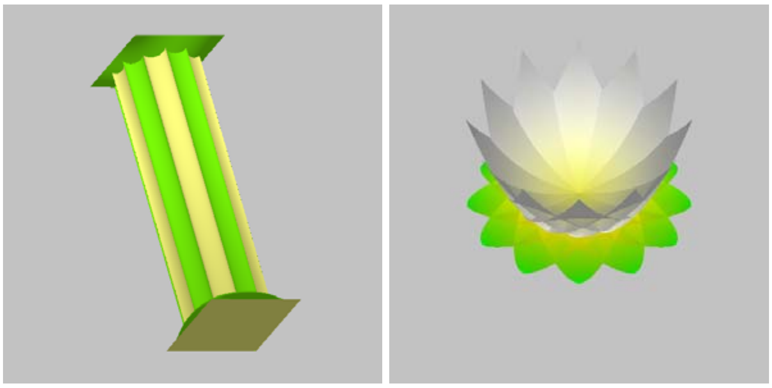
Method	"Teapot", min./%	"Duck", sec./%
Proposed	5,59/100%	20,39/100%
Nishita et al.	6,669/119,3%	169,154/824,7%
Efremov et al.	6,681/119,5%	116,233/570,0%
Martin et al.	5,613/100,4%	20,922/102,6%
Wang et al.	5,63/100,7%	23,968/117,5%

As we can see from Table 2 data, proposed method gives stabile visualization time reduction (in comparison with existing methods) in the frame of experiment. Proposed method gives visualization time reduction in comparison with Martin and Wang methods of 0,4% - 17,5% and in comparison with Nishita and Efremov methods approximately of 19%. Nishita and Efremov methods visualization time results on example "Duck" can testify that algorithm is unstable on existing object, or can denote potential and/or restrictions of the algorithm that were not described in [66] and [26]. Thereby, these achieved visualization times should be dropped off in further analysis.

The next experimental check-up of the scene visualization used the approach of 1 ray on pixel. The achieved size of the image in experiments equalled to 512x512 pixels. 2 objects, that were received after medical object modelling, were visualised in the experiment: fragment of the inner surface of the skull and pathology zone. All surfaces in the experiment were described with the help of Bezier surface. Achieved images were shown in further approbation part in Fig. 8, or in the Appendix 4, Fig. P4.4. – P4.6., and Fig. P4.8. Of the doctoral thesis.

Experimental check-up aim was to prove the effectiveness of new proposed method. Therefore the comparison with already known algorithm [66], [62], [112] and [26] was not made. Gained images successfully proved the effectiveness of the proposed method.

Visualization approach in ray tracing approbation of 3D objects that were modelled with B-spline surface. New developed B-spline surface high quality visualization method was proposed in this doctoral thesis, the proposed method is based on the fastest descent method. In order to evaluate the proposed method's value in practice comparative analysis was carried out with two algorithms: with proposed algorithm, what is based on gradient method (described in the 3.1.subchapter of the doctoral thesis) and Martin algorithm, that is described in work [62]. All these methods were realized in the form of computer program. Programming tool in experimental stage was programming environment Borland Delphi 7.0. To visualize the scene approach of 1 ray on pixel was used. The size of gained image was 512x512 pixels. 2 scenes were visualized experimentally: the first scene – column, and second scene – water lily. All surfaces in experimental scenes were described with the help of B-spline surface. Achieved images were shown on Fig. 4.



a) b)

Figure 4: Scenes that are achieved using the method that is based on the fastest descent method.

As it was described in the experiment before, the first criterion that was used to evaluate the proposed method is the quality of the image. As we can see from gained images, both proposed methods gives equivalent images and these images give better results regarding image quality position. The second criterion is the visualization time of the image. Image visualization times in the experiment are shown in Table 3.

Table 3

Image visualization time in seconds and percents

Method	"Column"	"Water lily"
Proposed, based on fastest descent method	31,156 / 51,3%	98,313 / 47,8%
Proposed, based on gradient method	74,906 / 123,3%	180,234 / 87,7%
Martin et al.	60,750 / 100%	205,600 / 100%

The Table 3 proves that the proposed method (based on gradient method) gives unstable visualization time reduction (in comparison with Martin method et al.[62]). At the same tame the proposed method that is based on the fastest descent method, gives stable and better result based on visualization time in our experiment, and give approximately double (in experiments 48,7% - 52,2%) visualization time reduction in comparison with method Martin et al.

In the next experimental check-up of the scenes visualisation there was used an approach of 1 ray on pixel. Achieved image size in the experiment is 512x512 pixels. 1 scene was visualised during this experiment that was obtained after modelling of real medical object: fragment of inner surface of the skull and pathology zone. Modelling methods of the real objects are described in the 3.5.subchapter of the doctoral thesis. All surfaces in the experimental scene were described with the help of B-spline surface. Achieved images were shown in Fig. 10.

The first criterion that we should evaluate is the quality of the obtained image. As we can see from obtained images, both proposed methods give equivalent images and these images give better results regarding image quality. The second criterion that what we should evaluate is visualization time of the obtained image. Experimental Image visualization times are shown in Table 4 middle column. As we can see from Table 4 middle column data, proposed method that is based on the fastest descent method gives visualization time reduction in comparison with proposed method that is based on gradient method, and existing Martin algorithm [62]. Table 4 right side column data testify that proposed method (based on the fastest descent method) gives visualization time reduction in the experiment of 30,02% in comparison with method Martin et al. [62].

Table 4

Image visualization time in seconds and percents

Method	Visualization time, min.	Visualization time, %
Proposed, based on the fastest descent method	17,683	69,98%
Proposed, based on gradient method	24,533	97,09%
Martin et al.	25, 267	100%

Visualization approach in ray tracing approbation of 3D objects that are modelled with NURBS surface. New developed NURBS surface high quality visualization method was proposed in this doctoral thesis, this method is described in the 3.3.subchapter of the doctoral thesis. A comparative analysis with algorithm Martin et al. that is described in work [62] was conducted in order to evaluate the proposed method. Both these methods were realized using computer program. CodeGear RAD Studio 2009 was used as programming tool, Delphi 2009 was used as programming environment.

Approach of 1 ray on pixel was used for scene visualization. The size of the obtained image is 512x512 pixels. 4 scenes were visualized during the experiment: the first scene – duck that what is taken from VRML programming language standard examples, the second scene visualized experimental object from the first scene, in total 27 VRML ducks, the third scene visualized experimental object – mobile phone and the fourth scene visualized practical object – modelled machine element. All surfaces of experimental scenes were described with the help of NURBS surface. Achieved images were shown in Fig. 5.

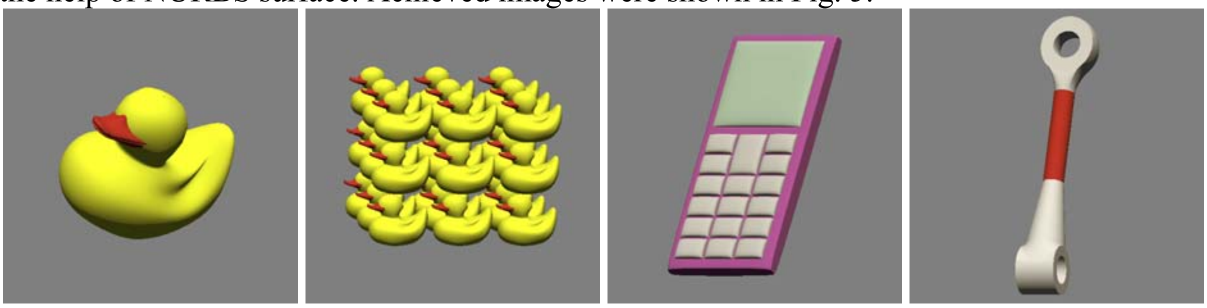


Figure 5. Scenes used in the experiment

As we can see from obtained images that are given in the Appendix 3 of the doctoral thesis the proposed method gives better results regarding image quality. Image visualization times of the experiment are shown in Table 5.

