DETERMINATION OF ELASTIC PROPERTIES OF STIFFENED COMPOSITE SHELLS BY VIBRATION ANALYSIS

RIBOTAS KOMPOZĪTMATERIĀLA ČAULAS ELASTĪGO ĪPAŠIBU NOTEIKŠANA IZMANTOJOT SVĀRSTĪBU ANALĪZI

S. Ručevskis

keywords: laminates, non-destructive testing, natural frequency, composites

Introduction

Composite laminates are being extensively used in aerospace industry, especially for the fabrication of high-performance structures. Determination of stiffness parameters for complex materials, such as fibre-reinforced composites, is much more complicated than for isotropic materials. A conventional way is testing the coupon specimens, which are manufactured by the technology similar to that used for the real large structures. In employing such a method, a question arises of whether the material properties obtained from the coupon tests are the same as those in the large structure. Therefore, the determination of realized material properties for composite laminates using non-destructive evaluation techniques has been widely investigated.

A number of various non-destructive evaluation techniques have been proposed for determining the material properties of composite laminates [1-3]. In the present study, attention is focused on the identification of elastic properties of laminated stiffened panels using the vibration test data. The modal vibration testing is a rapid and inexpensive method for obtaining data for the identification of elastic properties [4]. There is a great deal of information in the literature on the identification of elastic constants of laminated plates employing the vibration test data [5-13]. The problem associated with the vibration testing is converting the measured modal frequencies to elastic constants. A standard method for solving this problem is the use of a numerical-experimental model and optimization techniques [5-6, 9-12]. The identification functional represents the gap between the numerical model response and the experimental one. This gap should be minimized taking into account the side constraints on the design variables (elastic constants). The minimization problem is solved by using the non-linear mathematical programming techniques and sensitivity analysis [6, 9-12]. Similar identification functional has been employed in [14, 15], but the minimization method was different. Instead of the direct minimization of the functional, the experiment design and response surface approach are employed for approximation of the numerical (finite element) model. Such an approach can reduce the computational efforts significantly.

In order to reduce the computational efforts, methods based on the approximation concepts were used in the structural optimization for the first time [16]. The development of approximation functions has become a separate problem in the optimum structural design [17]. Approximating models can be built in different ways. The empirical model building theory is discussed in [18]. To construct a more general model of the original function, the methods of experiment design [19, 20] and approximate model building [21-23] can be employed. A simplified model, called meta-model [24], is elaborated using results of the numerical experiment at a sample point of the experiment design. The response analysis using the simplified model is computationally much cheaper than the solution employing the original model. Despite the great variety of literature available on the identification of elastic constants of laminated plates, the studies dedicated to the problems on estimation of elastic parameters of stiffened plates are very few. In [25], the determination of in-plane elastic constants of stiffened plates was performed. In that study, instead of a physical experiment, the numerical vibration data were used for determining the elastic constants.

In the present study, the identification of elastic properties of a curved stiffened panel from the measured eigenfrequencies is carried out. Six small panels with one stringer were cut out from a large stiffened panel. These small panels were tested for vibration in order to measure the eigenfrequencies and the corresponding eigenmodes. Using the vibration data measured, the identification of material properties was performed.

Curved Panel for Vibration Test

A curved stiffened panel with one stringer was cut out from an original 3-stringer panel (see *Figure 1*). The original panel was one-sixth (60°) of a cylinder 580 mm high and 415 mm wide, with a 1000-mm internal panel radius. The original panel was cut into six smaller panels with the following dimensions: four panels with length 290 mm, width 139 mm, and rib height 14.8 mm and two panels with length 290 mm, width 137 mm, and rib height 14.8 mm. The first four panels were used for the identification of material properties.

