

# Arrangement of Boundary Conditions for Hydrogeological Model of Latvia

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**Abstract** – In 2012, scientists of Riga Technical University have developed the regional hydrogeological model (HM) of Latvia (LAMO). LAMO covers 475km×300km area. It includes the territory of Latvia, the Gulf of Riga (active area of the HM) and border areas of the neighboring countries (presently, passive area of the HM). Both areas are separated by the border zone where outer boundary conditions for the active area must be fixed. LAMO simulates the steady state regime of the active groundwater zone that contains potable groundwater resources for Latvia. LAMO includes 25 geological layers and HM plane approximation step is 500 metres. The active LAMO volume is enveloped by its top and bottom surfaces, but the outer vertical surface of the border zone constitutes the shell of the HM. For the HM top and bottom surfaces and for the border zone, piezometric boundary conditions are arranged. Presence of the HM passive part makes the shell surface impermeable for the transboundary groundwater flow. This distortion of the natural groundwater regime must be compensated by a proper choice of boundary conditions that are fixed within the border zone. On the LAMO top surface, the digital relief (terrain) map is used as the boundary condition. The map includes elements of the hydrographical network (rivers, lakes, sea). The rivers are attached to various geological layers within the HM body. Special software has been developed to find the right attachment of over 200 rivers of Latvia to the HM. Lakes and the sea are joined with the HM through the aeration zone. The zone represents a formal aquitard that is considered in computing of infiltration flow for the HM. Because the digital relief is used as the boundary condition, the HM itself provides feasible infiltration flow distribution that can be adjusted by its calibration. Due to original methods of establishing the boundary conditions, LAMO has been created during a short period of time (2 years).

**Keywords** – hydrogeological model, boundary conditions, infiltration flow, calibration of models

## I. INTRODUCTION

The countries of the world and of the European Union are developing hydrogeological models (HM) where by means of computer modelling the information necessary for the groundwater management planning is obtained. In 2012, scientists of Riga Technical University have developed the regional HM of Latvia (LAMO). More detailed information regarding LAMO is given in publications [1, 2, 3, 4]. The present publication is focused on arrangement of the boundary conditions for LAMO. Due to original methods used for establishing the boundary conditions, it was possible to develop LAMO during a short period of time (2 years).

LAMO covers 475km×300km area (Fig.1). It includes Latvia and border territories of the neighbouring countries. For the current LAMO version, only Latvia and the Gulf of Riga constitute the active area of the HM (Fig.2), because, at

present, no agreement exists regarding cross border modelling. The active and passive areas are separated by the 4km wide border zone that is used for fixing boundary conditions for the outside of the active area.



Fig. 1: Location of Latvia's HM

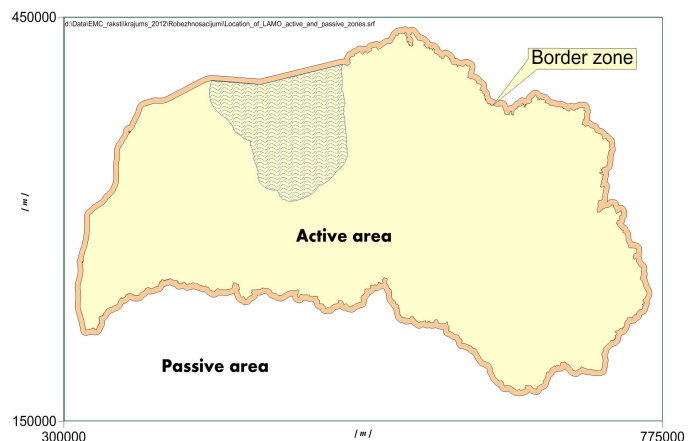


Fig. 2. Location of the active and passive areas of LAMO

To consider the problems of arranging boundary conditions for LAMO, the basic mathematics of the 3D steady state model must be introduced. By applying 3D finite difference approximation, the  $x, y, z$  – grid of the HM is built using  $(h \times h \times m)$  sized blocks ( $h$  is block plane size,  $m$  is the variable thickness of a geological layer). The model constitutes a rectangular  $p$ -tiered  $xy$ -layer system, where  $p$  is the number of geological layers. For LAMO,  $p=25$ ,  $h=500$  metres.

