Smoothing Approach of 3D Models Based on Interpolating Subdivision Surface

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Abstract. **The problem of free-form object modelling in 3D visualization is a topical task in computer graphics and solving it is important for practical use. This paper describes a new approach to free-form object surface reconstruction, based on initial 3D model smoothing using subdivision surface. There are many approaches to subdivision surface based on object surface approximation. In this paper, the proposed approach is based on interpolating subdivision surface.**

***Keywords*: interpolation, polygon, subdivision surface.**

I. Introduction

The problem of free-form 3D object modelling for 3D visualization is an important task in computer science and technology. It is important to solve this problem for practical application in various areas, for example in biomedical engineering.

The effective 3D modelling approach is the creation of a polygonal model, based on subdivision surface methods [1,2,4]. In this case, the resulting model can be used for visualization directly, using standard tools for computer graphics.

There are many approaches to subdivision surface based on surface approximation [1,2,4], but the interpolating subdivision surface is also used for some technical and scientific tasks. The 3D modelling approach based on interpolating analytical surfaces is proposed in [5].

This paper describes a new approach to free-form object construction, based on initial 3D model smoothing using subdivision surface. In this paper, the proposed approach is based on interpolating subdivision surface.

II. The Proposed Approach

Initial data in the proposed approach is a polygonal model of an object (or polyhedron), which is based on a set of vertices P_i and a set of normal vectors $\vec{n}_i$. The model uses two types of polygons – triangular (described as $P_iP_jP_k$) and quadrangular (described as $P_iP_jP_kP_l$). Fig.1 and Fig.2 illustrate both these cases.

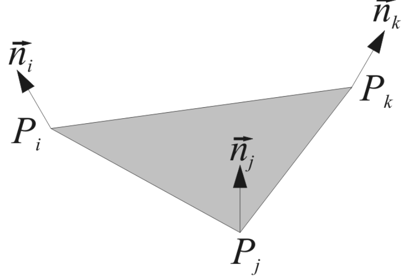

Fig. 1. Initial triangular polygon

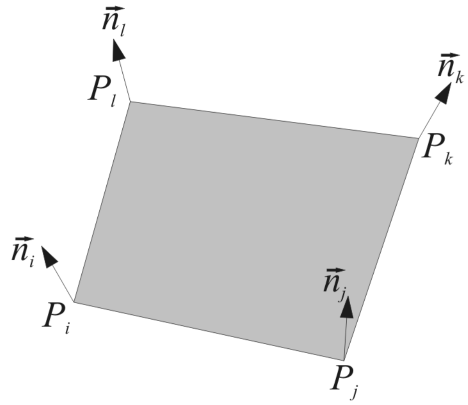

Fig. 2. Initial quadrangular polygon

The proposed approach is based on the creation of the interpolated polygonal surfaces. This approach can be described by the following three steps.

Step 1. The calculation of tangent vectors. The tangent vector m_{ij} from point P_i to point P_j must fulfill the following condition:

- tangent vector m_{ij} must be perpendicular to the normal vector n_i;
- tangent vector m_{ij} must belong to the plane, created by n_i and the vector between point P_i and P_j.

Under these conditions, the tangent vector can be calculated by the cross product of two vectors as follows:

$$\vec{m}_{ij} = h \cdot \left(\vec{n}_i \times \left(P_j - P_i\right)\right) \times \vec{n}_i \,. \tag{1}$$

where: h – is the length coefficient.

Fig.3 and Fig.4 illustrate a set of tangent vectors after step 1.