Table 5

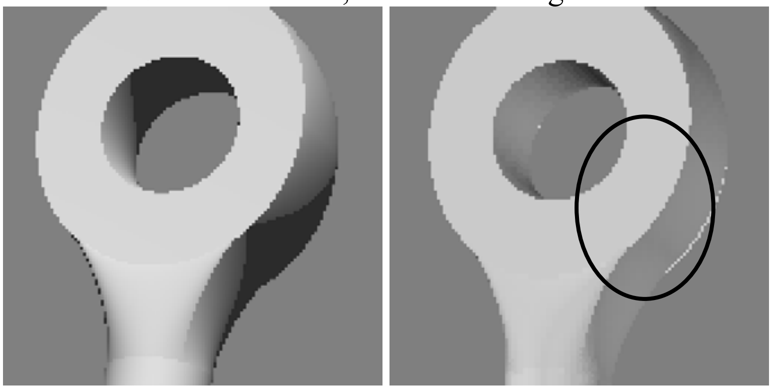
Image visualization time in seconds and percents

Scene	Proposed method	Martin et al. method
“Duck”	5,578/86,66%	6,437/100%
“27 ducks”	9,000/75,89%	11,860/100%
“Mobile phone”	4,297/79,25%	5,422/100%
“Machine element”	1,890/85,79%	2,203/100%

As we can see from Table 5 data the proposed method gives stable visualization time reduction (compare with existing methods) in experiment. Table 5 proves that proposed method gives time reduction from 13,3 – 24,1% in experiment in comparison with algorithm Martin et al.

The next experiment that was conducted in order to check-up the described experiment is the comparison with existing CAD systems. A comparison with two CAD systems was conducted: Autodesk AutoCAD 2010 and Siemens Solid Edge ST3. “Machine element” was chosen an object's example. To conduct object's visualization in CAD system

Autodesk AutoCAD 2010 this object was made with NURBS surfaces in visualization programs and was imported to Autodesk AutoCAD 2010. To ensure correct comparison the object was colored in one color and equivalent lightening settings were adjusted. Fragments that were made larger in the visualization result,are shown in Fig. 6.



a) b)

Figure 6. Objects “Machine element” visualization, using:
a) proposed method, b) Autodesk AutoCAD 2010

From Fig. 6 we can see that the proposed method gives better result regarding image quality, because the object has not any defects on surface borders. Image visualization times in the experiment are following: 1, 89 seconds using proposed method; 10, 24 seconds using Autodesk AutoCAD 2010 visualization. As we can see from given data, proposed method give visualization time reduction in experiment (the difference is 5, 4 times).

In order to develop object visualization in CAD system Siemens Solid Edge ST3 this object was modelled using Solid Edge ST3 tools. To ensure correct comparison the object was colored in one color and equivalent lightening settings were adjusted. To make comparison more accurate it is useful to make larger fragment of obtained image. Larger fragments are shown in Fig. 7.

a) b)

Figure 7. Object “Machine element” visualization using:
a) proposed method, b) Siemens Solid Edge ST3

As we can see from obtained images, the proposed method gives equivalent result regarding image quality, but as we can see the models differ, because of existing modelling restrictions in Siemens Solid Edge ST3 batch (we can not model by NURBS surface using its mathematical description). The model of the object transforms into polygonal model during visualisation stage. This restriction affects the visualization result and the size of the image is levelled out if the size in pixels is not big, for example 512x512 (as in experiment).

Image visualization times for experiment are following: 1, 89 seconds using proposed method; in Siemens Solid Edge ST3 batch image visualization time is not shown. The estimations with extra tools give the fastest result – 2,12 seconds that we can consider as a result. As we can see from given data the proposed method give small visualization time reduction in experiment (speed up is 14%).

3D modelling for medical object using local surface interpolation and its volume estimation approach approbation on medical object examples. Real object modelling method based on local surface interpolation, what, in this case, is based on Coons surface application was proposed in the doctoral thesis. Also an algorithm for achieved model's volume estimation was proposed. This algorithm is described in the 3.4.subchapter of the doctoral thesis. Evaluation by medical experts was conducted in order to evaluate the proposed modelling method value. To evaluate the proposed volume definition algorithm value a comparative analysis with existing algorithms (Trapezoidal method and Cavalieri method, and were described in work [94]) was conducted. All these algorithms were realized with the help of computer program. Programming environment Borland Delphi 7.0 was used as a programming tool. Point topological orthogonal data array in 3D space was used for the experiment. This data array is obtained from [85] gained results. Achieved image size in experiments is 512x512 pixels. Obtained images using visualization with the help of ray tracing was shown in Fig. 8.

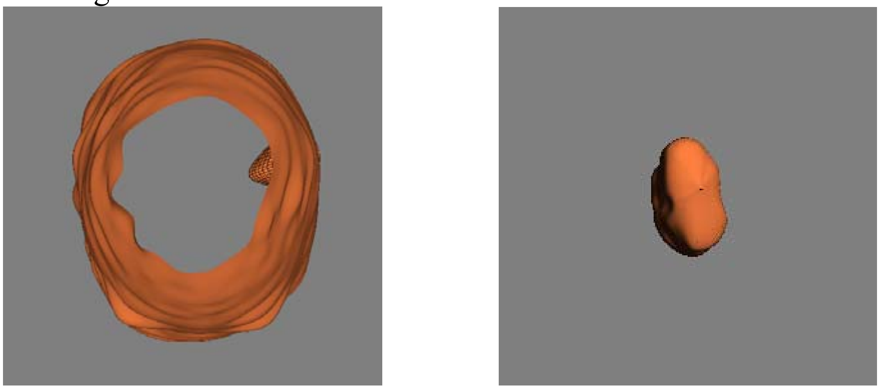


Figure 8. Object visualization using proposed method

After obtained model visualization was achieved positive experiment evaluation. Experimental check-up of the proposed method showed that used modelling method is very elastic and sensitive to small imprecision of input data. Object distortion involves input data deficits that involve medical image segmentation imprecision and are not connected with restrictions of the method.

The next task that was resolved in the frame of the doctoral thesis is 3D model volume estimation. Comparative analysis with existing algorithms (trapezoidal method and Cavalieri method, and these are reviewed in work [94]) was conducted in order to evaluate the proposed estimation algorithm value.

Table 6

3D model volume estimation results, conventional units and its fails in percents

Computation algorithm	**“Sphere model”**	**Skull inside surface fragment model**	**“Pathology zone model”**
Computation in analytic way	40.478.780,46 (Standard)	2.802.491,60 (Standard)	9.157,55 (Standard)
Graphical integration 10x10	40.475.553,47 ($7{,}97\times10^{-3}$%)	2.802.798,76 ($1{,}10\times10^{-2}$%)	9161,02 (0,0379%)
Graphical integration 100x100	40.478.746,86 ($8{,}30\times10^{-5}$%)	2.802.494,68 ($1{,}10\times10^{-4}$%)	9.157,58 ($3{,}79\times10^{-4}$%)
Trapezoidal and Cavaleri method	40.171.279,48 (0,760%)	2.825.806,26 (0,832%)	9.420,89 (2,88%)

Three 3D geometrical models were checked-up experimentally: the first model is model object – sphere model with foreknown volume value that is 40.478.780, 46 of conventional unit, the second and the third check-up model is the fragment of the inner surface of the skull model and pathology zone model that are the results of the modelling method experiment and they are shown in Fig. 8. In the experimental check-up were calculated model volumes using some following algorithms Model volume estimation results are shown in Table 6.