The modelling program controls the whole area of the HM that contains also the passive area that takes no part in simulation. The active HM volume is enveloped by the border zone. Its outer vertical surface represents an impermeable shell that blocks transboundary groundwater flow. The relief (ground surface) and the lower side of the model are its geometrical top and bottom, accordingly. The 3D-space volume enveloped by the boundary surfaces (top, bottom, shell) constitutes the active body of the HM. Vector  $\varphi$  of the piezometric head is the numerical solution of the boundary field problem which is approximated in nodes of HM  $xyz$ -grid by the following algebraic expression:

$$A\varphi = \beta - G\psi, \quad A = A_{xy} + A_z \quad (1)$$

where  $A$  is the symmetric sparse matrix of the geological environment which is presented by the  $xy$ -layer system containing horizontal ( $A_{xy}$  - transmissivity) and vertical ( $A_z$  - vertical hydraulic conductivity) elements of the HM grid;  $\psi$  - the boundary head vector:  $\psi_{rel}$ ,  $\psi_{bot}$ ,  $\psi_{out}$ ,  $\psi_{riv}$  - subvectors of  $\psi$  that represent boundary conditions the HM top, bottom, border zone, and rivers accordingly;  $G$  - the diagonal matrix (part of  $A$ ) assembled by elements, linking the nodes where  $\varphi$  must be found with the points where  $\psi$  is given (for  $\psi_{top}$ ,  $\psi_{bot}$ ,  $\psi_{out}$  these points are nodes of the HM grid);  $\beta$  - the boundary flow vector.

The elements  $a_{xy}$ ,  $a_z$  of  $A_{xy}$ ,  $A_z$  (or  $g_{xy}$ ,  $g_z$  of  $G$ ) are computed as follows:

$$a_{xy} = k \times m, \quad a_z = \frac{h^2 \times k}{m} \quad (2)$$

$$m_i = z_{i-1} - z_i > 0, \quad i = 1, 2, \dots, p$$

where  $z_{i-1}$ ,  $z_i$  are elevations, accordingly, of the top and bottom surfaces of the  $i$ -th geological layer;  $z_0$  represents the ground surface elevation  $\psi_{rel}$ -map with the hydrographical network included;  $k$ ,  $m$  are, respectively, elements of digital  $m$ ,  $k$ -maps of the computed layer thickness and permeability. If in some areas  $m_i = 0$ , then the  $i$ -th layer is discontinuous. To prevent the "division by zero", in the  $a_z$  calculation of (2),  $m_i = 0$  must be replaced by small  $\varepsilon > 0$  (for LAMO,  $\varepsilon = 0.02$  metres).

For LAMO the most important are boundary conditions  $\psi_{rel}$ ,  $\psi_{out}$ ,  $\psi_{riv}$ . Condition  $\psi_{bot}$  makes small impact on the HM, because the thick regional aquitard of D2nrz practically blocks vertical groundwater flow through the LAMO bottom surface. In LAMO, the  $\beta$ -vector is represented only by the withdrawal rates of well fields. It is not applied for usual simulation for recharge and evaporation flows on the HM top.

For LAMO outer surfaces, the  $\psi$ -conditions were applied, instead of the  $\beta$ -flows due to two main reasons:

- the  $\psi$ -conditions shortened the time needed for solving of (1), because they caused appearance of matrix  $D$  as the diagonal dominance factor of  $A$ ; [5, 6];
- numerical values of the  $\psi$ -conditions were known much better than the ones of the  $\beta$ -flows.

## II. THE DIGITAL RELIEF MAP AND THE HEAD DISTRIBUTION OF THE D2PR AQUIFER

Appliance of piezometric boundary condition  $\psi_{rel}$  (Fig. 3), on the HM top instead of the conventional recharge and discharge (evaporation) flows ( $\beta_{inf}$  - conditions for infiltration flow) has considerably reduced the effort of developing the HM, especially, of the large regional models where the groundwater infiltration distribution is very complex. It must account for numerous recharge and discharge areas. No modeller is able to guesswork the right  $\beta_{inf}$  distribution for a large 3D HM.