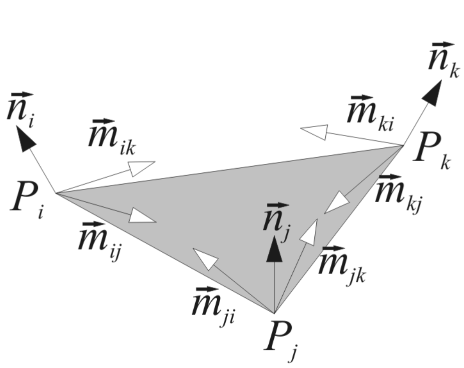

Fig. 3. The triangular polygon with a set of tangent vectors

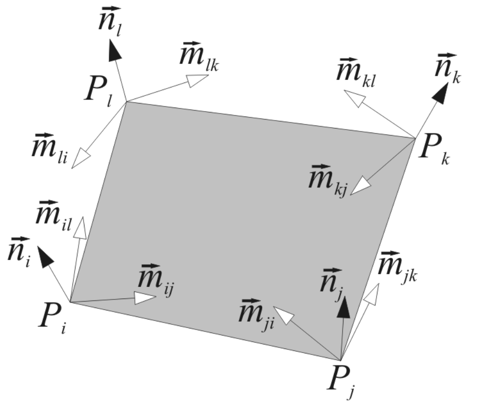

Fig. 4. The quadrangular polygon with a set of tangent vectors

Step 2. The calculation of the midpoint on the polygon edges. Let the polygon edge between vertices P_i and P_j be interpolated by a parametric curve. This curve is an edge of the triangular parametric surface, which interpolates the initial triangular polygon. In this case, the midpoint can be calculated as follows:

$$Q_{ij} = \frac{1}{2}\cdot\left(P_i + P_j\right) + \frac{1}{8}\cdot\left(\vec{m}_{ij} + \vec{m}_{ji}\right). \tag{2}$$

In the case of a triangular polygon, equation (2) can be used for each edge. The result of subdivision is a new polygonal mesh, created by 4 triangular polygons: $P_iQ_{ij}Q_{ik}$, $P_jQ_{jk}Q_{ij}$, $P_kQ_{ik}Q_{jk}$ and $Q_{ij}Q_{ik}Q_{ik}$. Fig.5 shows this case.

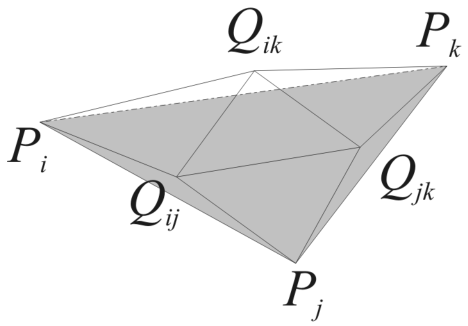

Fig.5. The resulting polygonal mesh of triangular polygon

In the case of the quadrangular polygon, it is necessary to calculate an additional vertex using Step 3.

Step 3. The calculation of the midpoint on the quadrangular polygon. Let the quadrangular polygon $P_iP_jP_kP_l$ be interpolated by a parametric surface. In this case, the midpoint can be calculated as follows:

$$\begin{aligned} Q_c = {} & \frac{1}{4}\cdot\left(P_i + P_j + P_k + P_l\right) + \frac{1}{64}\cdot\left(\vec{n}_i - \vec{n}_j + \vec{n}_k - \vec{n}_l\right) + \\ & + \frac{1}{16}\cdot\left(\vec{m}_{ij} + \vec{m}_{ji} + \vec{m}_{jk} + \vec{m}_{kj} + \vec{m}_{kl} + \vec{m}_{lk} + \vec{m}_{li} + \vec{m}_{il}\right) \end{aligned}. \tag{3}$$

In the case of the quadrangular polygon, the result of subdivision is a new polygonal mesh, created by 4 quadrangular polygons: $P_iQ_{ij}Q_cQ_{il}$, $P_jQ_{jk}Q_cQ_{ij}$, $P_kQ_{kl}Q_cQ_{jk}$ and $P_lQ_{il}Q_cQ_{kl}$. Fig. 6 shows this case.

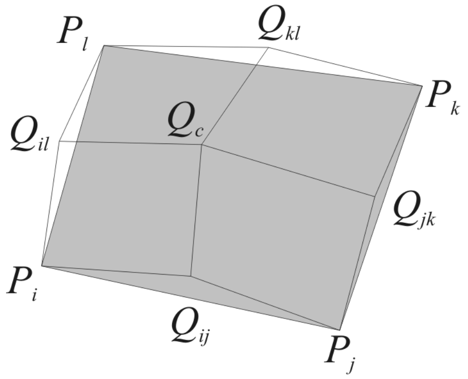

Fig.6. The resulting polygonal mesh of quadrangular polygon

III. Normal Vector Calculation

For 3D model visualization, the calculation of normal vector is necessary in each vertex. The normal vector for one vertex can be described by the following steps.