As we can see from Table 6 data the proposed algorithm in comparison to foreknown values, give absolutely accurate result in model object ("Sphere") example. Also Table 6 prove that the proposed volume estimation method ensures higher accuracy than foreknown methods (Trapezoidal and Cavaleri [94]) also using graphical integration algorithm. In case if in practical tasks the graphical integration ensures acceptable accuracy level then this algorithm can be used instead of analytical computing algorithm. Worse result in terms of accuracy of the existing methods (Trapezoidal and Cavaleri [94]) is connected with the fact that these methods do not take into account 3D model form between cuts.

Medical object 3D modelling using global surface interpolation and visualization. Real object modelling method based on global surface interpolation, what, in its turn, is based on B-spline surface application was proposed in this doctoral thesis. This method is described in the 3.5.subchapter of the doctoral thesis. Proposed modelling method was realized as computer program. Programming environment Borland Delphi 7.0 was used as programming tool at experimental stage.

Input data, that was used in experiment, is point topological orthogonal data array in the 3D space, that is obtained from real patient head brain medical image in graphical format and DICOM format, that are obtained with the help of computer tomography. This data array is obtained from work [85] achieved results. Modelling method result is a B-spline surface that describes one object surface. Object and/or scene that are developed with the help of B-spline data array can perform visualizing using computer graphics standard tool and high quality visualization method that are described in the 3.2.subchapter of the doctoral thesis. Obtained image size in experiments is 512x512 pixels. Obtained images are shown in Fig. 9.

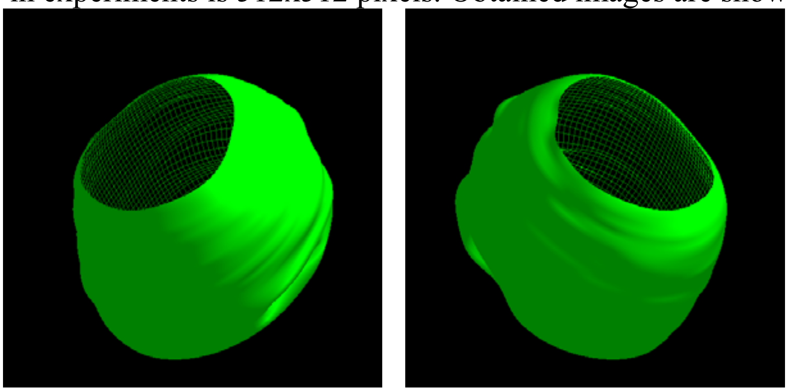


Figure 9. Objects "Fragment if the inner surface of the skull" model visualization.

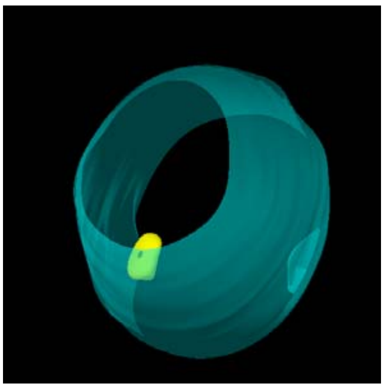


Figure 10. Medical object scene visualization using proposed visualization method.

Obtained visualization model gained positive rating from medical expert. During experimental check-up it was proven that used modelling method is not flexible enough and is not sensitive towards data inaccuracies. The use of the proposed method in medical object's modelling is restricted as the volume of calculations is too large and geometrical flexibility is not sufficient.

Obtained models also were used for proposed visualization method experimental check-up, and the results are shown in Fig. 10.

Medical object 3D modelling using curved line surface interpolation. Real object modelling method based on surface approximation, based on parametrical surface generation with parametrical curves was proposed in this doctoral thesis. This method is described in the 3.6.subchapter of the doctoral thesis. Proposed modelling method was realized with the help of computer program. Programming butch CodeGear RAD Studio 2009 was used as programming tool at experimental stage, programming environment Delphi 2009 was also used.

Input data, that was used in the experiment, is point topological not orthogonal data array in 3D space that is achieved with the help of computer tomography result processed data [54]. Modeling method work result is parametrical surface data array. One surface describes one object fragment between two cuts. Object and/or scene that are made with the help of parametrical surface data array and can perform visualizing using computer graphic standard tools, graphical library OpenGL. Obtained image size in experiments is 512x512 pixels. Obtained images were shown in Fig. 11.

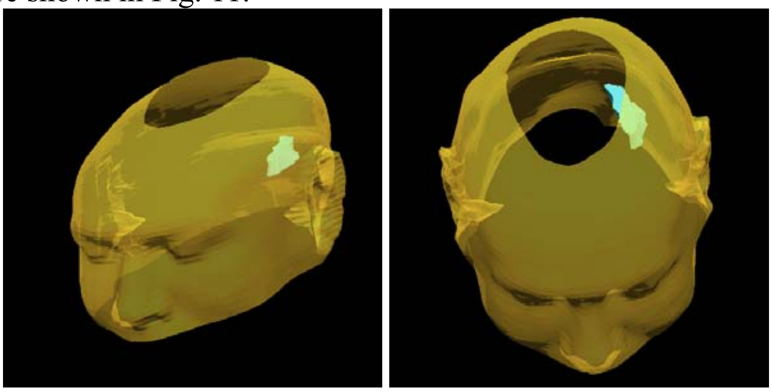


Figure 11. Medical object scene visualization

After obtained models visualization positive rating from medical expert was received. It was proved that the proposed method has sufficient flexibility and is sensitive towards input data inaccuracies. It is possible to use the proposed method for medical object modelling tasks as it ensures sufficient geometrical elasticity and obtained results correctly describe real medical object.

3. THE DOCTORAL THESIS MAIN RESULTS AND CONCLUSIONS

The aim of the doctoral thesis was to develop mathematical methods and algorithms for high quality visualization of parametrical free - form surface and real objects 3D geometrical modeling with possibility to compute the constructed model’s volume, practically realize it in software system and conduct experiment with the developed system. In order to achieve the aim of this work, the following tasks were solved:

- The choice of parametrical free - form surfaces visualization method was fortified, based on the research of the visualization methods. Visualization algorithms were researched in order to define their disadvantages and limitations.
- Based on prior task resolving results, demands were defined and a new algorithm for parametrical free - form surface visualization in ray tracing, was developed and, also experiments were conducted;

- Based on geometrical modeling method research results, existing surface modeling (interpolation) method usage possibilities in real object modeling tasks were evaluated;
- Experiments were conducted with real object 3D geometrical modeling methods on the examples of medical objects;
- Constructed 3D geometrical model volume estimation analytic method was developed and experiments were conducted.