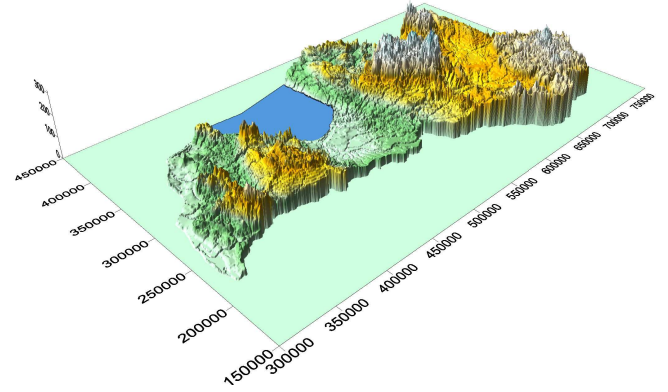


Fig. 3. Isometric image of the digital relief

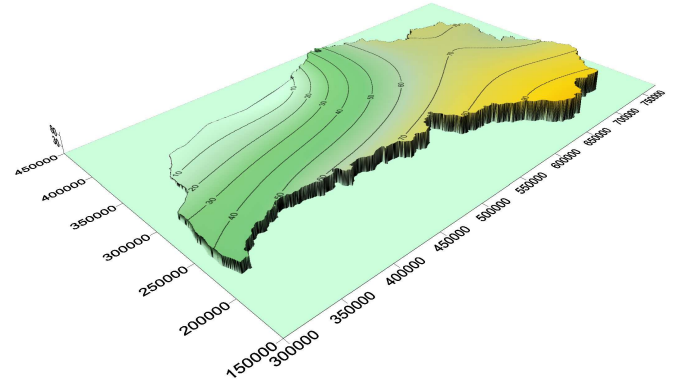


Fig. 4. Isometric image of the D2pr head distribution

The  $\psi_{rel}$ -map was obtained by using the data of the Geospatial Information Agency of Latvia. Methods used to create the map are described in [7].

If the  $\psi_{rel}$  - map is used, the flow  $q_{inf} = q_{aer}$  passes through the aeration (vadose) zone:

$$q_{aer} = G_{aer}(\psi_{rel} - \varphi_{Q2}) \quad (3)$$

where  $\varphi_{Q2}$  is the computed head (subvector of  $\varphi$ ) for the aquifer  $Q_2$ ;  $G_{aer}$  (diagonal submatrix of  $G$ ) contains the vertical links  $g_{aer}$  of the aeration zone connecting the fixed  $\psi_{rel}$  with the computed  $\varphi_{Q2}$ . The expression (3) gives the ordinary result of the HM, when a  $\psi$ -condition is applied. As a

rule, even the first run of the HM provides feasible results for  $q_{inf}$ .

The vertical links,  $g_{aer}$  of the diagonal matrix  $G_{aer}$ , are controlling the  $q_{aer}$  distribution. Values  $g_{aer}$  depend on  $h^2$ ,  $k_{aer}$  and  $m_{aer}$  (formula (2)) where  $h=500$ ,  $k_{aer}$  and  $m_{aer}$  are, accordingly, the permeability and thickness of the aeration zone. Initially,  $k_{aer}$  and  $m_{aer}$  are unknown. In nature,  $m_{aer}=\varphi_{rel}-\varphi_{Q2}$  if  $q_{aer}>0$ . If  $q_{aer}<0$  then  $m_{aer}$  ceases to exist, in the form when  $q_{aer}>0$ . Then the negative infiltration flow is caused mainly by lowlands, rivers and lakes. First, the following values of the aeration zone were tried:  $m_{aer}=0.02$  [m],  $k_{aer}=10^{-6}$  [m/day] (for recharge areas);  $k_{aer}=10^{-4}$  [m/day] (for areas of lakes and sea);  $k_{aer}=10^{-8}$  [m/day] (for areas of swamps). To avoid iterative changes of the HM geometry,  $m_{aer}=0.02$  was kept constant, until the calibrated state of  $q_{aer}$ -flow was achieved by adjusting the  $k_{aer}$ -distribution.

The piezometric head distribution  $\psi_{D2pr}$  of the D2pr aquifer (Fig. 4) is applied as the boundary condition on the LAMO bottom surface. The  $\psi_{D2pr}$ -map was obtained by using information of [8] about the factors that formed the head distribution for the D2pr aquifer. Influence of  $\psi_{D2pr}$  is small, because it is separated from the HM body by the thick regional D2nr aquitard. There are two reasons for application of this boundary condition: it may be useful if a modeller carries out research regarding the role of tectonic faults of the D2nr

aquitard; the D2pr aquifer contains drinking water at the North-East part of Latvia.