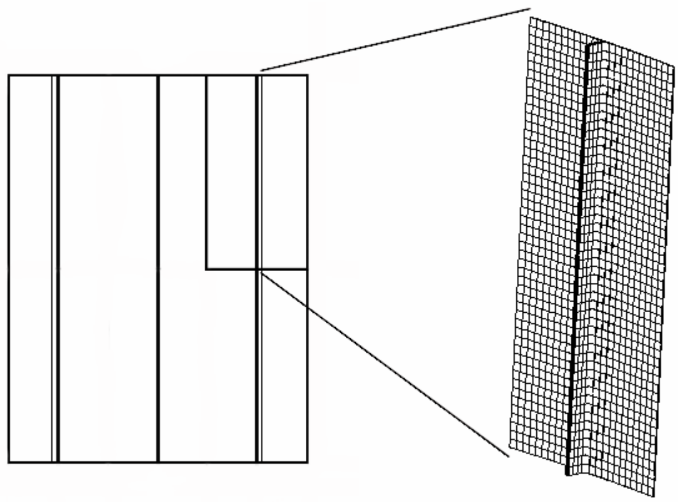

Fig. 1. One-stringer curved panel

Vibration Experiment

Panels were tested for vibration in order to measure the eigenfrequencies and the corresponding modes. The natural frequencies of the test panels were measured by a vibrograph [charge-coupled device (CCD) camera] using shearography technique. The shearography employs a single expanded beam of laser light which is reflected back from the specimen to the CCD camera. The camera includes an image shearing device, which brings two separate points of the object surface to meet in the image plane. The two overlapped portions of the sheared images interfere and produce a speckle pattern. When the object is deformed, the speckle pattern is slightly modified. A comparison of the two (stressed and unstressed) speckle patterns produces a fringe pattern which depicts the relative displacement of two neighbouring points. Since the magnitude of shearing is small, the fringe pattern approximately represents the first derivative of displacement with respect to the shearing direction, which may be either in-plane or out-of-plane. The experiments are performed under free boundary conditions on all edges of the panel so that to exclude the influence of boundary conditions on the results of identification. The specimens are hung by two corners using a band simulating free boundary conditions along the edges of the panel (see *Figure 2*). The panel is excited by a piezo-ceramic disc bonded to it. The excitation with small piezo-ceramic discs works via the radial expansion of the disc causing a bending moment to the panel surface. The piezo-ceramic disc is connected to an amplifier and the frequency is varied by a frequency generator. To enable a better scanning, the specimens are painted in white. A typical test procedure is as follows: the panel is excited continuously, and the laser measures its response. Then the experimental results are compared with the predicted frequencies, which are calculated by the finite element code employing the initial guess values of elastic constants. Such preliminary finite element calculations are necessary to be sure that all experimental frequencies are recorded in the range. Since not all the frequencies are observed experimentally, they are ranged according to the finite element solution. In total, four panels were measured.