Work theoretical acquisition and value

Main theoretical results achieved by resolving mentioned in the doctoral thesis problems:

- Based on existing algorithm analysis the parametrical free - form surface new visualization algorithm necessity is justified;
- Developed new methods for non rational parametrical free - form surface (Bezier and B-spline surface) visualization;
- The developed new method rational parametrical free - form surface (NURBS surface) visualization is a novel algorithm;
- Developed real object 3D geometrical modeling local surface interpolation method based on object interpolation with bicubic Coons surface set;
- Developed new 3D geometrical model volume estimation analytic method;
- Developed real object 3D geometrical modeling global surface interpolation method based on object interpolation with B-spline surface;
- Developed real object 3D geometrical modeling method based on surface development with parametrical curve based on geometrical curve approximation method;

Work practical value

The doctoral thesis also has practical value:

- Non rational (Bezier and B-spline) surface visualization algorithm effectiveness and preferential is shown compared to existing algorithms that are described in [66], [62], [112] and [26] in Bezier surface case, and [62] in B-spline surface case. Proposed algorithm ensures 3D scene visualization with good image quality and reduced image generation time;
- Rational (NURBS) surface visualization algorithm effectiveness is shown and it is compared with existing algorithm that is described in work [62]. Proposed algorithm ensures 3D scene visualization with good quality image and reduced image generation time;
- Proposed visualization methods were used in solving practical tasks. The comparison, using real object example (machine element), between the proposed method and existing CAD systems, Autodesk AutoCAD 2010 and Siemens Solid Edge ST3 proved the proposed method’s practical effectiveness.
- Developed real object 3D geometrical surface interpolation methods effectiveness is shown with experimental results.
- Developed 3D geometrical model for volume calculation analytic methods effectiveness is shown with experimental results, using medical objects as examples. Comparing analysis with existing volume measurement methods showed the proposed method’s preference from view of result accuracy;
- Developed real object 3D geometrical modeling effectiveness is shown based on surface development with curve approximation;
- Proposed modeling methods were used in to resolve practical tasks using medical object as examples. Positive evaluation was attained from medical experts. The positive evaluation proves the effectiveness of the proposed method.

Possible directions for future research

In the process of developing new 3D visualization methods, real object 3D modeling methods, as well as the process of conducting experiments, lead the author to believe that there is possibility of future development of the proposed methods. The main directions for future research are:

- Divided and parallel data processing principle realization in free - form surface visualization in the process of ray tracing computing;
- Use of visual effect in free - form surface visualization by ray tracing and its realization possibilities in ray tracing research;
- Speeding-up of data structure research and development in parametrical free - form surface visualization by ray tracing;
- Other visualization methods (at first radiation methods) combination possibility research with ray tracing with the aim to make larger 3D scene achieved image reality level;
- Real object 3D geometrical modeling methods in further development with the aim to perfect the quality of the constructed model.

4. APPROBATION OF THE WORK

The doctoral thesis main results are presented at 14 international conferences and reflected in 15 scientific papers.

4.1. Presentations on conferences

The doctoral thesis main results are presented at 14 international scientific conferences:

1. The 19th International Conference on Computer Graphics, Visualization and Computer Vision WSCG'2011, Plzen, Czech Republic, January 31 – February 3, 2011.
2. The 14th International Conference on Biomedical Engineering, Kaunas, Lithuania, October 28-29, 2010.
3. RTU 51. Starptautiskā zinātniskā konference. Rīga, 11.-15. oktobris, 2010.
4. The 9th International Conference on Global Research and Education Inter-Academia 2010, Riga, Latvia, August 9-12, 2010.
5. The 13th International Conference on Biomedical Engineering, Kaunas, Lithuania, October 29-30, 2009.
6. RTU 50. Starptautiskā zinātniskā konference. Rīga, 12.-16. oktobris, 2009.
7. The 17th International Conference on Computer Graphics, Visualization and Computer Vision WSCG'2009, Plzen, Czech Republic, February 2-5, 2009.
8. The 12th International Conference on Biomedical Engineering, Kaunas, Lithuania, October 23-24, 2008.
9. RTU 49. Starptautiskā zinātniskā konference. Rīga, 13.-15. oktobris, 2008.
10. The 4th International Conference on Computer Science and Information Systems, Athens, Greece, July 21-24, 2008.
11. The 18th International conference on Computer Graphics and Vision GraphiCon'2008, Moscow, Russia, June 23-27, 2008.
12. The 14th IEEE Mediterranean Electrotechnical Conference MELECON'2008, Ajaccio, France, May 5-7, 2008.
13. RTU 48. Starptautiskā zinātniskā konference. Rīga, 11.-13. oktobris, 2007.
14. RTU 47. Starptautiskā zinātniskā konference. Rīga, 12.-14. oktobris, 2006.

4.2. Papers

Researches main results are reflected in 15 scientific papers, including a book ”Recent Advances in Computing and Management Information Systems” chapter, one paper in international scientific journal ”International Journal of Advanced Materials Research” and 13 papers in accepted of Latvian Council of Science sources:

1. Sisojevs A., Glazs A. An Efficient Approach to Direct NURBS Surface Rendering for Ray Tracing// The 19th International Conference on Computer Graphics, Visualization and Computer Vision WSCG'2011 proceedings. – Plzen: University of West Bohemia, 2011. – pp. 9 – 12. (Database: Thompson Reuters ISI)
2. Krechetova K., Sisojevs A., Glazs A., Platkajis A. Medical Image Region Extraction and 3D Modeling Based on Approximating Curves// International Journal of Advanced Materials Research. – Trans Tech Publications: Switzerland, 2011. – Vol. 222, – pp. 285 – 288. (Database: Scientific.Net)
3. Sisojevs A., Glazs A. Racionālo parametrisko virsmu vizualizēšanas pieeja izmantojot staru trasēšanu// Rīgas Tehniskās universitātes zinātniskie raksti, Datorzinātne, sērija 5., sējums 42. – Rīga: RTU, 2010. – 13. – 18. lpp. (Database: EBSCOhost)
4. Sisojevs A., Glazs A. Volume estimation of 3D object modelled by parametrical iso-surfaces// Biomedical Engineering Proceedings of International Conference. – Kaunas: Kaunas University of Technology, 2010. – 220 – 223 p.
5. Sisojevs A., Glazs A. 3D objektu vizualizēšana izmantojot B-spline funkcijas un staru trasēšanu// Rīgas Tehniskās universitātes zinātniskie raksti, Datorzinātne, sērija 5., sējums 39. – Rīga: RTU, 2009. – 15. – 22. lpp. (Database: EBSCOhost)
6. Sisojevs A., Glazs A. Medical object 3D modelling using B-spline surface// Biomedical Engineering Proceedings of International Conference. – Kaunas: Kaunas University of Technology, 2009. – 168 – 171 p.
7. Sisojevs A., Glazs A. A New Approach for High-quality Visualization of Bézier Surfaces// Recent Advances in Computing and Management Information Systems. – Athens: Athens Institute for Education and Research, 2009. – 19 – 25 p.
8. Sisojevs A., Krechetova K., Glazs A. 3D Modeling of Free-Form Object (Interpolation, Visualization and Volume Estimation)// The 17th International Conference on Computer Graphics, Visualization and Computer Vision WSCG'2009 proceedings. – Plzen: University of West Bohemia, 2009. - 125 - 128 p. (Database: Thompson Reuters ISI)
9. Sisojevs A., Glazs A. Efficient approach to direct B–spline surface rendering by a ray tracing// The 17th International Conference on Computer Graphics, Visualization and Computer Vision WSCG'2009 proceedings. – Plzen: University of West Bohemia, 2009. - 13 - 16 p. (Database: Thompson Reuters ISI)
10. Sisojevs A., Glazs A. Brīvo formu virsmu interpolācija un to vizualizācija ar datorgrafikas standartlīdzekļiem// Rīgas Tehniskās universitātes zinātniskie raksti, Datorzinātne, sērija 5., sējums 35. – Rīga: RTU, 2008. – 10. – 17. lpp. (Database: EBSCOhost)
11. Sisojevs A., Glazs A. Efficient approach of medical objects 3D visualization// Biomedical Engineering Proceedings of International Conference. – Kaunas: Kaunas University of Technology, 2008. – 222 – 224 p.
12. Sisojevs A., Glazs A. A new approach to Bézier surface visualization by a ray tracing method// Conference Proceeding 18th International Conference on Computer Graphics & Vision GraphiCon 2008. – Moscow: Moscow State University, 2008. – 304 p.
13. Sisojevs A., Glazs A. An new approach of visualization of free – form surfaces by a ray tracing method// The 14th IEEE Mediterranean Electrotechnical Conference Proceedings. – Ajaccio: IEEE, 2008. – 872 – 875 p. (Databases: IEEE Xplore, Scopus)