### III. DESIGN OF BOUNDARY CONDITIONS FOR THE BORDER ZONE

Due to the presence of the HM passive area, the outer vertical surface of the boundary zone (shell) is impermeable for the transboundary groundwater flow. This factor distorts the natural groundwater regime in the close vicinity of the shell. Because the border zone width is 4,000 metres, the distortion is considerably smaller on the inner surface of the zone. There exists the flow passing through this surface. One must try to recover the natural groundwater regime on this border surface. It can be done by fixing auxiliary piezometric boundary conditions  $\psi_{out}$  on the middle line of the border zone. As the initial data for obtaining  $\psi_{out}$  for aquifers of the HM, three data sources were used that provided linewise information: 1. the  $\psi_{rel}$ -map; 2.  $\psi_{out}$  for the D2ar aquifer; 3. data extracted from the head distribution map of the prequaternary surface  $\varphi_{preQ}$ . The  $\varphi_{preQ}$ -surface can provide only fragments of  $\psi_{out}$  that can be observed from the bird's eye view. Special software was developed for extracting these data and for providing information that was necessary to design the interpolation tool that provided  $\psi_{out}$  for all aquifers of the HM. Principles used to create the tool were reported in [9].

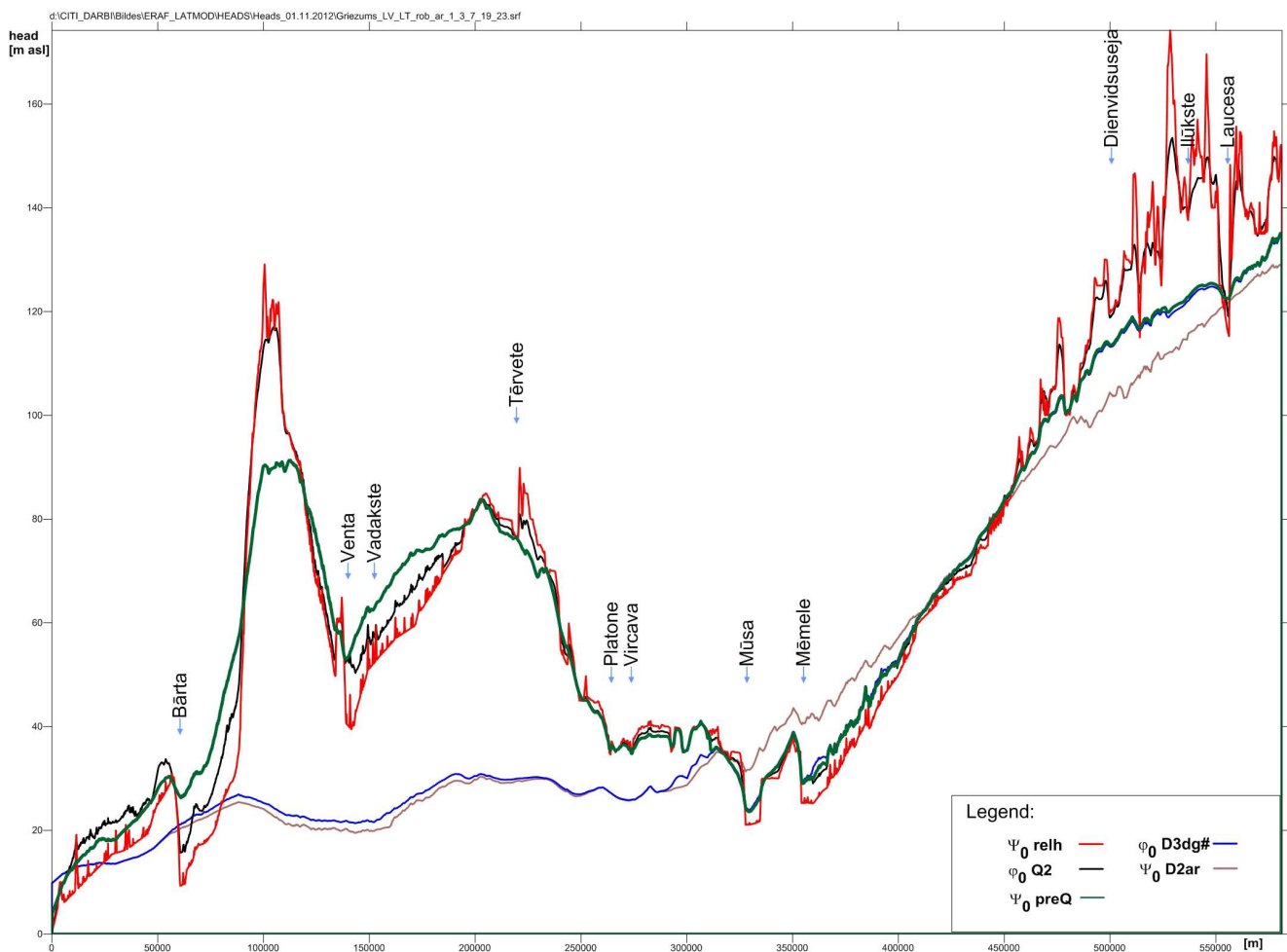


Fig. 5. Graphs of boundary conditions along the frontier of Lithuania / Latvia



The design of the interpolation tool is based on the fact that a set of vertical links  $a_z$  exists along the middle line of the border zone, which join the neighbouring aquifers. These links account both for the  $k$  and  $m$  parameters of the aquitards (formula (2)). If values of these links are considerably enlarged (at least 100 times), then this set of transformed vertical links behaves like a spatial interpolation device. It automatically provides the unknown components of  $\psi_{out}$ .

The above mentioned software detected these locations of the aquitards where they exist ( $m \neq 0$ ). For the  $m=0$  areas, no transformation of the  $a_z$ -values was done.

Therefore, the  $\psi_{out}$  boundary condition is obtained by the HM itself, as follows:

- the components originated from the  $\psi_{rel}$ ,  $\varphi_{D2ar}$ ,  $\varphi_{preQ}$ -maps are fixed as the initial ones;
- the other components of  $\psi_{out}$  are supported by the spatial interpolation tool that contains transformed  $a_z$ -links.