Step 1. The edge-vector calculation. These vectors can be calculated between vertex P and neighbouring vertices P_i as follows:

$$\vec{v}_i = \left(P_i + P\right). \tag{4}$$

Fig.7 illustrates this case.

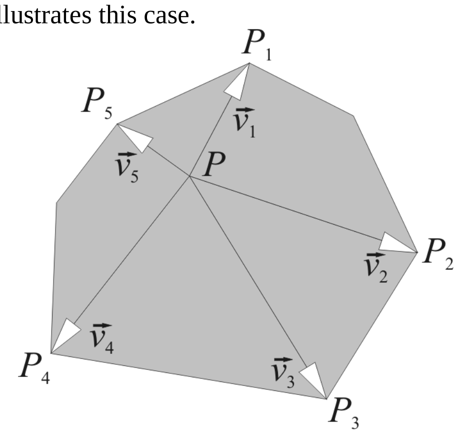

Fig.7. The edge-vectors

Step 2. The calculation of a set of side-based normal vectors. The normal vector to each polygon can be calculated as the cross product of two edge-vectors. It can be calculated as follows:

$$\vec{n}_i = \vec{v}_i \times \vec{v}_{i-1}. \tag{5}$$

Fig.8 illustrates this case.

Step 3. The resulting normal vector in the given vector can be calculated as a mean of a set of side-based normal vectors. It can be calculated as follows:

$$\vec{n}_p = \frac{1}{m}\cdot\sum \vec{n}_i. \tag{6}$$

Fig.9 illustrates this case.

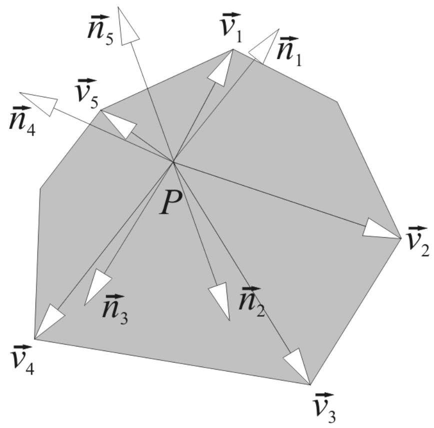

Fig.8. The set of side-based normal vectors

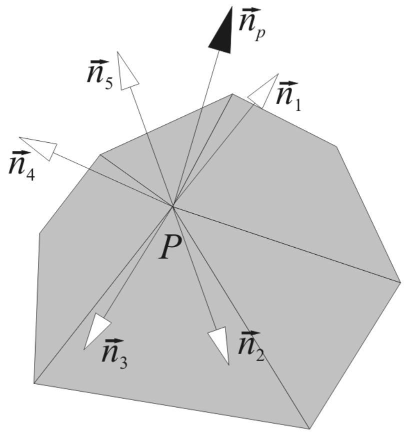

Fig.9. The resulting normal vector on the vertex

IV. Recursive Estimation

The resulting surfaces are defined recursively. Each iterative refinement replaces the current mesh with a smoother mesh. After many iterations, the surface will gradually converge onto a smooth limit surface.

The number of polygons increases four times in each iteration. In this case, the polygon count ratio can be described as follows:

$$S_{fin} = S_0 \cdot 4^R . \tag{7}$$

where: S_{fin} – the number of polygons in the final model;
S_0 – the number of polygons in the initial model;
R – the depth of recursion.

In case $R \to \infty$ the resulting object surface is equivalent to the object surface from [5].

V. Experiments

The proposed approach has been implemented and tested on the 3D model of a medical object. The obtained images are shown in Fig.10 and Fig.11.