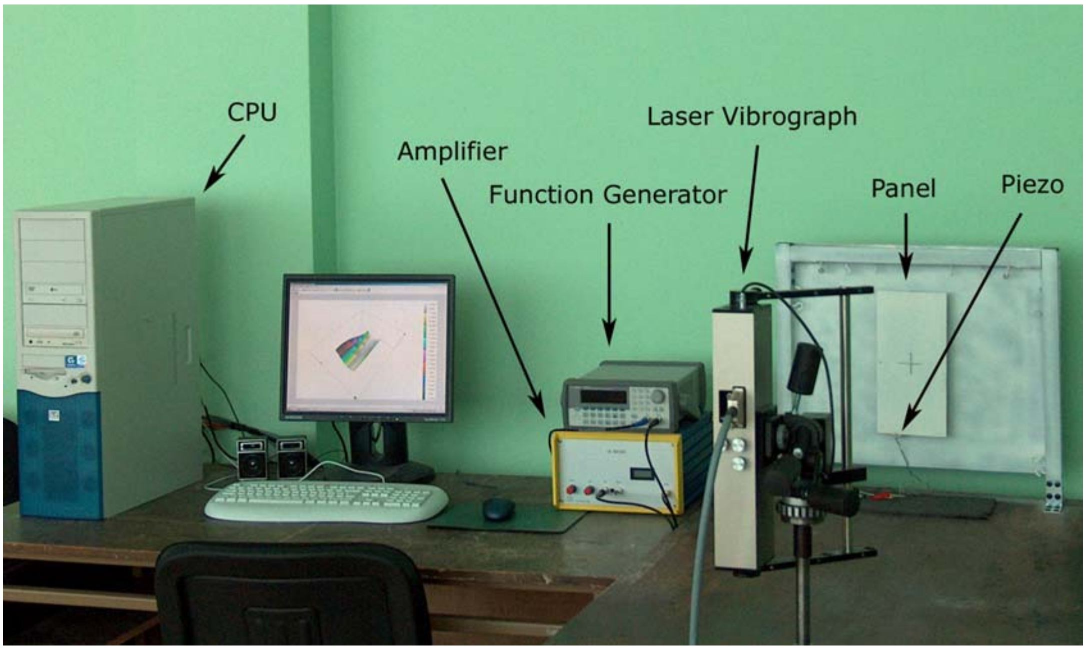

Fig. 2. Vibration experiment of a one-stringer curved panel

Finite Element Modelling

The geometry and the finite element (FE) model of a one-stringer curved panel are presented in *Figure 3*. For the skin, [+45/-45/0] laminate is considered. The ply thickness t = 0.125 mm is fixed due to the manufacturing technology. Therefore, the thickness of skin is h = 0.75mm. The laminate lay-up for the blade-type stringer is $[(+45/-45)_3/0_6]_2$, i.e., the stringer consists of 24 single layers, and the thickness of the stringer is b_w = 3 mm. The stringer flange is stepwise flattened for a better matching with the contour of the skin. The stringer flange consists of three steps: the inner flange step – laminate stacking sequence $[+45/-45]_3$, i.e., six layers with the thickness h_i = 0.75 mm, the middle flange step – laminate stacking sequence $[+45/-45]_2$, i.e., four layers with +{the thickness} h_m= 0.5 mm, and the outer flange step – laminate stacking sequence [+45/-45], i.e., two layers with +{the thickness} h_o= 0.25 mm. The density of the panels, measured by hydraulic weighting, is ρ = 1560.9 [kg/m^3].

(a) (b)

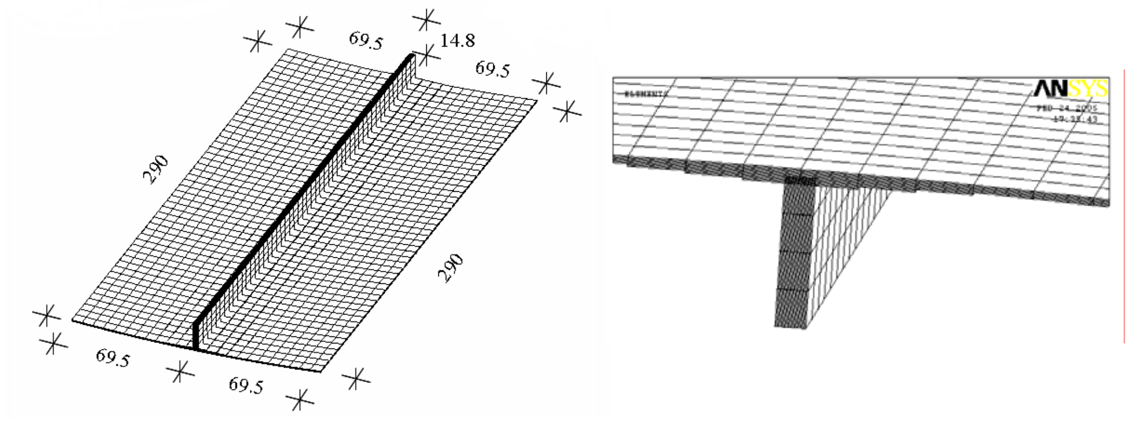

Fig. 3. a - geometry and FE model of the pane; b - FE model with zoom for stringer

The finite element solution is performed employing the ANSYS 9.1 software code. The finite element model of a one-stringer curved panel is modelled by using 1700 layered eight-node shear-deformable shell elements. Each node (5370) has six degrees of freedom, namely three displacements and three rotations.

Experiment Design and Identification Functional

The parameters to be identified are five elastic constants of a transversely isotropic laminate. Since, for a stiffened panel, some elastic constants are less sensitive to frequencies, two of the five independent elastic constants are fixed (G_{23} and ν_{12}) [26]:

G_{23} = 6.0 GPa, ν_{12} = 0.34

Thus, the identification of only three elastic constants $\boldsymbol{x} = (E_1, E_2, G_{12})$ of the single layer is carried out.

The identification process is carried out through minimization of an error function that expresses the relative difference between the measured $f_i^{\exp}$ and numerically calculated eigenfrequencies $f_i^{FEM}(\boldsymbol{x})$

$$\Phi(\boldsymbol{x}) = \sum_{i=1}^{m} w_i \left(\frac{f_i^{\exp} - f_i^{FEM}(\boldsymbol{x})}{f_i^{\exp}} \right)^2 \quad (1)$$

Here w_i are the weighting coefficients, w_i is equal to 1 for frequencies used in identification and equal to zero for unused frequencies.

It is suggested to minimize functional (1) according to the meta-model technology [24]. This technology employs the so-called numerical experiments creating the approximating relevance

$$f_i = f_i(\boldsymbol{x}), i = 1,...,m \tag{2}$$

Here m – the number of parameters to be identified, m=3 in our case.