Because the interpolator is not a part of the LAMO active volume, it can be used permanently. This feature is very useful during the HM calibration when the  $a_z$ -set gets changed and the  $\psi_{out}$  conditions follow these changes.

In Fig. 5., graphs of the outer boundary conditions are shown for the frontier line separating Latvia and Lithuania. There the graphs  $\psi_{0,relh}$ ,  $\psi_{0,preQ}$ ,  $\psi_{D2ar}$  represent the fixed conditions, but the conditions  $\varphi_{0,Q2}$ ,  $\varphi_{0,D3dg}$  are computed by the interpolation tool. The graphs of other computed conditions are not shown, in order not to make Fig. 5. too complex. The  $\psi_{0,relh}$  graph represents the slice along the frontier, for the  $\varphi_{relh}$  distribution. The graphs  $\psi_{0,preQ}$ ,  $\psi_{D2ar}$  are slices along the frontier from the head distributions of preQ surface and the D2ar aquifer accordingly. Both distributions have been prepared by skilled hydrogeologists by using borehole data.

#### IV. RIVERS AS BOUNDARY CONDITIONS

The considered-above boundary conditions  $\psi_{rel}$ ,  $\psi_{D2pr}$ ,  $\psi_{out}$  are fixed on the outer surfaces of the HM and they exist in the nodes of the HM grid. The condition  $\psi_{riv}$  that represents water levels of rivers is attached to inner nodes of the HM grid via the set of the river bed conductances  $G_{riv}$ . The flow  $q_{riv}$  caused by rivers is given by the matrix expression:

$$q_{riv} = G_{riv} (\varphi - \psi_{riv}) \quad (4)$$

where the diagonal matrix  $G_{riv}$  is a part  $G$ ;  $\varphi_{riv}$  is subvector of  $\varphi$ ,  $\varphi_{riv}$  represents these nodes where the corresponding components of  $\psi_{riv}$  are attached via the links belonging to  $G_{riv}$ . The vector  $\psi_{riv}$  is the subvector of  $\psi$ , but components of  $\psi_{riv}$  belong to a set of points located outside the HM grid. The value of a single element  $g_{riv}$  of  $G_{riv}$  is presented by the formula:

$$g_{riv} = h w_{riv} k_{riv} / m_{riv} \quad (5)$$

were  $h=500$  (plane step);  $w_{riv}$  – the width of the river that is known and it is applied for computing of  $g_{riv}$ ;  $k_{riv}$ ,  $m_{riv}$  – the permeability and thickness of the river bed layer accordingly; these parameters are unknown. Value  $m_{riv}=1$  was fixed and

$k_{riv}=0.002$  was found experimentally. Therefore, as the initial try, the formula (5) gives  $g_{riv}=w_{riv}$ . For LAMO, rivers are presented by their middle lines. The only exception is three artificial lakes of the Daugava River that have been formed by the Riga, Kegums, Plavinas hydroelectrical power stations.