14. Sisojevs A., Glazs A. Visualization of Free – form Surfaces by a Ray Tracing Method// Scientific Proceedings of Riga Technical University, Computer Science, part 5, volume 32. – Riga: RTU, 2007. – 7 – 13 p.
15. Sisojevs A. Staru trasēšana un bilineāru virsmu vizualizācija// Rīgas Tehniskās universitātes zinātniskie raksti, Datorzinātne, sērija 5., sējums 27. – Rīga: RTU, 2006. – 35. – 41. lpp.

5. BIBLIGRAPHY

[1] Adams B. et al. Efficient Raytracing of Deforming Point-Sampled Surfaces// Proceedings of the European Association for Computer Graphics 26th Annual Conference Eurographics 2005. – Dublin, Ireland: Eurographics, 2005. – pp. 677 - 684.

[2] Albert O., Geimer M., Müller S. Direct and Fast Ray Tracing of NURBS Surfaces// Proceedings of the 2006 IEEE Symposium on Interactive Ray Tracing. – Salt Lake City, USA: IEEE Computer Society, 2006. – pp. 161 – 168.

[3] Anderson C.W., Crawford-Hines S. Fast Generation of NURBS Surfaces from Polygonal Mesh Models of Human Anatomy// Technical Report CS-99-101, Department of Computer Science, Colorado State University, Fort Collins, CO, 1999.

[4] Balázs Á., Guthe M., Klein R. Efficient trimmed NURBS tessellation// Journal of WSCG – 2004. – Vol. 12, No. 1-3. – pp. 27 – 33.

[5] Barth W., Stürzlinger W. Efficient ray tracing for Bezier and B-spline surfaces// Computers&Graphics – 1993. – Nr. 17(4). – pp. 423 – 430.

[6] Benthin C. Realtime Ray Tracing on current CPU Architectures// Doctor's thesis, Computer Graphics Group, Saarland University, Saarbrücken, Germany. – 2006. – 210 p.

[7] Benthin C., Wald I., Slusallek P. Interactive Ray Tracing of Free-Form Surfaces// Proceedings of 3rd international conference Afrigraph'04. – Stellenbosch, South Africa: ACM Press, 2004. – 99 – 106 p.

[8] Benthin C. et al. Packet-based Ray Tracing of Catmull-Clark Subdivision Surfaces// SCI Institute, University of Utah, Technical Report #UUSCI-2007-011, 2007.

[9] Bigler J., Stephens A., Parker S.G. Design for Parallel Interactive Ray Tracing Systems// Proceedings of the IEEE Symposium on Interactive Ray Tracing. – Salt Lake City, USA: IEEE Computer Society, 2006. – pp. 187 – 196.

[10] Boulos S. et al. Interactive Distribution Ray Tracing// Technical Report, SCI Institute, University of Utah, No UUSCI-2006-022, 2006.

[11] Campagna S., Slusallek P. Improving Bézier clipping and Chebyshev boxing for Ray Tracing Parametric Surfaces// Proceedings of international conference "3D Image Analysis and Synthesis '96". – Erlangen, Germany: Erlangen University, 1996. – pp. 95 – 102.

[12] Campagna S., Slusallek P., Seidel H.-P. Ray tracing of spline surfaces, Bézier clipping, Chebyshev boxing, and bounding volume hierarchy – a critical comparison with new results// The Visual Computer. – 1997. – Vol.13, – pp. 265-282.

[13] Catmull E. E. A subdivision algorithm for computer display of curved surfaces// Doctor's thesis, Department of Computer Science, University of Utah, Salt Lake City, US. – 1974. – 83 p.

[14] Computer graphics: principles and practice (2nd ed.)./ J.D. Foley et al. – Boston, USA: Addison-Wesley Longman Publishing Co., 1990. – 1174 p.

[15] Coons S.A. Surfaces for Computer-Aided Design of Space Forms// Technical Report: TR-41, Massachusetts Institute of Technology, Cambridge, MA, 1967. – 105 p.

[16] Cox M.G. The numerical evaluation of B-splines// IMA Journal of Applied Mathematics. – 1972. Volume 10, – pp. 134–149.

[17] de Boor C. On calculating with B-splines// Journal Approximation Theory – 1972. Volume 6, – pp. 50 – 62.

[18] Dietrich A. et al. Realistic and Interactive Visualization of High-Density Plant Ecosystems// Proceedings of Eurographics Workshop on Natural Phenomena. – Dublin, Ireland: Eurographics, 2005. – pp. 73 – 81.
[19] Dietrich A. et al. The OpenRT Application Programming Interface – Towards A Common API for Interactive Ray Tracing// Proceedings of the 2003 OpenSG Symposium. – Darmstadt, Germany: Eurographics, 2003. – pp. 23-31.
[20] Dietrich A., Marmitt G., Slusallek P. Terrain Guided Multi-Level Instancing of Highly Complex Plant Populations// Proceedings of the IEEE Symposium on Interactive Ray Tracing. – Salt Lake City, USA: IEEE Computer Society, 2006. pp. 169 – 176.
[21] Dietrich A., Wald I., Slusallek P. Large-Scale CAD Model Visualization on a Scalable Shared-Memory Architecture// Proceedings of Vision, Modeling, and Visualization. – Erlangen, Germany: Erlangen University, 2005. – pp. 303 – 310.
[22] Djeu P. et al. Razor: An Architecture for Dynamic Multiresolution Ray Tracing// Technical Report TR-07-52, Department of Computer Science, The University of Texas at Austin, 2007.
[23] Dmitriev K.A. Efficiency issues on Ray Tracing Machine// Conference Proceeding 10th International Conference on Computer Graphics & Vision GraphiCon 2000. – Moscow: Moscow State University, 2000. – pp.99 – 103.
[24] Dmitriev K. et al. Interactive Global Illumination Using Selective Photon Tracing// Proceedings of the 13th Eurographics Workshop on Rendering. – Pisa, Italy: Eurographics, 2002. – pp. 25 – 36.
[25] Duckstein F., Kolla R. Ray tracing of parametric surfaces based on adaptive simplicial complexes// Technical report. – Würzburg: The University of Würzburg, 1998.
[26] Efremov A. Havran V. Seidel H.-P. Robust and Numerically Stable Bezier Clipping Method for Ray Tracing NURBS Surfaces// In Proceedings of the 21st Spring Conference on Computer Graphics SCCG’05 - Budmerice, Slovakia: Comenius University, 2005. – 127 – 135 p.
[27] Efremov A. Efficient Ray Tracing of Trimmed NURBS Surfaces// Master's thesis, Computer Graphics Group, Max-Planck-Institut für Informatik, Saarbrücken, Germany. – 2005. – 162 p.
[28] Fisher J., Lowther J., Shene C.-K. Curve and surface interpolation and approximation: knowledge unit and software tool// Proceedings of the 9th annual SIGCSE conference on Innovation and technology in computer science education. – Leeds, UK: SIGCSE, 2004. – pp. 146 – 150.
[29] Flórez J. et al. Improved Adaptive Antialiasing for Ray Tracing Implicit Surfaces
[30] Formella A., Gill C. Ray Tracing: A Quantitative Analysis and a New Practical Algorithm// The Visual Computer. – 1995. – Vol.11, – pp. 465 – 476.
[31] Formella A., Müller K. Bended Surfaces for Ray Tracing: A Fast View-Dependent Algorithm// . In H.P. Seidel, B. Girod, H. Niemann (Eds.), Proceedings of 3D Image Analysis and Synthesis '97. infix, Sankt Augustin, ISBN 3-89601-007-7, pp. 19-26, November 1997.
[32] Geimer M., Albert O. Interactive ray tracing of trimmed bicubic Bézier surfaces without triangulation// The 13th International Conference on Computer Graphics, Visualization and Computer Vision WSCG'2005 Full Papers Conference Proceedings. – Plzen: University of West Bohemia, 2005. – 71 – 78 p.
[33] Goral C.M. et al. Modeling the Interaction of Light between Diffuse Surfaces. Computer Graphics //Journal Computer Graphics. – 1984. - Vol. 18, Num. 3, - pp. 213 - 222.
[34] Guthe M., Balázs Á., Klein R. GPU-based trimming and tessellation of NURBS and T-Spline surfaces// Proceedings of ACM SIGGRAPH 2005. – New York: ACM, 2005. – PP. 1016 – 1023.
[35] Guthe M., Meseth J., Klein R. Fast and memory efficient view-dependent trimmed NURBS rendering// Proceedings of the 10th Pacific Conference on Computer Graphics and Applications. – Washington, USA: IEEE, 2002, pp. 204 – 213.