The numerical frequencies $f_i^{FEM}(\boldsymbol{x})$ are functions of elastic constants. These functions are obtained as approximation of the finite element solution, which is performed at the sample points of experiment design.

The previous investigations [26, 27] show that the second-order approximations should be used for building approximations of the identification functional $\Phi(\boldsymbol{x})$. To determine points for the finite element computations, the experiment design is planned using the criteria of D-optimality [28]. Unlike the classical D-optimal design, in the present method, plans of a Latin hypercube (LH) type [28] are employed. The number of sampling points is calculated by the following equation:

$$N = \prod_{i=1}^{n} \frac{(K+i)}{i} \tag{3}$$

where N is the number of sampling points, K is the number of variables, and n is the order of approximation function. The D-optimal plans for second-order approximations using three factors must consist of 4*5/2 = 10 sampling points. For a cubic approximation, 4*5*6/6 = 20 sampling points are needed. Since, in this case, the FEM calculations are not time consuming, the D-optimal Latin Hypercube sampling design [28] with N = 75 sample points and K = 3 variables is selected. The sampling points are distributed in the domain of interest, which is formed by the lower and upper limits of variables. The upper and lower limits for the variables are chosen by using the initial guess values of elastic constants. These values can be taken from the properties of a similar material or from the static test of the present material. In the process of search, the limits can be moved, if, for example, the identified constants are beyond the limits or if the search accuracy should be increased. The domain of interest shown in *Table 1* originally was selected by using the typical material properties of CFRP composites. In the process of search, the limits were moved to achieve the best accuracy. The numerical frequencies for this domain of 75 sample points were determined by the FEM analysis, and then this information was employed for approximating the functional $\Phi(\boldsymbol{x})$ by the EdaOpt in-house software code [29]. The same code was used for minimization of the identification functional. Exploiting the data obtained from FEM calculations, in the domain of interest, the EdaOpt software code determines a suitable model describing the behaviour of the system and builds the response surface using polynomial functions. Minimizing the functional $\Phi(\boldsymbol{x})$, the optimal response of the system and the best values of input variables (three elastic constants $\boldsymbol{x}$) are obtained.

Table 1. The domain of interest for identification

Property	**[GPa]**	
	Min	**Max**
E_1	105	125
E_2	20	35
G_{12}=G_{13}	3	8

Results of Identification

After the finite element calculations at the reference points of the experiment design have been performed, it is of interest to compare the mode shapes of experimentally measured and numerically calculated eigenfrequencies. Some typical vibration modes of both the experimentally measured and the respective numerically calculated eigenfrequencies (using the identified elastic properties for panel 2) of a curved one-stringer panel are presented in *Figure 4*.