In Fig. 6, the set of rivers and lakes included in LAMO is shown. To obtain the  $\psi_{rel}$ -conditions, the following items must be prepared:

- the xy –location of a river line;
- the long line profile (water levels of a river along its line);
- the width of a river along its line;
- the z-attachment of a river line.

The last item accounts for the fact that a river, on its run, may be joined with different geological layers. For example (Fig. 7), the River Gauja runs through the Quaternary, the lower and upper Devonian Layers. As about 200 rivers of Latvia are included in LAMO, no modeller is able to join them properly with a model. It was necessary to develop special software for preparing all data files that are needed for creating the  $\psi_{riv}$ -conditions.

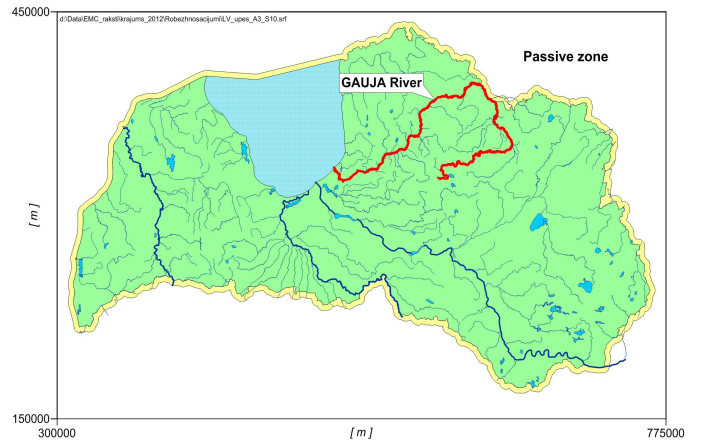


Fig. 6. Set of rivers and lakes included in the HM

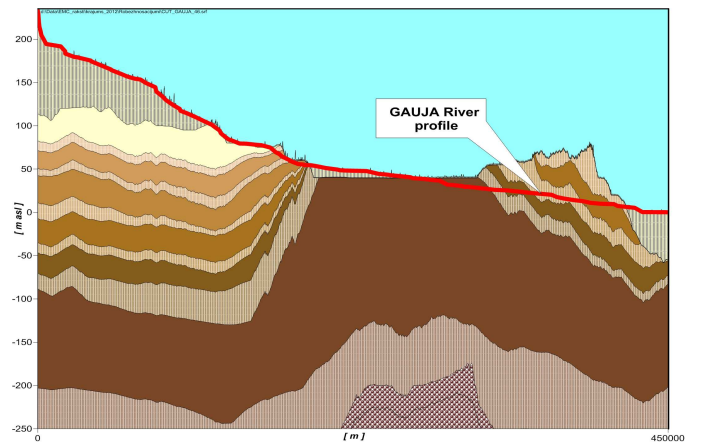


Fig. 7. Vertical cross section along the Gauja river

The program also performed the search for the river z-attachment. Presently, the empirical value  $k_{riv}=0.002$  is applied for all rivers. In nature,  $k_{riv}$  may be different not only

for each river, but also for their fragments. The task of finding more realistic  $k_{riv}$  distributions is very complex, because observed in nature river flows of (4) must be used. Knowledge of these flows provides more exact estimates of the interaction between groundwater and rivers.

## V. CONCLUSIONS

Scientists of Riga Technical University have developed the regional hydrogeological model of Latvia. Innovative methods used for arrangement of the boundary conditions of this model enabled to shorten the time and to ease efforts needed for creating this complex software tool. Application of the ground surface (terrain) map as the boundary condition on the model top enabled obtaining the infiltration flow distribution (the most important parameter) automatically, as the result provided by the model itself. Boundary conditions on the model outer vertical surface were also created by the model by transforming its outer surface into the interpolation tool that supported this boundary condition. Boundary conditions for rivers enabled to control river flows. Further efforts are needed to use this feature for more exact estimation of interaction between groundwater and rivers.

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