[36] Günther J. Realtime Caustics using Distributed Photon Mapping// Diploma work, Computer Graphics Group, Saarland University, Saarbrücken, Germany. – 2004. – 58 p.
[37] Günther J. et al., Efficient Acquisition and Realistic Rendering of Car Paint// Proceedings of Vision, Modelling, and Visualization (VMV) 2005. – Erlangen, Germany: VMV, 2005, pp. 487 - 494.
[38] Günther J. et al. Ray Tracing Animated Scenes using Motion Decomposition// Proceedings of the European Association for Computer Graphics 27th Annual Conference Eurographics 2006. – Vienna, Austria: Eurographics, 2006. – pp. 517 – 525.
[39] Günther J. Wald I., Seidel H.-P. Precomputed Light Sets for Fast High Quality Global Illumination// Proceedings of ACM SIGGRAPH 2005. – New York: ACM, 2005.
[40] Günther J. Wald I., Slusallek P. Realtime Caustics Using Distributed Photon Mapping// Proceeding of the 15th Eurographics Symposium on Rendering. – Norrköping, Sweden: Eurographics, 2004. – pp. 111 – 121.
[41] Havran V. Heuristic Ray Shooting Algorithms// Doctor's thesis, Department of Computer Science and Engineering, Faculty of Electrical Engineering, Czech Technical University, Prague. – 2000. – 220 p.
[42] Hearn D.D., Baker M.P. Computer Graphics with OpenGL, 3rd Ed., Prentice Hall, 2004. – 880 p.
[43] Hill F.S. Computer Graphics Using Open GL (2nd Edition). – New Jersey.: Prentice Hall PT, 2000. – 922 p.
[44] Himmelblau D.M. Applied Nonlinear Programming, – McGraw-Hill Book Company, 1972. – 416 p.
[45] Hurley J. Ray Tracing Goes Mainstream// Intel Technology Journal. – 2005. – Volume 9, Issue 2. – pp. 99 – 108.
[46] Immel D.S., Cohen M.F., Greenberg D.P. A radiosity method for non-diffuse environments// Journal ACM SIGGRAPH Computer Graphics. – 1986. – Volume 20, Issue 4. pp. 133 -142.
[47] Ize T. et al. An Evaluation of Parallel Grid Construction for Ray Tracing Dynamic Scenes// Proceedings of the IEEE Symposium on Interactive Ray Tracing 2006. – Salt Lake City, USA: IEEE, 2006. – pp.47 – 55.
[48] Joy K.I., Bhetanabhotla M.N. Ray tracing parametric surface patches utilizing numerical techniques and ray coherence// Journal ACM SIGGRAPH Computer Graphics. – 1986. - Volume 20, Issue 4. - pp. 279-285.
[49] Kajiya J.T. Ray Tracing Parametric Patches// Proceedings of the 9th annual conference on Computer graphics and interactive techniques. – Boston: SIGRAPH, 1982. – pp. 245 – 254.
[50] Kajiya J.T. The Rendering Equation// Journal ACM SIGGRAPH Computer Graphics. – 1986. - Volume 20, Issue 4. - pp. 143-150.
[51] Kobbelt L., Daubert K., Seidel H.-P. Ray Tracing of Subdivision Surfaces// Proceedings of the 9th Eurographics Workshop on Rendering. – Vienna, Austia: 1998. – pp. 69 – 80.
[52] Krechetova K., Sisojevs A., Glazs A., Platkajis A. Medical Image Region Extraction and 3D Modeling Based on Approximating Curves// International Journal of Advanced Materials Research. – Trans Tech Publications: Switzerland, 2011. – Vol. 222, – pp. 285 – 288.
[53] Krechetova K., Glaz A. Development of a new segmentation method for medical images// The 11th International conference on Biomedical Engineering proceeding. – Kaunas: Kaunas University of Technology, 2007. – 133 – 136 p.
[54] Krechetova K., Glaz A. Pathology zone volume estimation in 3D medical images of a brain// The 12th International conference on Biomedical Engineering proceeding. – Kaunas: Kaunas University of Technology, 2008. – 225 – 228 p.

[55] Krechetova K., Glaz A. Contour extraction and processing in CT images// The 13th International conference on Biomedical Engineering proceeding. – Kaunas: Kaunas University of Technology, 2009. – 172 – 175 p.

[56] Krečetova K., Glazs A. Patoloģijas zonu tilpuma noteikšana 3D medicīnas attēlos// Rīgas Tehniskās universitātes zinātniskie raksti, Datorzinātne, sērija 5., sējums 35. – Rīga: RTU, 2008. – 18. – 26. lpp.

[57] LaMothe A. Tricks of the 3D Game Programming Gurus–Advanced 3D Graphics and Rasterization – Indianapolis, USA: Sams Publishing, 2003. – 1728 p.

[58] Lischinski D., Gonczarowski J. Improved Techniques for Ray Tracing Parametric Surfaces// The Visual Computer. – 1990. – Vol.6, Issue3 – pp. 134 – 152.

[59] Lischinski D., Tampieri F., Greenberg D.P. A Discontinuity Meshing Algorithm for Accurate Radiosity// IEEE Computer Graphics and Applications. – 1992. – Vol. 12, Num. 6. – pp. 25–39.