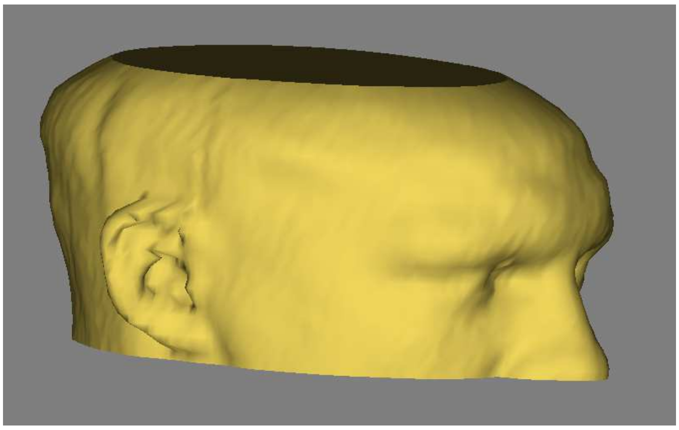

Fig.10. The initial model of the medical object

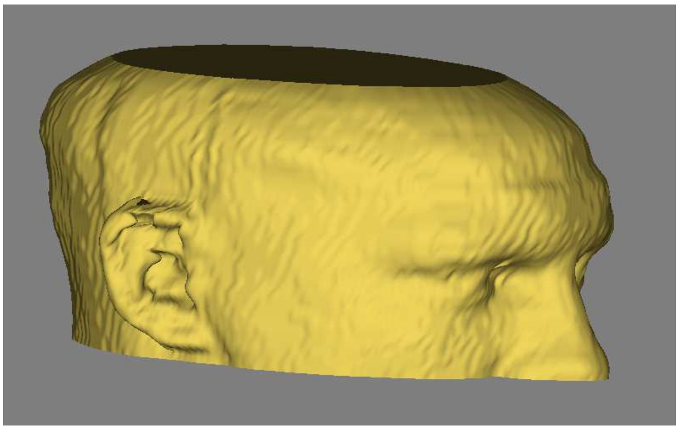

Fig.11. The smoothed model of the medical object

The enlarged fragment of the medical object is shown in Fig.12 and Fig.13.

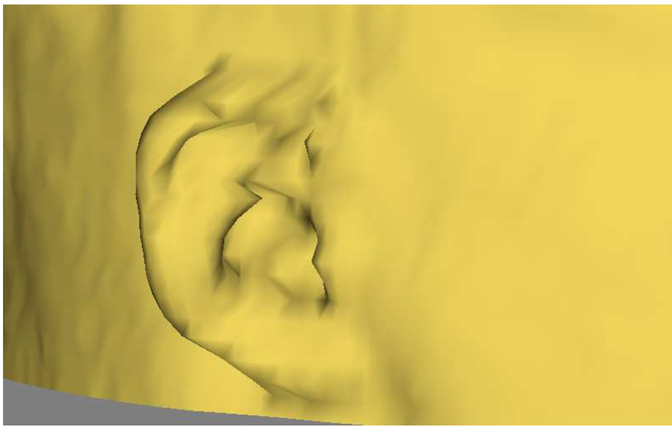

Fig.12. The enlarged fragment of the initial model of medical object

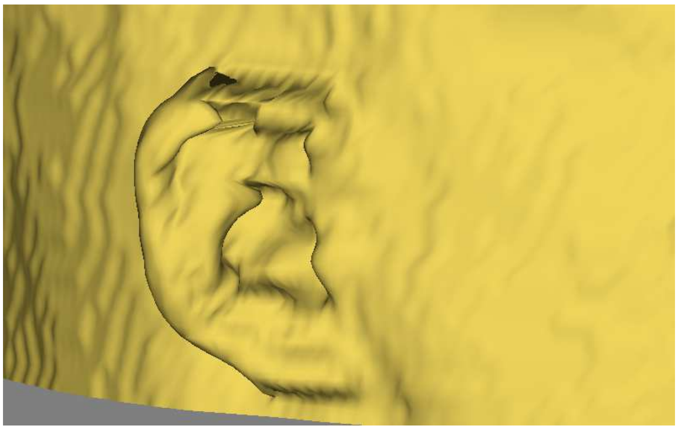

Fig.13. The enlarged fragment of the smoothed model of medical object.

VI. CONCLUSIONS

The main conclusions of this paper are the following:

- In this paper, a new approach to free-form object reconstruction has been proposed, which is based on initial 3D model smoothing using subdivision surface.
- The proposed approach is based on interpolating subdivision surface.