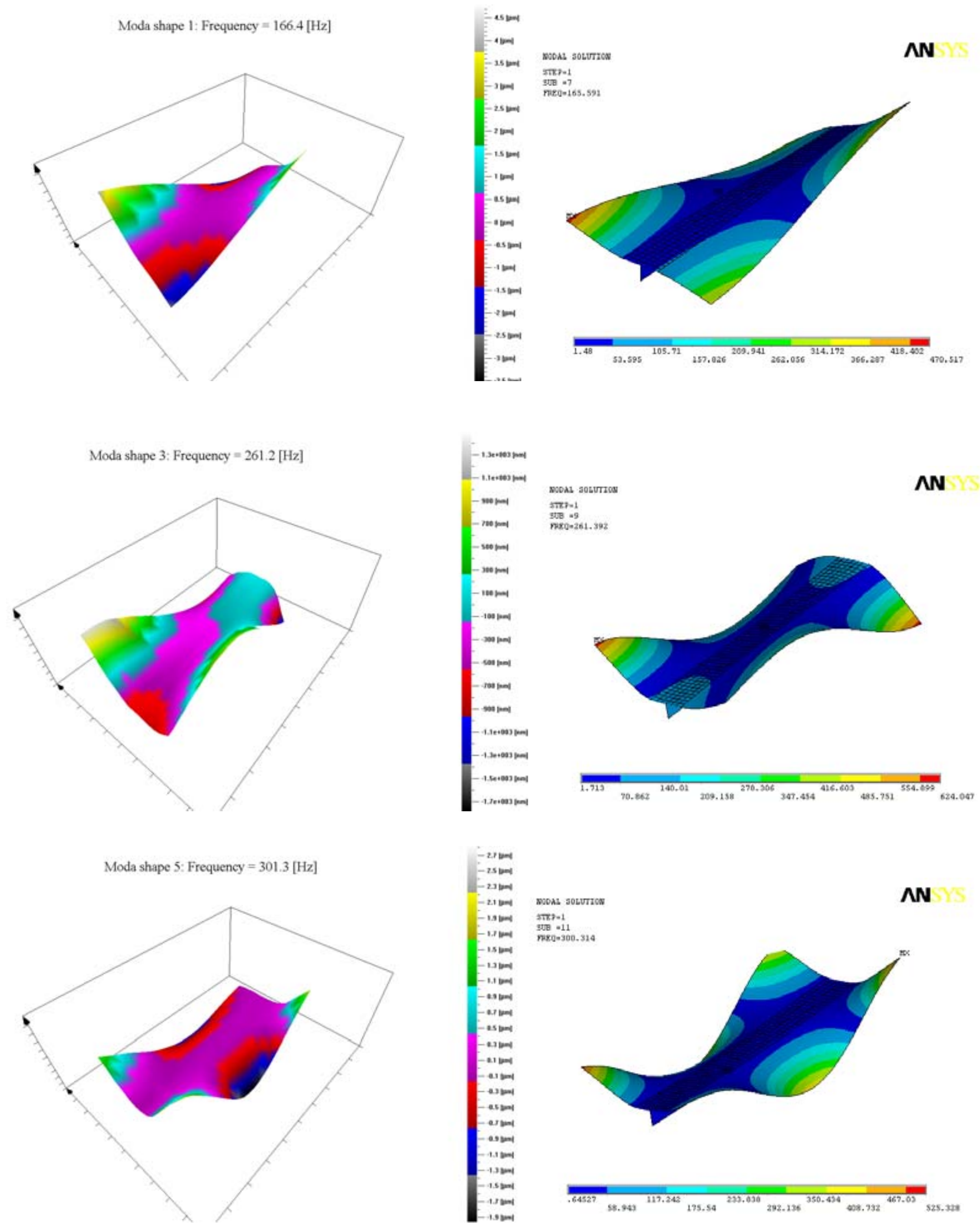

Fig. 4. Mode shape for the 1st, 3rd, 5th eigenfrequency: a - experimentally measured; b - numerically calculated

By minimizing functional (1), three elastic constants **x** are obtained. It should be noted that the number of frequencies, which are selected for identification, is different for each specimen. The experimentally measured frequencies, presented in Table 2, can be used for identification in any combination. A cross validation for all sample points was performed so that to achieve a better approximation of the original function and to select the most important (most sensitive to elastic constants) and reliable frequencies. The identification results are given in *Table 2*.

Table 2. Elastic constants obtained by identification

Property	Panel 1	Panel 2	Panel 3	Panel 4	Average
E_1, GPa	121.50	119.40	108.30	116.80	116.50
E_2, GPa	25.40	24.10	34.70	26.30	27.63
$G_{12}=G_{13}$, GPa	5.50	6.80	6.60	6.10	6.25

The results obtained were verified by comparing the experimentally measured eigenfrequencies with the numerical ones obtained by FEM at the point of optima (using the identified elastic properties). The residuals Δ_i (see *Table 3*) are calculated by the expression

$$\Delta_i = \frac{\left| f_i^{FEM}(\boldsymbol{x}^*) - f_i^{\exp} \right|}{f_i^{\exp}} \times 100 \tag{4}$$

Table 3. Flexural frequencies and residuals for panels 1 and 2

	Panel 1			Panel 2		
No.	$f_i^{\exp}$ (Hz)	$f_i^{FEM}(\boldsymbol{x}^*)$ (Hz)	Δ_i (%)	$f_i^{\exp}$ (Hz)	$f_i^{FEM}(\boldsymbol{x}^*)$ (Hz)	Δ_i (%)
1	166.4[a]	165.6	0.5	169.5[a]	165.9	2.1
2	236.8[a]	237.1	0.1	243.0[a]	243.7	0.3
3	261.2[a]	261.4	0.1	268.0[a]	267.5	0.2
4	289.5[a]	290.2	0.2	286.3[a]	288.6	0.8
5	301.3[a]	300.3	0.3	300.8[a]	301.8	0.3
6	411.5[a]	413.0	0.4	398.8[a]	410.3	2.9
7	431.3[a]	433.5	0.5	422.1[a]	433.5	2.7
10	570.2	584.6	2.5	577.6	591.6	2.4
12	741.0	762.3	2.9	730.5	765.2	4.8
13	759.3	784.5	3.3	763.3	789.0	3.4
14	939.0	949.4	1.1	921.9	956.8	3.8
16	-	1068.8	-	1069.0	1067.4	0.1
22	1402.0	1381.0	1.5	1401.0	1387.9	0.9
26	1614.0	1657.7	2.7	1618.0	1661.9	2.7
27	1705.0	1701.5	0.2	1702.0	1704.4	0.1
28	1745.0	1759.5	0.8	-	1783.6	-
29	-	1804.4	-	1809.0	1831.7	1.3