[60] Lohse M. Realtime Network-Integrated Multimedia Middleware, Services, and Applications// Doctor's thesis, Computer Graphics Group, Saarland University, Saarbrücken, Germany. – 2005. – 390 p.

[61] Marmitt G. et al. Fast and Accurate Ray-Voxel Intersection Techniques for Iso-Surface Ray Tracing// Proceedings of Vision, Modelling, and Visualization (VMV) 2004. – Stanford, USA: VMV, 2004, – pp. 429 – 435.

[62] Martin W. et al. Practical Ray Tracing of Trimmed NURBS Surfaces// Journal of Graphics Tools – 2000. – Volume 5, Issue 1. – pp. 27 – 52.

[63] Muuss M.J. Towards Real-Time Ray-Tracing of Combinatorial Solid Geometric Models// In Proceedings of BRL-CAD Symposium. – Aberdeen Proving Ground, MD, 1995.

[64] Müller K., Techmann T., Fellner D.W. Adaptive Ray Tracing of Subdivision Surfaces// Proceedings of the European Association for Computer Graphics 24nd Annual Conference Eurographics 2003. – Granada, Spain: Eurographics, 2003. – pp. 535 – 562.

[65] Nakamaru K., Ohno Y. Ray tracing for curves primitive// Journal of WSCG – 2002. – Vol. 10, No. 2. – pp. 311-316.

[66] Nishita T., Sederberg T.W., Kakimoto M. Ray tracing trimmed rational surface patches// Proceedings of the 17th annual conference on Computer graphics and interactive techniques. – Dallas: ACM Press, 1990. – 337 – 345 p.

[67] Pacheco X. Delphi for .NET Developer's Guide. – Indianapolis, USA: Sams Publishing, 2004. – 888 p.

[68] Phong B.–T. Illumination for computer generated pictures// Journal Communications of the ACM – 1975. – Volume 18, Number 6 – pp. 311–317.

[69] Piegl L., Tiller W. The NURBS book. (Monographs in Visual Communication).- Springer-Verlag, 1997. – 646 p.

[70] Pomi A. Realtime Interactive Mixed Reality Rendering in a Distributed Ray Tracing Framework// Doctor's thesis, Computer Graphics Group, Saarland University, Saarbrücken, Germany. – 2005. – 194 p.

[71] Purcell T.J. et al. Ray Tracing on Programmable Graphics Hardware// Proceedings of ACM SIGGRAPH 2002. – New York: ACM, 2002. – pp. 703 – 712.

[72] Reis G. et al. High-Quality Rendering of Quartic Spline Surfaces on the GPU// IEEE Transactions on Visualization and Computer Graphics – 2008. – Nr. 14(5). – pp. 1126 – 1139.

[73] Rogers D.F., Adams J.A. Mathematical Elements for Computer Graphic, 2nd Ed. – Boston: McGraw-Hill, 1990. – 611 p.

[74] Rogers D.F. Procedural Elements for Computer Graphics, 2nd Ed. – Boston: McGraw-Hill Book Company, 1997. – 752 p.

[75] Rubin S.M., Whitted T. A 3-Dimensional Representation for Fast Rendering of Complex Scenes// Journal ACM SIGGRAPH Computer Graphics. – 1980 – Volume 14, Issue 3. – pp. 110–116.

[76] Schmittler J. SaarCOR A Hardware-Architecture for Realtime Ray Tracing// Doctor's thesis, Computer Graphics Group, Saarland University, Saarbrücken, Germany. – 2006. – 202 p.
[77] Schollmeyer A., Froehlich B. Direct Trimming of NURBS Surfaces on the GPU// Journal ACM Transactions on Graphics (TOG) – 2009 – Volume 28, Issue 3, Article No. 47.
[78] Siliņa B. Rokasgrāmata matemātikā. – Rīga. Zvaigzne ABC, 2006. – 367. lpp.
[79] Sisojevs A., Glazs A. An Efficient Approach to Direct NURBS Surface Rendering for Ray Tracing// The 19th International Conference on Computer Graphics, Visualization and Computer Vision WSCG'2011 proceedings. – Plzen: University of West Bohemia, 2011. – pp. 9 – 12.
[80] Sisojevs A., Glazs A. Racionālo parametrisko virsmu vizualizēšanas pieeja izmantojot staru trasēšanu// Rīgas Tehniskās universitātes zinātniskie raksti, Datorzinātne, sērija 5., sējums 42. – Rīga: RTU, 2010. – 13. – 18. lpp.
[81] Sisojevs A., Glazs A. Volume estimation of 3D object modelled by parametrical iso-surfaces// Biomedical Engineering Proceedings of 14th International Conference. – Kaunas: Kaunas University of Technology, 2010. – 220 – 223 p.
[82] Sisojevs A., Glazs A. 3D objektu vizualizēšana izmantojot B-spline funkcijas un staru trasēšanu// Rīgas Tehniskās universitātes zinātniskie raksti, Datorzinātne, sērija 5., sējums 39. – Rīga: RTU, 2009. – 15. – 22. lpp.
[83] Sisojevs A., Glazs A. Medical object 3D modelling using B-spline surface// The 13th International conference on Biomedical Engineering proceeding. – Kaunas: Kaunas University of Technology, 2009. – 168 – 171 p.
[84] Sisojevs A., Glazs A. A New Approach for High-quality Visualization of Bézier Surfaces// Recent Advances in Computing and Management Information Systems. – Athens: Athens Institute for Education and Research, 2009. – 19 – 25 p.
[85] Sisojevs A., Krechetova K., Glazs A. 3D Modeling of Free-Form Object (Interpolation, Visualization and Volume Estimation)// The 17th International Conference on Computer Graphics, Visualization and Computer Vision WSCG'2009 Communication Papers proceedings. – Plzen: University of West Bohemia, 2009. - 125 - 128 p.
[86] Sisojevs A., Glazs A. Efficient approach to direct B–spline surface rendering by a ray tracing// The 17th International Conference on Computer Graphics, Visualization and Computer Vision WSCG'2009 proceedings. – Plzen: University of West Bohemia, 2009. - 13 - 16 p.
[87] Sisojevs A., Glazs A. Brīvo formu virsmu interpolācija un to vizualizācija ar datorgrafikas standartlīdzekļiem// Rīgas Tehniskās universitātes zinātniskie raksti, Datorzinātne, sērija 5., sējums 35. – Rīga: RTU, 2008. – 10. – 17. lpp.
[88] Sisojevs A., Glazs A. Efficient approach of medical objects 3D visualization// The 12th International conference on Biomedical Engineering proceeding. – Kaunas: Kaunas University of Technology, 2008. – 222 – 224 p.
[89] Sisojevs A., Glazs A. A new approach to Bézier surface visualization by a ray tracing method// Conference Proceeding 18th International Conference on Computer Graphics & Vision GraphiCon 2008. – Moscow: Moscow State University, 2008. – 304 p.
[90] Sisojevs A., Glazs A. An new approach of visualization of free – form surfaces by a ray tracing method// The 14th IEEE Mediterranean Electrotechnical Conference Proceedings. – Ajaccio: IEEE, 2008. – 872 – 875 p.
[91] Sisojevs A., Glazs A. Visualization of Free – form Surfaces by a Ray Tracing Method// Scientific Proceedings of Riga Technical University, Computer Science, part 5, volume 32. – Riga: RTU, 2007. – 7 – 13 p.

[92] Sisojevs A. Staru trasēšana un bilineāru virsmu vizualizācija// Rīgas Tehniskās universitātes zinātniskie raksti, Datorzinātne, sērija 5., sējums 27. – Rīga: RTU, 2006. – 35. – 41. lpp.