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Aleksandrs Sisojevs, Aleksandrs Glazs. 3D modeļa nogludināšanas pieeja, balstoties uz interpolācijas virsmas apakšiedalījumu
Brīvas formas objektu modelēšanas problēma 3D vizualizēšanā ir aktuāls uzdevums dažādās zinātnes un tehniskajās jomās. Šis risinājums ir svarīgs praktiskai pielietošanai daudzās nozarēs, piemēram, biomedicīnas inženierijā. Efektīvā un praktiskā pieeja 3D modelēšanā ir poligonālā modeļa izveide. Rezultātā modeļus iespējams vizualizēt, izmantojot datoru grafikas standarta rīkus, piemēram, ar grafisko bibliotēku OpenGL.
Eksistē dažas virsmas apakšiedalījumu pieejas, kuras balstās uz objekta virsmas aproksimāciju. Šajā darbā tika aprakstīta jauna pieeja brīvas formas objektu virsmas rekonstruēšanai, balstoties uz sākotnēja 3D modeļa nogludināšanu, izmantojot virsmas apakšiedalījumu. Ieejas datu struktūra ir trīsstūra un četrstūra poligonu kopa. Katrs sākotnējais poligons tiek aprakstīts ar virsotņu un normāles kopu. Darba gaitā piedāvātajā pieejā katrs poligons interpolējas ar jauno poligonu kopu (četri poligoni par vienu iterāciju). Tāpat tiek izskaitļotas normāles modeļa jaunajās virsotnēs. Iegūto poligonālo modeli var izmantot tālākai nogludināšanai, izmantojot to kā ieejas datus jaunā iterācijā. Gadījumā, ja nogludināšanu atkārto vairākas reizes (teorētiski – bezgalīgas reizes), tad sākotnējais poligons interpolējas ar Bezjē trīsstūri (sakrīt poligona virsotnes un virsmas stūrainās virsotnes). Piedāvātā pieeja tika realizēta, lai pārbaudītu tās darba spējas uz reāla objekta. Par reālo brīvas formas objektu tika ņemts medicīnas objekts, kas ir galvas virsmas fragments. Eksperimenta gaitā iegūtie rezultāti parādīja piedāvātās pieejas izmantošanas iespēju reālo uzdevumu risināšanai. Iegūtie medicīnas objekta attēli ir ekvivalenti pieejai [5], tajā pašā laikā piedāvātajā pieejā aprēķinu apjoms ir daudz mazāks.

Александр Сысоев, Александр Глаз. Подход сглаживания 3D моделей базирующийся на интерполяционном дроблении поверхности
Проблема моделирования объектов свободной формы для 3D визуализации является актуальной задачей во многих областях науки и техники. Её решение является важным для практического использования во многих областях, например в биомедицинской инженерии. Эффективный и практичный подход в 3D моделировании - это построение полигональных моделей. В этом случае результирующую модель можно визуализировать, используя стандартные инструменты компьютерной графики, например графическую библиотеку OpenGL.
Существует несколько подходов, которые базируются на аппроксимации поверхности объекта. В данной работе описан новый подход реконструкции поверхности свободной формы, базирующийся на сглаживании исходной 3D модели используя интерполяционное дробление поверхности. Структура входных данных – множество треугольных и четырехугольных полигонов. Каждый исходный полигон описан множеством вершин и нормалей. Во время работы предложеного подхода каждый полигон интерполируется новым множеством полигонов (четыре полигона за одну итерацию). Так же рассчитываются нормали в новых вершинах модели. Полученую полигональную модель можно использовать для дальнейшего сглаживания, используя её, как входные данные для новой итерации. В случае, если повторить сглаживание несколько раз (теоретически – бесконечное число раз), то изначальный полигон интерполируется треугольником Безье (вершины полигона и угловые вершины треугольника совпадают). Предложенный подход был реализован для проверки его работоспособности на примере реального объекта. В качестве реального объекта принят медицинский объект – фрагмент поверхности головы. Полученые в результате эксперимента результаты показывают возможность использования предложенного подхода в решении реальных задач. Полученые изображения медицинского объекта эквивалентны подходу [5], в то же самое время объем вычислений в предложеном подходе намного меньше.