[a] *the frequencies used in identification*

Conclusions

The identification of the realized material properties (elastic constants) was performed on a small stiffened panel cut out from a large three-stringer panel. The results obtained are stable only for the in-plane shear modulus. The elastic modulus in the fibre direction and the transverse modulus differ significantly from the nominal constants of carbon-fibre-reinforced composites [26]. These discrepancies are explained by the fact that the parameters of the real structure differ from the nominal values (layer thickness, layer angles, the material density is not so homogeneous in all parts of the stiffened panel, etc.) of laminated composites. These differences should be taken into account in designing the real structures by choosing the safety factors and calculating the limit and collapse loads of composite stiffened structures.

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Sandris Ručevskis
Institute of Materials and Structures, Riga Technical University
Kalku 1, Riga, LV - 1658
M.Sc.Ing
sandris_r@bf.rtu.lv

Ručevskis S. Ribotas kompozītmateriāla čaulas elastīgo īpašību noteikšana izmantojot svārstību analīzi
Lai spētu nodrošināt konstrukcijas augsta standarta drošību, materiālu īpašībām ir jābūt precīzi noteiktām. Ir zināms, ka nosakot materiāla elastīgās īpašības, izmantojot standarta paraugus, iegūtās materiāla īpašības var atšķirties no reālas kompozītmateriāla konstrukcijas īpašībām. Lai precīzāk spētu noteikt konstrukcijas elastīgās īpašības, pārbaudes paraugi ir izgriezti no lielas ribotas kompozītmateriāla čaulas. Ribotas kompozītmateriāla čaulas elastīgās īpašības tiek noteiktas, izmantojot identifikācijas metodi, kas balstīta uz skaitliskā eksperimenta plānošanas, atbildes virsmas un galīgo elementu metodes aprēķiniem. Identifikācijas rezultātā iegūtās materiāla elastīgās īpašības atšķiras no tipiskām oglekļšķiedras kompozītmateriāla elastīgajām īpašībām, kas tiek skaidrots ar to, ka reālas konstrukcijas slāņa biezums, šķiedru orientācijas leņķis un materiāla blīvums nesakrīt ar tipiska oglekļšķiedras kompozītmateriāla nominālajām vērtībām.

Ručevskis S. Determination of Elastic Properties of Stiffened Composite Shells by Vibration Analysis
To ensure the high reliability of a composite structure, the actual mechanical properties of a material must be accurately predicted. It is well known that the material properties determined from standard tests of small specimens, which are manufactured by using the same technology as for the real large structure, may differ from the actual material properties of a laminated composite structure. To determine more accurately the material properties of the structure, the specimens were cut out directly from a large stiffened composite panel. The elastic properties of a curved stiffened panel are found by an identification procedure based on the method of experiment design, the response surface approach, and the finite element method. The identification results obtained from the vibration tests of the small panel slightly differ from the typical material properties of CFRP composites, which can be explained by the fact that single ply thickness, material density, and layer angles of the real structure are slightly different from the nominal values.

Ручевскис С. Определение эластичных свойст у ребристых композитных оболочек используя вибрационный анализ.

Чтобы обеспечить высокие требования безопастности, необходимо точно определить эластичные свойства материала. Известно, что полученные характеристики композитных материалов с тестовых образцов могут отличатся от реальных свойств конструкции. Чтобы определить точные эластичные свойства конструкции, образцы вырезают из больших ребристых композитных оболочек. Для определение эластичных свойст у ребристых композитных оболочек используют метод индификации, которой базируется на планирование численного эксперимента и расчёта методом конечных элементов. Результат идентификации вибрационного теста, показывают незначительные различия с характеритстиками материала, которые могут обьяснятся различной толщиной слоя, плотностью материала и углом ориентации волокон конструкции с номинальным значением материала.