[93] Slusallek P. Vision An Architecture for Physically-Based Rendering// Doctor's thesis, Der Technischen Fakultät der Universität Erlangen-Nürnberg, Erlangen, Germany. – 1995. – 288 p.

[94] Smitha S., Revathy K., Kesavadas C. Segmentation and Volume Estimation of Brain tissues from MR Images// The International MultiConference of Engineers and Computer Scientists Proceeding. – Hong Kong: International Association of Engineers, 2006. – p. 543 – 547.

[95] Stephens A. et al. An Application of Scalable Massive Model Interaction using Shared-Memory Systems// Proceeding of the Eurographics Symposium on Parallel Graphics and Visualization. – Braga, Portugal: Eurographics, 2006. – pp. 19 – 26.

[96] Strothotte T., Schlechtweg S. Non-Photorealistic Computer Graphics: Modeling, Rendering, and Animation. – Morgan Kaufmann Publisher, 2002. – 496 p.

[97] Stürzlinger W. Ray Tracing Triangular Trimmed Free - form Surfaces// IEEE Transactions on Visualization and Computer Graphics. – 1998. – Volume 4, Issue 3. – pp. 202 – 214.

[98] Šteiners K. Augstākā matemātika VI// Rīga: Zvaigzne ABC, 2001. – 208.lpp.

[99] Taha H.A. Operations Research: an Introduction, 7th Ed. – New Jersey: Prentice Hall, 2003. – 848 p.

[100] Toth D.L. On ray tracing parametric surfaces// Journal ACM SIGGRAPH Computer Graphics. – 1985. – Volume 19, Issue 3. – pp. 171 – 179.

[101] Váša L. Methods for Size Reduction of Dynamic Meshes// Doctor's thesis, Department of Computer Science and Engineering, Faculty of Applied Sciences, University of West Bohemia, Pilsen, Czech Republic. – 2008. – 174 p.

[102] Wald I. et al. Applying Ray Tracing for Virtual Reality and Industrial Design // Proceedings of the IEEE Symposium on Interactive Ray Tracing 2006. – Salt Lake City, USA: IEEE, 2006. – pp.177 – 185.

[103] Wald I. et al. Faster Isosurface Ray Tracing using Implicit KD-Trees// IEEE Transactions on Visualization and Computer Graphics – 2005. – Nr. 11(5). – 562 – 572 p.

[104] Wald I. et al. Interactive Global Illumination using Fast Ray Tracing// Proceedings of the 13th Eurographics Workshop on Rendering. – Pisa, Italy: Eurographics, 2002. – pp. 15 – 24.

[105] Wald I. et al. Interactive Ray Tracing on Commodity PC clusters State of the Art and Practical Applications//Lecture Notes in Computer Science – 2003. – Volume 2790. – pp. 499 –508.

[106] Wald I. et al. Interactive Rendering with Coherent Ray Tracing// Proceedings of the European Association for Computer Graphics 22nd Annual Conference Eurographics 2001. – Manchster, UK: Eurographics, 2001. – pp. 153 – 164.

[107] Wald I., Benthin C., Slusallek P. Interactive Global Illumination in Complex and Highly Occluded Environments// Proceedings of the 14th Eurographics Workshop on Rendering. – Leuven, Belgium: EuroGraphics, 2003. – pp. 74 – 81.

[108] Wald I., Günther J., Slusallek P. Balancing Considered Harmful – Faster Photon Mapping using the Voxel Volume Heuristic// The European Association for Computer Graphics 25th Annual Conference EuroGraphics 2004. – Oxford, UK: EuroGraphics, 2004. – pp.595 – 603.

[109] Wald I., Slusallek P., Benthin C. Interactive Distributed Ray Tracing of Highly Complex Models// Proceedings of the 12th Eurographics Workshop on Rendering. – London, UK: EuroGraphics, 2001. – pp. 274 – 285.

[110] Wald I., Slusallek P. State of the Art in Interactive Ray Tracing// In State of the Art Reports, The European Association for Computer Graphics 22nd Annual Conference EuroGraphics 2001. – Manchster, UK: EuroGraphics, 2001. – pp.21 – 42.
[111] Wald I. Realtime Ray Tracing and Interactive Global Illumination// Doctor's thesis, Computer Graphics Group, Saarland University, Saarbrücken, Germany. – 2004. – 311 p.
[112] Wang S.-W., Shih Z.-C., Chang R.-C. An Efficient and Stable Ray Tracing Algorithm for Parametric Surfaces// Journal of Information Science and Engineering – 2001. – Nr.18. – pp. 541 – 561.
[113] Whitted T. An Improved Illumination Model for Shaded Display// Communications of the ACM. – 1980. Volume 23, Issue 6, - pp. 343 – 349.
[114] Woodward C. Ray tracing parametric surfaces by subdivision in viewing plane// Theory and practice of geometric modeling, Springer-Verlag New York, Inc., New York, NY, USA. – 1989. pp. 273 – 287.
[115] Woop S., Schmittler J., Slusallek P. RPU: A Programmable Ray Processing Unit for Realtime Ray Tracing// ACM Transactions on Graphics (TOG). – 2005. Volume 24, Issue 3, – pp. 434 – 444.
[116] Wright-jr. R.S., Lipchak B. OpenGL SuperBible, 3rd Edition. – Indianapolis, USA: Sams Publishing, 2004. – 1200 p.
[117] Yang C.-G. On speeding up ray tracing of B-spline surfaces// Journal Computer Aided Design – 1987 – Volume 19, Issue 3, pp. 122-130.
[118] Yuan F. Windows Graphics Programming: Win32 GDI and DirectDraw. – New Jersey.: Prentice Hall PT, 2000. – 1280 p.
[119] Архангельский А.Я. Приемы программирования в Delphi на основе VCL. – М.: Бином, 2006. – 944 с.
[120] Боресков А.В. Разработка и отладка шейдеров. – СПб.: БХВ – Петербург, 2006. – 496 с.
[121] Гофман В. Э., Хомоненко А.Д. Delphi 5 – СПб.: БХВ – Петербург, 2001. – 800 с.
[122] Дарахвелидзе П. Г., Марков Е. П., Котенок О. А. Программирование в Delphi 5. – СПб.: БХВ – Петербург, 2001. – 784 с.
[123] Дякин В.Н., Матвейкин В.Г., Дмитриевский Б.С. Оптимизация управления промышленным предприятием. – Тамбов: Изд-во ТГТУ - Тамбов, 2004. – 84 с.
[124] Евченко А.И. OpenGL и DirectX: программирование графики. Для профессионалов. – СПб.: Питер, 2006. – 350 с.
[125] Краснов М.В. DirectX. Графика в проектах Delphi. – СПб.: БХВ – Петербург, 2005. – 416 с.
[126] Краснов М.В. OpenGL. Графика в проектах Delphi. – СПб.: БХВ – Петербург, 2002. – 352 с.
[127] Поляков А.Ю., Брусенцев В.А. Программирование графики: GDI+ и DirectX. – СПб.: БХВ – Петербург, 2005. – 368 с.
[128] Шикин Е.В., Боресков А.В. Компьютерная графика. Динамика, реалистические изображения. – М.:ДИАЛОГ – МИФИ, 1997. – 288 с.
[129] Шикин Е.В., Боресков А.В. Компьютерная графика. Полигональные модели. – М.:ДИАЛОГ – МИФИ, 2000. – 464 с.